IWASAWA 2017

To commemorate the 100th anniversary of Kenkichi Iwasawa's birth.

MAIN CONFERENCE (JULY 24–JULY 28)

Lecture Hall, Graduate School of Mathematical Sciences
the University of Tokyo

July 24 (Mon)

10:00–11:00	Christopher Skinner *Iwasawa theory and modular forms*
11:20–12:20	Xin Wan *Iwasawa main conjecture for non-ordinary modular forms*
14:00–15:00	Kazuya Kato *Adelic period domains, heights of motives, and Iwasawa theory*
15:20–16:20	Otmar Venjakob *Towards regulator maps for Lubin-Tate extensions*
16:40–17:40	John Coates, Ehud de Shalit, and Ralph Greenberg *Memories of Kenkichi Iwasawa*
18:00–	Reception party

July 25 (Tues)

10:00–11:00	Samit Dasgupta *On the Gross-Stark conjecture and refinements I*
11:20–12:20	Mahesh Kakde *On the Gross-Stark conjecture and refinements II*
14:00–15:00	Henri Darmon *Singular moduli for real quadratic fields: a rigid analytic approach*
15:20–16:20	David Burns *On higher rank Euler and Kolyvagin systems*
16:40–17:40	Poster session

July 26 (Wed)

10:00–11:00 Sarah Zerbes
An Euler system for $GSp(4)$

11:20–12:20 Kazim Büyükboduk
Non-ordinary symmetric squares and Euler systems of rank 2

Evening Yakata-bune (boat trip) dinner

July 27 (Thu)

10:00–11:00 Haruzo Hida
Cyclicity of adjoint Selmer groups and fundamental units

11:20–12:20 Preston Wake
Massey products, pseudo-representations, and the Eisenstein ideal

14:00–15:00 Yasushi Mizusawa
Pro-p link groups in Iwasawa theory

15:20–16:20 Werner Bley
Solomon's construction in the elliptic rank 2 *case*

16:40–17:40 Takenori Kataoka
Fitting ideals in equivariant Iwasawa theory

July 28 (Fri)

10:00–11:00 Karl Rubin
Heuristics for the growth of Mordell-Weil ranks in big abelian extensions of number fields

11:20–12:20 Tadashi Ochiai
Iwasawa Main conjecture for p-adic family of elliptic modular forms

14:00–15:00 Ming-Lun Hsieh
Hida families and p-adic triple product L-functions

Christopher Skinner

Iwasawa theory and modular forms

Abstract: This talk will address some of the recent progress toward proving main conjectures of Iwasawa theory for modular forms (including one and two variable main conjectures) with an emphasis on the non-ordinary cases. It will also describe some of the applications to special value formulas in cases of analytic rank zero or one.

Xin Wan

Iwasawa main conjecture for non-ordinary modular forms

Abstract: We explain some recent progresses in proving Iwasawa main conjecture for modular forms which are non-ordinary at p (especially the lower bound for Selmer groups). The method combines together the automorphic congruences and the use of Euler systems.

Kazuya Kato

Adelic period domains, heights of motives, and Iwasawa theory

Abstract: The adelic period domain is the restricted product of Hodge theoretic period domains and p-adic period domains. Principal adeles in it are motives over the number field. How these motives distribute in the adelic period domain is an interesting problem. Tamagawa number conjecture formulated by S. Bloch and me is regarded as such problem. I hope to discuss such problems and the relations to heights of motives.

Otmar Venjacob

Towards regulator maps for Lubin-Tate extensions

Abstract: We report on a joint project with Peter Schneider: Let L_∞ be a Lubin-Tate tower over a finite extension L of $\mathbb{Q}_p$ with Galois group $\Gamma_L = G(L_\infty/L)$. We extend Kisin-Ren's theory of (φ_L, Γ_L)-modules and of Wach-modules for L-analytic, crystalline representations of G_L, the absolute Galois group of L. If χ_{LT} and χ_{cyc} denote the Lubin-Tate and p-cyclotomic character of G_L, respectively, then, for a Galois-stable O_L-lattice T in a crystalline Galois representation V over L such that $V(\chi_{cyc}\chi_{LT}^{-1})$ is L-analytic and with non-negative Hodge-Tate weights, we suggest a definition of a regulator map from the Iwasawa cohomology of T over L_∞ into $D_{cris,L}(V)$ tensor the locally L-analytic distribution algebra of Γ_L.

