Research Report

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History of Western Philosophy from the quantum theoretical point of view; [Ver. 5]

by

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History of Western Philosophy from the quantum theoretical point of view; [Ver. 5]

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Abstract: In this text I will concentrate only on the scientific aspects of the "History of Western Philosophy". Thus, the title of this text could have been "Philosophy of Science and its History". Recently I proposed "quantum language" which was characterized as the metaphysical and linguistic turn of quantum mechanics. This turn from physics to language does not only realize the remarkable extension of quantum mechanics but also yield the quantum mechanical world view. And thus, the turn makes us expect that Western philosophies (i.e., Parmenides, Zeno, Plato, Anselmus, Thomas Aquinas, Descartes, John Locke, Leibnitz, Berkeley, Hume, Kant, Wittgenstein, Hempel, etc.) can be fully understood in the quantum mechanical worldview. Upon reading this text, the reader will be convinced that the history of Western philosophy can be characterized as a history of the search for quantum language (≈ the language of science), if limited only to the scientific point of view. That is, the philosophy from Greece to the present day was the history of the growth of quantum language. Therefore, quantum language has power to solve many unsolved philosophical problems. In this text, I will prove most all the famous unsolved problems of traditional philosophies, e.g., (a): Zeno's paradox (Greek philosophy), (b): the problem of universals (Scholastic philosophy), (c): the mind-body problem and the subjectivity problem (Descartes-Kant epistemology), (d): How is analytic philosophy related to Descartes-Kant epistemology? (Analytic philosophy), (e): Why do logic and statistics both work in our world? (Analytic philosophy), etc.

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Preface:

0.1: Philosophy has progressed toward quantum language

First, let me say that, this text will concentrate only on the scientific part of the philosophy.

Quantum language (i.e., QL (=MT = measurement theory), the linguistic Copenhagen interpretation of quantum mechanics, the quantum mechanical worldview) proposed by myself is a scientific language that is inspired by quantum mechanics, and moreover, it has a great power to describe classical systems as well as quantum systems.

My lecture for graduate students in the faculty of science and technology in Keio university has been continued, with a gradual improvement, for about 20 years.

Figure 0.1: The location of QL in the history of western philosophy

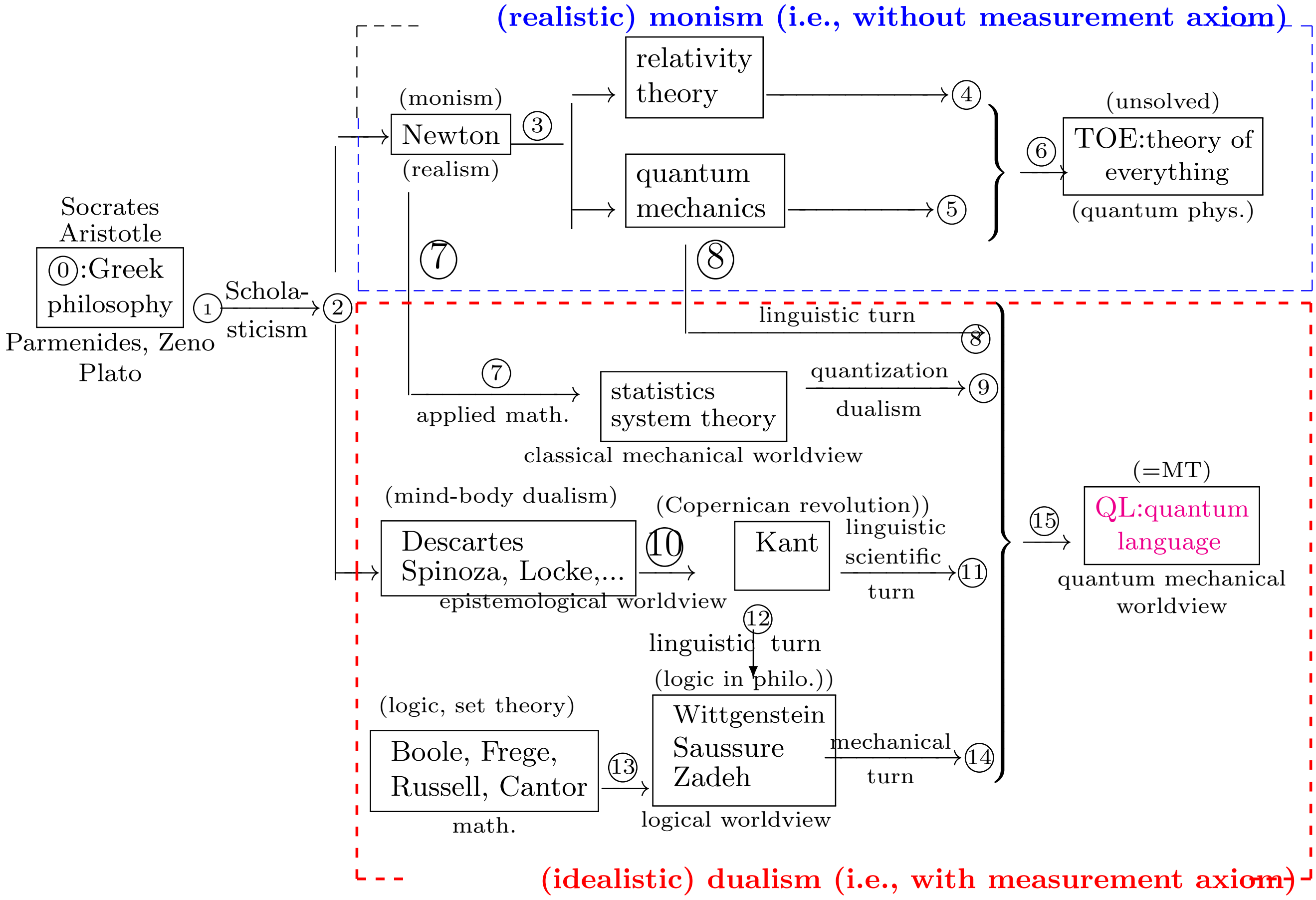


Figure 0.1 (in Preface): The history of the world-descriptions
Philosophy (≈ dualistic idealism) has progressed toward QL
(i.e., ⓪→ ①→ ②→ ⑩→ ⑫→ ⑭→ ⑮)

The main part of quantum language [③ - ⑧ - ⑮] and statistics[⑦ - ⑨- ⑮] in Figure 0.1 (in Preface) were already studied in the following:

(A_1) ref. [74]: S. Ishikawa, "Linguistic Copenhagen interpretation of quantum me-

chanics: Quantum Language [Ver. 5]", Dept. Math. Keio University, 2019, KSTS/RR-19/003, 473 pages
(http://www.math.keio.ac.jp/academic/research_pdf/report/2019/19003.pdf)

Therefore, in this text I concentrate on part [⓪ - ① - ② - ⑩- ⑫- ⑭- ⑮], which is almost equal to the history of western philosophy. This part [⓪ - ① - ② - ⑩- ⑫- ⑭- ⑮] was not detailed in my lecture, but it will be a good preparation to read (A_1) (since (A_1) is mathematically graduate level). That is, this text is written as the supplementary reader in my lecture, and thus it can be read without the mathematical preparation. Even so, this text is not an adjunct to ref. [74]. The content of this text is at the heart of the philosophy of science, and I am convinced that the level of the content is worthy of a graduate course.

Note that Figure 0.1 (in Preface) asserts that

(A_2) **statistics, quantum mechanics, Scholasticism, Descartes=Kant philosophy and analytic philosophy are each one aspect of quantum language (or, immature states of quantum language) .**

Also, I think that the following is one of the most fundamental problems of western philosophy:

The progress problem (B) : Has philosophy made progress?

(B) Do the time series [⓪ - ① - ② - ⑩- ⑫- ⑭- ⑮] in Figure 0.1 (in Preface) mean the progress of philosophy?

The same thing, in other words

(B_1) What is the goal of the pseudoscientific time series [⓪ - ① - ② - ⑩- ⑫- ⑭] ?

(B_2) Or is it literature that has no goal, but merely reflects the prevailing conditions of the time?

Thoughout this text, I conclude that

(B_3) Has this pseudo-scientific timeline already come to an end? Will it be perpetuated in the future?

(B_4) In Eastern philosophy, we seldom hear the question, "Has philosophy progressed? What is the difference between the two?

In this text I conclude that

(C): My answer; the progress of philosophy (≈ dualistic idealism)

(C_1) If "to make progress" is defined by "to come near quantum language" (i.e., "becoming more and more like quantum language")† we can say that the time series [⓪ - ① - ② - ⑩- ⑫- ⑭] can be regarded as progress, that is,

This will be proved throughout this text.

† This definition is somewhat ambiguous, but I consider it sufficient for the purposes of this text.

Here, in this text, it suffices to consider that "idealism"≈"linguistic theory"≈"metaphysics" (*cf.* Note 1.4). Thus, we conclude that

(C_2) a scientific perfection of dualistic idealism is realized by quantum language (QL).

which is surely one of the most important assertions in western philosophy.

Also recall that quantum language is intended as a language of science. Therefore,

(C_3) the **scientific purpose** of philosophy of dualistic idealism was to make the language of science.

The conventional theory is to think of statistics as the "language of science," but in this text, we propose quantum language. This is because statistics is in the halfway position of being both idealistic and monistic, acting as a kind of applied mathematics.

0.2: Almost all unsolved problems in traditional philosophy can be solved in QL: Wittgenstein's dream came true

It should be noted that most unsolved problems in the history of western philosophy are caused by insufficient understanding of idealism and dualism. Thus,

- if the (C_2) is true (i.e., if quantum language is overwhelmingly powerful compared to other immature philosophies), I can expect that most unsolved problems in dualistic idealism can be solved in the framework of quantum language.

In this text, I will show that the following unsolved philosophical problems are easily solved as corollaries of the (C_2):

(D_1): The list of unsolved problems of dualistic idealism to be solved in this text

Wittgenstein's dream came true

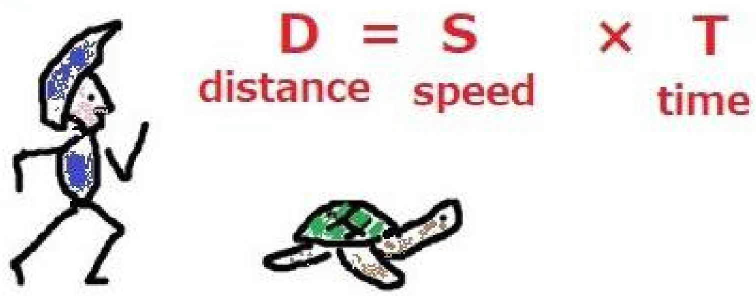
It is said that

- Wittgenstein carried out one of the most ambitious attempts in the history of philosophy to solve all the problems of philosophy at once in [125]: "Tractatus Logico-Philosophicus (abbreviated as TLP)".

However, 100 years later, we can't say that he succeeded in his challenge. However, as the following list shows, QL realized his dream.

Almost famous unsolved problems concerning dualistic idealism can be solved in QL

- What is probability (or, measurement, causality)? *cf.* Sec. 1.2.1)
- What is "Copenhagen interpretation"? (*cf.* Sec. 1.3)
  (In fact, answering this question is the main purpose of this text. This should be realized when the reader finishes reading this text.)
- Zeno paradox (Flying arrow, Achilles and a tortoise), (*cf.* ref. [47], or Sec. 2.4)



("to solve Zeno paradox" = "to use DST-formula" (*cf.* Sec. 2.4.3)
- the measurement theoretical understanding of Plato's allegory of the sun , (*cf.* Sec. 3.3.2)

- Plato's Idea theory≈Zadeh's fuzzy theory≈Sausuure's linguistic theory (*cf.* Sec. 4.1.3)
- Syllogism does not always holds in quantum systems (*cf.* Sec. 4.3.3)
  Syllogism always holds in classical and quantum systems (*cf.* Remark 12.9 in Sec. 12.1)
  (The two above seem contradictory, but they are not contradictory because they are formulated differently.)
- Only the present exists (*cf.* Sec. 6.1.2: Augustines)
- Anselmus' proof of God's existence is a wordplay (*cf.* Sec. 6.4.3)
- We proposed the definition of "universal" and clarified the meaning of "the problem of universals" (*cf.* ref. [83], Sec. 6.5).

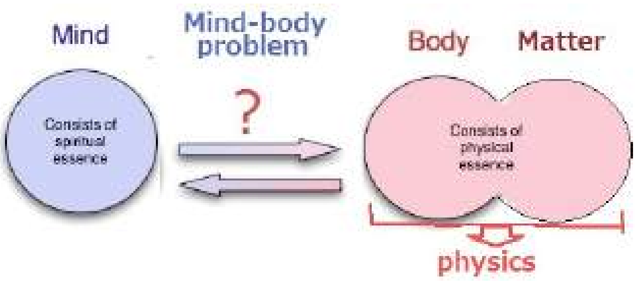
  Thus, the problem of universals is regarded as a kind of "monism vs. dualism".
- What is Geocentrism vs. Heliocentrism? Ultimately, this is a dispute between different worldviews. (*cf.* Sec. 7.4)

- Two (scientific or non-scientific) interpretations of "I think, therefore I am" .(*cf.* Sec. 8.2.2)
- Leibniz-Clark correspondence (i.e., what is space-time?), (*cf.* Sec. 9.2)
- The existence theorem of the kinetic function (*cf.* Sec. 9.2)

- Brain in a vat argument (*cf.* Sec. 9.4.2)
- The solution of Hume's problem of induction (*cf.* Sec. 9.6)

- The grue paradox does not exist in quantum language (*cf.* Sec. 9.7)
- What is causality? (*cf.* Sec. 10.2)
- Kantian synthesis of continental rationalism and British empiricism is pseudoscientific and is rewritten as QL synthesis. (*cf.* ref, [84], Sec. 10.3)

- The mind-body problem (i.e., How are mind and body connected?), (*cf.* ref. [70], Sec. 12.8)
- The subjectivity problem (≈ the problem of qualia (*cf.* Sec. 9.4.1)

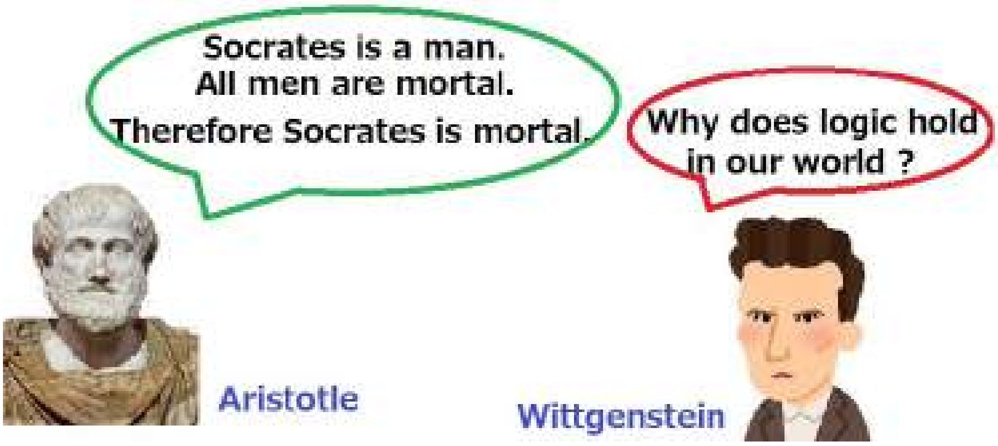


- What is Peirce's abduction? (*cf.* Sec. 11.4.1)
- Five-minute hypothesis (*cf.* Sec. 11.5.2)

- McTaggart's paradox (*cf.* Sec. 11.5.3)
- quantitative representation of "Signifier" and "signified" (*cf.* Sec. 12.1)
- The quantum linguistic understanding of "Tractatus Logico-Philosophicus (=TLP)" and "Zadeh's fuzzy sets", in other words, QL can answer the question, "What is a proposition?" (*cf.* ref. [81], or Sec. 12.1)

That is, Wittgenstein took over the baton of "dualistic idealism (i.e., the mainstream of philosophy)" from Kant. This is essentially important, since Wittgenstein's theory (i.e., TLP) must belong to dualistic idealism in order to assert (C) (or, Figure 0.1 (in Preface)).

- The proposal of QL logic (= fuzzy logic); Why does logic work in our world? (*cf.* Sec. 12.1)
  This question is proved within quantum language as a set of "why does statistics work in this world?". Statistics is regarded as a kind of "multi-valued logic"," which allows for more elaborate arguments than ordinary logic, which is a "binary logic," but both are kinds of measurement.



- Solving analytic-synthetic distinction problem (Carnap-Quine controversy) (*cf.* Sec. 12.3)
- Lewis Carroll's paradox (*cf.* Sec. 12.5)
- Flagpole problem, (*cf.* Sec. 12.6)
- Hempel's raven paradox (*cf.* Sec. 12.7)

These are quantum linguistic solutions to unsolved problems in philosophy, but of course, these are just a few of the most representative ones, and there are many more that could be solved.

For the quantum linguistic solutions of unsolved problems in quantum mechanics, statistical mechanics, statistics and probability theory, see List (D_2) in Chap. 13: "Post-script."

0.3: Why is philosophy strange?

As will be mentioned in Chap 1, QL is composed with two axioms (Axiom 1 concerning measurement and Axiom 2 concerning causal relation) and the manual to use Axioms 1 and 2 (called "Copenhagen interpretation", or precisely, "linguistic Copenhagen interpretation"). That is, (*cf.* refs [48, 74]),

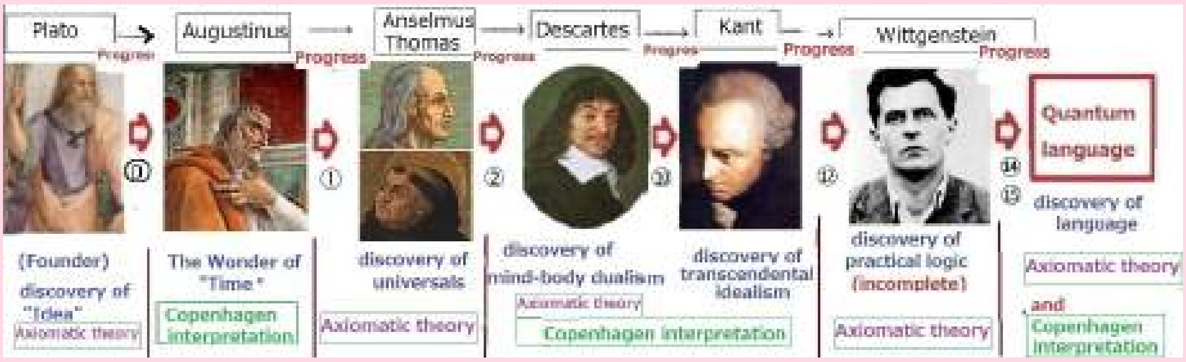
$$
\text{(E)} \quad \overset{\text{(=quantum language)}}{\boxed{\boxed{\text{measurement theory}}}}
$$

$$
= \underset{\text{(Axiomatic theory)}}{\boxed{\begin{array}{l}\text{Axiom 1: measurement} \\ \text{Axiom 2; causality}\end{array}}} + \underset{\text{(Empirical theory)}}{\overset{\text{[the manual to use Axioms 1 and 2]}}{\boxed{\text{linguistic Copenhagen interpretation}}}} \qquad (0.1)
$$

Throughout this text, I assert the following.

(F) The scientific part of the history of Western philosophy was the history of the construction of QL (= Axioms 1 and 2 + the linguistic Copenhagen interpretation).

More precisely, we say that



As will be shown throughout this text, the following can be seen from the figure above.

(F_1) Western philosophy is the history of the debate on Plato's "Idea"

(F_2) The history of Western philosophy has always been a mechanism to keep things fresh (to avoid getting stuck in a rut) by alternating between the "axiomatic" and "Copenhagen Interpretation" aspects.

The discovery of QL changes the view of modern philosophy as follows. In other words, rejecting **"Kant's synthesis"** and advocating the following **"QL synthesis"**.

(G)

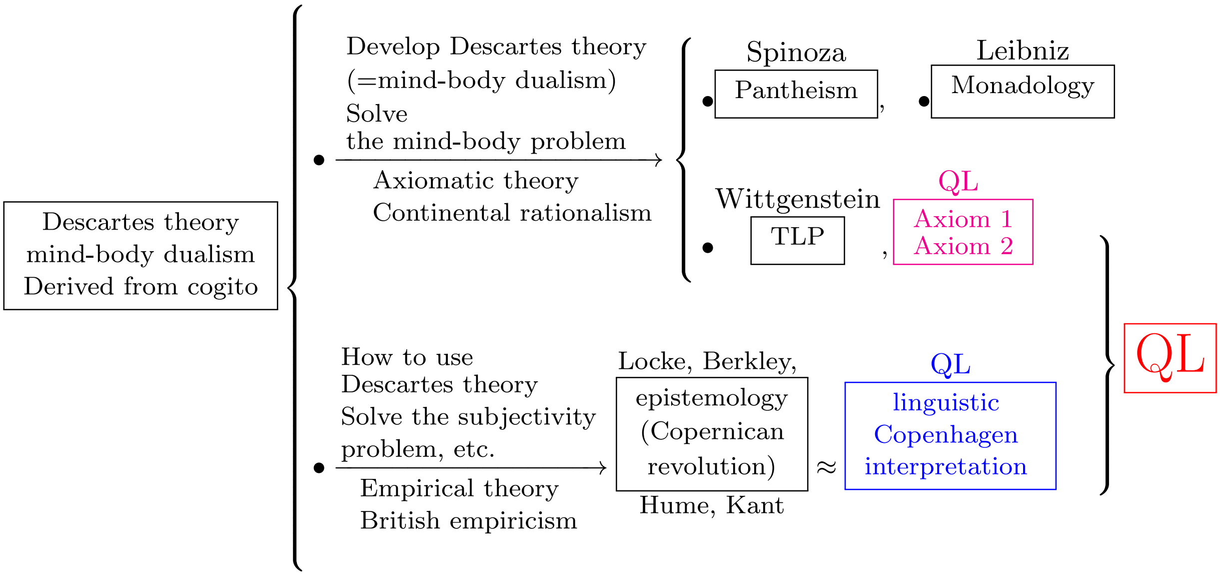


Figure 9.1 (in Chap. 9): QL synthesis

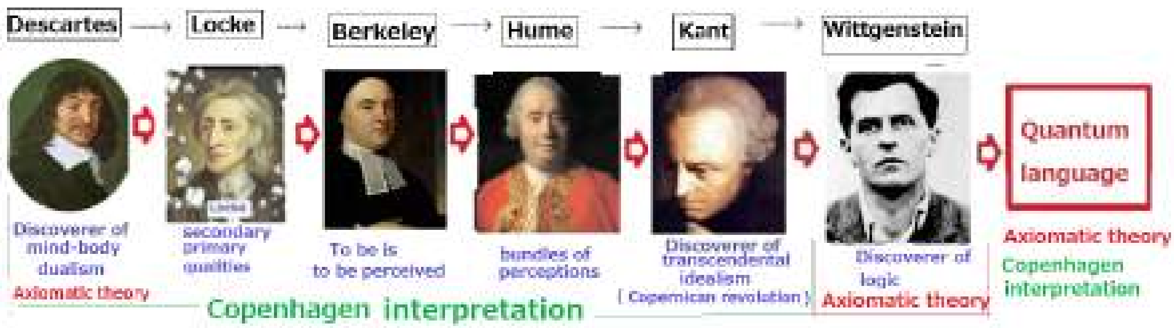
That is,

(H) Western philosophies (i.e., the dualistic idealism from Plato to Wittgenstein) is incomplete and immature, namely, pseudoscientific. And thus, they can only be understood from the perspective of quantum language. That is,

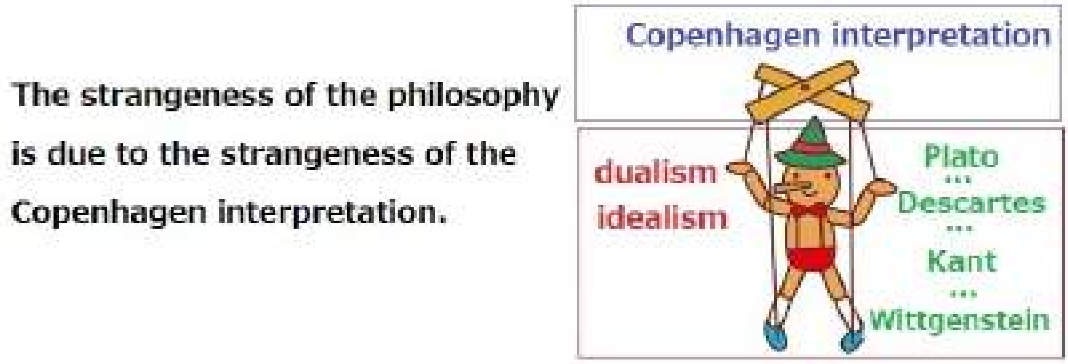
Without seeing the finished product, it is impossible to know exactly what it is.

(I_1) Axiomatic theory (i.e., Axioms 1 and 2) in QL is too difficult mathematically. Thus, philosophies that were studied by Anselmus, Thomas Aquinas, Spinoza, Leibniz, Wittgenstein, etc.) were very incomplete. However, their high aspirations were admired by philosophy enthusiasts.

(I_2) The Copenhagen interpretation of quantum mechanics is tricky and exciting, and one does not have to be an expert to enjoy it. Thus, philosophies that were studied by Augustinus, Descartes, Locke, Berkeley, Hume, Kant could introduce the general public to the pleasures of philosophy. Therefore, I agree to the established opinion such that the mainstream of western philosophy (after Descartes) is as follows.



(I$_3$) The readers will understand the question "Why is always philosophy strange?" That is, the strangeness of philosophy (i.e., dualistic idealism) is due to the strangeness of the Copenhagen interpretation. In other words, it is due to the strangeness of quantum mechanics (e.g. Schrödinger's cat, etc.).



Thus,

(J) Western philosophy has evolved, with refreshing changes, into a quantum language. This refresh was also made possible by the alternating emergence of axiomatic theories and Copenhagen interpretation.

I think this is why Western philosophy has flourished (and not fallen into a rut).

0.4: Supplementary information

Finally, I'd like to add a few things
This text is intended as a supplement to the main text of the graduate course, ref. [74], i.e.

(A$_1$) ref. [74]: S. Ishikawa, "Linguistic Copenhagen interpretation of quantum mechanics: Quantum Language [Ver. 5]", Dept. Math. Keio University, 2019, KSTS/RR-19/003, 473 pages
(http://www.math.keio.ac.jp/academic/research_pdf/report/2019/19003.pdf)

and thus, this text can be read without the knowledge of quantum theory. Also, the mathematical arguments were left to ref. (A$_1$). The low level of mathematics does not imply a low level of philosophy, and I am proud to say that the level of philosophy of science in this text is about the same as that of a graduate school.

This text is the revised version of ref. [82]:

(K) ref. [82]: S. Ishikawa, *History of Western Philosophy from the quantum theoretical point of view* [Ver. 4], Research Report (Department of mathematics, Keio university, Yokohama), (KSTS-RR-21/001, 2021, 306 pages) (http://www.math.keio.ac.jp/academic/research_pdf/report/2021/21001.pdf)

In [ver. 4] of this (K), I wrote in a way that could be read as "ref. [74] in (A_1) is primary and ref. [82] in (K) is secondary," but in this text, I emphasized that "ref. [74] and this text are equivalent in scholarly value".

Philosophy, like literature, may be written for the general public as intellectual entertainment, but this does not necessarily diminish its academic value.

In this text, for this reason, the use of mathematical formulas has been kept to a minimum. This is because I believe that quantum mechanics without formulas is incomprehensible, but quantum language without formulas is comprehensible. This is because quantum language without formulas is mostly Copenhagen interpretation.

In order to make the book as enjoyable to read as a picture book, we have used many figures and pictures. Moreover, as English is not my first language, I believe that the figures and pictures help me to convey my true meaning.

I hope many readers will enjoy this text.[*2].

December 2022
Shiro Ishikawa

[*2] For the further information concerning quantum language (e.g. an improved version of this text may be published), see my home page: https://ishikawa.math.keio.ac.jp/indexe.html

Contents

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Chapter 1

The outline of quantum language (=measurement theory)

Abstract: Readers who are completely unfamiliar with quantum languages (or, quantum mechanics) would be rather encouraged to skip this chapter and start reading in the next chapter.
I have written this text in such a way that no prior knowledge of "quantum language" is required to read Chapters 2 and beyond. If there are serious readers who are concerned about this, it is enough to read through Section 1.1sim1.3. If you are a graduate student, however, please read Section 1.4 "Appendix. Mathematical Foundations of Axioms 1 and 2" and "Quantum Language (ref. [74])" are recommended for graduate students.

Many parts of philosophy are enlightening, literary, and literary, and the domain of the scientific part is not that large. However, I believe:

(♯) Western philosophy has always been dominated by scientific thought. To put it more bluntly,

Western philosophy's scientific thinking = dualistic idealism = quantum language

=the central thema of philosophy of science

The purpose of this text is to explain this (♯).
Suppose there were aliens whose civilization is more advanced than ours. They would be more advanced than we are when it comes to math, physics, statistics, and AI. But they might not have any concept of "ethics" at all. Then again, they may not have any so-called philosophy. But I think they do have a "theory of dualistic idealism". Because I think that

[The theory of dualistic idealism] = [quantum language]
≈[quantum mechanics] ≈ [statistics]

The contents of this chapter are as follows

Sec. 1.1: About worldviews
Sec. 1.2: quantum language (Axiom 1 (measurement) and Axiom 2 (causality)
Sec. 1.3: Copenhagen interpretation
Sec. 1.4: Appendix . Mathematical Foundations of Axioms 1 and 2

1.1 About Worldviewism

Philosophy is usually based on worldviewism. Here, worldviewism is a way of stating philosophy in the following form:

Worldviewism:

(A) [Worldview] (the world is so) $\xrightarrow{\text{therefore}}$ [Consideration, discussion] (Let's think so)

For example, let's take a very good worldview (Newtonian mechanics and statistics) to illustrate the above.

(i) [Newtonian mechanics] (Kinetic equation + universal gravitation; realistic worldview)

$\xrightarrow{\text{therefore}}$
- heliocentrism *cf.* Chap.7
- Heavy and light balls fall the same way *cf.* Sec.7.5
- ...

or,

(ii) [Fisher statistics (*cf.* Note 1.3)] (The world can be represented in a sample space with parameters; idealism)

$\xrightarrow{\text{therefore}}$
- The person in the distance wearing a skirt must be a woman. *cf.* Sec.1.4.4
- I found fish fossils in this mountain, so I estimated that this place used to be a sea *cf.* Sec.11.4
- ...

