Modeling and analysis of crack evolution based on the Griffith fracture criterion

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For brittle elastic materials, like for instance ceramics, the Griffith fracture criterion is well suited in order to decide whether a crack will propagate for given external forces. This criterion is formulated in form of a complementary condition and states that the crack driving force is always less than or equal to a critical material dependent quantity (the fracture toughness) and that the crack can propagate only if the crack driving force is equal to the fracture toughness. In this context the crack driving force is given by the so-called energy release rate, which is the variation of the elastic energy with respect to the crack length.

In this lecture, first the Griffith fracture criterion and the notion of energy release rate is introduced for elastic materials. We then discuss the analytical properties of the energy release rate.

The main part of the lecture will be devoted to setting up a crack evolution model that is based on the Griffith fracture criterion. This will result in a time-dependent variational inequality for determining the crack tip position in dependence of time-dependent loadings. Since the elastic energy is nonconvex with respect to the crack length variable, solutions may occur that are discontinuous with respect to time. Hence, a weak formulation is needed that closes the crack evolution model with suitable jump criteria. We will present two different approaches to derive such jump criteria, discuss the existence of solutions of the corresponding models and illustrate the different predictions with examples. Finally we discuss a numerical algorithm that allows us to compute selectively the different types of solutions.

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