John Coates, Ehud de Shalit, and Ralph Greenberg

Memories of Kenkichi Iwasawa

Samit Dasgupta, Mahesh Kakde

On the Gross-Stark conjecture and refinements I (Samit Dasgupt), II (Mahesh Kakde)

Abstract: In 1980, Gross conjectured a formula for the expected leading term at $s = 0$ of the Deligne–Ribet p-adic L-function associated to a totally even character ψ of a totally real field F. The conjecture states that after scaling by $L(\psi\omega^{-1}, 0)$, this value is equal to a p-adic regulator of p-units in the abelian extension of F cut out by $\psi\omega^{-1}$.

In the first talk, we describe the statement of Gross's conjecture including a precise definition of Gross's regulator. We then sketch an outline of our proof of Gross's conjecture (joint w/ Kevin Ventullo). We conclude by presenting a refinement of Gross's conjecture giving an analytic formula for the entire characteristic polynomial of Gross's regulator matrix, rather than just its determinant. This refinement, which is still open, is joint work of Dasgupta and Michael Spiess.

In the second talk, we describe details of the proof of Gross-Stark. The basic technique follows Ribet's method, as employed by Dasgupta-Darmon-Pollack in the proof of the rank 1 case. We highlight the new ideas necessary for the higher rank case, including insights drawn from the Phd thesis of Ventullo.

Henri Darmon

Singular moduli for real quadratic fields: a rigid analytic approach

Abstract: I will describe a class of rigid meromorphic functions on the p-adic upper half plane, and present a conjecture that certain of their values at real quadratic irrationalities belong to class fields of real quadratic fields. This circle of ideas leads to a generalisation of the "differences of singular moduli" $j(\tau_1) - j(\tau_2)$, whose factorisations exhibit patterns similar to those explored in a seminal work of Gross and Zagier.

David Burns

On higher rank Euler and Kolyvagin systems

Abstract: We discuss some recent developments regarding the theory of higher rank Kolyvagin systems introduced by Mazur and Rubin. In particular, we shall describe how a natural higher rank Kolyvagin derivative operator construction produces Kolyvagin systems from (general) Euler systems in the case of arbitrary core rank and also how such Kolyvagin systems control the structure of associated Selmer groups. This is joint work with Ryotaro Sakamoto and Takamichi Sano.

Sarah Zerbes

An Euler system for $GSp(4)$

Abstract: I will outline the construction of a new Euler system for Galois representations associated to cohomological cuspidal automorphic representations of the symplectic group $GSp(4)$. This is joint work with David Loeffler and Chris Skinner.

Kazim Buyukboduk

Non-ordinary symmetric squares and Euler systems of rank 2

Abstract: We will report on joint work with A. Lei, D. Loeffler and G. Venkat, where we develop a signed-splitting procedure for non-integral Beilinson-Flach classes associated to the symmetric squares of p-non-ordinary forms. The novelty in this procedure is that it works even though the cyclotomic deformations of

Beilinson-Flach classes do not possess an interpolation property that covers the full critical range. This is consistent with Perrin-Riou's conjectures on rank 2 Euler systems; as a matter of fact, our methods are inspired by the implications of her conjecture. In certain cases, we are able to prove that the integral classes we obtain indeed lift to a non-trivial rank 2 Euler system, confirming Perrin-Riou's predictions. This has applications to the Iwasawa theory of symmetric squares.
(The speaker's research was partially supported by TUBITAK grant 113F059.)