This text will focus on the following three types of worldviews (in particular, the idealistic worldview).

(B) Worldview Type

worldview:
- (i): realistic worldview (=Aristotelian worldview) (physics, Aristotle philosophy, etc.)
- (ii): idealistic worldview (Plato worldview) (Statistics, Plato, Descartes, Kant , etc.)
- (iii): mathematics(=mathematical thinking) (logical worldview) (set thoretical worldview) (Set theory, etc.)[*1]

Let me explain [(i): realistic worldview (=realism)], [(ii): idealistic worldview (=idealism)] and [(iii): mathematics].

(i) A realist worldview (=realism)] is "a statement that describes the laws of the (concrete) world as accurately as possible" and we can assume physics. As a matter of course, "the world comes first, the worldview comes after". this realistic worldview can be pointed out by experiments to be wrong.

$$\overset{\text{the truth that " the world is so"}}{\boxed{\text{realistic worldview}}} \xrightarrow{\text{therefore}} \overset{\text{Let's consider so}}{\boxed{\text{consideration, discussion}}}$$

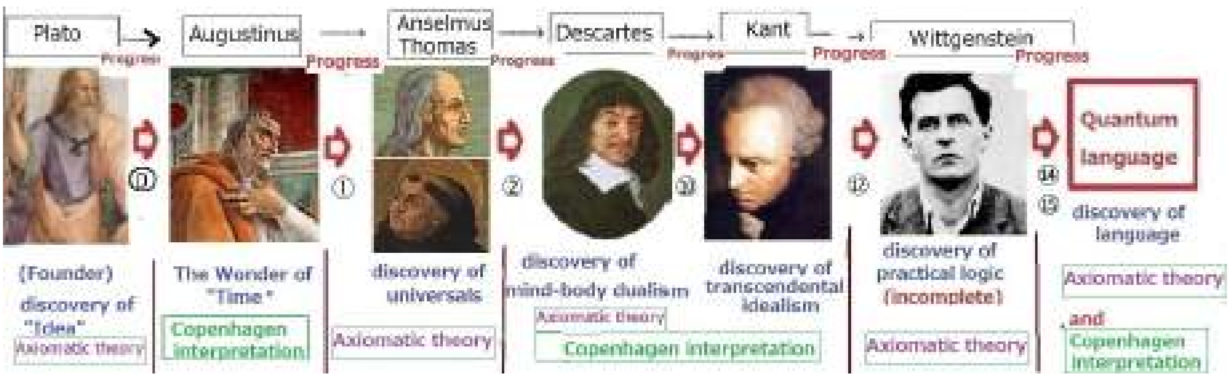
(ii) [Idealistic worldview (=idealism)] is "a way of trying to understand the world through sutra-like statements. We can assume statistics (which can be applied to medicine, economics, engineering, or anything else). Thus, "the worldview comes first and the world comes after". The Idealistic worldview (since it is worded like a sutra) makes no sense to determine right or wrong by experimentation, but "usefulness (usefulness?)" is evaluated by "usefulness". In fact, statistics is very useful, as it is said to be the most powerful science.

$$\overset{\text{the fiction (spell) that "the world is so"}}{\boxed{\text{idealistic worldview}}} \xrightarrow[\text{habitual thinking}]{\text{therefore}} \overset{\text{Let's consider so}}{\boxed{\text{consideration, discussion}}}$$

(iii) We do not consider [math] to be a worldview, but for convenience we will assume its position here. I wrote an excuse in Note1.1.

The worldview that this text mainly deals with is "quantum language (= quantum mechanical worldview)," which is a dualistic version of "statistics. Therefore, quantum language is a dualistic idealism, the same as the mainstream of Western philosophy: Plato → Augustine → Anselmus → Thomas Aquinas → Descartes → Kant → Wittgenstein.

Thus, throughout this text, I will explore the relationship between the genealogy of idealism and quantum language in the figure below.



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*1 It is impossible to consider mathematics as a kind of worldview, but I will classify it here in the sense of "mathematical thinking." (*cf.* Sec.4.3.4: Set-theoretic Worldview, Logical Worldview)

Now, the following are nearly identical.

- To understand the physics (worldview) of Newtonian mechanics
- To master the language of Newtonian mechanics
- To understand the logic (propositions) of Newtonian mechanics

Thus, "worldview" and "language" are almost synonymous.

Wittgenstein said in [ref. [125]: "Tractatus Logico-Philosophicus (abbreviated as TLP)"(1921)],

(C) **"The limits of my language mean the limits of my world."**

This implies that

(D) **What cannot be spoken in my language (i.e., my worldview) does not exist.**

Namely,

(E_1) **Without a worldview, there is no logic or proposition.**

In the same sense,

(E_2) **Without a worldview, nothing can be said (or, only chit-chat can be said).**

♠**Note 1.1.** (i): The (scientific) worldview is made up of statements that relate the world to mathematics, but as Hempel's paradox "All crows are black" shows, the relation between set theory and the world must be somewhat unnatural. The question, "Can mathematics itself be a worldview? is a somewhat tricky question. In this text, mathematics alone is not considered a worldview. However, since we usually think of "natural numbers" and "space" as existing in the real world when we actually do mathematics, it is not necessarily wrong to think that mathematics and the world are connected even without a worldview.
(ii): The question, "Is there a complete worldview? is a serious question (cf. Sec. 12.2.3). Wittgenstein's paradox). However, since we cannot move on if we dwell on these fundamental issues, we will proceed in the spirit of "keeping a lid on what smells bad.

♠**Note 1.2.** [the definition of "existence"]
The word "existence" is polysemous and difficult to use. For example.

- Does God exist?
- Do Atoms Exist?
- Does Beauty exist?
- Does Mathematics Exist?
- Does Self exist?
- Does the mind exist?
- Does the past exist?
- Does "what is in front of us" exist?

These show the difficulty to use the word "existence".

In this text, I assume that

- **When we are discussing a certain worldview, we assume that the keywords for that worldview exist.**

For example, with Descartes' mind-body dualism, "mind," "body," and "thing" exist.

1.2 The overview of quantum language(=measurement theory)

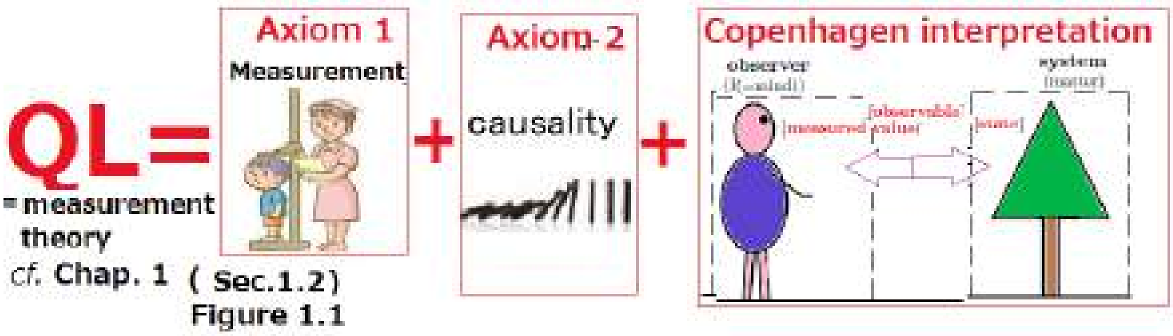
1.2.1 Axiom 1; What is probability (or, measurement)?) and Axiom 2 (What is causality?)

The idea of quantum language (also known as "measurement theory") is inspired and structured by quantum mechanics, in which microphenomena are analyzed. Quantum language is a language, by which we cannot only describe quantum mechanics but also most sciences (e.g., economics, psychology, engineering, etc.). However, it should be noted that quantum language is not almighty, for example, the theory of relativity is beyond the description of quantum language. Nevertheless, I believe that quantum language will be widely accepted as **the language of science**.

The framework of measurement theory (=quantum language) is simple, that is, it is composed with two axioms (Axiom 1 concerning measurement and Axiom 2 concerning causal relation) and the manual to use Axioms 1 and 2 (called the linguistic Copenhagen interpretation). That is, (*cf.* refs [48, 74]),

$$\underset{\text{(=quantum language)}}{\boxed{\text{measurement theory}}} = \underset{\text{[Axiom 1]}}{\boxed{\text{measurement}}} + \underset{\text{[Axiom 2]}}{\boxed{\text{causal relation}}} + \underset{\text{[the manual to use Axioms 1 and 2]}}{\boxed{\text{linguistic Copenhagen interpretation}}} \tag{1.1}$$

That is,

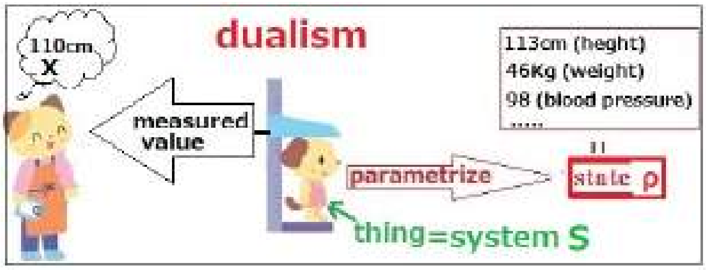


Although you do not need to fully understand Axiom 1 and Axiom 2 to read this text, I, for completeness, mention them as follows. (For the mathematical foundations of Axioms 1 and 2, see Section 1.4 (Appendix: the mathematical foundations of quantum language) later. The following axiom can be regarded as the mathematical generalization of the probabilistic interpretation of quantum mechanics (due to M. Born (*cf.* ref. [6])).

Axiom 1 [measurement] (in Section 1.2.1)

(This will be explained more precisely in Section 1.4)

With any system S, a C^*-algebraic basic structure $[\mathcal{A} \subseteq B(H)]$ can be associated in which measurement theory of that system can be formulated. When the observer takes a measurement $\mathsf{M}(\mathsf{O}{=}(X, 2^X, F), S_{[\rho]})$ $\Big($i.e., a measurement of an observable (or, by a measuring instrument) $\mathsf{O}{=}(X, 2^X, F)$ for a system $S_{[\rho]}$ i.e. a system S with a state ρ) $\Big)$, the probability that a measured value x $(\in X)$ is obtained by the measurement is given by $\rho(F(\{x\}))(\equiv {}_{\mathcal{A}^*}(\rho, (F(\{x\}))_{\mathcal{A}})$.



Max Born (1882–1970)

♠**Note 1.3.** [The relation between Quantum Language and Statistics]
(i): In the above, let $\rho(F(\{x\}))$ be denoted by $P_\rho(\{x\})$. And let $(X, 2^X, P_\rho(\cdot))$ be regarded as a sample space with parameter. If we start from this point (i.e., $(X, 2^X, P_\rho(\cdot)$), we will be in statistics. This is because

"statistics = theory of probability space with parameters"

That is,

quantum language		statistics
$\mathsf{M}(\mathsf{O}{=}(X, 2^X, F), S_{[\rho]})$)	$\xrightarrow{\hspace{3cm}}$ Elimination of observable O	$(X, 2^X, P_\rho(\cdot))$
philosophy		applied math

The elimination of an observable O implies the elimination of a philosophy. That is,

- Philosophy = "The Theory of Plato ideas (= observable)"

(ii): Let me be a little more specific. Statistics is mostly a theory of normal distributions, so let's use the following example of a normal distribution.

$$N_{\mu,\sigma}[\Xi] = \int_\Xi \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-\frac{(x-\mu)^2}{2\sigma^2}]dx \quad (\Xi \in \mathcal{B}(\mathbb{R}) = \text{the Borel field of the real line } \mathbb{R})$$

Then, we have the sample space with the paramers $\mu, \sigma (\in \mathbb{R} \times \mathbb{R}_+)$ such that

$$(\mathbb{R}, \mathcal{B}(\mathbb{R}), N_{\mu,\sigma}[\,\cdot\,])$$

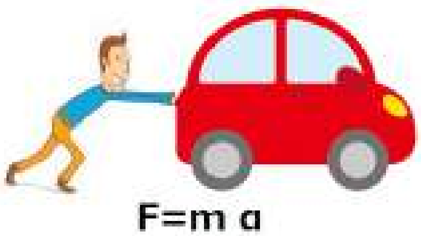
This is the starting point of statistics.

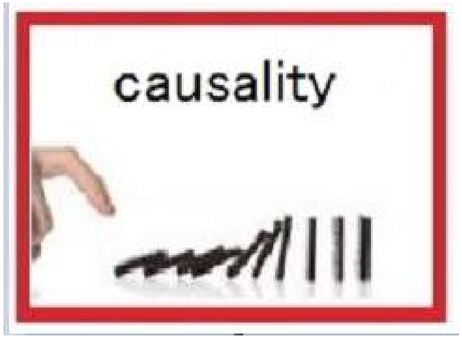
The following is a mathematical generalization of Heisenberg's kinetic equation ($\approx$ Schrödinger equation).

Axiom 2 Causality [Axiom 2; causality]

(This will be explained more precisely in Section 1.4)

Let T be a **tree** (i.e., semi-ordered tree structure). For each $t(\in T)$, a C^*-algebraic basic structure $[\mathcal{A}_t \subseteq B(H_t)]$ is associated. Then, the **causal chain** is represented by a **sequential causal operator** $\{\Phi_{t_1,t_2} : \mathcal{A}_{t_2} \to \mathcal{A}_{t_1}\}_{(t_1,t_2)\in T^2_{\leqq}}$, where it holds that $\Phi_{t_1,t_2}\Phi_{t_2,t_3} = \Phi_{t_1,t_3}$ $(t_1 \le t_2 \le t_3)$..





Note that this axiom is the mathematical generalization of Schrödinger equation (i.e., Heisenberg's kinetic equation).

Here, note that

(A) the above two axioms are kinds of spells (i.e., incantation, magic words, metaphysical statements), and thus, it is impossible to verify them experimentally. **What we should do is not to understand the two, but to learn the spells (i.e., Axioms 1 and 2) by rote.**

In this sense, quantum language is a kind of metaphysics ($\approx$ idealism). Therefore,

(B) **The formation of quantum languages depends on human marvelous language ability**

♠**Note 1.4.** (i): In this text, we assume that [metaphysics] = [idealism] = [theory not testable by experiment]. For example, most philosophy, mathematics, statistics, etc. are metaphysics.
On the other hand, [realism] = [theory that can be tested]. Examples include Newtonian mechanics, relativity, and quantum mechanics. Thus, quantum mechanics is a realism, but quantum language, which is a verbalization of quantum mechanics (or, the Copernican revolution), is a metaphysics*2.

*2 For the Copernican revolution, see Sec.10.2 and Sec.11.7.

(ii): If metaphysics did something wrong in the history of science, it is because metaphysics attempted to answer the following questions seriously in ordinary language:

($\sharp_1$) What is ◯◯ (e.g., ◯◯ = measurement, probability, causality, space-time, etc.)?

Although the question ($\sharp_1$) looks attractive, it is not productive. What is important is *to create a language* to deal with the keywords. So the ($\sharp_1$) has to be replaced by the following ($\sharp_2$).

($\sharp_2$) How should ◯◯ (e.g., ◯◯ =measurement, probability, causality) be used?

The problem ($\sharp_1$) will now be solved in the sense of ($\sharp_2$). If there are some failure in the history of philosophy, philosophers failed to propose a suitable language. It should be noted that Newton's success is due to the proposal of "the language called Newtonian mechanics". Recall that Newton mechanics does not answer the question "What are mass, force, acceleration?", but "How should they (i.e., mass, force, acceleration) be used?" or equivalently, "How do they relate to each other?". That is,

"[mass] · [acceleration] = [force]."

Thus, I think that Newton's most important accomplishment is the following three (which are equivalent):

($\flat_1$) to create physics called Newtonian mechanics.
($\flat_2$) to create the worldview called Newtonian mechanics.
($\flat_3$) to create the language called Newtonian mechanics.

where these are equivalent: i.e. ($\flat_1$)≈($\flat_2$)≈($\flat_3$).
Similarly I think that the greatest task of philosophy is the following three: from the idealistic point of view,

($\natural_1$) to create a philosophy called □ □
($\natural_2$) to create a worldview called □ □
($\natural_3$) to create a language called □ □

where these are equivalent: i.e. ($\natural_1$)≈($\natural_2$)≈($\natural_3$). Our purpose is to show that □ □ = quantum language.

Thus we can answer the following problem from the quantum linguistic point of view:

Problem 1.1. Scientifically please answer to the problem:

($\sharp$) What is measurement, probability, causality?

[Answer]:
As mentioned in Note 1.4, the question "What are measurement, probability, and causality?" should be interpreted as the question "How do we use the terms measurement, probability, and causality?" If so, it suffices to propose a worldview [=language) with the words measurement, probability, and causality as keywords. But if so, since we have already proposed "quantum language (= Axiom 1(measurement) + Axiom 2 (causality)]" and know that "measurement, probability, and causality are keywords of quantum language", we already know that the answer to the question ($\sharp$) is a "quantum language". Of course, the most important thing is "confirmation of the great descriptive power of quantum language", which I think you can understand by reading this text and ref. [74].

If there is a reader who does not accept this, he/she should create a "language more powerful than quantum language". □

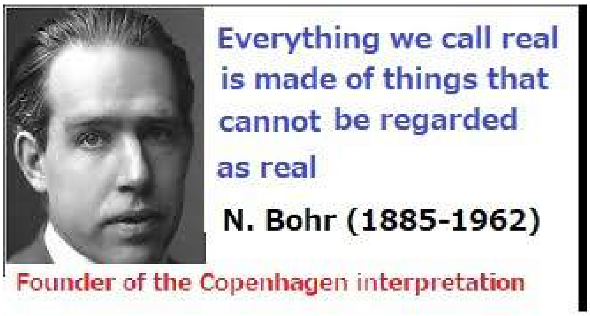
♠**Note 1.5.** (i): The above problem is the most difficult unsolved problem in the history of philosophy. The readers may be surprised at how easily this can be solved. I think many readers already know how powerful it is to create a proper language as the great success of Newtonian mechanics. Again, recall that Newtonian mechanics never answer to "What are forces, accelerations, and masses?". What Newtonian mechanics answers to is "How are used the terms: forces, accelerations, and masses?"

(ii): Recall the quotes in "Tractatus Logico-Philosophicus" (by Wittgenstein) such that

(♯) **The limits of my language mean the limits of my world.**

which says that "world=language". However, he could not create "his language".

1.3 Linguistic Copenhagen interpretation (or in short, Copenhagen interpretation)



1.3.1 What is the Copenhagen interpretation?

In the previous section, an overview of quantum language [Axiom 1 (measurement) and Axiom 2 (causality)] was outlined.
(A)

$$\underset{\text{(=measurement theory)}}{\boxed{\text{quantum language(=QL)}}} = \underset{\text{[Axiom 1]}}{\boxed{\text{measurement}}} + \underset{\text{[Axiom 2]}}{\boxed{\text{causal relation}}} + \underset{\text{[the manual to use Axioms 1 and 2]}}{\boxed{\text{(linguistic) Copenhagen interpretation}}} \tag{1.1}$$

In this section, the "Copenhagen interpretation ((linguistic) Copenhagen interpretation)" will be explained. Before doing so, let us reiterate the following

(B_1) Axioms are a kind of incantation (spell, magic word, metaphysical statement) and cannot be experimentally verified

Further,

(B_2) Quantum language is a language, and you may not be able to use it well at first. You can only acquire the ability to use it through practice and trial and error.

♠**Note 1.6.** In Mermin's book [101], he said

- If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be **"Shut up and calculate"**
- Stop being bothered!

Also, D. Howard said in [34]:

- Even within the Copenhagen School, there was a wide range of opinion on the Copenhagen interpretation. For example, there was disagreement about "wave function collapse" which is supposed to be the central theme of the Copenhagen interpretation.

 For further information, see my homepage

(See ref. [65] for my opinion on "wave function collapse.")

This means that

"What is the Copenhagen interpretation?" is unsolved problem.

We believe that this is one of the most important unsolved problems in science. Thus, I can say that one of the purposes of this text is to answer the unsolved problem: "What is the Copenhagen interpretation?". I assert that the linguistic Copenhagen interpretation is the true Copenhagen interpretation (*cf.* ref. [74]). That is, we assert that the Copenhagen interpretation is justified in philosophy (i.e., language) and not in physics.

Thus. in this text, "Copenhagen interpretation" is identified with "linguistic Copenhagen interpretation".

If so,

(C_1) It is essential to acquire a habitual thinking to master the axioms (Axioms 1 and 2). In order to master the quantum language as quickly as possible, you will need a good manual for mastering the axioms (Axioms 1 and 2)

Thus, we get the following definition,

Definition 1.2. [Copenhagen Interpretation (=Linguistic Copenhagen Interpretation)]

(C_2) **Copenhagen Interpretation**
$\underset{\text{def.}}{:=}$ **Manual for using spells (Axiom 1 and Axiom 2)**

The above can be rephrased as drawing a line between "good use (i.e., scientific use)" and "bad use (i.e., pseudo-scientific use)" as follows.

(C_3) **Rules for drawing the line between scientific and pseudoscientific propositions**

Checking the validity of this definition is one of the key objectives of this text.

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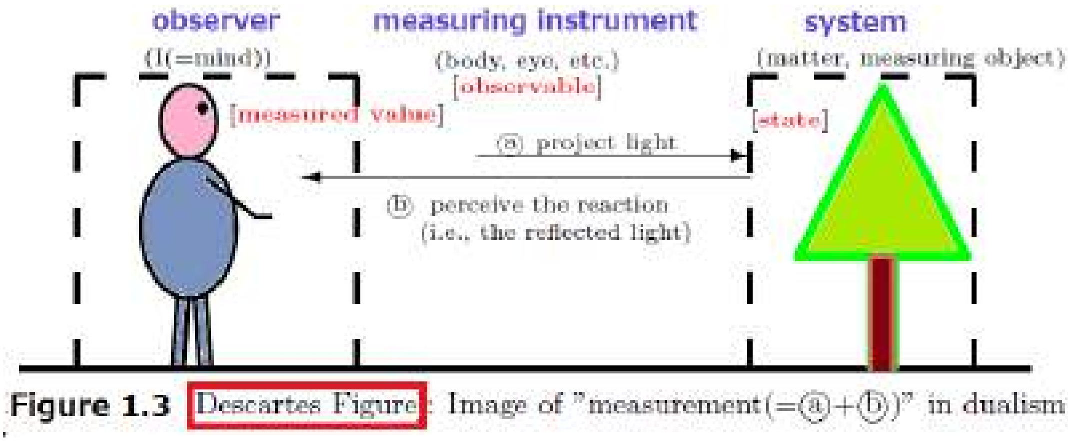
1.3.2 Descartes figure

Now, let's go on to explain the Copenhagen Interpretation.

Since Axiom 1 includes the term "measurement", the concept of "measurement" should be, at first, understood in dualism (i.e., "observer" and "measuring object") as illustrated in Figure 1.3.

Figure 1.3. [Descartes Figure]: Image of "measurement(=ⓐ+ⓑ)" in dualism



In the figure, "measurement" is characterized as interaction between "observer" and "system"(matter or object to be measured, measuring object), composed of

(D_1) ⓐ projection of light onto the object (i.e., someone, not necessarily an observer, shines the light.)
ⓑ perception of the reaction of the object (i.e., the observer receives the reaction.)

However, I want to emphasize that the interaction cannot be represented by kinetic equations. Therefore,

(D_2) in measurement theory (= quantum language), we use the term "measurement" instead of "interaction". Therefore, we won't say the above (D_1) outright.

After all, we think that:

(D_3) **there is no measured value without observer.**

Thus, measurement theory is composed of three keywords:

measured value ,	observable (≈ measuring instrument) ,	state .
(observer, brain, mind)	(telescope, thermometer, eye, ear, body, polar star)	(matter, measuring object)

In view of the above figure, it might be called "ternary relation (or, trialism)" instead of "dualism". But, following the convention, we use *"dualism"* throughout this text.

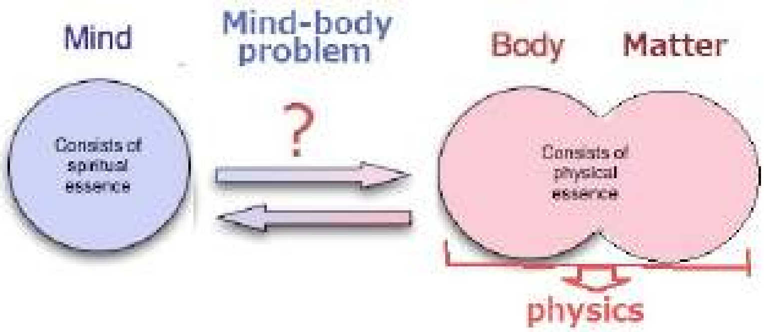
♠**Note 1.7.**
(i): Descartes' dualistic idealism has the following form:

$$[A](\textbf{mind}) \quad \longleftarrow \underset{\textbf{(medium)}}{[B(\textbf{body, sensory organ})]} \longrightarrow \quad [C](\textbf{matter})$$

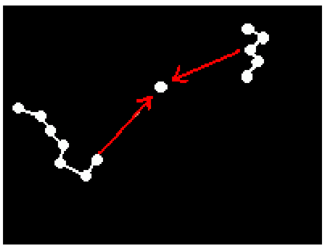
This corresponds to measurement theory (= quantum language) as follows.

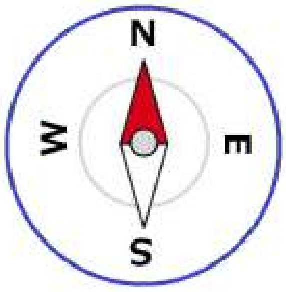
\	[A](= mind)	[B](Mediating of A and C) (body)	[C](= matter)
quantum language	**measured value**	**observable**	**state [system]**

(ii): If the interaction of (D_1) could be described by the equation of motion, the relationship between "observer" and "object" would become obvious, and we would end up with a monism of "observer + object". Thus, we assume that (D_1) can not be described by kinetic equation.



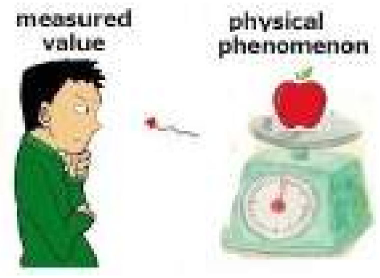
(iii): The concept of "observable" (which can be identified with "measuring instrument") is not easy. For example, telescopes, glasses and eyes are a type of measuring instrument. A directional magnet is, of course, a measuring instrument. If so, then the polar star is also a type of measuring instrument.





(iv): The following argument, called the Bohr-Einstein debates, is extremely profound (*cf.* Sec.9.3.3 Bohr-Einstein debates).

- For example, assume that an apple is put on food scale. We are not sure if he is trying to weigh an apple or make sure the food scale isn' t broken (see Figure below). Thus, the difference between "body" and "matter" is arbitrary. This is determined by the observer. Both "body" and "matter" obey the laws of physics. However, the laws governing "mind" are assumed to be unknown. Maybe there is no such thing. This is the position of monism. On the other hand, the dualist position is that measurement is essential.



Einstein is the former position (i.e., monism) and Bohr is the latter (i.e., dualism). It's the biggest scientific debate of the 20th century, but it's not yet settled. My standing point is as follows.

- These two (monism and dualism) are not to be unified. As mentioned in Figure 0.1 in Preface, they should be compatible.

(v): By the way, Merleau-Ponty (1908 - 1961) might think in the following manner.

- I shake hands with my right hand and the left hand. In this case, if I regard the right hand as the measuring instrument, I feel the existence of my left hand. On the contrary, if I regard the left hand as the measuring instrument, I feel the existence of my right hand.

Thus, "observable" is a treasure trove of wordplay. In fact, as will be shown throughout this text, I think that

- The history of Western philosophy is the story of the maturation of "Plato's Idea" into "observables" through twists and turns.

////

1.3.3 The linguistic Copenhagen interpretation [(E_0)-(E_7)]

The (linguistic) Copenhagen Interpretation is "a manual for using Axiom 1 (measurement) and Axiom 2 (causality). If that were the case (if it were a manual), wouldn't we have to list all sorts of miscellaneous things and "there would be no end to the explanations"? Even car driving manuals are endless in detail. There is no such thing as a complete rulebook for baseball or soccer, either. The author believes that the Copenhagen Interpretation may have such a fear (*cf.* Sec. 12.2.3 : Wittgenstein's paradox).

Now, below [(E_0)∼(E_7)], I will briefly explain the Copenhagen interpretation. The most important of these, and especially important, is,

(E_4) Only one measurement is permitted.

(E):The linguistic Copenhagen interpretation

With Descartes figure 1.3 (in Sec. 1.3, i.e.

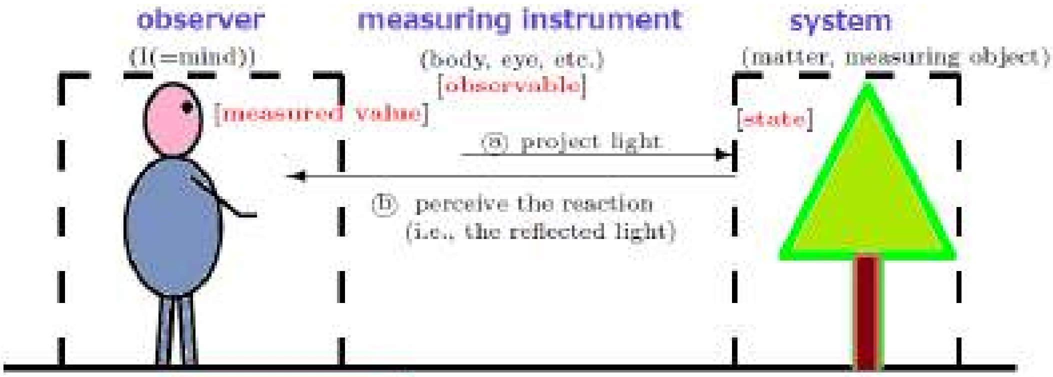
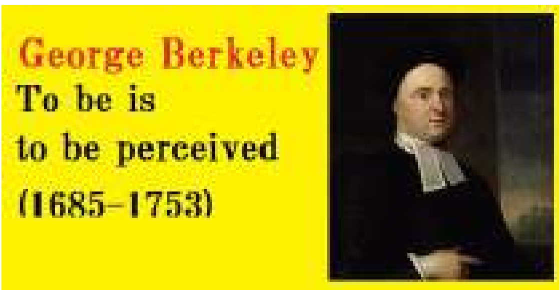
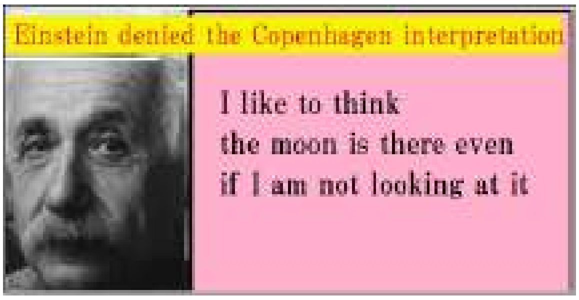


Figure 1.3 Descartes Figure : Image of "measurement(=ⓐ+ⓑ)" in dualism

and the following (E_0)-(E_7) in mind,
describe every phenomenon in terms of Axioms 1 and 2!