Haruzo Hida

Cyclicity of adjoint Selmer groups and fundamental units

Abstract: Generalizing a result of Cho-Vatsal, we prove cyclicity over the Hecke algebra of the adjoint Selmer group of each universal deformation of an induced irreducible representation of a finite order character of the Galois group over F for a real quadratic field F under mild conditions.

Preston Wake

Massey products,pseudo-representations,and the Eisenstein ideal

Abstract: In his landmark 1976 paper "Modular curves and the Eisenstein ideal", Mazur studied congruences modulo p between cusp forms and an Eisenstein series of weight 2 and prime level N. In joint work with Carl Wang Erickson, we use deformation theory of pseudorepresentations to answer a question of Mazur on the rank of the corresponding Hecke algebra. The answer is intimately related to the algebraic number theoretic interactions between the primes N and p, and is given in terms of Massey products in Galois cohomology. We will highlight parallels with Iwasawa theory and present related open questions.

Yasushi Mizusawa

Pro-p link groups in Iwasawa theory

Abstract: As an analogue of a link group, I will propose the Galois group of the maximal pro-p-extension of a number field with restricted ramification which is cyclotomically ramified at p, i.e., tamely ramified over the intermediate cyclotomic $\mathbb{Z}_p$-extension of the number field. Based on the analogies between Iwasawa theory and Alexander-Fox theory, I will discuss such pro-p Galois groups with some related topics.

Werner Bley

Solomon's construction in the elliptic rank 2 case

Abstract: Let $L = k$ be a finite abelian extension of an imaginary quadratic number field k. Let p be a rational prime which does not split in $k = \mathbb{Q}$ and let p denote the prime of $\mathcal{O}_k$ lying over p. We assume that p splits completely in $L = k$. We then generalize a construction of Solomon and obtain in this way a pair of elliptic p-units in L. We then conjecturally express their valuations in terms of a p-adic logarithm of an explicit elliptic unit and provide numerical

evidence for the conjecture. We will then study the relation of our conjecture to a conjecture independently formulated by Mazur/Rubin and Sano and the relation to the relevant case of the equivariant Tamagawa number conjecture. This is joint work wit Martin Hofer.

Takenori Kataoka

Fitting ideals in equivariant Iwasawa theory

Abstract: In equivariant Iwasawa theory, the Iwasawa modules which we are interested in are often modified appropriately, and the modification prevents us from computing the (initial) Fitting ideals of the Iwasawa modules themselves. However, Greither and Kurihara gave an explicit description of them for the cyclotomic $\mathbb{Z}_p$ extensions of totally real fields in the commutative case. In this talk, I will discuss a generalization of the algebraic theory behind their work, and applications of the generalization to other situations in Iwasawa theory.

Karl Rubin

Heuristics for the growth of Mordell-Weil ranks in big abelian extensions of number fields

Abstract: I will discuss some questions about the distribution of modular symbols attached to elliptic curves. By expressing L-values in terms of modular symbols, we obtain heuristics for Mordell-Weil ranks. For example, these heuristics predict that every elliptic curve over $\mathbb{Q}$ has finite Mordell-Weil rank over the compositum of all $\mathbb{Z}_p$-extensions of $\mathbb{Q}$. This is joint work with Barry Mazur.

Tadashi Ochiai

Iwasawa Main conjecture for p-adic family of elliptic modular forms

Abstract: I will try to explain Iwasawa Main Conjecture for p-adic family of modular forms. First, we review Hida family (ordinary case) and the formulation of Iwasawa Main Conjecture in this setting as well as known results. Next, we discuss Coleman family as a non-ordinary generalization of Hida family and we discuss Iwasawa Main conjecture. We will also explain some difficulties compared to ordinary case and give some partial results in this non-ordinary context.

Ming-Lun Hsieh

Hida families and p-adic triple product L-functions

Abstract: In this talk, I will report on the recent progress on the construction of p-adic (twisted) triple product L-functions attached to primitive Hida families on $GL(2)$ and discuss some arithmetic applications to anticyclotomic Iwasawa theory for modular forms.

Preparatory Lecture Series (July 19–July 22)

Lecture Hall, Graduate School of Mathematical Sciences
the University of Tokyo

July 19 (Wed)

11:00–	Registration
13:00–14:00	D. Loeffler 1
14:30–15:30	A. Nickel 1
16:00–17:00	M. Kurihara 1

July 20 (Thu)

10:00–11:00	S. Kobayashi 1
11:30–12:30	D. Loeffler 2
14:30–15:30	A. Nickel 2
16:00–17:00	M. Kurihara and T. Sano 2

July 21 (Fri)

10:00–11:00	T. Sano 3
11:30–12:30	S. Kobayashi 2
14:30–15:30	D. Loeffler 3
16:00–17:00	A. Nickel 3

July 22 (Sat)

10:00–11:00	S. Kobayashi 3
11:30–12:30	D. Loeffler 4

David Loeffler
Euler systems

Abstract: Euler systems are among the most important tools in Iwasawa theory: they serve to 'control' the sizes of arithmetically inte resting groups, such as ideal class groups or Selmer groups of elliptic curves. The aim of these lectures will be to give an introduction to what Euler systems are and some of the tools used to construct them.

I'll begin by introducing the language of Galois representations and their cohomology, and the definition of an Euler system; and I'll explain the statements of the general theorems showing that Euler systems give bounds for Selmer groups

of Galois representations. I will then introduce the basic building blocks used in most of the known Euler system constructions: 'Siegel units', which are special rational functions on modular curves which are related to weight 2 Eisenstein series. I'll explain the construction and properties of Siegel units, and then describe how these are used to construct some of the known Euler systems, such as the Beilinson–Kato Euler system for a modular form, and the Beilinson–Flach Euler system for Rankin–Selberg convolutions. Finally, I'll conclude with a brief sketch of the explicit reciprocity laws relating Euler systems to values of L-functions.

Andreas Nickel

Non-abelian Stark-type conjectures and noncommutative Iwasawa theory

Abstract: In this Lecture Series we give an introduction to non-abelian Stark-type conjectures and their relation to noncommutative Iwasawa theory. There will be three parts (each part is roughly one lecture):

1. Noncommutative algebra. After some motivating examples we elaborate the necessary tools from noncommutative algebra. Namely, we recall the notion of Fitting ideals over a commutative ring and generalize this concept to certain orders in separable finite dimensional K-algebras. The examples we have in mind are p-adic group rings and Iwasawa algebras of one-dimensional p-adic Lie groups. When the relevant order is not commutative, then certain 'denominator ideals' appear in the theory which play a crucial role in the second part of the course. We also discuss some useful properties of noncommutative Fitting invariants.

Prerequisites: basic knowledge in noncommutative algebra (notions of separable algebras, orders, (completed) group rings; Wedderburn's theorem is essential), some representation theory of finite groups might be useful.

2. Non-abelian Stark-type conjectures. Let L/K be a Galois extension of number fields with Galois group G. To each integer $r \leq 0$ one can define certain Stickelberger elements $\theta(r)$ in the center of the complex group algebra $\mathbb{C}[G]$. These elements are constructed via values at $s = r$ of the complex Artin L-series attached to the characters of G. It is known that $\theta(r)$ always has rational coefficients and in fact (almost) integral coefficients whenever G is abelian. In this case Brumer's conjectures (resp. the Coates–Sinnott conjecture) asserts that $\theta(0)$ annihilates the class group of L (resp. that $\theta(r)$ annihilates the higher K-theory $K_{-2r}(\mathcal{O}_L)$ of the ring of integers $\mathcal{O}_L$ in L if $r < 0$).

When G is non-abelian, it is known that $\theta(r)$ does not always have integral coefficients. Nevertheless, we formulate a conjecture on the integrality of Stickelberger elements. We then generalize the conjectures of Brumer and Coates–Sinnott to arbitrary (finite) Galois extensions. We also report on known cases of these conjectures.