(E_0) (i); If you don't measure it, you don't know anything. Don't talk about things that cannot be measured. This seems to be Berkeley's saying "To be is to be perceived." On the other hand, Einstein, a monistic realist of the anti-Copenhagen interpretation, said "The moon is there even when we are not looking".

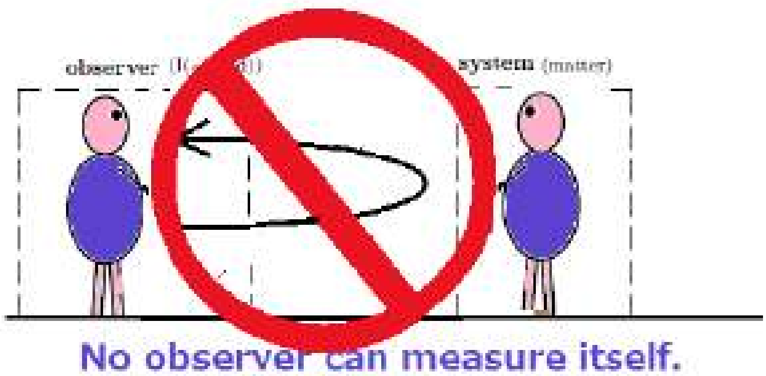




(ii); [Popper's Falsifiability; ref. [105]]
In order to guarantee the objectivity of a scientific theory, there must be a possibility that the hypothesis will be disproved by experiment or observation. That is, truth must always be subjected to experiments that deny its truth. And if the denying experiment is confirmed, then the truth must be rejected.

♠**Memo 1.1.** It is impossible to consider Popper's falsifiability as a "foundational principle of science". This is because statistics (and quantum language) do not have disfalsifiabilit. Therefore, it is more acceptable for scientists to consider disprovability as one rule in the Copenhagen interpretation (*cf.* Sec. 12.7.3).

(E_1) Consider the **dualism** composed of "observer" and "matter (= object to be measured)", where "observer" and "matter (= measuring object)" must be absolutely separated. Figuratively speaking, "Audience should not go on stage", or

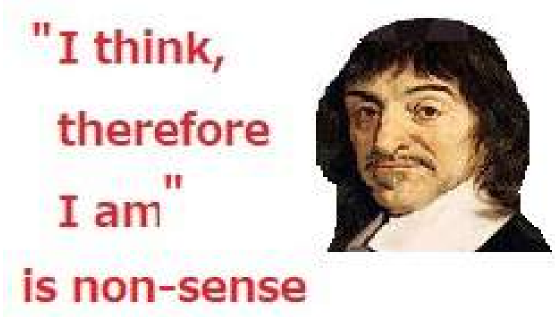


"the observer cannot measure the observer himself"

or

"The measurement is not dependent on the observer"

That is, the following qualia problem is non-sense.



To be more specific, the words "I", "Here", "Now" are forbidden . Hence, "I think, therefore I am" is non-sense.

♠**Note 1.8.** Consider the followings:

($\sharp_1$) I measure my body temperature with a thermometer.
($\sharp_2$) I feel my body feverish.

and

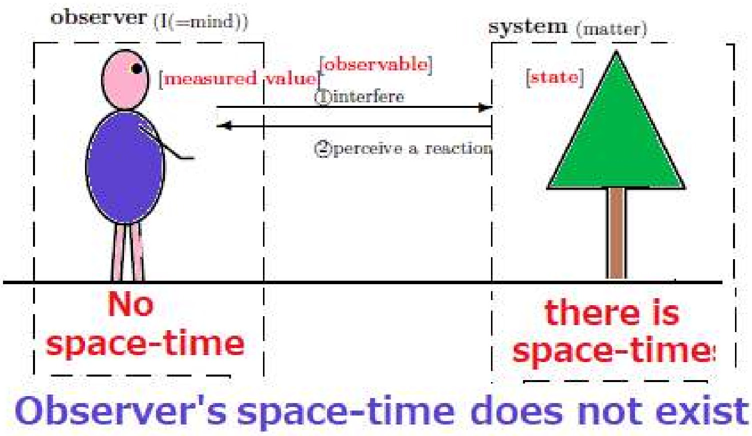
($\flat_1$) The doctor measures my body temperature with a thermometer.
($\flat_2$) The doctor feels my body feverish.

In terms of measurement, ($\sharp_1$) and ($\flat_1$) are the same. On the other hand, ($\sharp_2$)and ($\flat_2$) are different. Thus, in the strictest sense, we consider that ($\sharp_2$) cannot be regarded as a measurement. However, the ($\flat_2$) seems to be a measurement. This example will help you understand that cogito proposition "I think, therefore I am" in Chapter 8.

(E_2) While "matter" is in the space-time, the observer is not (Sec.9.2: Leibniz's relationalism concerning space-time). That is, observer's space-time does not exist. Thus, the question: "When, where and by whom was the measured value obtained?" is out of the scope of measurement theory. Thus, words such as "now," "here," and "I" should not be used in a scientific proposition. If you are going to use it, you need to be very careful. That is,

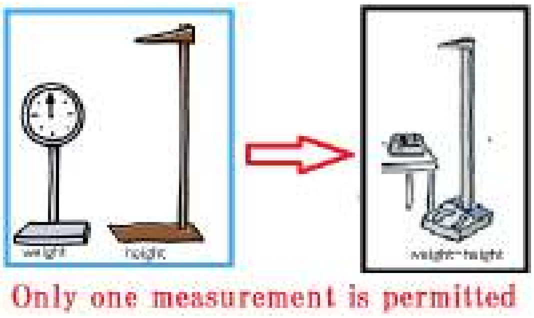
there is no tense either in measurement theory or in science.

The "tense" is a treasure trove of word play (*cf.* Augustinus "Only the present exists"(Sec. 6.1), McTaggart's paradox, Russell's five-minute hypothesis in Chap. 11).

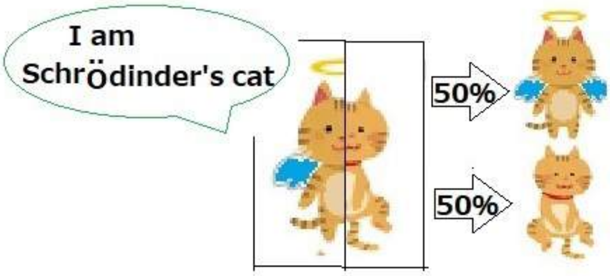


(E_3) In measurement theory, "observable(=measuring instrument ≈ body)" is the most important than "measured value(≈mind)" and "state(≈matter)" in (D_3). The prototype of observables is Plato's Idea. Also, statistics is not philosophical because it does not have "observables" (*cf.* Note 1.3).

(E_4) **Only one measurement is permitted**.



Thus, the state after measurement (or, wave function collapse, the influence of measurement) is meaningless.



For the virtual wave function collapse, see:

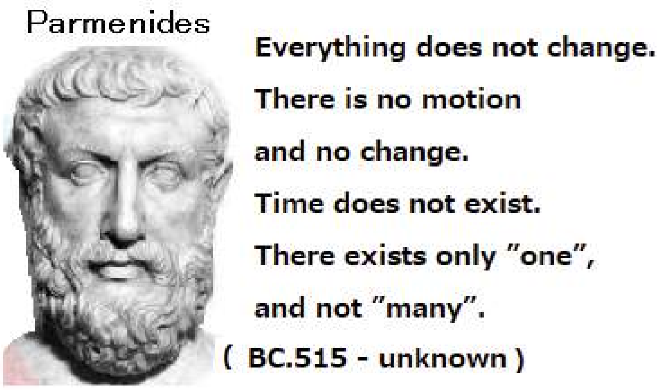
- ref.[65] S. Ishikawa, *Linguistic interpretation of quantum mechanics; Projection Postulate,* JQIS, Vol. 5, No.4, 150-155, 2015, DOI: 10.4236/jqis.2015.54017
(`http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=62464`)

♠**Note 1.9.** Ref.[65] is powerful as follows. The Schrödinger cat is the most famous paradox in quantum mechanics. However, we are not bothered by this paradox since the state after measurement is not described in quantum language.

(E_5) There is no probability without measurement. Also, the measurement cannot be measured.
There is no logic without measurement. See Figure 0.1 in Preface such as

Analytic philosophy (logic) —⑭ (See Chapter 12)→
statistics (probability) —⑨→
} —⑮→ QL (= Measurement theory)

(E_6) There is one state and it never moves. Therefore, there is no time (time is just an ordered parameter cf. Sec. 9.2). Therefore, we always use the Heisenberg picture (basically we do not use the Schrödinger picture), etc. It is still surprising that Parmenides mentioned almost all of the Copenhagen interpretations 2500 years ago (cf. Sec. 2.3).



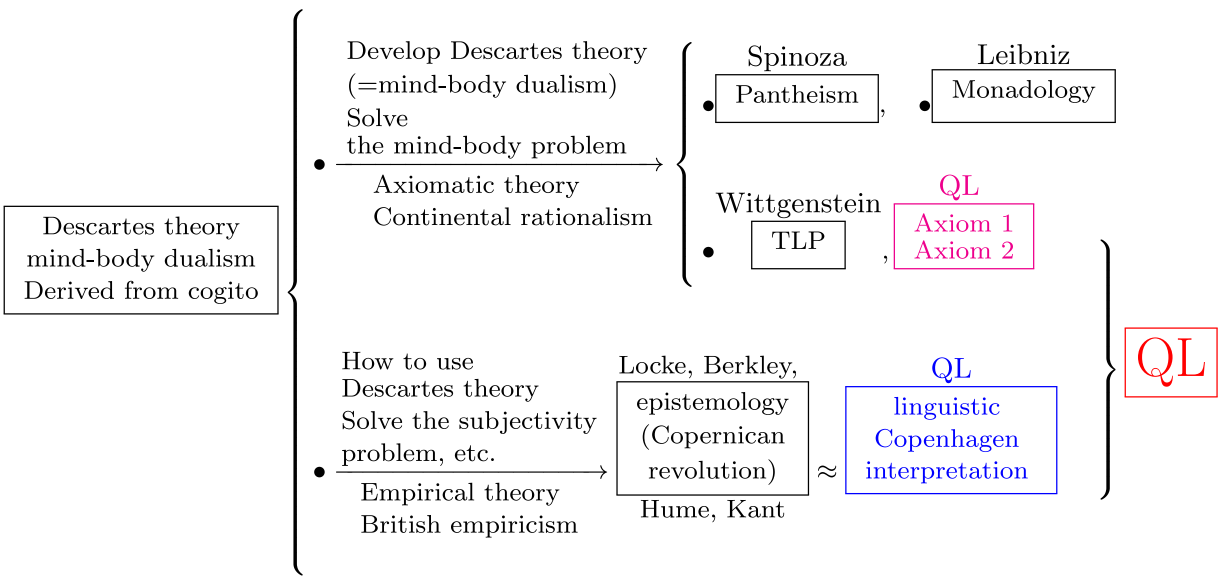
and so on.

If we believe that quantum language is the final destination of dualistic idealism (cf. ⑪ and ⑮ in Figure 0.1 (in Preface)), it seems natural to think as follows

(E_7) most maxims of the philosophers (particularly, the dualistic idealism) can be regarded as expressions in linguistic Copenhagen interpretation.

♠**Note 1.10.** To become skillful in the Copenhagen interpretation, it is not enough to memorize (E_0)~(E_7), but rather to do a lot of exercises. This may be the quickest way to read ref. [74].

Remark 1.4. **[Why "philosopher's golden rule = Copenhagen Interpretation"?]**
Why "philosopher's golden rule = Copenhagen Interpretation"? I insist on the diagram below. That is, we reject the pseudoscientific "Kantian synthesis" and insist on the following "QL synthesis"(*cf.* Chap. 8).



For example, Descartes' "I think therefore I am", Locke's "Secondary quality", Berkeley's "To be is to be perceived", Kant's "Copernican revolution", Wittgenstein's

"The limits of my language mean the limits of my world", "What we cannot speak about we must pass over in silence" are the products of the pursuit of dualistic idealism. If so, it is a matter of course that the ideas of British empiricism and the Copenhagen interpretation become similar, so [QL integration] is natural (*cf.* Sec. 10.3).

The above says that

(F_1) Both Cartesian-Kantian philosophy and the linguistic Copenhagen interpretation are products of the pursuit of dualistic idealism.

Then, it is natural to consider

maxims of philosophers $\approx$ the linguistic Copenhagen interpretation.

(F_2) : Now we can answer the following interesting question:

Why is philosophy (= dualistic idealism) preposterous?

Because

The linguistic Copenhagen interpretation is preposterous.

1.3.4 Descriptive power of quantum language

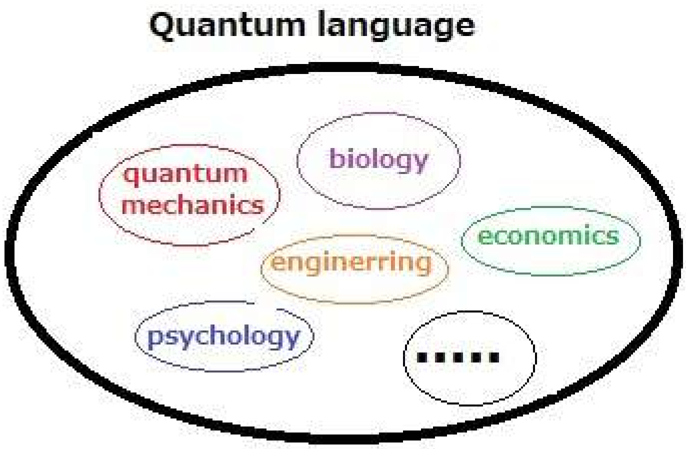
We assert the following.

Assertion 1.5. [descriptive power of quantum language](*cf.* ref. [74])
The descriptive power of quantum language (=QL) is quite large. The following can be said about it.

Descriptive power of quantum language
≈descriptive power of statistics
+ descriptive power of quantum mechanics

Therefore, we assert that

(G_1) **quantum language is a language, by which most sciences (e.g., economics, psychology, engineering, etc.) are described**



Quantum language is a language, by which we cannot only describe quantum mechanics but also most sciences (e.g., economics, psychology, engineering, etc.). Thus, as a modified version of "statistics is the language of science", we consider that

(G_2) **Quantum language is the language of science,**
or,
to do sciences is to describe phenomena by quantum language

However, it should be noted that quantum language is not almighty, for example, the theory of relativity is beyond the description of quantum language.

1.4 Appendix: The mathematical foundations of quantum language

Literary enjoyment is also one of the charms of philosophy (e.g., references [23, 116]). If you want to read this book as literary, it might be recommended to skip this section. Therefore, I have tried to write this book in such a way that you can enjoy it even if you skip this section (the mathematics part).

However, since we are interested in the "scientific side of philosophy", it is necessary to mention some scientific theory.

Although we will explain "quantum language (= measurement theory") in this section, I may have omitted too much. This section alone may not be sufficient to read the mathematical part of this text. For the precise explanation, see the references [46, 74]. Also, ref. [48] is easy to read.

We have two formulations of QL such that

$$\text{(A) the formulation of QL} \begin{cases} (\mathrm{A}_1)\text{: the } C^*\text{-algebraic formulation (in Section 1.4)} \\ \\ (\mathrm{A}_2)\text{: the } W^*\text{-algebraic formulation } (\textit{cf.} \text{ ref. [74])} \end{cases}$$

(A_1) is simple, on the other hand, (A_2) is rather mathematical. In this text, we usually use the C^*-algebraic formulation (A_1).

1.4.1 Mathematical preparations (the C^*-algebraic formulation of QL)

Following refs. [46, 48, 49, 74] (all our results until present are summarized in ref. [74]), we will review quantum language, which has the following form:

$$\underset{\text{(= measurement theory)}}{\boxed{\boxed{\text{Quantum language}}}} = \underset{\text{(Axiom 1)}}{\boxed{\text{measurement}}} + \underset{\text{(Axiom 2)}}{\boxed{\text{causality}}} + \underset{\text{(how to use Axioms 1 and 2)}}{\boxed{\text{Copenhagen interpretation}}} \tag{1.2}$$

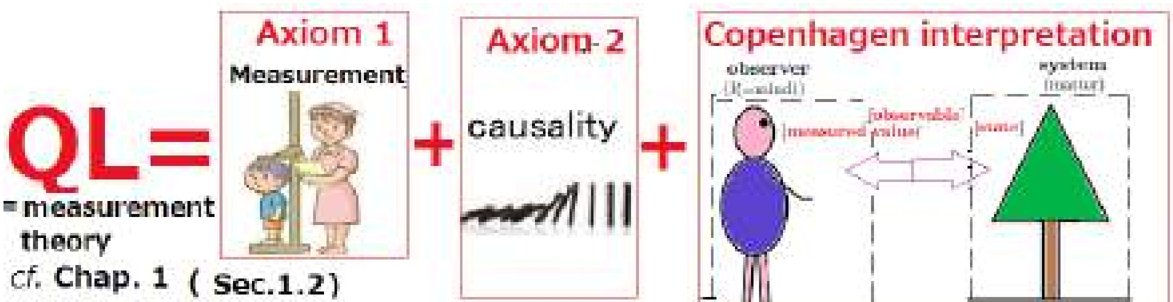


Figure 1.1

which asserts that "measurement" and "causality" are the most important concepts in science.

Consider an operator algebra $B(H)$ (i.e., an operator algebra composed of all bounded linear operators on a Hilbert space H with the norm $\|G\|_{B(H)} = \sup_{\|u\|_H=1} \|Gu\|_H$. Let $\mathcal{A}(\subseteq B(H))$ is a C^*-algebra, (i.e., norm-closed subalgebra of $B(H)$) (*cf.* refs. [121], [118],

[128]). Our purpose of this section is not to explain QL in general situation but to explain QL in an understandable setting. Thus, from here, we devote ourselves to the following simple cases:

$$
\text{(B)} \quad \text{QL} = \begin{cases} (\mathrm{B}_1)\text{: quantum QL} \quad (\text{when } \mathcal{A} = B(\mathbb{C}^n), \text{ where } H = \mathbb{C}^n) \\ \qquad \text{i.e. the } C^*\text{-algebra composed of all } (n \times n) \text{ complex matrixes} \\ \\ (\mathrm{B}_2)\text{: classical QL} \quad (\text{when } \mathcal{A} = C(\Omega)), \\ \qquad \text{i.e. the space of all continuous functions on a compact space } \Omega \end{cases}
$$

The pair $[\mathcal{A}, B(H)]$ is called a C^*-algebraic basic structure. Let $\mathcal{A}^*$ be the dual Banach space of $\mathcal{A}$. That is, $\mathcal{A}^* = \{\rho \mid \rho \text{ is a continuous linear functional on } \mathcal{A}\}$, and the norm $\|\rho\|_{\mathcal{A}^*}$ is defined by $\sup\{|\rho(G)(= {}_{\mathcal{A}^*}\langle \rho, G\rangle_{\mathcal{A}})| \mid G \in \mathcal{A} \text{ such that } \|G\|_{\mathcal{A}}(= \|G\|_{B(H)}) \leq 1\}$. Define the **mixed state** ρ $(\in \mathcal{A}^*)$ such that $\|\rho\|_{\mathcal{A}^*} = 1$ and $\rho(L) \geq 0$ for all $L \in \mathcal{A}$ such that $L \geq 0$. And define the mixed state space $\mathfrak{S}^m(\mathcal{A}^*)$ such that

$$
\mathfrak{S}^m(\mathcal{A}^*)=\{\rho \in \mathcal{A}^* \mid \rho \text{ is a mixed state}\}.
$$

A mixed state $\rho(\in \mathfrak{S}^m(\mathcal{A}^*))$ is called a **pure state** (or simply, **state**) if it satisfies that "$\rho = \theta\rho_1 + (1-\theta)\rho_2$ for some $\rho_1, \rho_2 \in \mathfrak{S}^m(\mathcal{A}^*)$ and $0 < \theta < 1$" implies "$\rho = \rho_1 = \rho_2$". Put

$$
\mathfrak{S}^p(\mathcal{A}^*)=\{\rho \in \mathfrak{S}^m(\mathcal{A}^*) \mid \rho \text{ is a pure state}\},
$$

which is called a *state space*. It is well known (*cf.* ref. [118]) that

$$
\text{(C)} \begin{cases} [\text{Case } (\mathrm{C}_1)]; \quad \mathfrak{S}^p(B_c(H)^*) = \{\rho = |u\rangle\langle u| \text{ (i.e., the Dirac notation)} \mid \|u\|_H = 1\} \\ \\ [\text{Case } (\mathrm{C}_2)]; \quad \mathfrak{S}^p(C(\Omega)^*) = \{\rho = \delta_{\omega_0} \mid \delta_{\omega_0} \text{ is a point measure at } \omega_0 \in \Omega\} \approx \Omega. \end{cases}
$$

In [Case (C_2)], under the following identification:

$$
\mathfrak{S}^p(C(\Omega)^*) \in \delta_\omega \leftrightarrow \omega \in \Omega,
$$

ω and Ω is respectively called a state and a state space.

Definition 1.6. [**Observable, Image observable**] According to the noted idea (*cf.* refs. [14], [33]), an *observable* $\mathsf{O} =(X, \mathcal{P}(X), G)$ in $\mathcal{A}$ is defined as follows:

(i) X is a finite set, $\mathcal{P}(X)(= 2^X$, the power set of X).

(ii) [Additivity] G is a mapping from $\mathcal{P}(X)$ to $\mathcal{A}$ satisfying: (a): for every $\Xi \in \mathcal{P}(X)$, $G(\Xi)$ is a non-negative element in $\mathcal{A}$ such that $0 \leq G(\Xi) \leq I$, (b): $G(\emptyset) = 0$ and $G(X) = I$, where 0 and I is the 0-element and the identity in $\mathcal{A}$ respectively. (c):[additivity]

$$
G(\Xi_1) + G(\Xi_2) = G(\Xi_1 \cup \Xi_2)
$$

for all $\Xi_1, \Xi_2 \in \mathcal{P}(X)$ such that $\Xi_1 \bigcap \Xi_2 = \emptyset$.

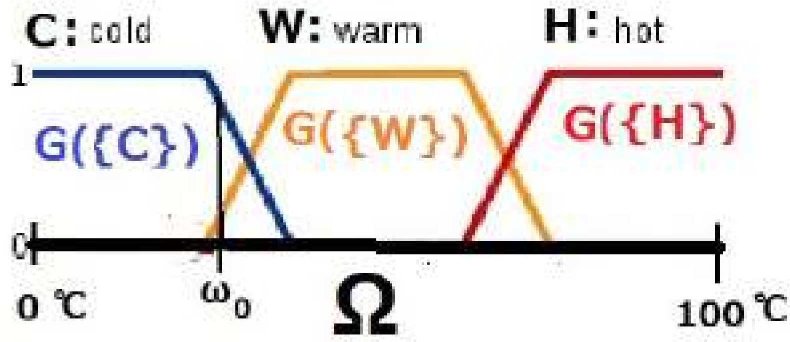
If $G(\Xi) = G(\Xi)^2$ $(\forall \Xi \in \mathcal{P}(X))$, then $\mathsf{O} =(X, \mathcal{P}(X), G)$ in $\mathcal{A}$ is a projective observable (or, crisp observable). Also, $\mathsf{O} =(X, \mathcal{P}(X), G)$ in $\mathcal{A}$ is also called an X-valued observable. Let Y be a finite set, and let $\Theta : X \to Y$ be a map. Then, $\Theta(\mathsf{O}) =(Y, \mathcal{P}(Y), G(\Theta^{-1}(\cdot))$ in $\mathcal{A}$ is also an observable in $\mathcal{A}$ (which is called an image observable).

♠**Note 1.11.** We have simplified the setup considerably.

- C^*-algebra$\mathcal{A}$ is assumed to have the identity I (In particular, for classical systems, the state space Ω is assumed to be a compact space). In the case that Ω is a locally compact space (e.g., $\Omega = \mathbb{R}$), it is usual to use $C_0(\Omega) = \{f : \Omega \to \mathbb{C}, \mid f \text{ is continuous}, \lim_{\omega\to\infty} f(\omega) = 0\}$. In this text, we use the compactification $\overline{\Omega}$(e.g., $\overline{\Omega} = \Omega \bigcup \{\infty\}$).
- We assume that X is a finite set. This may be too elementary.
- Perhaps we should have adopted the W^*-algebraic formulation. In general, the exact measurement is impossible in C^*-algebraic formulation (*cf.* Sec. 1.4.5).

Despite the above difficulties, the C^*-algebraic formulation was adopted since it was handy.

Example 1.7. C^*-algebraic formulation, [Classical QL]:



$H = L^2(\Omega, m)$, where m is the Lebesgue measure, $\mathcal{A} = C(\Omega)$ $(\subseteq B(H))$,
$[C(\Omega), B(L^2(\Omega, m))]$: C^*-algebraic basic structure.
$X = \{C, W, H\}$, Observable $O = (X, 2^X, G)$, where $[G(\{x\})] : \Omega \to \mathbb{R}$ is continuous $(\forall x \in X)$, such that $0 \le G(\{x\})](\omega) \le 1$ $(\forall x \in X, \omega \in \Omega$,

$$[G(\{C\})](\omega) + [G(\{W\})](\omega) + [G(\{H\})](\omega) = 1 \qquad (\forall \omega \in \Omega)$$

Also, a state is defined by $\delta_{\omega_0} \approx \omega_0 (\in \Omega)$.

1.4.2 Axiom 1 [Measurement] and Axiom 2 [Causality]

With any *system* S, a a C^*-algebra $\mathcal{A}(\subseteq B(H))$ can be associated in which the measurement theory (2) of that system can be formulated. A *state* of the system S is represented by an element $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$ and an *observable* is represented by an observable $\mathsf{O} = (X, \mathcal{P}(X), G)$ in $\mathcal{A}$. Also, the *measurement of the observable* O *for the system* S *with the state* ρ is denoted by $\mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]})$ $\big($ or more precisely, $\mathsf{M}_{\mathcal{A}}(\mathsf{O} = (X, \mathcal{P}(X), G), S_{[\rho]})$ $\big)$. An observer can obtain a measured value x $(\in X)$ by the measurement $\mathsf{M}(\mathsf{O}, S_{[\rho]})$.

The Axiom 1 presented below is a kind of mathematical generalization of Born's probabilistic interpretation of quantum mechanics. And thus, it is no longer a law of physics, but an axiom of metaphysics

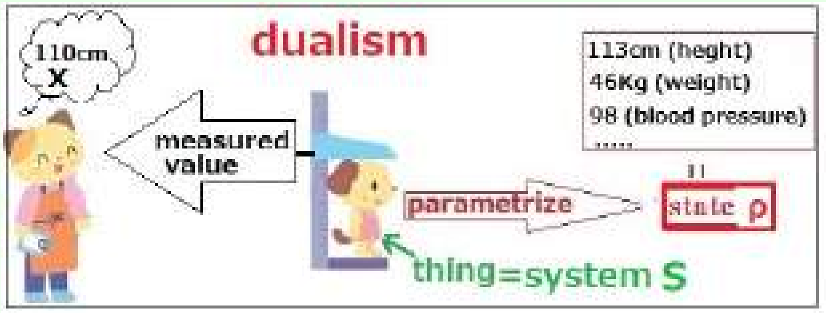
Now we can present Axiom 1 as follows.

Axiom 1 [State, Measurement] in the C^*-algebraic formulation

•[General (i.e., quantum and classical) case]:

(i) [State]: With any *system* S, a C^*-algebra $\mathcal{A}(\subseteq B(H))$ can be associated. A *state* of the system S is represented by an element $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$.

(ii) [Measurement]: An *observable* is represented by an observable $\mathsf{O} = (X, \mathcal{P}(X), G)$ in $\mathcal{A}$. And, the *measurement of the observable* O *for the system* S *with the state* ρ is denoted by $\mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]})$ (or more precisely, $\mathsf{M}_{\mathcal{A}}(\mathsf{O} = (X, \mathcal{P}(X), G), S_{[\rho]})$). An observer can obtain a measured value x $(\in X)$ by the measurement $\mathsf{M}(\mathsf{O}, S_{[\rho]})$. The probability that a measured value x $(\in X)$ obtained by the measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O} = (X, \mathcal{P}(X), G), S_{[\rho]})$ is given by $\rho(G(\{x\}))(\equiv {}_{\mathcal{A}^*}\langle \rho, G(\{x\})\rangle_{\mathcal{A}})$.

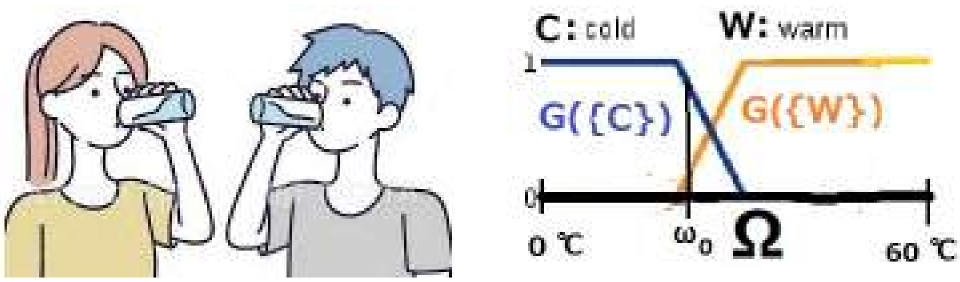


The above includes both classical and quantum cases. In the classical case, the above axiom can be written as follows.
•[Classical case, i.e. the case that $\mathcal{A} = C(\Omega)$]: The above (i) and (ii) can be summarized for the classical system as follows

(i) [Classical state]: A classical state of the system S is represented by an element $\omega(\in \Omega)$.

(ii) [Classical measurement]: The probability that a measured value $x(\in X)$ is obtained by the measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}(= (X, \mathcal{P}(X), G)), S_{[\omega]})$ is given by $[G(\{x\})](\omega)$.

Example 1.8. C^*-algebraic formulation, [Classical QL]:
For simplicity, assume that $\Omega = [0, 60]$ in Example 1.7. Thus, the observable is represented by $\mathsf{O} = (X = \{C, W\}, 2^X, G)$



Similar to Example 1.7, we have a measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O} = (X = \{C, W\}, 2^X, G), S_{[\omega_0]})$. Thus Axiom 1 says that

($\sharp_1$) When an observer measures the observable $\mathsf{O} = (X = \{C, W\}, 2^X, G)$ for a system S with the state $\omega_0 (\in \Omega)$, the probability that a measured value $x (\in X)$ is obtained is given by $[G(\{x\})](\omega_0)$.

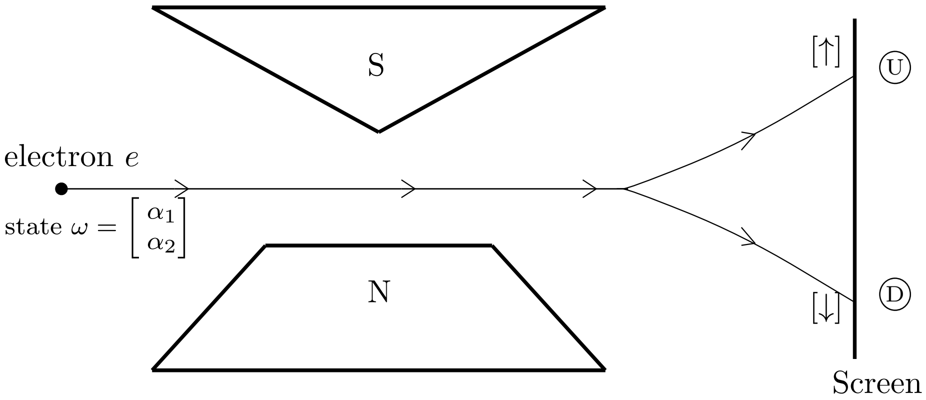
Using everyday language, we say:

($\sharp_2$) Suppose 100 subjects are asked to drink this glass of ω_0 °C water. One person is randomly selected from these 100 and asked, "Was this water cold, warm?

The probability that this subject answers $\left\{ \begin{array}{l} \text{"it was cold"} \\ \text{"it was warm"} \end{array} \right\}$ is $\left\{ \begin{array}{l} [G(\{C\})](\omega_0) \\ [G(\{W\})](\omega_0) \end{array} \right\}$

////

Example 1.9. [Quantum measurement(Schtern–Gerlach experiment (1922))] Assume that we examine the beam (of silver particles(or simply, electrons) after passing through the magnetic field. Then, as seen in the following figure, we see that all particles are deflected either equally upwards or equally downwards in a 50:50 ratio.



Stern–Gerlach experiment (1922)

Consider the two dimensional Hilbert space $H = \mathbb{C}^2$, And therefore, we get the non-commutative basic algebra $B(H)$, that is, the algebra composed of all 2×2 matrices. Thus, we have the quantum basic structure:

$$[\mathcal{C}(H) \subseteq B(H) \subseteq B(H)] = [B(\mathbb{C}^2) \subseteq B(\mathbb{C}^2) \subseteq B(\mathbb{C}^2)]$$

since the dimension of H is finite.

The spin state of an electron P is represented by $\rho (= |\omega\rangle\langle\omega|)$, where $\omega \in \mathbb{C}^2$ such that $\|\omega\| = 1$. Put $\omega = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$ (where, $||\omega||^2 = |\alpha_1|^2 + |\alpha_2|^2 = 1$).

Define $\mathsf{O}_z \equiv (Z, 2^Z, F_z)$, the spin observable concerning the z-axis, such that, $Z = \{\uparrow, \downarrow\}$ and

$$F_z(\{\uparrow\}) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad F_z(\{\downarrow\}) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{1.3}$$

$$F_z(\emptyset) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad F_z(\{\uparrow, \downarrow\}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Here, Born's quantum measurement theory (the probabilistic interpretation of quantum mechanics) says that

($\sharp$) When a quantum measurement $\mathsf{M}_{B(\mathbb{C}^2)}(\mathsf{O}, S_{[\rho]})$ is taken, the probability that

$$\text{a measured value } \begin{bmatrix} \uparrow \\ \downarrow \end{bmatrix} \text{ is obtained is given by } \begin{bmatrix} \langle \omega, F^z(\{\uparrow\})\omega \rangle = |\alpha_1|^2 \\ \\ \langle \omega, F^z(\{\downarrow\})\omega \rangle = |\alpha_2|^2 \end{bmatrix}$$

That is, putting ω $(= \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix})$, we says that

When the electron with a spin state ρ progresses in a magnetic field,

the probability that the Geiger counter $\begin{bmatrix} Ⓤ \\ Ⓓ \end{bmatrix}$ sounds

$$\text{is give by} \begin{bmatrix} \begin{bmatrix} \overline{\alpha}_1 & \overline{\alpha}_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = |\alpha_1|^2 \\ \\ \begin{bmatrix} \overline{\alpha}_1 & \overline{\alpha}_2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = |\alpha_2|^2 \end{bmatrix}$$

Also, we can define $\mathbf{O}^x \equiv (X, 2^X, F^x)$, the spin observable concerning the x-axis, such that, $X = \{\uparrow_x, \downarrow_x\}$ and

$$F^x(\{\uparrow_x\}) = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}, \quad F^x(\{\downarrow_x\}) = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}.$$

And furthermore, we can define $\mathbf{O}^y \equiv (Y, 2^Y, F^y)$, the spin observable concerning the y-axis, such that, $Y = \{\uparrow_y, \downarrow_y\}$ and

$$F^y(\{\uparrow_y\}) = \begin{bmatrix} 1/2 & i/2 \\ -i/2 & 1/2 \end{bmatrix}, \quad F^y(\{\downarrow_y\}) = \begin{bmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{bmatrix},$$

where $i = \sqrt{-1}$.

Here, putting

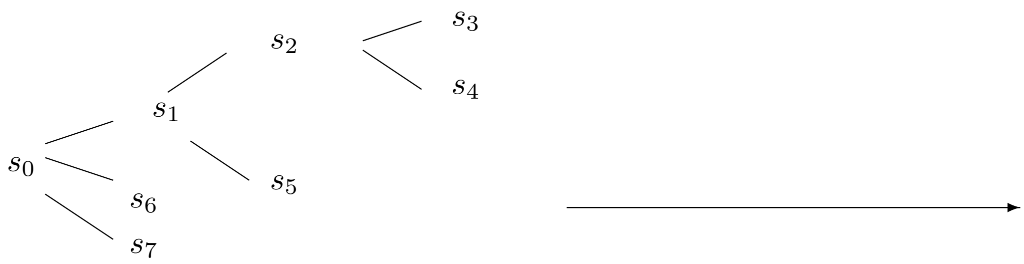
$$\hat{S}_x = F_x(\{\uparrow\}) - F_x(\{\downarrow\}), \quad \hat{S}_y = F_y(\{\uparrow\}) - F_y(\{\downarrow\}), \quad \hat{S}_z = F_z(\{\uparrow\}) - F_z(\{\downarrow\})$$

we have the following commutation relation:

$$\hat{S}_y\hat{S}_z - \hat{S}_z\hat{S}_y = 2i\hat{S}_x, \quad \hat{S}_z\hat{S}_x - \hat{S}_x\hat{S}_z = 2i\hat{S}_y, \quad \hat{S}_x\hat{S}_y - \hat{S}_y\hat{S}_x = 2i\hat{S}_z$$

Next, we explain Axiom 2 (which is rarely used in this text).

Let $(\mathbb{T}, \leq)$ be a tree-like semi-ordered set. The following two are typical.



Tree $\mathbb{T}$ real line (i.e., linear ordered set $\mathbb{T}$)

Assume that, for each $t \in \mathbb{T}$, a C^*-algebra $\mathcal{A}_t$ is associated.

A continuous linear operator $\Phi_{1,2} : \mathcal{A}_2 \to \mathcal{A}_1$ is called a *homomorphism* (or, *causal operator*), if it satisfies that

($\sharp_1$) $\Phi_{1,2}(L) \geq 0$ ($\forall L \in \mathcal{A}_2$ such that $L \geq 0$)
($\sharp_2$) $\Phi_{1,2}(I_2) = I_1$, where I_1 and I_2 is identity maps in $\mathcal{A}_1$ and $\mathcal{A}_2$ respectively.
($\sharp_3$) $\Phi_{1,2}(L_2 \cdot M_2) = \Phi_{1,2}(L_2)\Phi_{1,2}(M_2)$ and $\Phi_{1,2}(L_2^*) = (\Phi_{1,2}(L_2))^*$ ($\forall L_2, M_2 \in \mathcal{A}_2$))

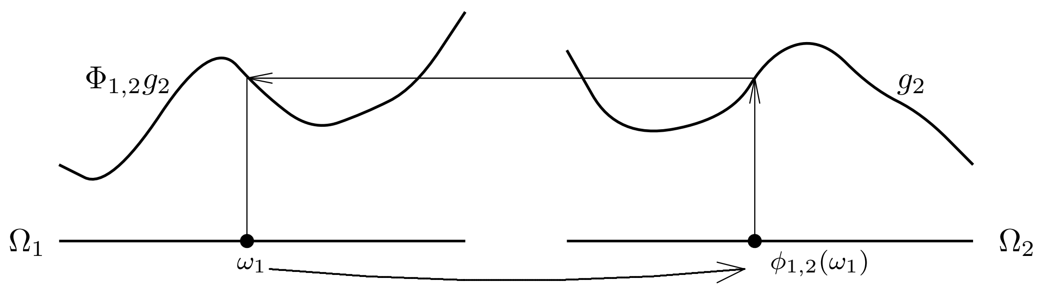
The dual operator $\Phi^*_{1,2} : {\mathcal{A}_1}^* \to {\mathcal{A}_2}^*$ of a causal operator causal operator $\Phi_{1,2} : \mathcal{A}_2 \to \mathcal{A}_1$ satisfies

$$\Phi^*_{1,2}(\mathfrak{S}^p(\mathcal{A}^*_1)) \subseteq \mathfrak{S}^p(\mathcal{A}^*_2) \tag{1.4}$$

In classical case (i.e., $\mathcal{A}_t = C(\Omega_t) = C(\Omega)$, $t \in \mathbb{T}$), a homomorphism $\Phi_{t_1,t_2} : C(\Omega_{t_2}) \to C(\Omega_{t_1})$ is characterized as follows:

(D) there exists a continuous map $\phi_{t_1,t_2} : \Omega_{t_1} \to \Omega_{t_2}$ such that

$$[\Phi_{t_1,t_2}(g_{t_2})](\omega_{t_1}) = g_{t_2}(\phi_{t_1,t_2}(\omega_{t_1})) \qquad (\forall \omega_{t_1} \in \Omega_{t_1}, \forall g_{t_2} \in C(\Omega_{t_2})), \tag{1.5}$$



Causal map $\phi_{1,2}$ and causal operator $\Phi_{1,2}$

Thus, the family of continuous maps $\{ \phi_{t_1,t_2} : \Omega_{t_1} \to \Omega_{t_2},\ t_1, t_2 \in T\ (t_1 \leq t_2) \}$ is a classical causal relation.

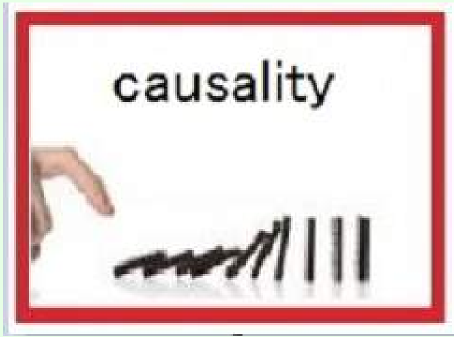
Now we can propose Axiom 2 (i.e., causality). (For details, see ref. [74].)

Axiom 2 [Causality] in the C^*-algebraic formulation

•[General case (i.e., quantum case ($\mathcal{A}_{t_i} = B_c(H_{t_i})$) and classical case ($\mathcal{A}_{t_i} = C(\Omega_{t_i})$))]:
Assume $[t_1 \le t_2]$.
($\sharp_1$): Causality is represented by a causal operator $\Phi_{t_1,t_2} : \mathcal{A}_{t_2} \to \mathcal{A}_{t_1}$
In the same sense,
($\sharp_2$): Causality is represented by a dual causal operator $\Phi^*_{t_1,t_2} : \mathcal{A}^*_{t_1} \to \mathcal{A}^*_{t_2}$

•[Classical case ($\mathcal{A}_{t_i} = C(\Omega_{t_i})$)]
Assume $[t_1 \le t_2]$:

($\flat_1$): Classical causality is represented by a causal operator $\Phi_{t_1,t_2} : C(\Omega_{t_2}) \to C(\Omega_{t_1})$
In the same sense,
($\flat_2$): Classical causality is represented by a dual causal operator $\Phi^*_{t_1,t_2} : C(\Omega_{t_1})^* \to C(\Omega_{t_2})^*$
Also,
($\flat_3$): Classical causality is represented by a continuous map $\{ \phi_{t_1,t_2} : \Omega_{t_1} \to \Omega_{t_2}, (t_1 \le t_2) \}$



Remark 1.10. [Heisenberg picture (= moving observation) and Schrödinger picture (= moving state)]
(i): The above ($\sharp_1$) and ($\flat_1$) are called the Heisenberg pictures. And ($\sharp_2$), ($\flat_2$) and ($\flat_3$) are called the Schrödinger pictures.

(ii):In most cases, the Heisenberg picure (=observable-moving) and Schrödinger picture (=state-moving) are equivalent. However, in complex situations (especially in quantum systems), the Schrödinger picture (=state-moving) is sometimes powerless. Therefore, it is recommended to always use the Heisenberg picture according to the Copenhagen interpretation (E_6) "the state is fixed and does not move".

Remark 1.11. So far, the following three theories (= scientific language) have been formulated.
(i): Quantum language (including classical and quantum systems): This is the theory (=

scientific language) with the most powerful descriptive power.

$$\boxed{\boxed{\text{QL}}} = \underset{\text{(Axiom 1)}}{\boxed{\begin{matrix}\text{state}\\ \text{measurement}\end{matrix}}} + \underset{\text{(Axiom 2)}}{\boxed{\text{causality}}} + \overset{\text{(how to use Axioms 1 and 2)}}{\boxed{\begin{matrix}\text{Copenhagen}\\ \text{interpretation}\end{matrix}}} \tag{1.6}$$

(ii): Quantum language of classical systems: only the classical part of quantum language, but as descriptive as statistics

$$\boxed{\boxed{\text{classical QL}}} = \underset{\text{(Axiom 1[Classical case (i), (ii)])}}{\boxed{\begin{matrix}\text{state}\\ \text{measurement}\end{matrix}}} + \underset{\text{(Axiom 2[Classical case])}}{\boxed{\text{causality}}} + \overset{\text{(how to use Axioms 1 and 2)}}{\boxed{\begin{matrix}\text{Copenhagen}\\ \text{interpretation}\end{matrix}}} \tag{1.7}$$

(iii): Abstract Classical Mechanics: Embodies the spirit of "Think of Newtonian mechanics as a model for all motion! and is roughly equivalent to dynamical systems theory (which does not include the concept of probability).

$$\boxed{\boxed{\text{abstract classical mechanics}}} = \underset{\text{(Axiom 1 [Classical case] (i))}}{\boxed{\text{state}}} + \underset{\text{(Axiom 2 [Classical case])}}{\boxed{\text{causality}}} \tag{1.8}$$

1.4.3 Product observable, Product measurement

Definition 1.12. [(i): Product observable (= simultaneous observable), product measurement (= simultaneous measurement)]: Let $\mathsf{O}_i = (X_i,\ \mathcal{P}(X_i)\ ,\ G_i)$ $(i = 1, 2, ..., N)$ be commutative observables in $\mathcal{A}$. Define a product observable (= simultaneous observable) $\times_{i=1,2,...,n} \mathsf{O}_i = (\times_{i=1}^n X_i,\ \mathcal{P}(\times_{i=1}^n X_i),\ \times_{i=1,2,...,n} G_i)$ such that

$$[\underset{i=1,2,...,n}{\times} G_i](\underset{j=1,2,...,n}{\times} \Xi_j) = \underset{j=1,2,...,n}{\times} G_j(\Xi_j) \quad (\forall \Xi_j \in \mathcal{P}(X_j), j = 1, 2, ..., n)$$

Also, $\mathsf{M}_{\mathcal{A}}(\ \times_{i=1,2,...,n} \mathsf{O}_i = (\times_{i=1}^n X_i,\ \mathcal{P}(\times_{i=1}^n X_i),\ \times_{i=1,2,...,n} G_i),\ S_{[\rho]})$ is called a product measurement (= simultaneous measurement) of $\mathsf{M}_{\mathcal{A}}(\ \mathsf{O}_i = (X_i, \mathcal{P}(X_i), G_i)\ , S_{[\rho]})$ $(i = 1, 2, ..., n)$.

[(ii): quasi-product observable, quasi-product measurement]: Let $\mathsf{O}_i = (X_i,\ \mathcal{P}(X_i)\ ,\ G_i)$ $(i = 1, 2, ..., N)$ be commutative observables in $\mathcal{A}$. Define a quasi-product observable $\times^{qp}_{i=1,2,...,n} \mathsf{O}_i = (\times_{i=1}^n X_i,\ \mathcal{P}(\times_{i=1}^n X_i),\ \times^{qp}_{i=1,2,...,n} G_i)$ such that

$$[\overset{qp}{\underset{i=1,2,...,n}{\times}} G_i](X_1 \times X_2 \times ... \times X_{j-1} \times \Xi_j \times X_{j+1} \times ... \times X_n) = G_j(\Xi_j) \quad (\forall \Xi_j \in \mathcal{P}(X_j), j = 1, 2, ..., n)$$

Also, $\mathsf{M}_{\mathcal{A}}(\ \times^{qp}_{i=1,2,...,n} \mathsf{O}_i = (\times_{i=1}^n X_i,\ \mathcal{P}(\times_{i=1}^n X_i),\ \times^{qp}_{i=1,2,...,n} G_i),\ S_{[\rho]})$ is called a quasi-product measurement of $\mathsf{M}_{\mathcal{A}}(\ \mathsf{O}_i = (X_i, \mathcal{P}(X_i), G_i)\ , S_{[\rho]})$ $(i = 1, 2, ..., n)$.

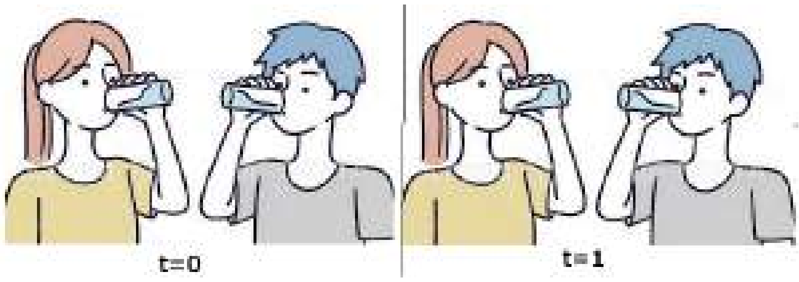
[(iii): Tensor C^*-algebra, tensor product observable, tensor product measurement]: Let $\mathsf{O}_i = (X_i,\ \mathcal{P}(X_i)\ ,\ G_i)$ be observables in $\mathcal{A}_i$, $(i = 1, 2, ..., n)$. Define a tensor product observable $\bigotimes_{i=1,2,...,n} \mathsf{O}_i = (\times_{i=1}^n X_i,\ \mathcal{P}(\times_{i=1}^n X_i),\ \bigotimes_{i=1,2,...,n} G_i)$ in the tensor C^*-algebra $\bigotimes_{i=1}^n \mathcal{A}_i$ such that

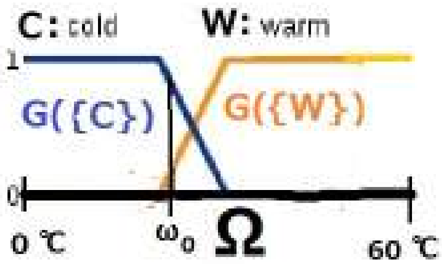
$$[\bigotimes_{i=1,2,...,n} G_i](\underset{j=1,2,...,n}{\times} \Xi_j) = \bigotimes_{j=1,2,...,n} G_j(\Xi_j) \qquad (\forall \Xi_j \in \mathcal{P}(X_j), j = 1, 2, ..., n)$$

Also, $\mathsf{M}_{\bigotimes_{i=1}^n \mathcal{A}_i}(\ \bigotimes_{i=1,2,...,n} \mathsf{O}_i = (\times_{i=1}^n X_i,\ \mathcal{P}(\times_{i=1}^n X_i),\ \bigotimes_{i=1,2,...,n} G_i),\ S_{[\bigotimes_{i=1}^n \rho_i]})$ is called a tensor product measurement of $\mathsf{M}_{\mathcal{A}_i}(\ \mathsf{O}_i = (X_i, \mathcal{P}(X_i), G_i)\ , S_{[\rho_i]})$ $(i = 1, 2, ..., n)$.

.

Example 1.13.





Assume that

($\sharp$) water at ω°C at time $t = 0$ becomes $\phi_{0,1}(\omega)$°C at time $t = 1$, where $\phi_{0,1} : [0, 60] \to [0.60]$ is defined such that $\phi_{0,1}(\omega) = \frac{\omega}{2}$ $(0 \le \omega \le 60)$.

Let's measure whether the water is cold or warm at time 0 and time 1. Using the notation in Examples 1.8, we consider two measurements $\mathsf{M}_{C(\Omega)}(\mathsf{O} = (X = \{C, W\}, 2^X, G), S_{[\omega_0]})$ and $\mathsf{M}_{C(\Omega)}(\mathsf{O} = (X = \{C, W\}, 2^X, G), S_{[\phi_{0,1}(\omega_0)]})$ $\big(\approx \mathsf{M}_{C(\Omega)}(\Phi_{1,0}\mathsf{O} = (X = \{C, W\}, 2^X, \Phi_{1,0}G), S_{[\omega_0]})\big)$.

However, the Copenhagen interpretation says that "Only one measurement is permitted". Thus, combine the two measurements above into one product measurement (= simultaneous measurement) as follows

$$\mathsf{M}_{C(\Omega)}(\mathsf{O} \times \Phi_{1,0}\mathsf{O} = (X^2 = \{C, W\}^2, 2^{X^2}, G \times \Phi_{1,0}G), S_{[\omega_0]})$$

Then, Axiom 1 says that

($\flat_1$) The probability that the measured value $(x_0, x_1) \in \{C, W\}^2$ is obtained by the above measurement is given by

$$[G(\{x_1\})](\omega_0) \cdot [G(\{x_2\})](\omega_0/2)$$

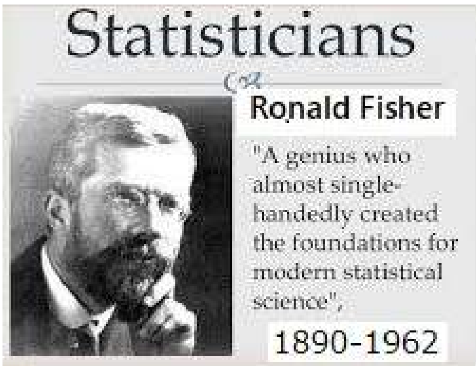
Or, using ordinary language, we see

($\flat_2$) the probability that one of the subjects (chosen at random) answers it was

$$\left\{ \begin{array}{l} \text{"cold" at time 0 and "cold" at time 1} \\ \text{"cold" at time 0 and "warm" at time 1} \\ \text{"warm" at time 0 and "warm" at time 1} \\ \text{"warm" at time 0 and "cold" at time 1} \end{array} \right\}$$

$$\text{is equal to} \quad \left\{ \begin{array}{l} [G(\{C\})](\omega_0) \cdot [G(\{C\})](\omega_0/2) \\ [G(\{C\})](\omega_0) \cdot [G(\{W\})](\omega_0/2) \\ [G(\{W\})](\omega_0) \cdot [G(\{W\})](\omega_0/2) \\ [G(\{W\})](\omega_0) \cdot [G(\{C\})](\omega_0/2) \end{array} \right\}$$

1.4.4 Inference; Fisher inference and Bayesian inference in classical QL



We begin with the following notation:

Notation 1.14. [$\mathsf{M}_{C(\Omega)}(\mathsf{O}, S_{[*]})$ and $\mathsf{M}_{C(\Omega)}(\mathsf{O}, S_{[*]}(\rho))$] : Consider a classical measurement $\mathsf{M}_{C(\Omega)}$ (O:=$(X, \mathcal{F}, F)$, $S_{[\omega_0]}$). Here,

($\sharp_1$) It is usual that a state $\omega_0(\in \Omega)$ is unknown when a measurement $\mathsf{M}_{C(\Omega)}$ (O:=$(X, \mathcal{F}, F)$, $S_{[\omega_0]}$) is taken.

That is because

($\sharp_2$) we usually take a measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}, S_{[\omega_0]})$ for the purpose to know a state $\omega_0(\in \Omega)$

Thus,

(E_1) when we want to emphasize that "we don't know the state ω_0 at all", we usually use the notation $\mathsf{M}_{C(\Omega)}$ (O:=$(X, \mathcal{F}, F)$, $S_{[*]}$) instead of $\mathsf{M}_{C(\Omega)}$ (O:=$(X, \mathcal{F}, F)$, $S_{[\omega_0]}$)
(E_2) when we want to emphasize that "we don't know the state ω_0, but know that the distribution is $\rho(\in \mathfrak{S}^m(C(\Omega)^*))$(see Sec. 1.4.1), we usually use the notation $\mathsf{M}_{C(\Omega)}$ (O:=$(X, \mathcal{F}, F)$, $S_{[*]}(\rho)$) instead of $\mathsf{M}_{C(\Omega)}$ (O:=$(X, \mathcal{F}, F)$, $S_{[\omega_0]}$)

////

Theorem 1.15. [Inference (I); Fisher's maximal likelihood method]
Assume that a measured value $x(\in X)$ is obtained by a measurement $\mathsf{M}_{C(\Omega)}$ (O:=$(X, 2^X, F)$, $S_{[*]}$). Then, the unknown state $[*]$ can be inferred to be $\omega_0(\in \Omega)$ such that

$$[F(\{x\})](\omega_0) = \max_{\omega \in \Omega}[F(\{x\})](\omega)$$

[Inference (II); Bayesian inference]
Assume that a measured value $x(\in X)$ is obtained by a measurement $\mathsf{M}_{C(\Omega)}$ (O:=$(X, 2^X, F)$, $S_{[*]}(\rho_0)$). Then, the distribution $\rho_1(\in \mathfrak{S}^m(C(\Omega)^*)$ of unknown state $[*]$ can be inferred as follows.

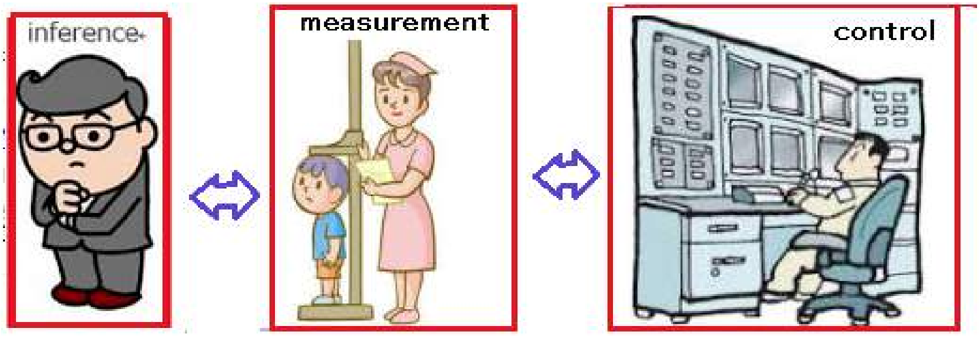
$$\rho_1(D) = \int_D [F(\{x\})](\omega)\rho_0(d\omega) \qquad (\forall D \subseteq \Omega, \text{where } D \text{ is any open set in } \Omega))$$

For the proof, see ref. [44] or §5.2 in ref. [74]). □

Remark 1.16. [The relation between statistics (inference) and dynamical system theory (control)]
$\mathsf{M}_{C(\Omega)}$ (O:=$(X, \mathcal{F}, F)$, $S_{[\omega_0]}$) has two aspects as follows.

-
- [inference]:
  when we know an observable O and a measured value $x(\in X)$, infer the state $\omega_0(\in \Omega)$!
- [control]:
  when we know an observable O and we want to get a measured value $x(\in X)$, how should the state $\omega_0(\in \Omega)$ be set up?

Inference and control are in different moods, but the substance (= Fisher's maximum likelihood method) is the same.

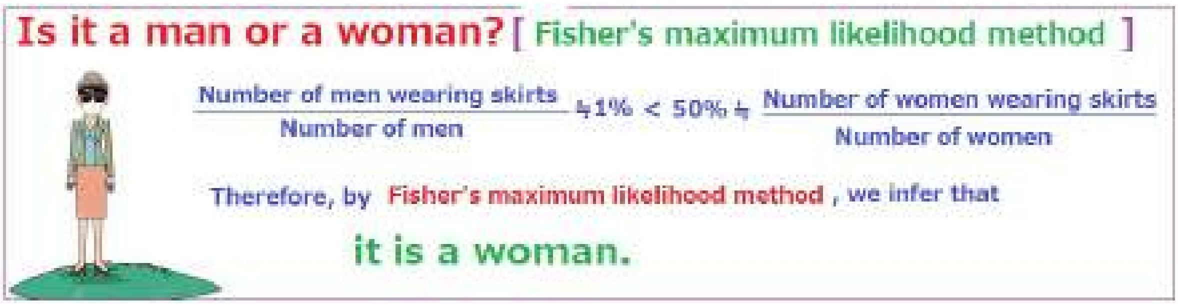


1.4.4.1 Exercise: Fisher's maximum likelihood method and Bayesian inference (ref. [44])

Problem 1.17. (i): I can't tell if it is a man or a woman, but I see someone in the distance. He/she is wearing a skirt. I need you to estimate whether this person is male or female.
(ii):Actually, this area is very male dominated.

$$\text{Number of men} : \text{Number of women} = 49 : 1$$

If so, is he/she wearing a skirt male or female?



Answer (i): Fisher inference

Let $\Omega = \{\omega_M, \omega_W\}$ be a state space, where ω_M [resp. ω_W] is implied by "men" [resp.