Prerequisites: basic knowledge in number theory, Artin L-series; when you like to fully understand the case $r < 0$, you also need to know about higher (Quillen) K-theory.

3. Noncommutative Iwasawa theory. We first recall some basic facts on the structure of Iwasawa algebras of 1-dimensional p-adic Lie groups due to Ritter and Weiss. We introduce p-adic Artin L-series and state a variant of the main conjecture of equivariant Iwasawa theory. This has been proven under a suitable '$\mu = 0$'-hypothesis by Ritter-Weiss and Kakde, independently. We then sketch how the main conjecture implies a large part of the non-abelian Brumer conjecture (the latter is joint work with Henri Johnston).

Prerequisites: completed group rings (Iwasawa algebras), basic knowledge in Iwasawa theory, some Galois cohomology.

Masato Kurihara and Takamichi Sano

Generalized Stark elements of arbitrary weights for $\mathbb{G}_m$

Abstract: The classical Stark conjecture predicts the existence of certain algebraic elements, whose images under the regulator maps are the leading terms of Artin L-functions at $s = 0$. These elements, often referred to as 'Stark elements', lie in the higher exterior powers of the unit groups with rational coefficients. The Stark conjecture is regarded as a generalization of the classical class number formula due to Dirichlet, and a typical example of the Stark elements is given by a cyclotomic unit.

In 1996, Rubin proposed a conjecture, which is known as the 'Rubin-Stark conjecture', on some delicate integral property of the Stark elements. This conjecture asserts that the Stark elements in general do not lie in the higher exterior powers of the unit groups over integral group rings, but certain modified exterior powers, which we call 'Rubin lattices'. Rubin then gave an insightful idea on the construction of Euler systems from the Stark elements. This pioneering work is now regarded as one of the origins of the theory of 'higher rank Euler systems'.

Recently, more detailed arithmetic theory of Stark elements have been developed by the two speakers of this lecture and D. Burns. In particular, we formulated a conjectural refinement of the Rubin-Stark conjecture, which relates Stark elements with Fitting ideals of certain integral cohomology (Selmer) groups. We also have developed the theory of Stark elements for arbitrary integers $s = n$.

In this lecture, we begin with basics on Stark-type conjectures, and explain the fundamental properties of (Rubin-)Stark elements. Then we will talk on our recent works with Burns, which we mentioned above, in particular, several refinements of the Rubin-Stark conjecture. We introduce the 'generalized Stark elements' for arbitrary integers $s = n$, and discuss their arithmetic and p-adic properties. These elements form a 'p-adic family', which can be interpreted as a conjectural generalization of classical Kummer's congruences. We will also talk on the relation with the equivariant Tamagawa number conjecture.

Shinichi Kobayashi

Local Iwasawa theory of modular forms for the anti-cyclotomic $\mathbb{Z}_p$-extension

Abstract: Iwasawa main conjecture describes a relation between p-adic L-function and the characteristic ideal of Selmer group. These two ingredients of different natures are considered to have the same origin, called zeta element. Zeta element has the property of Euler system to bound the size of Selmer group on the one hand, and it is related to p-adic L-function by the Perrin-Riou map and the explicit reciprocity law on the other hand. We call the latter theory local Iwasawa theory. In this lecture, as introduction, we first recall the classical local Iwasawa theory connecting cyclotomic units and the Kubota-Leopoldt p-adic L-function. Then we explain the local Iwasawa theory of modular forms for the anti-cyclotomic $\mathbb{Z}_p$-extension. Such theory was developed when the modular form is ordinary at p by Castella-Hsieh but in this lecture, we explain an intrinsic formulation that works both ordinary and non-ordinary cases.

1. Introduction, anti-cyclotomic extensions and the Serre-Tate local moduli
2. p-adic interpolation of generalized Heegner cycles
3. Perrin-Riou theory for a relative Lubin-Tate group of height 1