For further information, see my homepage

"women"]. The observable $O = (\{y,n\}, 2^{\{y,n\}}, F)$ in $C(\Omega)$ is defined by

$$[F(\{y\})](\omega_M) = \frac{\text{Number of men wearing skirts}}{\text{Number of men}} < 0.01, \quad [F(\{n\})](\omega_M) = 1 - [F(\{y\})](\omega_M)$$
$$[F(\{y\})](\omega_W) = \frac{\text{Number of women wearing skirts}}{\text{Number of women}} > 0.5, \quad [F(\{n\})](\omega_W) = 1 - [F(\{y\})](\omega_W)$$

Assume that a measured value y is obtained by the measurement $\mathsf{M}_{C(\Omega)}$ $(\mathsf{O}{:=}(\{y,n\}, 2^{\{y,n\}}, F)$, $S_{[*]})$, that is, He/she is wearing a skirt. Then,

Note that

$$[F(\{y\})](\omega_W) = \max\{[F(\{y\})](\omega_M), [F(\{y\})](\omega_W)\}$$

Thus, Fisher's maximal likelihood metheod says that

- the person is a woman.

Answer (ii): Bayesian inference

Consider the measurement $\mathsf{M}_{C(\Omega)}$ $(\mathsf{O}{:=}(\{y,n\}, 2^{\{y,n\}}, F)$, $S_{[*]}\rho_0)$. Note that

$$\rho(\{\omega_M\}) = 49/50, \qquad \rho(\{\omega_W\}) = 1/50$$

Thus, Bayesian inference says that

$$\frac{\text{Probability of being male}}{\text{Probability of being female}} = \frac{0.01}{0.5} \times \frac{49/50}{1/50} = \frac{49}{50}$$

That is,

$$\text{Probability of being male} = \frac{49}{99}, \qquad \text{Probability of being female} = \frac{50}{99}$$

Thus, the probability of being a woman is a little higher. □

1.4.5 Precise measurement, Tensor precise measurement

The C^*-algebraic formulation is easier than the W^*-algebraic formulation. The in this text, C^*-algebraic formulation is adopted. However, C^*-algebraic classical measurement has the weakness that "precision measurement cannot be formulated naturally." Having said that, it is useful to have precision measurements, so we define them here for convenience.

Definition 1.18. [precision measurement]: Let Ω be a compact space, and let $(\Omega, \mathcal{B}(\Omega), \nu)$ be a measure space such that

(i) $\mathcal{B}(\Omega)$ is the Borel field (i.e., it is a smallest σ-field that contains all open sets), and $\mu(\Omega) = 1$
(ii) $\nu(\{\omega\}) = 0 \;\; (\forall \omega \in \Omega)$
(iii) $\nu(D) > 0 \;\; (\forall D \subseteq \Omega, D$ is open$)$

The exact observable $\mathsf{O}_{E_\Omega} = (\Omega, \mathcal{B}(\Omega), E_\Omega)$ is defined by

$$[E_\Omega(D)](\omega) = \begin{cases} 1 & (\omega \in D) \\ 0 & (\omega \notin D) \end{cases} \qquad (\forall D \in \mathcal{B}(\Omega))$$

Putting $X = \Omega$, O_{E_Ω} is sometimes denoted by $(X, \mathcal{B}(X), E_\Omega)$.

////

We see

(F$_1$) the measured value $x (\in X = \Omega)$ obtained by the precise measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}_{E_\Omega} = (X, \mathcal{B}(X), E_\Omega), S_{[\omega]})$ surely belongs to any open set including ω, that is, $x (\in X = \Omega) = \omega$.

Definition 1.19. [Tensor precise measurement]: As above, let $(\Omega, \mathcal{B}(\Omega), \nu)$ be a compact measure space. and let $\mathsf{O}_{E_\Omega} = (\Omega, \mathcal{B}(\Omega), E_\Omega)$ be the precise observable in $C(\Omega)$[*3] Let Λ be a infinite set. Let $\bigotimes_{\lambda \in \Lambda} C(\Omega)$ be the infinite tensor space of $C(\Omega)$. Then, we see:

$$C(\Omega^\Lambda) = \bigotimes_{\lambda \in \Lambda} C(\Omega)$$

Also, the tensor precise observable in $C(\Omega^\Lambda)$ is defined by

$$\bigotimes_{\lambda \in \Lambda} \mathsf{O}_{E_\Omega} = (\Omega^\Lambda, \mathcal{B}(\Omega^\Lambda), E_{\Omega^\Lambda})$$

Putting $X = \Omega$, we sometimes denote $\bigotimes_{\lambda \in \Lambda} \mathsf{O}_{E_\Omega} = (X^\Lambda, \mathcal{B}(X^\Lambda), E_{\Omega^\Lambda})$.

////

We have the following theorem.

Proposition 1.20. [infinite tensor precise measurement]:

[*3] It is not guaranteed that $E_\Xi \in C(\Omega)$

(F$_2$) The infinite tensor precise measurement$\mathsf{M}_{C(\Omega^\Lambda)}(\bigotimes_{\lambda\in\Lambda}\mathsf{O}_{E_\Omega} = (X^\Lambda, \mathcal{B}(X^\Lambda), E_{\Omega^\Lambda}), S_{[(\omega_\lambda)_{\lambda\in\Lambda}]})$ is taken. Then, the probability that the measured value $(x_\lambda)_{\lambda\in\Lambda}(\in X^\Lambda = \Omega^\Lambda)$ belongs to any open set which includes $(\omega_\lambda)_{\lambda\in\Lambda}(\in \Omega^\Lambda)$ is equal to 1. In the same sense, the measured value $(x_\lambda)_{\lambda\in\Lambda}(\in X^\Lambda = \Omega^\Lambda)$ is surely equal to $(\omega_\lambda)_{\lambda\in\Lambda}$.

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1.4.6 Infinite Tensor Product Observable, Infinite Tensor Product Measurements

In the C^*-formulation of the measurement, the infinite tensor product measurement requires some ingenuity.

This is explained below. The same argument can be made for quantum systems, but we will limit our discussion to classical systems.

Let Λ be an arbitrary set. For each $\lambda(\in \Lambda)$, consider the measures $\mathsf{M}_{C(\Omega_\lambda)}(\mathsf{O}_\lambda(= (X_\lambda, \mathcal{P}(X_\lambda), F_\lambda)), S_{[\delta_{\omega_\lambda}]})$. Then, these infinite tensor product measures $\bigotimes_{\lambda\in\Lambda}\mathsf{M}_{C(\Omega_\lambda)}(\mathsf{O}_\lambda S_{[\delta_{\Omega_\lambda}]})$. That is,

$$\bigotimes_{\lambda\in\Lambda}\mathsf{M}_{C(\Omega_\lambda)}(\mathsf{O}_\lambda, S_{[\delta_{\omega_\lambda}]})$$
$$=\mathsf{M}_{C(\times_{\lambda\in\Lambda}\Omega_\lambda)}(\bigotimes_{\lambda\in\Lambda}\mathsf{O}_\lambda, S_{[\otimes_{\lambda\in\Lambda}\delta_{\omega_\lambda}]})$$
$$=\mathsf{M}_{C(\times_{\lambda\in\Lambda}\Omega_\lambda)}(\bigotimes_{\lambda\in\Lambda}\mathsf{O}_\lambda = (\times_{\lambda\in\Lambda} X_\lambda, \mathcal{P}_0(\times_{\lambda\in\Lambda} X_\lambda), \bigotimes_{\lambda\in\Lambda} F_\lambda), S_{[\otimes_{\lambda\in\Lambda}\delta_{\omega_\lambda}]})$$

Let me explain what this means. In the above,

($\sharp_1$) $C(\Omega^\Lambda)$ and $\otimes_{\lambda\in\Lambda}\delta_{\omega_\lambda}$ are not a problem. because Ω is compact and Ω^Λ is also compact (by Tychonoff's theorem (*cf* ref. [128])).

($\sharp_2$)] $\mathcal{P}_0(\times_{\lambda\in\Lambda} X_\lambda)$ is determined by devising the following.

$$\mathcal{P}_0(\times_{\lambda\in\Lambda} X_\lambda)$$
$$=\Big\{ \times_{\lambda\in\Lambda}\Xi_\lambda \;\Big|\; \Xi_\lambda \subseteq X_\lambda \ (\lambda\in\Lambda), \quad \{\lambda\in\Lambda \mid \Xi_\lambda \neq X_\lambda\} \text{ is finite} \Big\}$$

Here, we have the following, the generalization of Axiom 1.

Axiom' 1 [Infinite tensor product measurement]: C^*-algebraic formulation

Suppose that the infinite tensor measurement $\bigotimes_{\lambda\in\Lambda}\mathsf{M}_{C(\Omega_\lambda)}(\mathsf{O}_\lambda, S_{[\delta_{\omega_\lambda}]})$ $(= \mathsf{M}_{C(\times_{\lambda\in\Lambda}\Omega_\lambda)}(\bigotimes_{\lambda\in\Lambda}\mathsf{O}_\lambda = (\times_{\lambda\in\Lambda} X_\lambda, \mathcal{P}_0(\times_{\lambda\in\Lambda} X_\lambda), \bigotimes_{\lambda\in\Lambda} F_\lambda), S_{[\otimes_{\lambda\in\Lambda}\delta_{\omega_\lambda}]}))$ yields the measured value $(x_\lambda)_{\lambda\in\Lambda}$ (where $x_\lambda \in X_\lambda$ $(\forall\lambda\in\Lambda)$). The probability that this measured value belongs to $\times_{\lambda\in\Lambda}\Xi_\lambda$ $(\in \mathcal{P}_0(\times_{\lambda\in\Lambda} X_\lambda))$ is

$$\times_{\lambda\in\Lambda}[F_\lambda(\Xi_\lambda)](\omega_\lambda).$$

Chapter 2

Ancient Greek philosophy (before Socrates)

Readers can start reading from this chapter (i.e., you can skip Chapter 1).
In Ancient Greek philosophy (before Socrates), the phrase "the arche (= the first thing of all things) is ○○" is standard. Here "○○" is, for example, as follows.

Thalēs······water	Democritus······atom
Pythagoras······Number is within all things	Hērakleitos······motion, fire
Parmenides······logic, motion	Zeno······logic, motion

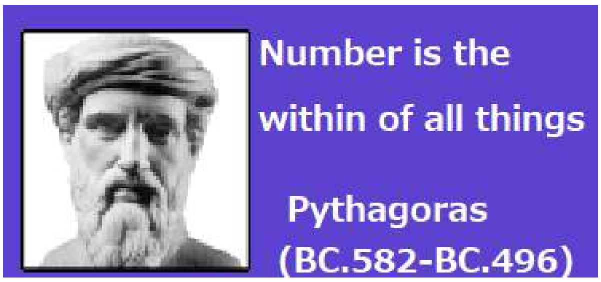
Worldviewism:

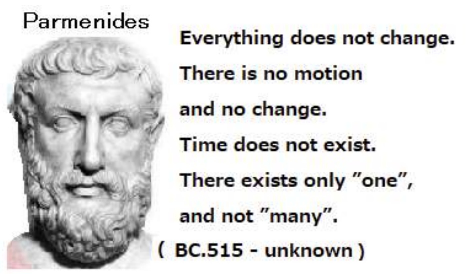
the arche is so [Worldview] $\xrightarrow{\text{therefore}}$ Let's think so [Consideration, discussion]

This argument has shaped Western philosophy for 2,500 years.

Firstly we examine Pythagoras' saying " Number is within all things". But unfortunately, this golden rule has never been followed in all the history of philosophy.

- The reason that quantum language has a strong descriptive power is due to the fact that quantum language adhered to Pythagoras' saying.





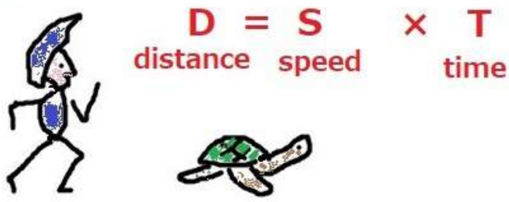
Also, we note that Parmenides' claim is very similar to the Copenhagen interpretation of quantum mechanics. And thus,

- we treated Parmenides like the ancestor of the Copenhagen interpretation

Next, we examine Zeno's paradoxes (the flying arrow, Achilles and the tortoise) from a quantum linguistic perspective and present a new view of Zeno's paradoxes.

And we conclude that

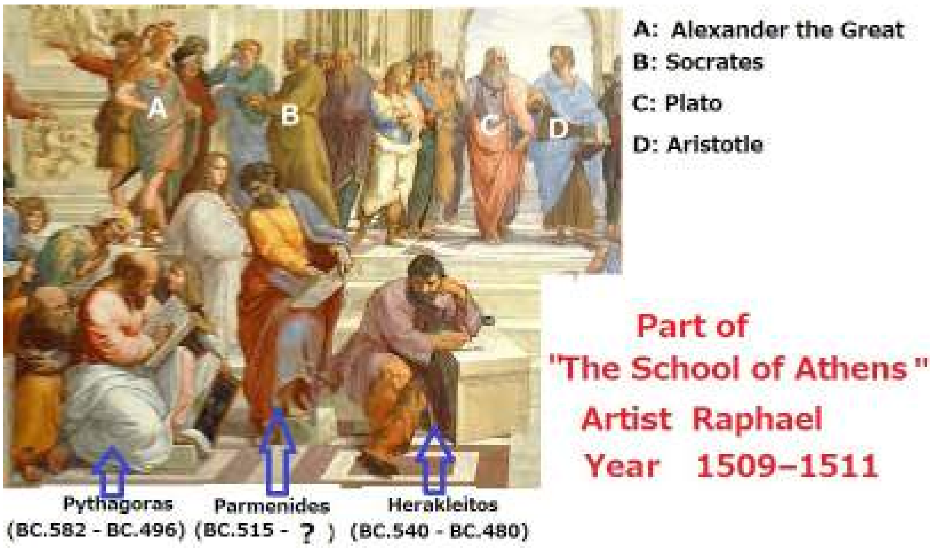
- The confusion about Zeno's paradox lies in the lack of a world description method. This paradox can be easily solved under the worldview of the motion function method, which is a subclass of quantum language. It is not a mathematical problem about geometric series, as is often claimed.



This success has led us to worldviewism (i.e., the spirit that "Start from worldview!"). Throughout this text, we always argue that

without worldview, there is no logic, no proposition.

It should be noted that this spirit is hardly accepted even today. That is because this is the reason that Zeno's paradoxes can not be solved in analytic philosophy.



2.1 Thalēs (BC.640 - BC.546)

2.1.1 Thalēs: the first philosopher: " the arche (= the first thing of all things) is water"

Every race had its own "mythology". Myths are the literature that explains the world by reason of God. Myths have been handed down from generation to generation from our ancestors by oral and written word. There is a great deal of maritime trade, and many ethnic groups come together to interact, but it is rare to find a region that is not united by force of arms because it is not a vast plain. This situation was realized on the eastern Mediterranean coast (now called Greece, Turkey, Syria, and Israel (Egypt)) around 1000 BC. In this region, various civilizations and cultures have merged to create a new culture. Particularly noteworthy was the realization of unification not by force of arms, but by culture. In other words, "philosophy as a synthesis of several myths" and "the alphabet as a synthesis of several letters" were born. That is,

$$
\text{(A)} \begin{cases} \text{integration of several myths} & \Longrightarrow \text{philosophy} \\[2ex] \text{integration of the several characters} & \Longrightarrow \text{Phoenician alphabet} \Longrightarrow \text{alphabet} \end{cases}
$$

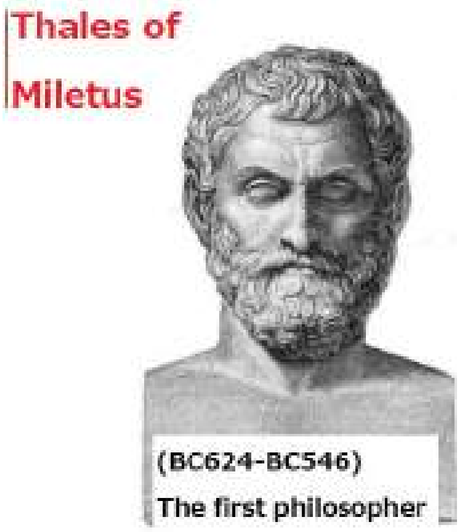
The alphabet is a phonetic alphabet because it was created with the intention of being a common letter between different ethnic groups. Egypt had an advanced civilization (such as the pyramids), but it was so unified in thought that philosophy did not develop.

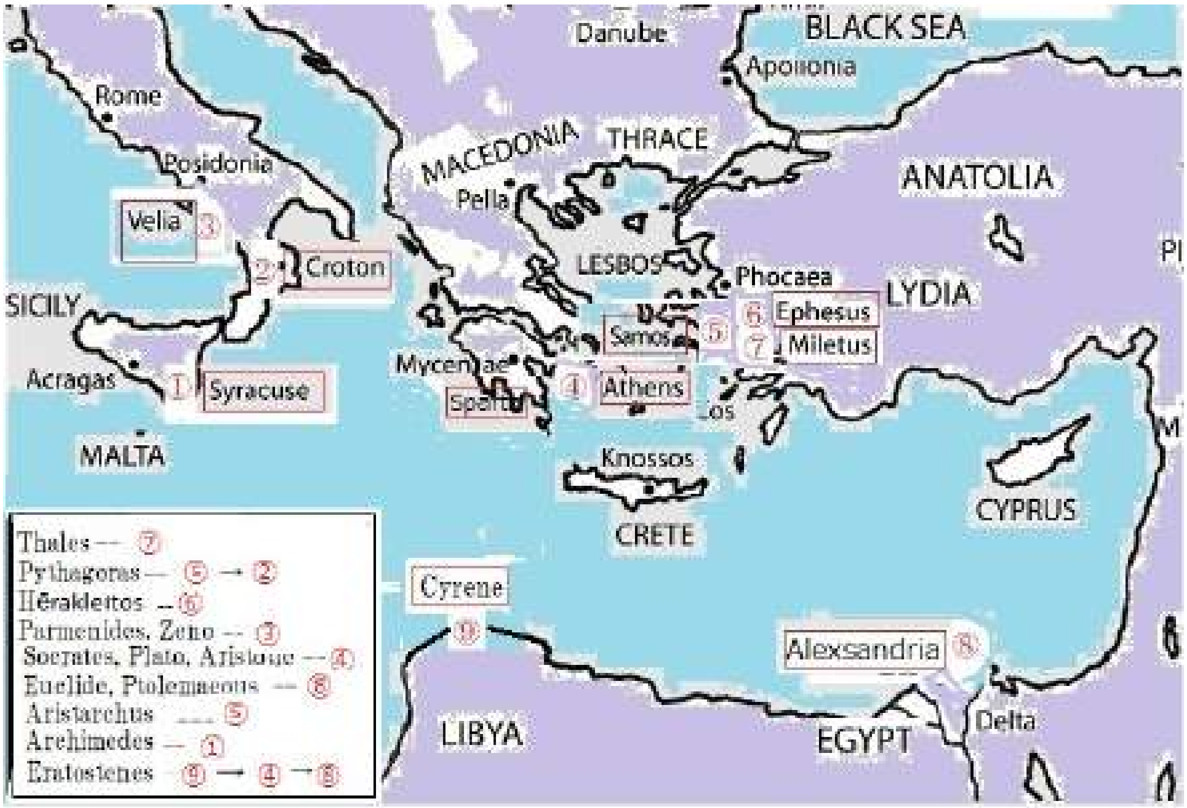
In Aristotle's book "Metaphysics," Thales Is called the "first philosopher". For example, to the question "why does an earthquake occur?" Most myths will say "because God is angry". Thalēs appeared in Miletus in ancient Greece. He said, in accordance with his worldview,

(B) The arche (= the first principle of all things) is water. Therefore, the earthquake is caused by the vibration of the water

$$
\underset{\text{the arche is so}}{\boxed{\text{Worldview}}} \xrightarrow{\text{therefore}} \underset{\text{Let's think so}}{\boxed{\text{Consideration, discussion}}}
$$

This may be childish, but is an explanation that does not brought out the "God" . This is the reason why Aristotle said Thalēs was "the first philosopher."





♠**Note 2.1.** S. Weinberg (1933 -2021), An American physicist, won a Nobel Prize of physics in 1979, said in his book [123] "To explain the world; The discovery of modern science" as follows:

(♯) *It seems to me that to understand these early Greeks, it is better to think of them not as physicists or scientists or even philosophers, but as* ***poets****.*

I agree with him. As mentioned earlier, I believe that the main purpose of the ancient Greek philosophers was not to pursue truth but to provide a common topic (i.e., a common myth, a universal myth) in order to avoid conflict between different peoples. This paper, however, may be a bit more radical. As far as to describing the world (not ethics), Plato, Descartes, Kant, etc. all should be regarded as poets or fiction writers. As we will discuss later, I believe that the purpose of Kant's Critique of Pure Reason is not the pursuit of truth, but a proposal for a symbiosis with science. That is because the rule of philosophy is as follows:

($\sharp_1$) Only discussion, no experimentation.
($\sharp_2$) Winners are determined by the popularity of the general public (or philosophical enthusiasts).


In short, I think most of the philosophy is "enlightenment and ethics". However, this text focuses on the scientific part of philosophy.

♠**Note 2.2.** Who is the first philosopher? Of course there may be a lot of opinions for this question. As mentioned above, Aristotle said Thalēs was "the first philosopher." Also, A.N.Whitehead (1861 - 1947) said that

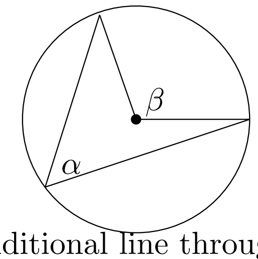
($\sharp$) **Western philosophy is characterized as a series of footnotes to Plato**

Although I do not know Whitehead's intension, I want to think that this means "Plato is the first philosopher", which is the same as the spirit of this text.

2.1.2 Thalēs' ability at math

By the statement:"the first principle of all things is water", we cannot judge Thalēs' knowledge, However, the following is known as Thalēs' theorem, which shows his ability of math:

Theorem 2.1. [central angle $= \beta$, inscribed angle$= \alpha$] $\Longrightarrow \beta = 2\alpha$

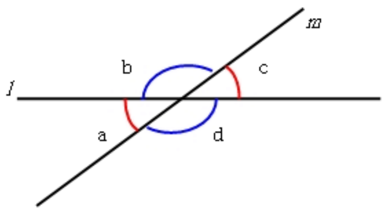


Proof. It suffices to draw the additional line through the center. □

Although it is a problem within the scope of junior high school mathematics, since training in elementary geometry is neglected in middle and high schools, only about 60 percent of graduate students in mathematics at an average level can answer this proof immediately.

However, if the next theorem is the discovery of Thales, we can trust Thales' mathematical ability.

Theorem 2.2. • *The vertical angles are equal. That is, in the figure below, it holds that $a = c$, $b = d$*



The model answer may be as follows.

$$a + b = 180^\circ, b + c = 180^\circ \tag{2.1}$$

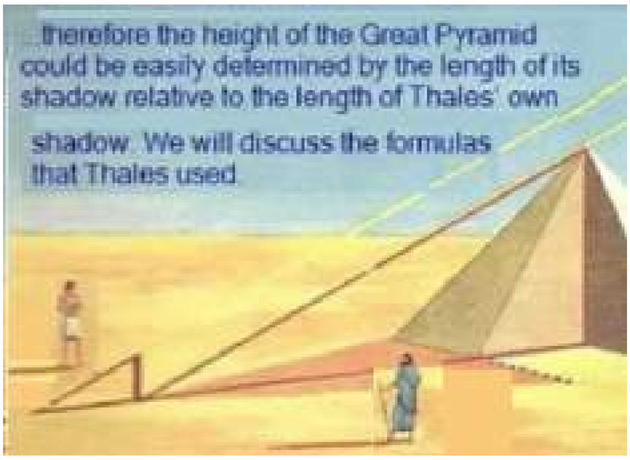
thus,

$$a = 180^\circ - b = c$$

////

However, I think this is too obvious and difficult to prove. The fact that Thales said that "the vertical angles are equal" means that he was aware that "it is worth speaking." This fact makes us believe in Thales' (or at that time) mathematical ability. This tradition of ancient Greek philosophy may have given rise to Euclid's Elements.

For further information, see my homepage



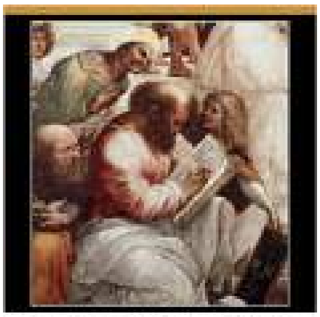
♠**Note 2.3.** (i): When Thales visited Egypt, there is a story that the king of Egypt was impressed by Thales' measuring the height of the pyramid in the way of measuring called triangulation. But, I think it is unreliable. Three great pyramids in the Egyptian Giza desert (deceased persons are Khufu, Khafre, Menkaure) erecting time of is the 2500 B.C. Thus I guess that the triangulation was known in those days (2000 years before Thalēs) in Egypt. If so, Thalēs' theorem should be doubt whether it is due to Thalēs. However, even as a true prover was unknown, the ability of mathematics at the time (i.e., the discovery of the concept of "proof") should be surprising. This led to Euclid's "Elements" (due to Euclid (275 BC - 330 BC)).
(ii): Anyone who is interested in the history of mathematics will be interested in the "origin of area". However, I don't know anything about this.

KSTS/RR-22/001
December 20, 2022

2.2 Pythagoras (BC.582 - BC.496)

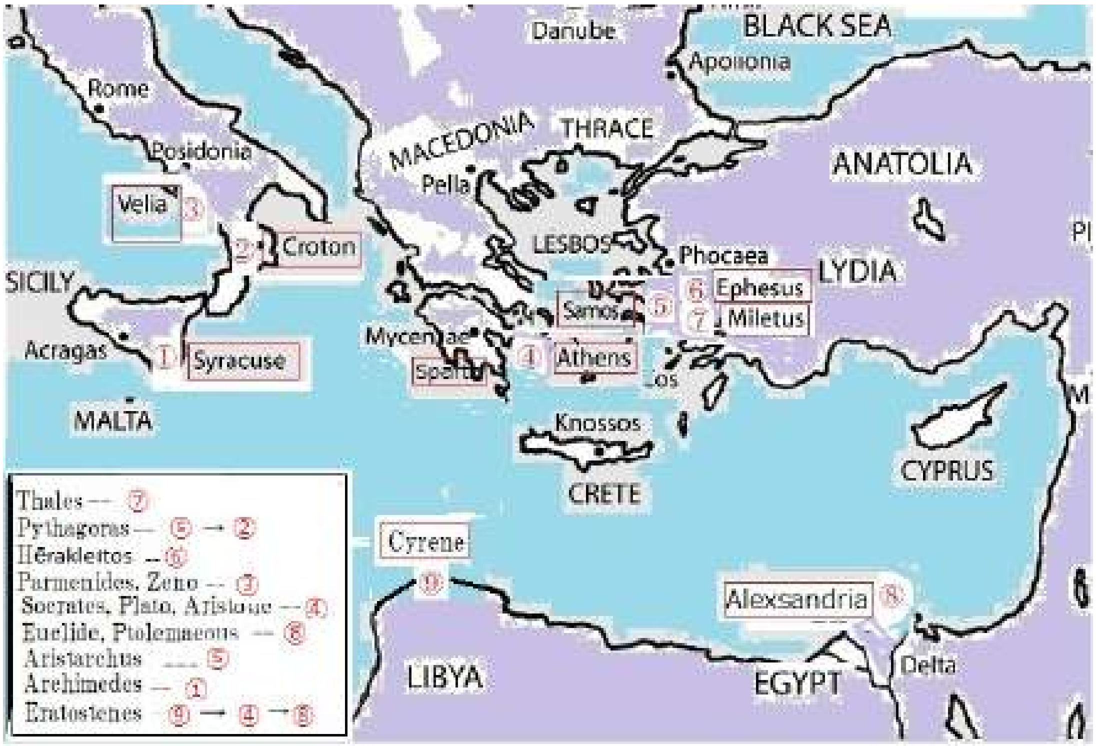
2.2.1 The mathematical ability of Pythagorean religious organization

Celebrate the sunrise



Pythagolas in the School of Athens by Raphael (AD.1511)

Pythagoras appears to have been the son of a gem-engraver on the island of Samos. Although contemporary scholars disagree about the origins of Pythagoras, they do agree that, around 530 BC, he travelled to Croton in southern Italy, where he founded a school for mathematics. Pythagoras was a leader in the mathematics study group, which may be regarded as a kind of religious organization called Pythagorean religion.



As the mathematical achievements, the followings are known:

- the discovery of irrational numbers, the Pythagorean theorem, the construction of a regular pentagon

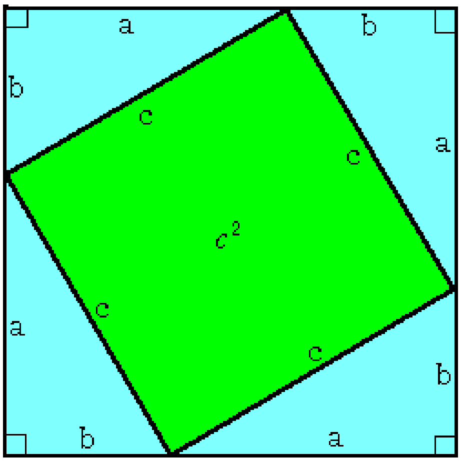
and so on.

Theorem 2.3. Finding of irrational members, e.g., $\sqrt{2}$ is an irrational number.

Theorem 2.4. (Pythagorean theorem): In △ABC, the followings are equivalent:

($\sharp_1$) $\angle A = 90°$
($\sharp_2$) $AB^2 + CA^2 = BC^2$

Proof. The proof of Pythagoras himself is unknown, but the next proof will be learned in junior high school.



Thus,

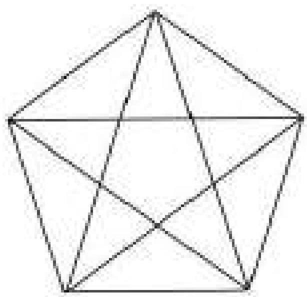
$$4 \times \frac{ab}{2} + c^2 = (a+b)^2$$

And thus, $a^2 + b^2 = c^2$.

////

♠**Note 2.4.** However, this proof presupposes the additivity of the area. Therefore, there may be room for consideration.

Construction 2.5. the construction of a regular pentagon

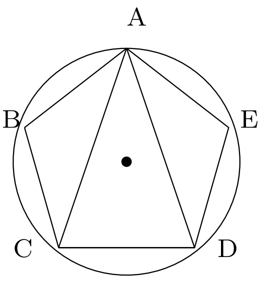


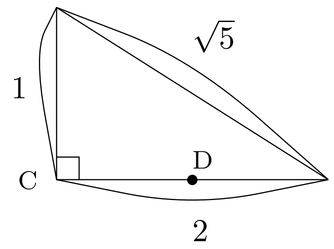
Pythagorean Emblem

Explanation: In a regular pentagon as shown in the figure below (left), put AB = BC = CD = DE = EA = 1. Then, we see

$$AC = AD = \frac{1+\sqrt{5}}{2}$$

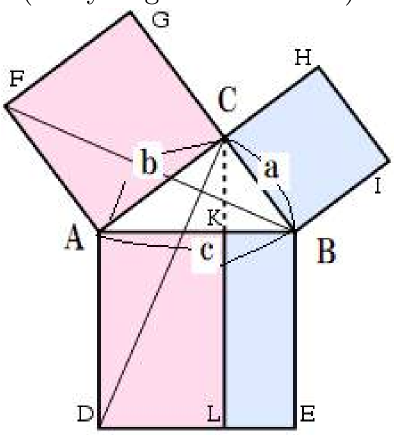
Hence, it suffices to construct $\frac{1+\sqrt{5}}{2}$. By the Pythagorean theorem, $\sqrt{5}(=\sqrt{1^2+2^2})$ can be constructed as follows (the figure below (right)). Thus, we easily get $\frac{1+\sqrt{5}}{2}$.



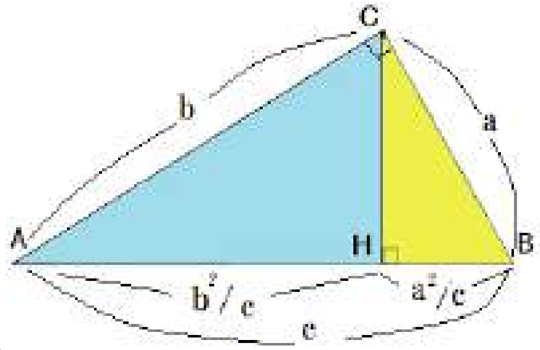


□

♠**Note 2.5.** (i):If you are a university student in mathematics, you must know at least three proofs (of Pythagorean theorem). Supplement the two proofs as follows:.



(a)



(b)

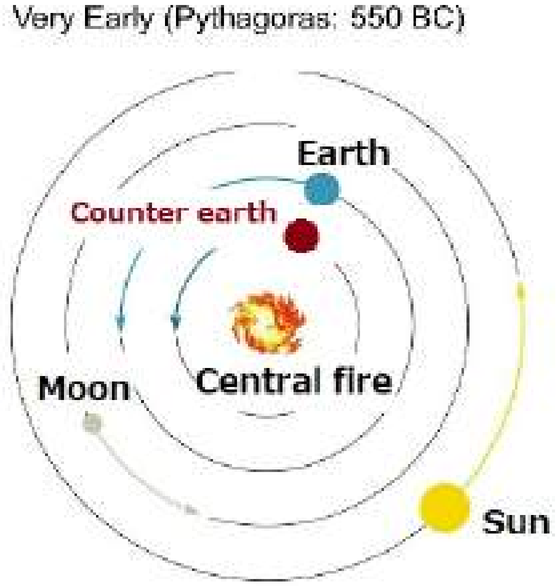
where (a) is due to Euclid ((b) is due to Einstein?).
(ii): It could have been something extra, but I wrote the explanation of the construction of a regular pentagon for beginners. Even university students in the department of mathematics sometimes don't know this.

♠**Note 2.6.** The above two (the discovery of irrational numbers and the Pythagorean theorem) are one of the most important discoveries in mathematics. If the following episodes are true, we can trust his mathematical ability.

($\sharp_1$) Pythagoras was killing the disciple, who found the irrational number, in order to hide the existence of irrational numbers.
($\sharp_2$) When Pythagoras discovered the Pythagorean theorem, he celebrated it, offered the sacrifice of the bull.

The two theorems, about 2000 years from the originally discovered to the scientific revolution (Descartes' Analytical Geometry, etc.; 17th century), not been used even once with an essential meaning. Nevertheless, their importance had been recognized in Pythagorean organization. This suggests a very high mathematical level of Pythagorean organization. Even if I had been a member of the Pythagorean school, I don't think I could have recognized the importance of these two theorems to the extent that they deserve in ($\sharp_1$) and ($\sharp_2$) above.

♠**Note 2.7.** (i): Pythagoras is said to have known that the earth was round. And he consider the following astronomical system:



Pythagorean astronomical system

I love this astronomical system and could watch it for an hour and never get tired of it. par At the time, astronomy might be believed to be a part of mathematics. Greek astronomy develops in the following way.

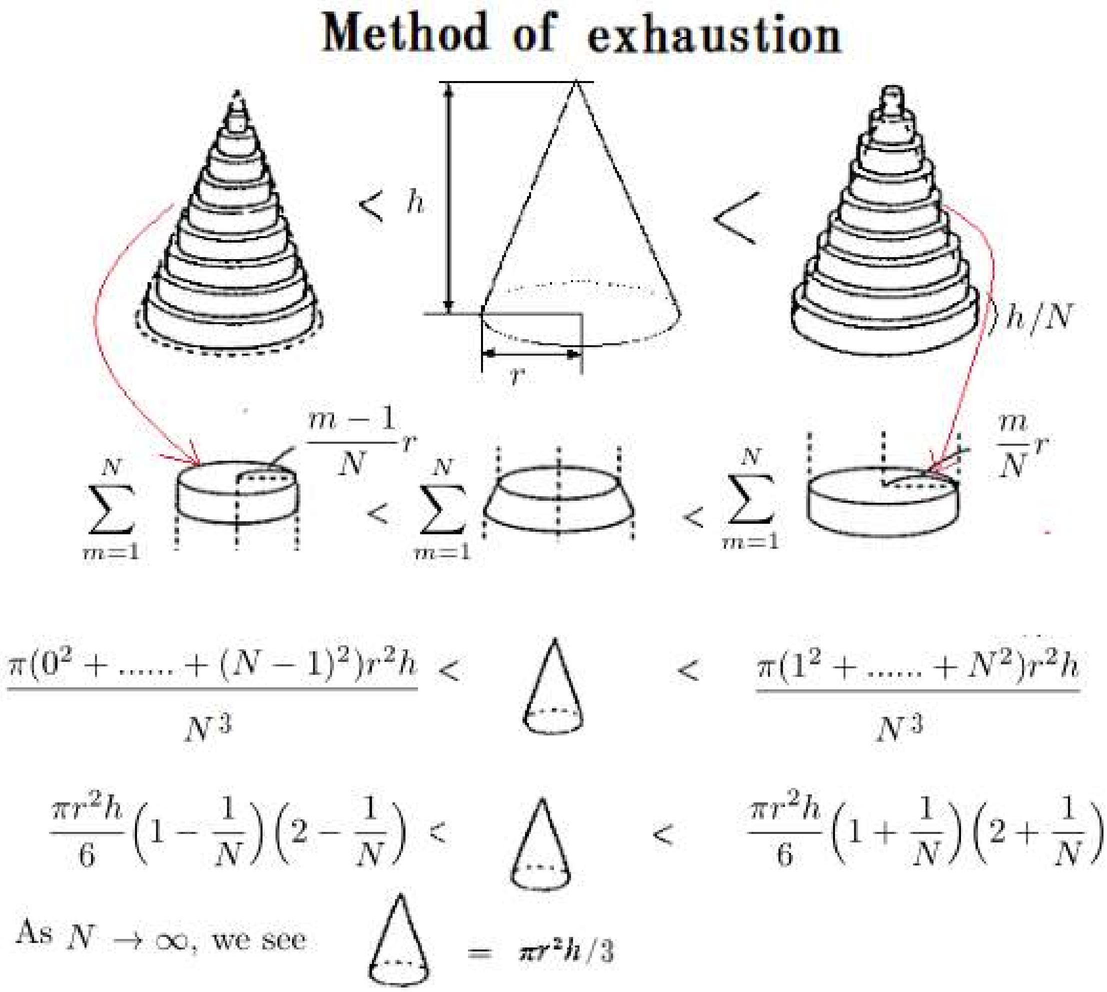
$(\flat_1)$ [Pythagoras] (the earth is round; BC582-496) $\longrightarrow$ [Eudoxus] (geocentrism 1; BC400-347) $\longrightarrow$ [Aristotle] (geocentrism 2; BC384-322) $\longrightarrow$ [Ptolemaios] (geocentrism 3; AD83-168)

(Although there are several geocentrism k $(= 1, 2, 3)$, the explanation is omitted.)

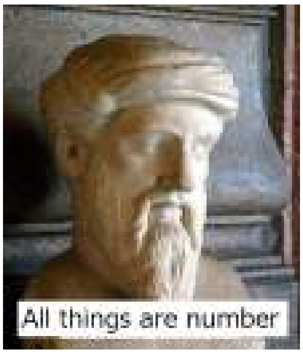
where Eudoxus(BC.400 – BC.347) is a Greek astronomer and mathematician (called the greatest of ancient Greek mathematicians), a student of Plato. He proposed a kind of geocentrism and the method of exhaustion (as the quadrature). It is said that many of his theorems are written in Euclid's Elements.
Even in ancient China which had the great culture, the prevailing belief was that the Earth was flat and square, while the heavens were round, until the introduction of European astronomy in the 17th century
(ii): Also, ancient Greek mathematics should be praised. That is,

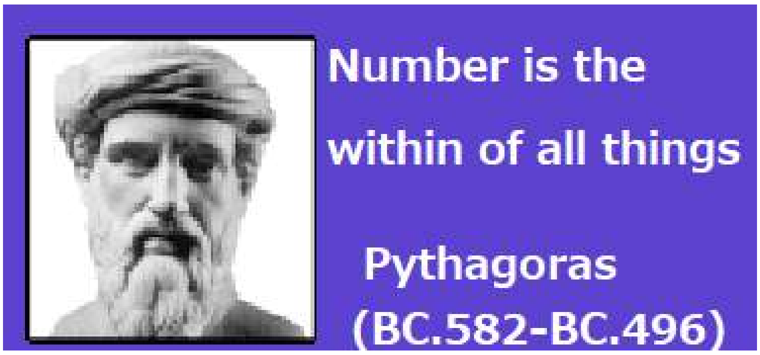
$(\flat_2)$ [Pythagoras] (Pythagorean theorem; BC582-496) $\longrightarrow$ [Eudoxus] (method of exhaustion; BC400-347) $\longrightarrow$ [Euclid] (Elements; BC330-275) $\longrightarrow$ [Archimedes] (quadrature; BC287-212)

The above two $(\flat_1)$ and $(\flat_2)$ are the greatest achievements in ancient Greek philosophy.


Method of exhaustion
< h
<
h/N
r
$\frac{m-1}{N}r$
$\frac{m}{N}r$
$\sum_{m=1}^{N}$ < $\sum_{m=1}^{N}$ < $\sum_{m=1}^{N}$
$\frac{\pi(0^2+......+(N-1)^2)r^2h}{N^3}$ < <
$\frac{\pi(1^2+......+N^2)r^2h}{N^3}$
$\frac{\pi r^2h}{6}\left(1-\frac{1}{N}\right)\left(2-\frac{1}{N}\right)$ < <
$\frac{\pi r^2h}{6}\left(1+\frac{1}{N}\right)\left(2+\frac{1}{N}\right)$
As $N\to\infty$, we see
$= \pi r^2h/3$

2.2.2 The arche (= the first principle of all things) is number





The main spirit of Pythagorean religious organization is "the first principle of all things is number". Now let us explain the following principle (called Pythagoreanism in this text):

(A):Pythagoreanism

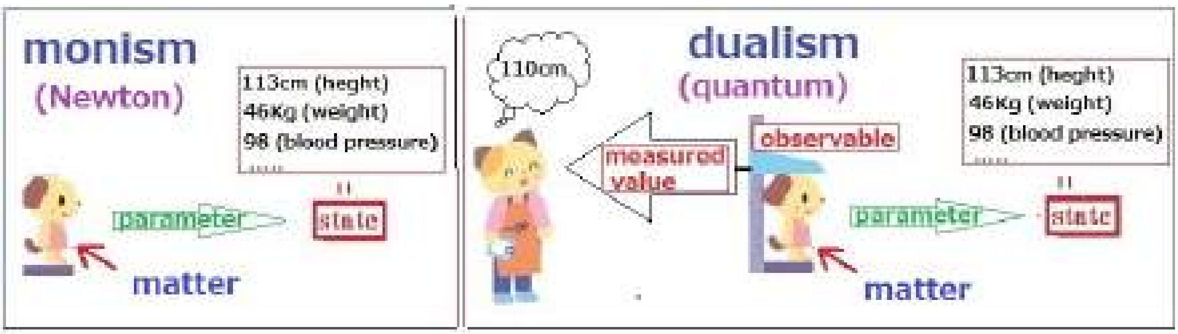
the first principle of all things is number.

That is, "Describe the world using mathematics!", or

"Number is within of all things"

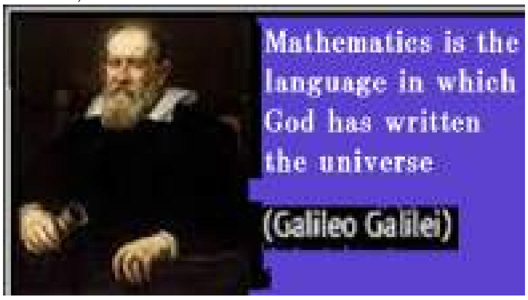
"To make a scientific theory is to parameterize things"

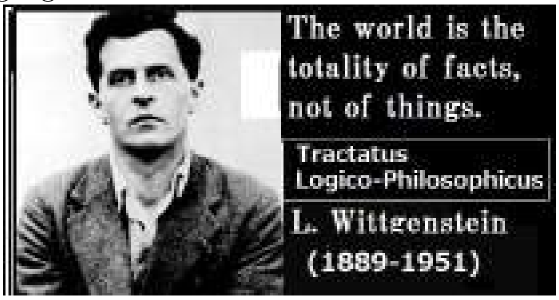
Although this spirit was never realized in Western philosophy, I believe that this is the most important claim throughout Western philosophy. This spirit was first realized in physics (i.e., Newtonian mechanics and quantum mechanics) as follows:



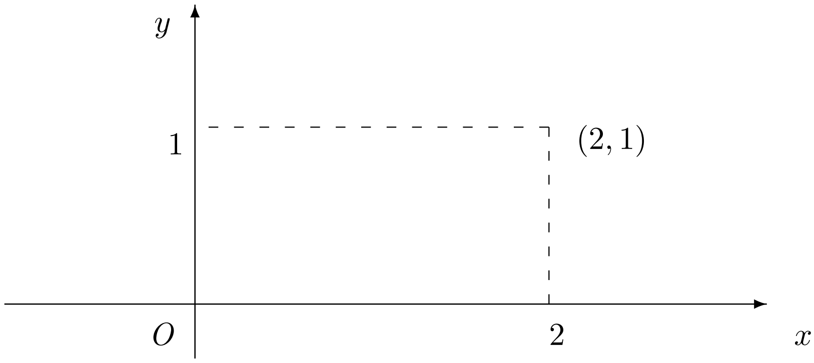
The phrase: "The arche (= first principle) of all things is ◯◯" is a fashion in those days. Note that "water", "fire" etc. are visible, but "number" is not.

♠**Note 2.8.** (i): After about 2000 years from Pythagoras, Galileo was talking about a similar thing. That is, *the universe is written in the language of mathematics.*
In fact, Galileo wrote the universe in the language of mathematics.





(ii): Descartes discovered that number is the within of all points of space as the Cartesian coordinates.



(iii): At the beginning of his book "ref. [125]: Tractatus Logico-philosophicus", Wittgenstein said

- (1.1): The world is the totality of facts, not of things.

Since "fact"="state" (or, precisely, "state"="quantification of fact"), he was merely emphasizing the orthodoxy of Pythagoreanism. But unfortunately he didn't perform parameterization. Thus, his work had become literature.

♠**Note 2.9.** As mentioned in Note 2.7, Pythagoras is said to have known that the earth was round. It may be a joke, but apparently he thought the earth was round because the most beautiful shape was a sphere. It is natural that the pure Pythagoreanism such as

(♯) The world is written in only the language of mathematics.

is not true. I believe that the world and mathematics are not directly related (though religious people might not think so). If it can be written in only the language of mathematics, it is just mathematics. However, this pure Pythagoreanism has carried over to the present day and is inhibiting the healthy development of our worldview (e.g., see "The theory of probability" in [74], or "Analytic philosophy" in Chap. 12).

Hence we have the following problem (i.e., the problem of worldview), which is the main problem in this text:

Problem 2.6. The problem of worldview is as follows.

- A scientific worldview has the form: "scientific worldview= mathematics + α". If so,

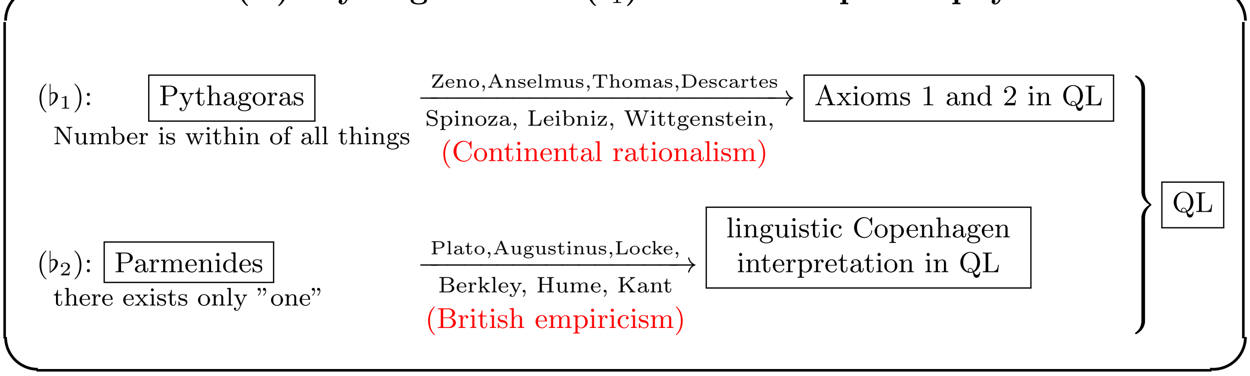
what is α?

As mentioned later, let us say here conclusion now. For example, α is "motion", "causal relation", "probability", "measurement", etc. From the quantum theoretical point of view, that is, in this text, we devote ourselves to "measurement (Axiom 1) and causality (Axiom 2) (*cf.* Sec.1.2.1).

♠**Note 2.10.** (i): In this period, the distinction between mathematics and science may not have been clear. The clear answer to the question "What is mathematics? we had to wait for Cantor's set theory and axiomatic set theory by Zermelo and Frenkel (around 1900 A.D.)
(ii): The reader may wonder, "Why bother to raise something like Problem 2.6 here?" I think
(♯) In analytic philosophy, mathematics (number theory, set theory) is too strong (so strong that it could be called a "set-theoretic worldview"), and "alpha" is weak.
As will be discussed in Chapters 11 and 11, Wittgenstein's attempt to strengthen the "alpha" in "TLP), ref.[125]" was not successful. As a foreshadowing of this, I wrote Problem 2.6 here

In science, Pythagoreanism flourished in the early modern period (during the age of the Scientific Revolution with Galileo-Newtonian mechanics and Bernoulli's law of large numbers, etc.). In philosophy, however, Pythagoreanism did not gain a mainstream position throughout the history of Western philosophy, as follows

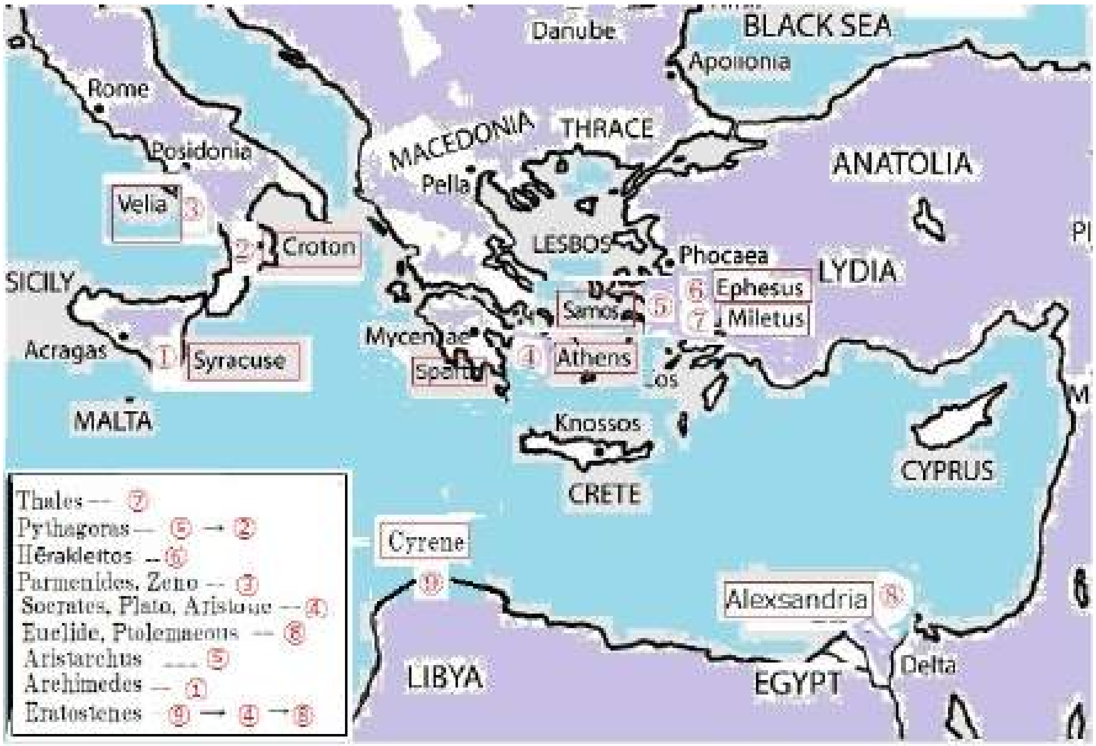
(B) :Pythagoreanism ($\flat_1$) in Western philosophy



Looking at (B) above, we can see that Pythagoreanism ($\flat_1$) never flourished in the mainstream of Western philosophy. In fact, the universal problem (by Amselmus and Thomas) was understood only by some philosophers. Spinoza and Leibniz, who tried to deepen Descartes' theory, failed theoretically, but were diverted to the role of reinforcing ethics (or God). Wittgenstein also failed as a theory (though he succeeded in literature and fashion). In short, Pythagoreanism ($\flat_1$) was too difficult. However, Locke, Hume, and Kant, who tried to apply Descartes' theory, succeeded in their own way.

2.3 Hērakleitos and Parmenides

2.3.1 Hērakleitos(BC.540 - BC.480)



Hērakleitos said the following.

(A):Hērakleitos(BC.540 - BC.480) in Ephesus

The arche (= the first principle of all things) is fire

And, further,

Everything flows.

Although "Everything flows" and "You cannot step into the same river twice" are interesting, everyone may be able to say similar thing. Hence, in this text, we interpret

"Everything flows" as follows.

(B_1) "motion" is the most fundamental key-word in science.

If so, we can relate the (B_1) to Parmenides (or even Newton).
That is,

(B_2) Both Hērakleitos and Parmenides said that

the arche of all things is motion

which will be discussed in the following section:

2.3.2 Parmenides; the ancestor of the Copenhagen interpretation

If so, we can relate the (B) to Parmenides.

2.3.3 Parmenides; The ancestor of the Copenhagen interpretation

In the same period of Hērakleitos, Parmenides said the exact opposite of words of Hērakleitos. That is,

(C):Parmenides(BC.515 - unknown)

Parmenides said:

(C_1) Everything does not change. There is no motion and no change. Time does not exist. There exists only "one", and not "many".

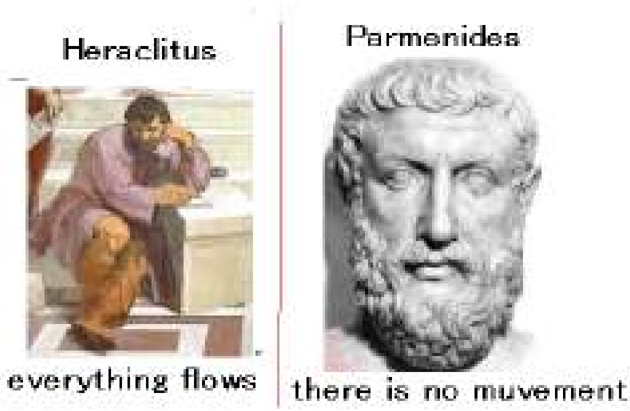
Also,

(C_2) We should not rely on our senses to understand the world, but should think logically with reason. Even if it appears to be moving, it's just that the human being has the sense to see it. It does not guarantee the existence of the movement.

(Notice): Remark the similarity between (C_1) and the Copenhagen interpretation (*cf.* Sec. 1.3), i.e.

- **only one measurement is permitted**
- **State does not move**

Also, in case of quantum mechanics, its object is too small, is not seen. Thus, we cannot rely on the sense, but only calculation. We can completely consent to Parmenides' assertion (C_2) in case of quantum mechanics.



Thus, it is no exaggeration to say that Parmenides is **the founder of the Copenhagen interpretation.**

($\flat_1$): **Pythagoras** — Number is within of all things $\xrightarrow{\text{Zeno, Anselmus, Thomas, Descartes}}$ Spinoza, Leibniz, Wittgenstein, (Continental rationalism) → **Axioms 1 and 2 in QL**

($\flat_2$): **Parmenides** — there exists only "one" $\xrightarrow{\text{Plato, Augustinus, Locke,}}$ Berkley, Hume, Kant (British empiricism) → **linguistic Copenhagen interpretation in QL**

(Axioms 1 and 2 in QL + linguistic Copenhagen interpretation in QL) = **QL**

The Parmenides genealogy has always been involved in the Copenhagen Interpretation and has formed the mainstream of Western philosophy all the way up to Kant.

♠**Note 2.11.** It is certain that the Copenhagen interpretation of quantum mechanics was born in Niels Bohr institution of the university of Copenhagen.

However, I may have an opinion such that

($\sharp$) It is not too much to say that Parmenides and Kolmogorov are the founders of the linguistic Copenhagen interpretation of quantum mechanics.

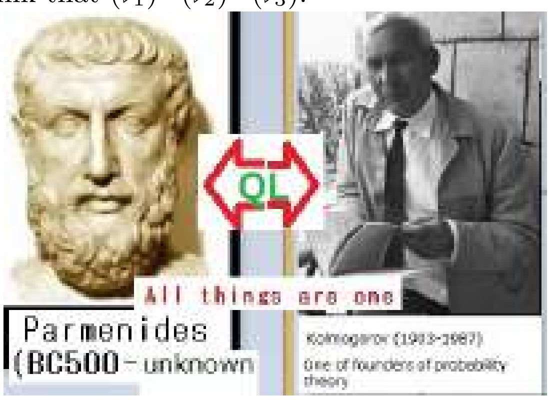
(where Kolmogorov is the founder of the modern theory of probability (*cf.* ref. [91]). It is known that Kolmogorov's extension theorem is the most fundamental in the theory of probability.) That is because

($\flat_1$) Parmenides says that there exists only "one", and not "many", there is no motion .

($\flat_2$) Linguistic Copenhagen interpretation says that only one measurement is permitted .

($\flat_3$) Kolmogorov's extension theorem says that only one probability space is permitted .

Roughly speaking, I think that ($\flat_1$)=($\flat_2$)=($\flat_3$).



If Parmenides didn't think motion was important, there's no way he could say "motion doesn't exist". Thus, I consider that Parmenides believed in the importance of motion. Parmenides' assertion is similar to Hērakleitos', that is,

(D) "motion" is the most fundamental key-word in science.

The following (E_1) and (E_2) are my fiction about the difference between Hērakleitos and Parmenides:

(E_1) Since Hērakleitos said "The first principle of all things is fire", he seems to premise the realistic world. Thus, his motion is similar to the motion in physics.
(E_2) Parmenides might study the abstract motion in the linguistic worldview. For example, his motion is "vegetable growth", "increase of the population", "economic growth", "Achilles' motion" and so on.

♠**Note 2.12.** Many would think the following.

($\sharp_1$) Hērakleitos is ordinary, and thus understandable, hence, scientific, therefore, realistic.
($\sharp_2$) Parmenides is ridiculous, and thus incomprehensible, hence, philosophical, therefore, idealistic (= linguistic).

Table 2.1:(= Table 4.1) Philosophical controversy that has been ongoing for 2,500 years [(monistic) realistic worldview] vs. [(dualistic) linguistic/idealistic world view]

dispute ＼ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	Ockham	Anselmus
ⓓ: space-time	Newton (Clarke)	Leibniz
ⓔ: quantum theory	Einstein	Bohr
ⓕ:philosophy of science	Carnap	Quine
ⓖ: fuzzy sets	Kalman	Zadeh

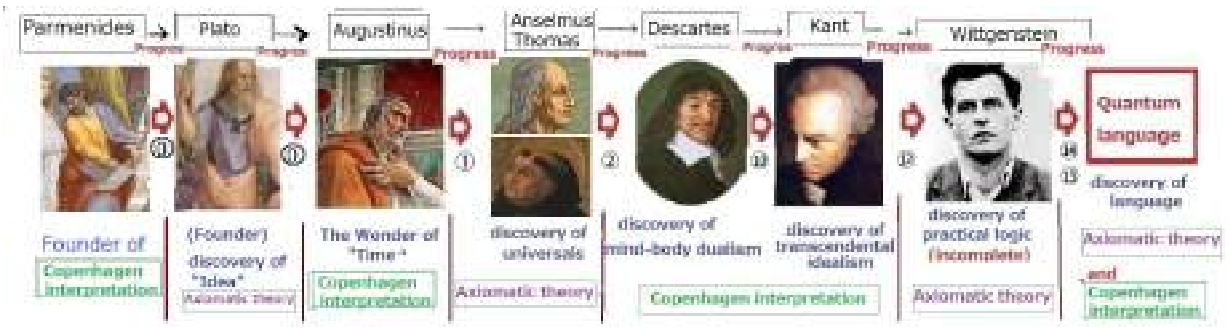
For the precise argument, see Table 4.1.

ⓐ∼ⓖ In the table above, we can think of it as

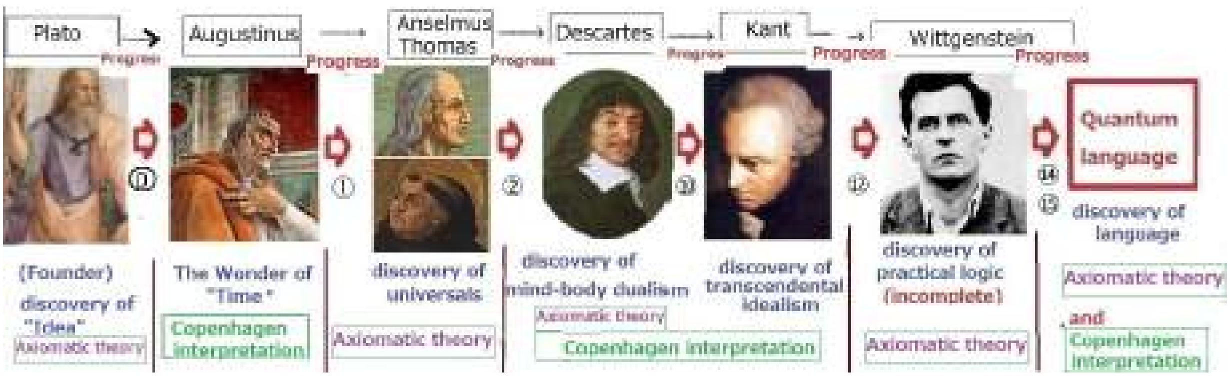
- The dualistic idealism (Platonism) says things that are absurd and incomprehensible, and the monistic realism (Aristotelianism) says things that are obvious and uninteresting.

The reason why I think this way is that, as I have said many times before, a curious Copenhagen interpretation sits at the core of the idealistic worldview.

Now, since Parmenides is the "founder of the Copenhagen Interpretation," it is reasonable to claim the following as the mainstream of Western philosophy.



But Parmenides was too much of a genius to be generally understood. There is a partial impression that he was a philosopher who dreamed up a bunch of nonsensical ideas. It is difficult to say that Parmenides is the source of Western philosophy. Therefore, in this text, the following will be considered "the main stream of Western philosophy.



Of course, in making this claim, we must show that

- the central theme of Western philosophy has always been Plato's Idea for 2500 years.

This text does this. Even so, Parmenides' achievement should always be honored.

2.3.4 Motion function method as a worldview

As mentioned in the previous section, Pythagoras said "The arche (= the first principle of all things) is number", Hence,

(F) Mathematics (or, parameterization) is indispensable to describe the world. However, we need words (or, concepts) to connect mathematics and the world.

And further, we want to consider the following fiction:

(G) As an influential candidate of the key-words, Parmenides (and Hērakleitos) thought of "motion"

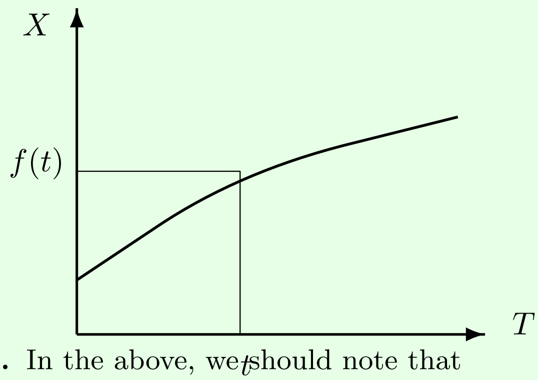
As a kind of mechanical worldviews, we introduce the following "motion function method", which is assumed to be due to Parmenides in this text though the true discoverer cannot be specified, (*cf.* Note 2.15).

(H): (Scientific linguistic) motion function method (due to Parmenides?)

Principle 2.7. [The motion function method]

Let T be time axis, and let X be space axis. A function $f : T \to X$ is called a motion function.
Then, the motion function method (in the mechanical worldview) is proposed as follows:

(H_1) **"motion" should be described by the motion function $f : T \to X$.**



♠**Note 2.13.** In the above, we should note that

($\sharp_1$) "Moving feeling" is erased since the above graph is fixed.

If it is so, as Parmenides says, we think that

($\sharp_2$) if we devote ourselves to logic or mathematics without relying on the sense, then we cannot look at "motion".

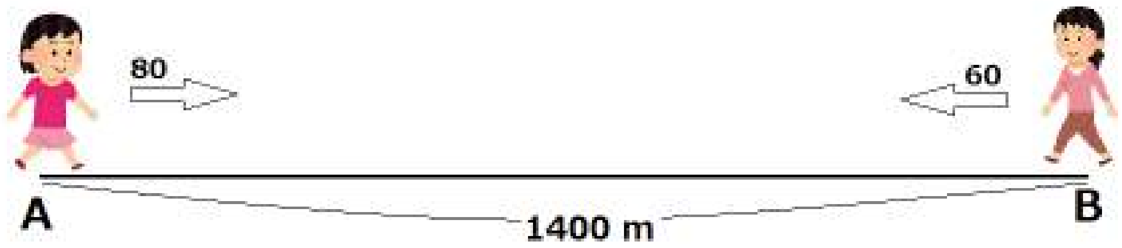
Also, the motion function method belongs to the realistic worldview as well as the linguistic worldview. In fact, it is easy to see that the motion function method can be derived from Newton mechanics. However, motion functions are not limited to those derived from Newtonian mechanics. Thus, in this text we usually consider that it belongs to the linguistic worldview.

($\sharp_3$) For the derivation of motion function method from quantum language, see ref. [47], or Chap. 14 in ref.[74].

The motion function method is easy, and it is usually studied in elementary school as the DST formula:

$$\underset{\text{(distance)}}{\boxed{\mathbf{D}}} = \underset{\text{(speed)}}{\boxed{\mathbf{S}}} \times \underset{\text{(time)}}{\boxed{\mathbf{T}}} \qquad (\clubsuit)$$

Exercise 2.8. An A spot and a B spot are 1400 meters away. Amy left the A spot for a B spot by 80 m per minute. Betty left the B spot for A spot at the same time by 60 m per minute. How many minutes later will Amy and Betty meet?



[Answer] Amy's motion function $f_A : \mathbb{R}$(time axis) $\to \mathbb{R}$(one dimensional space axis) is defined by $f_A(t) = 60t$, Betty's motion function $f_B : \mathbb{R}$(time axis) $\to \mathbb{R}$(one dimensional space axis) is defined by $f_B(t) = 1400 - 80t$, Thus, solving $f_A(t) = f_B(t)$, we see

$$60t = 1400 - 80t \quad \text{then,} \quad t = 10$$

Hence, after 10 minutes later, the two meet. □

♠**Note 2.14.** Some may think that to consider two motion functions f_A and f_B is not consistent with Parmenides' saying: *there exists only "one" and not "many"*(*cf.* Sec.1.2.1: linguistic Copenhagen interpretation). However, if so, it suffices to consider the following motion function:

($\sharp$) $(f_A, f_B) : \mathbb{R}$(time axis) $\to \mathbb{R}^2$(two dimensional space axis).

This technique is common practice in quantum mechanics and analytical mechanics.

♠**Note 2.15.** (a): Note that the motion function method is derived from quantum language (*cf.* ref. [45, 46, 74]). Namely,

$$\text{[quantum language]} \xrightarrow[\text{derivation}]{\text{Sec. 9.2.3 in this text}} \text{[motion function]}$$

That is,

($\sharp$) **the motion function method is one aspects of the quantum mechanical worldview (i.e., quantum language)**

Although I do not know, from the historical point of view, the discoverer of the motion function method, I want to assume that Parmenides is the main character, since he was a teacher of Zeno (*cf.* next section). Strictly speaking, the discovery might not be in Ancient Greece since the complete understanding of the concept of "function" is after Leibniz. However, we think that the spirit of the motion function method was understood by Pythagoras, Aristotle, Archimedes, etc.

(b): Of course, the above "motion function method" is incomplete and temporary. The motion function $f : T$(time) $\to X$(space) is not sufficient without the answers to the questions "What is time?" and "What is space?" (the Leibniz=Clarke correspondence (*cf.* Sec. 9.2.2). For the quantum linguistic answers to this questions, see ref. [74]

(c): The reader should want to ask the following questions.

- What is the motion function method? Isn't it in physics? Please answer briefly (without quantum language).

To put it simply, it is not in physics, but

a kind of ”habitual thinking” (≈ ”idealistic (or, linguistic) worldview”).

Such an idea may have come from D. Hume and I Kant (*cf.* Sec. 9.6, Chap.10).

♠**Note 2.16.** As the scientific worldviews before Newtonian mechanics, the most important is
”the motion function method” and ”Archimedes' principle of leverage and buoyant force” Some may have a question such as

- Why isn't the importance of the motion function method emphasized?
  Why can't the discoverer of the motion function method be specified?

Although I have no clear answer, I consider as follows:

- The realistic worldview (i.e., physics) was usually discovered by one genius, for example, Archimedes, Newton, Maxwell, Einstein and so on. On the other hand, the idealistic worldview is discovered by plural persons. For example, the discoverer of the theory of probability (e.g., Pascal, J. Bernoulli, Laplace, Kolmogorov, etc.) cannot be specified. Probability theory was imperceptibly formed by a number of people. In this text (*cf.* ref. [74]), we want to regard quantum theory, discovered by Heisenberg, Schrödinger and Born, as a kind of mechanical worldview (and not realistic worldview).

- The realistic worldview (i.e., physics) was usually discovered by one genius, for example, Archimedes, Newton, Maxwell, Einstein and so on. On the other hand, the idealistic worldview is discovered by plural persons. For example, the discoverer of the theory of probability (e.g., Pascal, J. Bernoulli, Laplace, Kolmogorov, etc.) cannot be specified. Probability theory was imperceptibly formed by a number of people. In this text (*cf.* ref. [74]), we want to regard quantum theory, discovered by Heisenberg, Schrödinger and Born, as a kind of mechanical worldview (and not realistic worldview).

Finally, let us discuss the importance of the motion function.

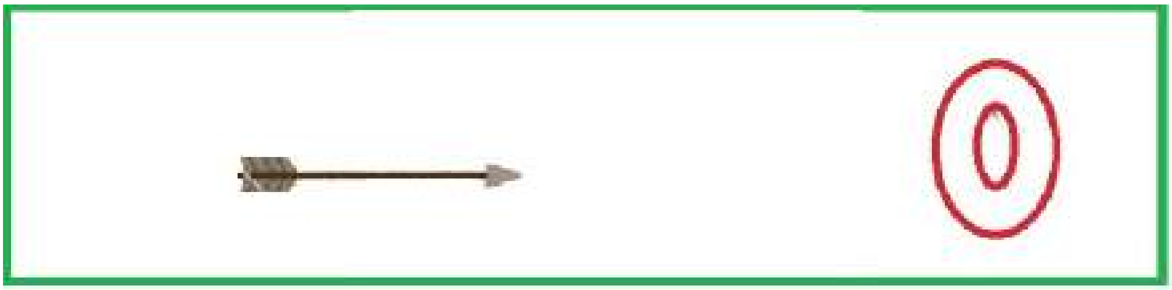
- There are only two ways to think about motion. That is, ”motion function method” or ”Newtonian mechanics (differential equation method). When we think about motion in our daily lives, 99% of the time we use the motion function method. We rarely use ”Newtonian mechanics. Newtonian mechanics” is used only by some people in the science field.

2.4 Zeno (BC490 - BC430): The Motion Paradox

In this section, from the quantum linguistic point of view, we study Zeno's paradoxes, the oldest paradox in science.

2.4.1 What is Zeno's paradoxes? Without a worldview, we cannot say anything

Zeno is a disciple of Parmenides. Thus, Zeno's Paradoxes may a collaboration between the two. Although Zeno's paradox has some types (i.e., "flying arrow", "Achilles and a tortoise", "dichotomy", "stadium", etc.), I think that these are essentially the same problem. And I think that the flying arrow expresses the essence of the problem exactly and is the best masterpiece in Zeno's paradoxes. As we will see in the next section, "Achilles and a tortoise" is a trick question.



Now we present Zeno's paradoxes (i.e., flying arrow) as follows: Please enjoy the literary pleasure.

Paradox 2.9. **[Zeno's paradoxes: The literature-like antinomy]**

The literature-like proof of [Flying arrow is at rest]

- Consider a flying arrow. In any one instant of time, the arrow is not moving. Therefore, if the arrow is motionless at every instant, and time is entirely composed of instants, then motion is impossible. □

The literature-like proof of [Flying arrow is not at rest]

- We have to accept that an arrow passes there. However, "to pass there" is not equivalent "to exist there". What is "to pass there"? "To pass there" is both "to exit there" and "not to exist there". Therefore, flying arrow is not at rest.

□

Therefore,

if we believe in such "logic" as above, we fall into a paradox.

Thus, our present problem is "How do we define 'logic'?"

Now we can answer the question "What is Zeno's paradoxes?". That is, we consider that

(A) **Zeno's paradoxes say "Don't trust on 'logic' unconditionally", that is, "Start from a worldview and not logic".**

since Paradox 2.9 shows that "antinomy" happens in the logic in ordinary language. If so, we have to obey the worldviewism in Sec. 1.1. that is,

- First declare the worldview, and discuss the world in the worldview. Namely,

$$\underset{\text{premise}}{\overset{\text{world is so}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{subject}}{\overset{\text{conclusion}}{\boxed{\text{discussions, calculation, practical logic, properties}}}}$$

Therefore, to solve Zeno's paradoxes is to solve the following problem

Problem 2.10. Propose a certain worldview, in which Zeno's paradoxes (e.g., Flying arrow) can be discussed.

[Answer]; This is answered in Answer 2.11 below.
Slogan-wise, we say

(B) **"Without a worldview, there is no logic (or precisely, practical logic)".**

Again see "the worldviewism (A)" in Sec. 1.1.

♠**Note 2.17.** The great tradition of philosophy from Plato to Kant was always based on the worldviewism(*cf.* Chaps. 3-10). I believe that the above (B) is the fundamental spirit of philosophy. As mentioned in Chaps. 11 and 12 later, the spirit of analytic philosophy is "Start from logic" or "Think logically !". Thus, I am somewhat skeptical of analytic philosophy

2.4.2* The solution* about Zeno's paradoxes (e.g., Flying arrow) in the motion function method

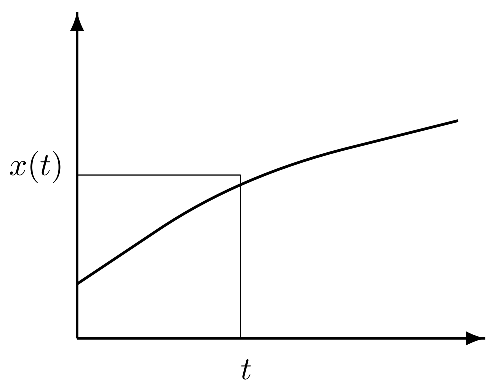
If we obey the motion function method, we can easily solve Zeno's paradoxes (e.g., Flying arrow) as follows.

Answer 2.11. [An answer to Problem 2.10(scientific answer)] Under the motion function method (*cf.* Section 2.3.4), we discuss "Flying arrow" as follows.

- Consider the motion function $x(t)$, that is, for each time t, the position $x(t)$ of the arrow is corresponded. It is obvious that

  (♯) "for each time t, the position $x(t)$ of the arrow is corresponded" does not imply that the motion function $x(t)$ is a constant function.

  Therefore, the arrow is not necessarily at rest.



□

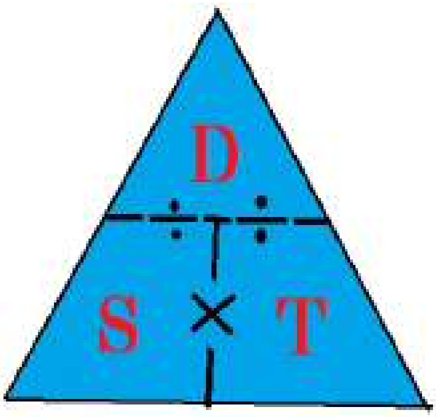
♠**Note 2.18.** Recall that we were confused in Paradox2.9[Zeno's paradoxes]. However, we could easily solve it in Answer 2.11. Thus we should be surprised at the power of the motion function method. If a certain worldview is determined, Zeno's paradoxes can be solved. Thus, the motion function method is not necessarily determined uniquely. For example, it is a good exercise to solve Zeno's paradoxes under Newtonian mechanics or the theory of relativity.

2.4.3 DST formula

In this section, we defined "motion function method", which was a metaphysical method. However, it is not difficult, and it is similar to the "DST formula" learned in elementary school. The DST formula was the following formula.

$$\underset{\text{(distance)}}{\boxed{\mathbf{D}}} = \underset{\text{(speed)}}{\boxed{\mathbf{S}}} \times \underset{\text{(time)}}{\boxed{\mathbf{T}}} \qquad (\clubsuit)$$

This "DST formula" may be the most difficult formula/concept in elementary school mathematics. It took me a long time to be able to use this formula when I was a child. Rather, I sometimes get confused even now. The following memory method, which was not available when the author was in elementary school, seems to be quite popular nowadays.



Here, the Pythagorean Theorem is a mathematical formula. The law of conservation of momentum is a formula of Newtonian mechanics. Now let's ask,

(C) In which field is the DST formula (♣) a formula?

This problem is equivalent to the following problem:

(D) Solve Zeno's paradoxes!

That is, as the answer for (C), we think that

- the DST formula (♣) is a formula in quantum language.

Thus, I think the DST formula (♣) is one of the most difficult formulas taught in junior high school, high school, and university.

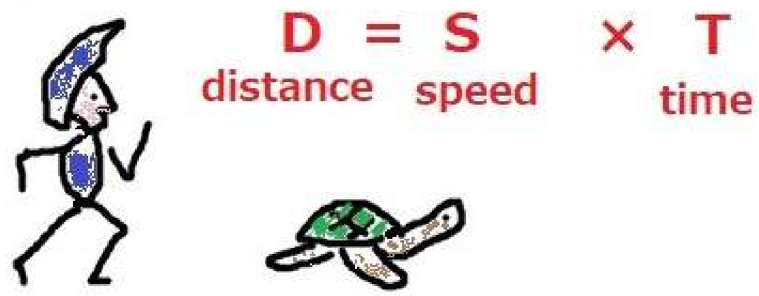
2.4.4 Appendix: The discussion about Zeno's paradoxes (e.g., Achilles and a tortoise]) in the motion function method

The idea of associating Zeno's paradox ("Achilles and a tortoise") with an infinite series misses the essence of Zeno's paradox, because Parmenides and Zeno's interest should be a worldview. That is, "Flying arrow" is the most important paradox in Zeno's paradoxes. However, since geometric series' method is most famous, I add "Achilles and a tortoise" as an appendix (*cf.* ref. [47], or Chap. 14 in ref. [74]). Readers should also taste the literary pleasure in the following.

Paradox 2.12. [Zeno's paradoxes(the literature-like answer)] [Achilles and a tortoise]

Zeno's paradox (Achilles and a tortoise) is as follows.

- Consider the competition of Achilles (a quick runner) and a tortoise (a late runner). Consider the competition of Achilles and a tortoise. Achilles' starting point will be behind the turtle's starting point. Suppose that both started simultaneously. If Achilles tries to pass a tortoise, Achilles has to go to the place in which a tortoise is present now. However, then, the tortoise should have gone ahead more. Achilles has to go to the place in which a tortoise is present now further. Even if Achilles continues to do this infinite times, he will not be able to keep up with the turtle.



[The scientific answer to Zeno's paradox (Achilles and a tortoise) by the motion function method]

For example, assume that the velocity $v_q(= v)$ [resp. $v_s(= \gamma v)$] of the quickest [resp. slowest] runner is equal to $v(> 0)$ [resp. γv $(0 < \gamma < 1)$]. And further, assume that the position of the quickest [resp. slowest] runner at time $t = 0$ is equal to 0 [resp. a (> 0)]. Thus, we can assume that the position $q_1(t)$ of the quickest runner and the position $q_2(t)$ of the slowest runner at time t (≥ 0) is respectively represented by the following motion function:

$$\begin{cases} q_1(t) = vt \\ q_2(t) = \gamma vt + a \end{cases} \tag{2.2}$$

Thus, it suffices to calculate formula (2.2).

Although it can be solved by various method, I present two methods as follows(i.e., (i) or (ii)):

[(i): Algebraic calculation of (2.2)]:
Solving $q_1(s_0) = q_2(s_0)$, that is,

$$vs_0 = \gamma vs_0 + a$$

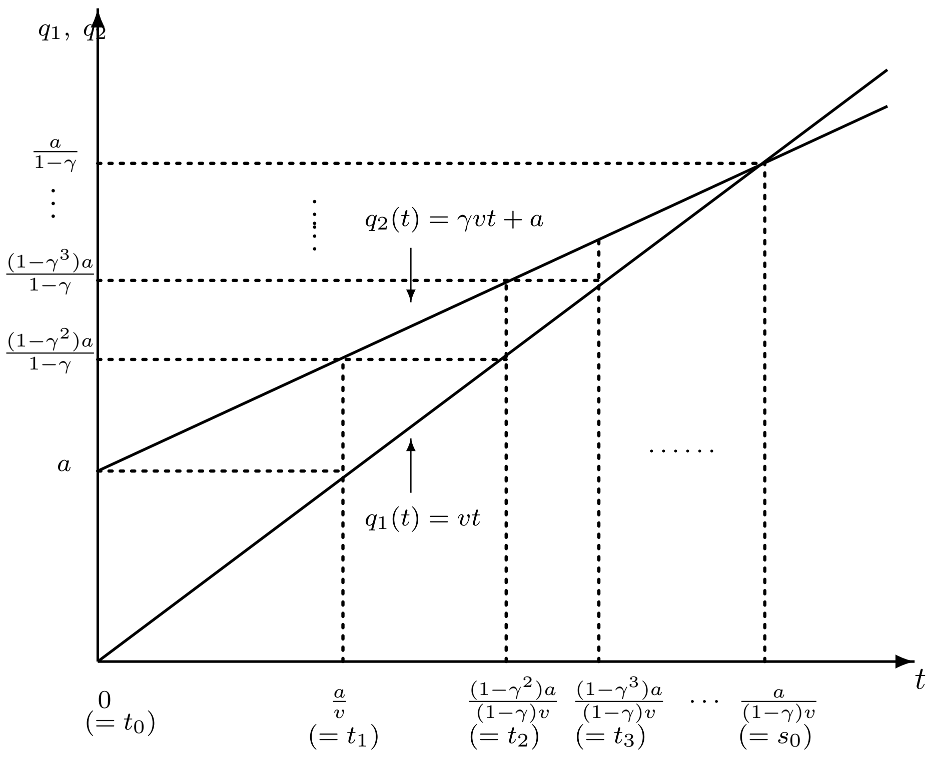
we get $s_0 = \frac{a}{(1-\gamma)v}$. That is, at time $s_0 = \frac{a}{(1-\gamma)v}$, the fast runner catches up with the slow runner.
[(ii): Iterative calculation of (2.2)]:
Define t_k $(k = 0, 1, ...)$ such that, $t_0 = 0$ and

$$t_{k+1} = \gamma vt_k + a \quad (k = 0, 1, 2, ...)$$

Thus, we see that $t_k = \frac{(1-\gamma^k)a}{(1-\gamma)v}$ $(k = 0, 1, ...)$. Then, we have that

$$\begin{aligned}\big(q_1(t_k), q_2(t_k)\big) &= \big(\frac{(1-\gamma^k)a}{1-\gamma}, \frac{(1-\gamma^{k+1})a}{1-\gamma}\big) \\ &\to \big(\frac{a}{1-\gamma}, \frac{a}{1-\gamma}\big) \end{aligned} \tag{2.3}$$

as $k \to \infty$. Therefore, the quickest runner catches up with the slowest at time $s_0 = \frac{a}{(1-\gamma)v}$.



Graph: $q_1(t) = vt$, $q_2(t) = \gamma vt + a$

[(iii): Conclusion]: After all, by the above (i) or (ii), we can conclude that

($\sharp$) the quickest runner can overtake the slowest at time $s_0 = \frac{a}{(1-\gamma)v}$.

♠**Note 2.19.** (a): Note that the above (ii) [= the solution using a geometric series] is another solution of (i). Of course, there was no need to use geometric series. The point is that there is a difference in the position of whether one considers the Achilles and a turtle paradox to be a mathematical problem or a philosophical problem. Many philosophers have gotten

into dead ends by confusing the two positions. Philosophers should have considered it as a problem of world description.
(b): From the philosophical point of view, "flying arrow" is definitely better than "Achilles and a tortoise". However, as far as quizzes go, "Achilles and a tortoise" is well done. "Achilles and a tortoise" is well done in the sense that the trick is designed to make it easy for solvers to fall into it. In fact, for 2,500 years, most solvers have fallen for this trick.

♠**Note 2.20.** (i): As mentioned in Preface, the purpose of this text is to understand the history of western philosophy from the quantum linguistic point of view. Thus,

(♯) We aren't interested about how Zeno himself considered his paradoxes. The established theory may say that Zeno might study the infinite division of time in physics (and space)

However, if so, Zeno's paradoxes are the problem in physics and not philosophy. Then the problem should be entrusted to physicists. However, in this text, we assume that Zeno's paradoxes belong to philosophy and not physics. Also, for the answer to "What is space-time in quantum language?", see Sec. 9.2.2: (Leibniz=Clarke correspondence).
(ii): Zeno's paradox attracted the interest of many philosophers because, in the author's opinion, the kinetic function method is a metaphysical method. However, Zeno's paradox is mentioned by many philosophers, including Aristotle, Spinoza, Bertrand Russell, and Bergson. But I always think, "Even a wise philosopher looks like a fool when talking about Zeno's paradox." So, readers may think I am a fool. I don't mind if they think so, but I would like them to continue reading to the next chapter, not to stop reading here.

♠**Note 2.21.** It would not be interesting to solve an unsolved problem that is 2,500 years old if this did not affect various issues in modern philosophy. I have some doubts about analytic philosophy, as I will discuss in Chapters 11 and 12.

- The reason for this is that Zeno's paradox cannot be solved in analytic philosophy.

More specifically, it is because the problems in the list (D_1) of Preface and the list (D_2) of Postscript cannot be solved by analytic philosophy.
Since it was "solving Zeno's paradox" that started my questioning of analytic philosophy, I devoted a considerable number of pages of this chapter to explaining "Zeno's paradox".

Chapter 3

The Big Three in Greek Philosophy (Socrates, Plato)

By the appearance of The Big Three in Greek Philosophy (Socrates, Plato, Aristotle), the origin of western philosophy was formed as follows.

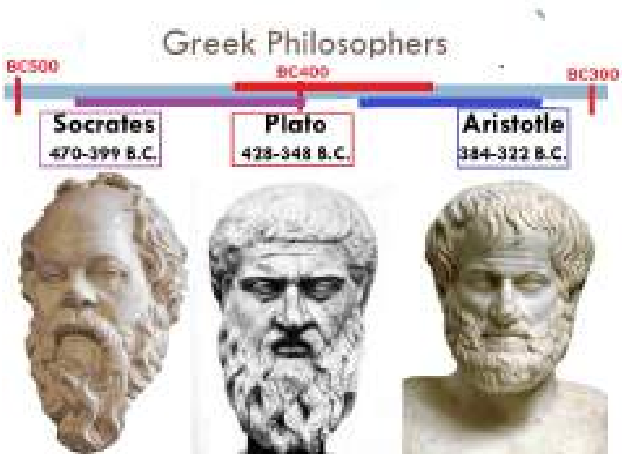
($\flat_1$) Ethics, etc.(Socrates)
($\flat_2$) worldview [*1]

- ($\flat_{21}$) : dualism and idealism (Plato, Augustinus, Anselmus, Thomas Aquinas, Descartes, Locke, Berkeley, Hume, Kant,Wittgenstein[*2],...,quantum language))
- ($\flat_{22}$) : monism and realism (Aristotle → Newton)

Whitehead(1861 - 1947) said:

- **Western philosophy is characterized as a series of footnotes to Plato**

I agree to him, thus in this text I will devote myself to the above ($\flat_{21}$).



Socrates was the teacher of Plato, and Plato was the teacher of Aristotle.

[*1] Here in the text, we will concentrate only on the scientific part of Plato
[*2] I am of the opinion that Wittgenstein should belong to dualistic idealism. (*cf.* Chaps. 11,12)

3.1 Protagoras and Socrates

3.1.1 Ethics

The philosophy of worldview aimed at the following problems

(A) How is the world described? How is the world understood?
By what kind of language is the world described?

But, there is another philosophy (i.e., philosophy of ethics) different from the worldview.

Ethics, morals [How should we live?]

Many people in the sciences (no, even the liberal arts) may think that ethical philosophy is "debating skills". In fact, **Protagoras (490 BC. - 420 BC.)**, a central figure of the Sophists, preached that **"man is the measure of all things"**. He argued for relativism, which holds that there is no such thing as objective truth. In other words, he argued that only the subjective judgment of each person is essential. Since the common sense of capitalism is

"the average of subjective value" = "price"

one might say that the sophist's claim is reasonable and modern common sense.

However, **Socrates** (BC.469 - BC.399) had objected to this idea. He investigated that

(B) How should we live?

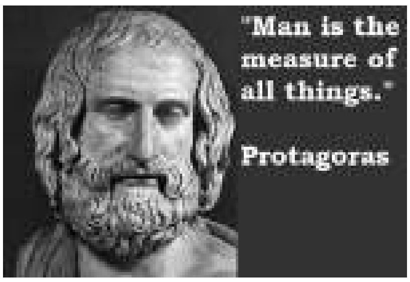
And, he clarified the following words:

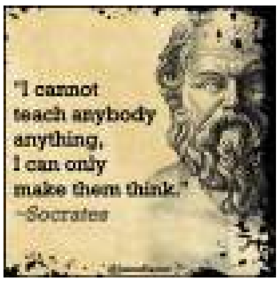
(C) "goodness", "happiness", "virtue", "justice", "courage", "love" ······

That is, Socrates asserted that the investigation of the above words is also the central theme of philosophy. In the following dispute:

"relativism (rational sophists) " vs. "absolutism (a man of faith: Socrates) "

Socrates advocated the ethical philosophy.





Note that three philosophers (Socrates (BC.469 - BC.399), Buddha (BC.565 - BC.486), Confucius (BC.551 - BC.479) and Mozi (BC.470 - BC.390)) were contemporary, and investigated the same problem (B). In this sense, we can say that

(D_1) The mathematics of Pythagoras was unparalleled in the world. Compared to this, Socrates was common sense and mediocre.

It is a matter of course that

(D_2) If these words mentioned in the above (C) didn't spread, the human race might have been ruined. At least, we wouldn't be able to form "human society". Maybe the mankind perished.

Therefore, I cannot overemphasize the importance of ethical philosophy. Also, the philosophy of ethics is worldwide. When it isn't so, we're in trouble. That is, when it isn't so, "world peace" isn't achieved. As emphasized throughout this text, I believe that

- **The main theme of philosophy is ethics (and not worldview). Even if we didn't have a philosophy of world description, we would have done it reasonably well.**

Hence I agree that Socrates is called **the father of philosophy**.

♠**Note 3.1.** In general, Ethical philosophy does not have any truth or universality. Society of ants has "ethics and morals of ants", and apes must have "ethics and morals of apes". The ethical philosophy of the Neanderthals must have been quite different from ours. Therefore, our ancestors must have destroyed the Neanderthals. Communication and contact with civilized aliens will happen in the future. However, it is too optimistic to expect the aliens to be unconditionally friendly at this time. I remembered that the late Dr. Hawking had emphasized the same opinion.

3.1.2 Magic proposition: I know I know nothing

♠**Note 3.2.** Socrates did not leave a book. The "Sophists vs. Socrates" is Plato's fiction. Since Plato is a disciple of Socrates, it's not fair. For example. The strongest logic, **"I know that I know nothing"** goes something like this.

- Sophists: something is asserted ······
- Socrates: deny Sophists' assertion
- Sophists and Socrates: debate (called Socratic Method)
  ······ Sophists and Socrates tell eloquently, and thus, they get tired. ······
- At that time, Socrates says "Your ignorance is now revealed. **I know that I know nothing**, but you do not know that you know nothing. Thus, I am superior to you".

This is Socrates' strongest logic **"I know that I know nothing"**. If we, without sticking to an established theory, read Plato's novel which makes Socrates a main character, we may have a variety of opinions on the following issues.

- Which is playing with sophistry, Socrates or sophists?

I may feel that Socrates uses more sophistry. □

In physics, you can draw conclusions with experiments, in mathematics you can prove them, and in engineering you just need to employ something useful. However, ethical philosophy does not solve the problem in that way. So Socrates (= Plato) came up with the magic word **"I know that I know nothing"** (**self-referential proposition**, anti-Copenhagen interpretive proposition) as a way to end the discussion.

(E) Since Socrates is the main character of Plato's novel, he must always be undefeated.

Suppose Socrates had said to Protagoras (whose name he borrowed on behalf of the Sophists)

(F) "I know that I am ignorant, while you do not know that you are ignorant. You, on the other hand, do not know that you are ignorant."

If this is a scientific proposition, then Socrates must provide a way to experimentally verify the following

(G_1) Socrates knows that Socrates himself knows nothing.
(G_2) Protagoras does not know that Protagoras himself knows nothing

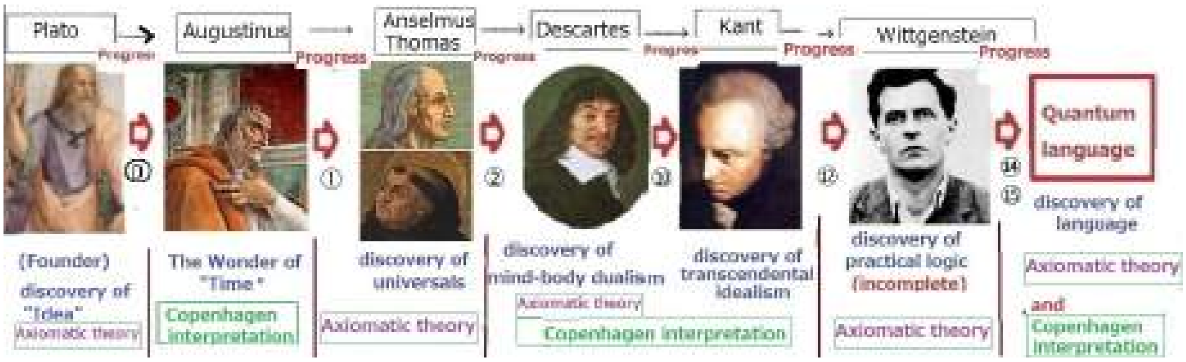
If Socrates cannot present a method to experimentally verify this, then it is a pseudo-scientific proposition. Therefore, Protagoras must have a "point" to make, but due to the circumstances described in (E) above, Protagoras was not given an opportunity to argue his case.

In the field of argumentation, pseudo-scientific propositions are often effective. Since the meaning of pseudo-scientific statements is unclear, listeners (i.e., readers) often take them as profound propositions, and they often become "decisive words". Descartes also used the pseudo-scientific proposition "I think, therefore I am" with success. A good philosopher, a good man of letters, and a good sophist have much in common. This is not surprising. Philosophy is a kind of literature, not science. Therefore, it is not only good, but also recommended for philosophy to make skillful use of pseudo-scientific propositions in order to advance the argument.

As described above, philosophy and pseudoscience have a very high affinity. This high affinity led to the emergence of the "Copenhagen Interpretation" in philosophy. This is because,

(H) the Copenhagen Interpretation is "a rule for drawing a line between scientific and pseudo-scientific propositions (taboo language) (cf. Sec. 1.3 (C_3)).

Recall the following:



where,

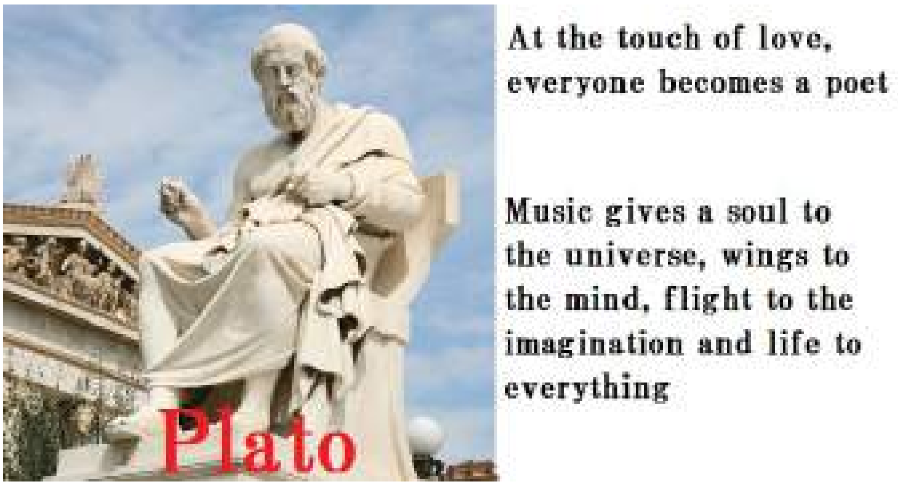
- Axiomatic theory $\begin{cases} \text{Plato} \cdots \text{ Idea} \\ \text{Anselmus, Thomas} \cdots \text{ universal} \\ \text{Descartes} \cdots \text{ I think therefore I am} \cdots \text{ mind-body problem} \\ \text{Wittgenstein} \cdots \text{ practical logic} \end{cases}$

- Copenhagen interpretation $\begin{cases} \text{Augustinus} \cdots \text{ Only the present exists.} \\ \text{Descartes} \cdots \text{ I think therefore I am} \cdots \text{ subjectivity problem} \\ \text{Kant} \cdots \text{ thing-in-itselt} \end{cases}$

I believe that the history of Western philosophy is the history of the highest intellects of the time, who have been thinking about these pseudo-scientific terms and propositions for more than 2,000 years.

I will demonstrate this throughout this text.

3.2 Plato (BC.427 - BC.347)



3.2.1 The theory of Ideas – Asserted fiction –

In the binary opposition (in ethics):

(A) "relativism (rational sophists)" vs. "absolutism (a man of faith: Socrates)"

Plato, a student of Socrates, established "the theory of Ideas" as the foundation of absolutism in order to support Socrates.

If the propositions such as "Man's life is heavier than the Earth", "Love is forever", "Love always overcomes money", etc. are the objective truth, the occupation of the insurance company does not hold. However, Socrates wanted to believe so. To help Socrates, Plato proposed the occult heavenly world (i.e., the world of Idea). That is,

(B) the theory of Ideas is a reckless attempt to derive ethics (i.e., "How should we live?") from worldview (i.e., "How is the world?"), that is,

$$\underset{\text{the fiction (spell) that "the world is so"}}{\boxed{\text{idealistic worldview}}} \xrightarrow[\text{habitual thinking}]{\text{therefore}} \underset{\text{Let's consider so}}{\boxed{\text{consideration, discussion}}}$$

This method (= the form of philosophy) has been accepted as the standard form of "how to tell philosophy" in the history of two thousands hundreds of years.

(C): The fiction called "the theory of Ideas"

Theory 3.1. [Theory of Ideas]:
The theory of Ideas is as follows

- In the real world, it is impossible to say that "love is better than money. However, there exists another world (i.e., the Idea world) in which "love is better than money" can be believed as an objective truth. And the existence of the Ideal world can be realized through the Idea.

This is the theory of Ideas.

Then, the real world is a shadow picture, hence, in the real world,

(D_1) love sometimes loses money
(D_2) We can't live on justice alone.
(D_3) Good man is sometimes unhappy.

That is, Plato wanted to say that

"love always overcomes money" is the objective truth in the world of Idea, therefore, "Believe in love!"

Whether you believe in this reasoning or not, this idea, i.e. the discovery of two key words "Idea world" and "reality world", was the beginning of "dualistic idealism" and has always been the mainstream of Western philosophy, despite the twists and turns that followed. The theory of ideas, which was supposed to be a logistic support of ethical philosophy (Socrates), became the mainstream of the philosophy of worldview.

(E) Our human DNA prefers logic (or reasoning) as if the philosophy of ethics were derived from the philosophy of worldview. That is,

"the world is so", therefore "we should live so"

namely,

(F_1) (=(B))

the fiction (spell) that "the world is so" [idealistic worldview] —therefore / habitual thinking→ Let's consider so [consideration, discussion]

Also, for example,

- invisible man exists [idealistic worldview] (fictional)premise —therefore→ don't steal a glance [practical logic, ethics·morals] main subject

and

- ultimate beauty exists [idealistic worldview] premise, fiction —therefore→ she is an embodiment of beauty. [practical logic, ethics, morals, aesthetic sense] main assertion

This is, of course, irrational since this is a reckless attempt such that the problem "How should we live?" is answered from the objective point of view. However, the human brain does not operate on logic alone. "Logic" cannot function without some kind of sensuous common soil. Logic alone is not enough, what is important is **"logic in a common sensory soil" (= "practical logic")**.

In other word, **"without common sensory soil, logic cannot work"**, therefore **"the first thing to do is to form a common sensory soil**. If so, Plato's way of telling philosophy (F_1) may not be reckless. That is because the (F_1) means

(F_2) [fictional worldview] (world is so; introduction·preface·fiction; **common sensory soil**) $\xrightarrow{\text{therefore}}$ [practical logic, Ethics · morals] (we should live so; main subject)

That is, "logic" depends on "worldview". Slogan-wise, we say

(G) (=Sec.2.4 (B)) **"Without a worldview, there is no logic (= practical logic)".**

Recall that this slogan plays an essential role in the solution of Zeno's paradox (*cf.* Sec.2.4 (B), Sec. 1.1).

♠**Note 3.3.** Here, "logic in the common soil of the senses" has a similar meaning to "logic under a certain worldview". In a broader sense, this "logic" is the logic usually used by ordinary people, such as the logic of Newton mechanics, the logic of a court case, the logic of politics, the logic of family life, the logic of insurance solicitation, and so on. One of the various kinds of logic is the logic of mathematical logic (= symbolic logic), which is so universal that even aliens may know it. However,

(♯) this does not mean that mathematical logic experts can master the logic of other disciplines well.

Naturally, a good understanding of economics means that one can use the logic of the common soil of economics. Judges and lawyers don't have to learn mathematical logic. It is natural to expect that logic under the theory of Ideas promote our understanding of Socrates and Plato ethics.

As mentioned in Chap. 12, we have two types of logic (i.e., symbolic logic and practical logic), that is,

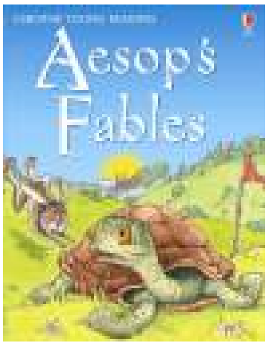
- { symbolic logic (=mathematical logic), i.e., logic in mathematics
  practical logic (= non-mathematical logic), i.e., logic induced by worldview

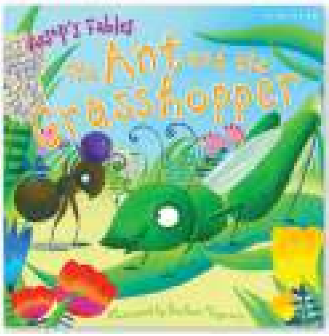
Practical logic plays an important role in worldviews. On the other hand, symbolic logic plays an important role in mathematics. I am skeptical of analytic philosophy, in which symbolic logic and practical logic may be confused. I feel that the above spirit (♯) is scarce in analytic philosophy. Analytic philosophy is not mathematics.

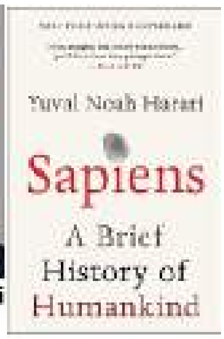
♠**Note 3.4.** (i): Aesop (BC.620 - BC.510): Idea theory is similar to Aesop's fable in some ways. It is natural to be hesitant to teach morality to others face-to-face. I think Aesop was preaching ethics indirectly through the animals. Aesop's fables were already well known before the late 5th century BC. Of course, Plato must have been aware of the persuasive power of Aesop's fables. And further, Plato might think that

A worldview is an abstraction of many allegories

In this sense, Plato might discover the power of abstraction.







(ii): In the book "Sapiens: A Brief History of Humankind" (*cf.* [30], 2016), Harari, the author, describes three revolutions which brought about dramatic changes to human societies.

- ① : the Cognitive revolution, about 70,000 years ago
- ② : the Agricultural revolution, about 11,000 years ago
- ③ : the Scientific revolution, beginning a mere 500 years ago

The above ① implies that

- we can rule the world because we are the only animals that can create and believe in fictions like God, the state, money and human rights.

Buddha (BC.565 BC.486), Confucius (BC.551 BC.479), and Mozi (BC.470 BC.390) were active on similar themes at the same time as Socrates and Plato. The reason is probably because the technology to record it as a character has been established.

////

Plato's say is as follows: No matter how much we argue, the issue of "relativism (rational sophistry) vs. absolutism (man of faith: Socrates)" can't be concluded. It is not a truth that we seek. There may not be the truth.

What people want is an "asserted fiction", not the truth.

And this is the philosophy. I think that's what Plato would have thought. In a simple analogy, it's next.

Say!, "I love you," even if it's a lie.

♠**Note 3.5.** S. Weinberg (1933 -2021), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said in his book [123] "To explain the world; The discovery of modern science" as follows:

(♭) [in Chapter 1] *There is an important feature of modern science that is almost completely missing in all the thinkers I have mentioned, from Thales to Plato: none of them attempted to verify or even (aside perhaps from Zeno) seriously to justify their speculations.*

Plato's philosophy is a representative of idealism (*cf.* for the definition of "idealism", see Sec. 1.1), which is a completely different style of philosophy from (experiment-oriented) science. As noted above, Plato's philosophy is a philosophy that begins with "asserted fiction". Also, it is a time of F. Bacon (i.e., the father of British Empiricism) of the scientific revolution (the 17th century) that the importance of observation was, for the first time, emphasized. See Section 7.2
Recall the rule of philosophy ($\approx$ idealism) is as follows:

($\sharp_1$) Only discussion, no experimentation.
($\sharp_2$) Winners are determined by the popularity of the general public (or philosophical enthusiasts).

♠**Note 3.6.** In order to avoid eternal argument, Socrates invented "Socratic method", that is, the magic phrase "I know that I know nothing". Plato also invented the theory of Ideas.
For completeness, we add the following:

($\sharp_1$) I think that Plato did not believe in the existence of the world of Idea. If he believed in it, he was not a philosopher but a founder of religion. He also understood that the theory of Ideas is sophistry, and there is no truth in ethics.

Even so, there may be a reason to consider that

($\sharp_2$) Something such as the prototype of the sense of ethics of the human commonness may be printed in the arrangement of a human DNA,

However, this idea may not be within philosophy.

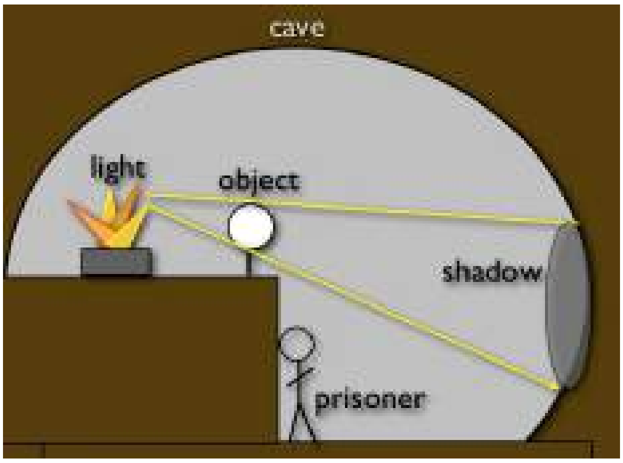
3.3 Three allegories about the Idea (the cave, anamnesis, the sun)

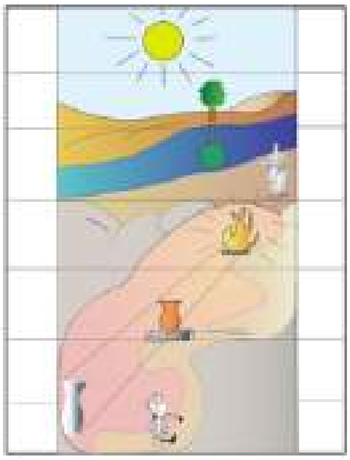
The "Ideal World" is like heaven. With religion, you can repeat it over and over until they believe it. Plato, however, tried to explain this with "three allegories (the allegory of the cave, the allegory of anamnesis, and the allegory of the sun).

This Platonic contrivance was "Idea", which became the "magic word" that has always remained at the center of Western philosophy's 2,000-plus year history.

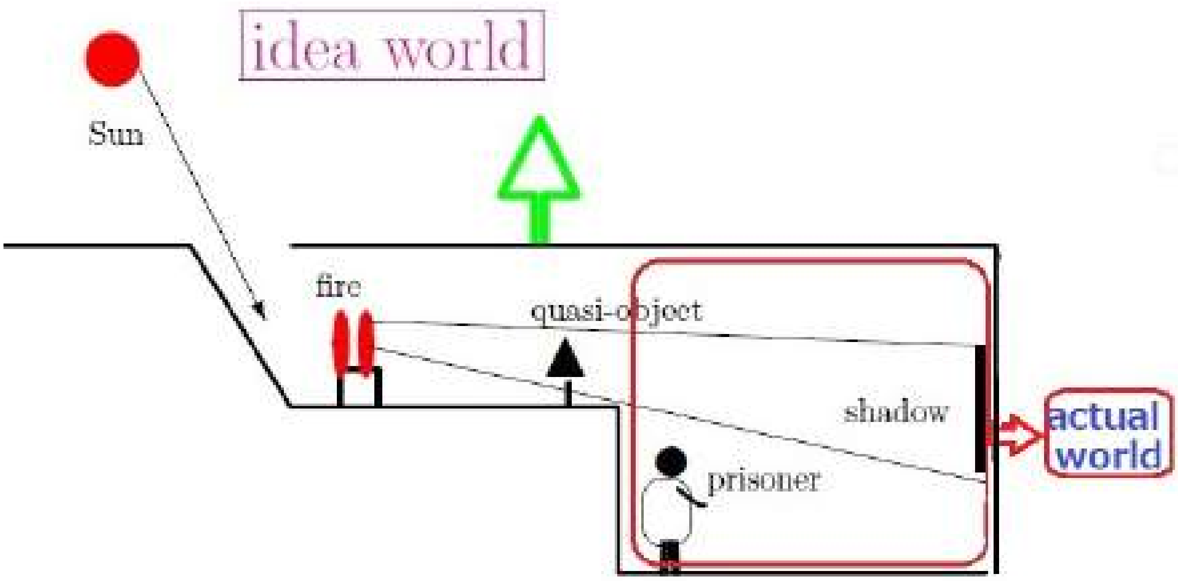
3.3.1 Allegory of the cave

The allegory of the cave was presented by Plato in his work "Republic" in order to promote the understanding of "the theory of Ideas".





Allegory 3.2. [Allegory of the cave]



A group of prisoners are looking at the shadows on the back wall of a cave. The shadows just are reality since they have never experienced anything other than shadows. The shadows represent physical reality. One day, one of the prisoners gets free. Namely, he is the philosopher, the lover of wisdom. Let's assume he is Socrates. He turns around and the first thing he sees is objects of stone and wood made to resemble the shapes of real things such as a tree. Further along, generating the light that hits these objects that then produce shadows on the back wall of the cave, is a fire. Beyond the fire is the entrance/exit of the cave. The philosopher exits the cave and is temporarily blinded by the light. The first thing he sees is a real tree. Finally, the philosopher sees the sun which Plato called The Idea of the Good. The freed prisoner (who is assumed to be Socrates) would think that the world outside the cave was superior to the world he experienced in the cave. And he would want to bring his fellow cave dwellers out of the cave and into the sunlight. The returning prisoner, whose eyes have become accustomed to the sunlight, would be blind when he re-enters the cave, just as he was when he was first exposed to the sun. The prisoners would infer from the returning man's blindness that the journey out of the cave had harmed him and that they should not undertake a similar journey. Hence, there is a possibility that the prisoners would therefore reach out and kill anyone who attempted to drag them out of the cave. In fact Socrates was killed.

Remark 3.3. Descartes' dualistic idealism has the following form:

$$[\text{A}](\textbf{mind}) \quad \underset{\textbf{(medium)}}{\longleftarrow [\text{B : body}] \longrightarrow} \quad [\text{C}](\textbf{matter})$$

This suggests a correspondence between the Allegory of the Cave and measurement theory (= quantum language), as follows.

\	[A](= mind)	[B](Mediating of A and C) (body)	[C](= matter)
Plato (Allegory of the cave)	actual world	sunlight	/ [Idea world]
quantum language	**measured value**	**observable**	**state** **[system]**

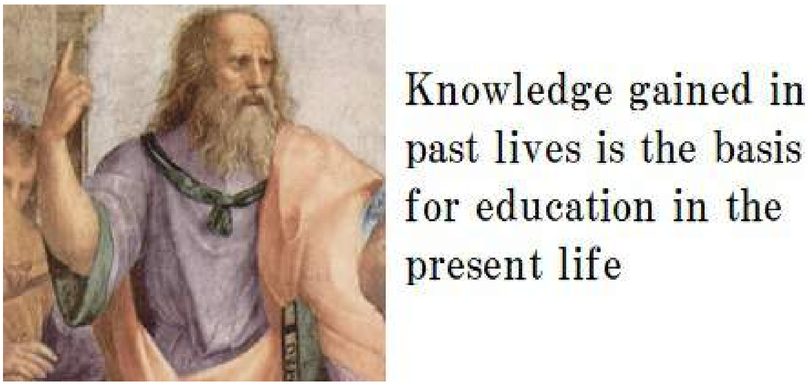
In essence, he brought up the instrument of visualization (the sun = Idea) to make us believe in an "ideal world".

3.3.2 The theory of anamnesis

Plato uses various metaphors to explain the Idea Theory. Besides the "Allegory of the Cave" in the previous section, they are the "The theory of anamnesis" (this section) and the "Allegory of the Sun" (next section).

As a mediator between the real world and the idea world, Plato presents the concept of "anamnesis". Namely,

- We had seen the Idea before we were born. But we forget that when we are born. Therefore, to know an Idea is to recall the Idea. In other words, learning is nothing less than recalling (= anamnesis).



In the history of western philosophy, the theory of anamnesis is located as follows:

[modern philosophy] Descartes ⟶ { [tabula rasa, experience] British Empiricism [Locke, Berkeley, Hume] ; [a priori, nativism] Continental Rationalism [Leibniz] } ⟶ *3 ["Critique of Pure Reason"] Kant philosophy [summing-up, compromise]

Plato [anamnesis] ⟶ Continental Rationalism

However, I think that "tabula rasa vs. nativism" is pseudoscientific (*cf.* Sec. 10.3), and thus, the above is not our interest.

Remark 3.4. Descartes' dualistic idealism has the following form:

$$[\text{A}](\textbf{mind}) \quad \longleftarrow \underset{\textbf{(medium)}}{[\text{B : body}]} \longrightarrow \quad [\text{C}](\textbf{matter})$$

Adding the recall theory to the table in Remark 3.3, we obtain the following

*3 In our style, "Kant's Synthesis" can be rewritten as "QL Synthesis"(*cf.* Chap. 10).

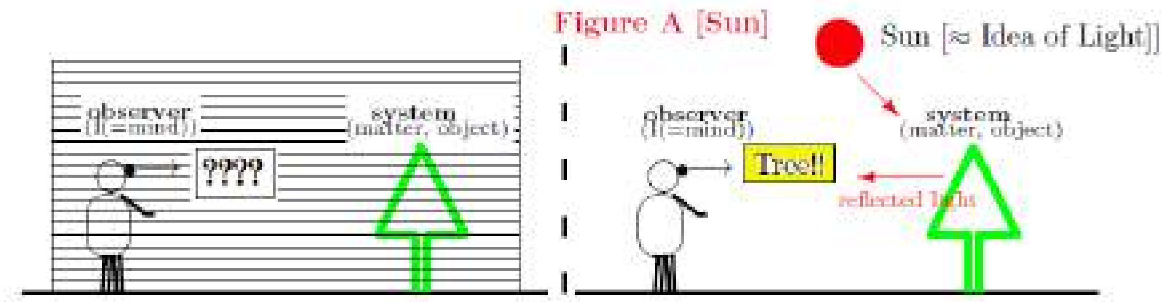
\	[A](= mind)	[B](Mediating of A and C) (body)	[C](= matter)
Plato (Allegory of the cave)	actual world	sunlight	/ [Idea world]
Plato (anamnesis)	actual world	anamnesis	/ [Idea world]
quantum language	**measured value**	**observable**	**state** **[system]**

In short, in order to make the "Idea World" seem familiar, Plato said, "Before we were born, we lived in the Idea World. we just forgot about it."

3.3.3 The allegory of the sun: Measurement theoretical aspect of Idea theory

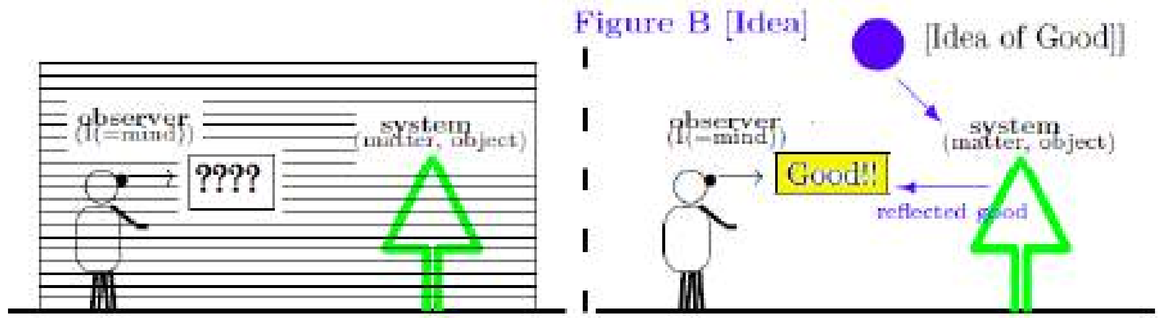
The analogy of the sun is found in "Republic", written by Plato. Upon being urged by Glaucon (Plato's elder brother) to define goodness, a cautious Socrates professes himself incapable of doing so. Instead he draws an analogy and offers to talk about **"the child of goodness"**. For the answer to "Why a child?", see Note 3.7.

Allegory 3.5. [The Allegory of the Sun]: The Allegory of the Sun explains what the "Idea of Good" is all about. No matter how much you open your eyes, you cannot see anything "visible" in the "visible" world, such as a flower, a tree, or a dog, without the light of the sun.



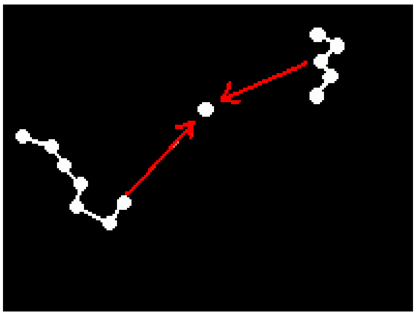
In the beginning, when it's dark as shown on the left figure, you can't see anything, even if your vision is normal. However, by developing the skills to use a measuring instrument called the "Sun [= Idea of Light]," you can see that it is a "tree". Thus, in this fable, the ternary relation ("the beholder," "the sun (the mediating thing)," "the thing to be seen") are clear and very easy to understand.

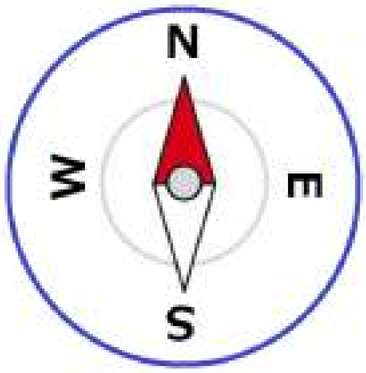
In the same way, things that exist in the invisible realm, such as virtue and courage, cannot be known unless one has the skill to use the measuring instrument called the "idea of the good", even if one has reason.



In other words, if you learn how to use a measuring instrument called the sun, you can make the object "visible". In the same way, the mastery of the measuring instrument, which is called the "idea of Good," makes clear virtue and courage.

Remark 3.6. The polar star can be regarded as a measuring instrument such as a kind of compass (*cf.* Note 1.7 (iii)). Thus, it is reasonable to regard the sun (Idea) as a measure of "Good".





That is, I want to think:

$$\text{"Idea"} = \text{"instrument to make Ideal world visible"}.$$

Therefore, three key-words in Plato philosophy:

[A](**mind**) ⟵ [B] ⟶ [C](**matter**)
(medium)

correspond to as follows:
Adding the allegory of the sun to the table in Remark 3.4, we obtain the following

\	[A](= mind)	[B](Mediating of A and C) (body)	[C](= matter)
Plato (Allegory of the cave)	actual world	sunlight	/ [Idea world]
Plato (anamnesis)	actual world	anamnesis	/ [Idea world]
Plato (Sun)	actual world	Idea (Sun)	/ [Idea world]
quantum language	**measured value**	**observable**	**state [system]**

Of course, this is somewhat too forced. That is because the theory of Idea was not put forward with the intention of being a theory of measurement (or epistemology).

3.3.4* The measurement theoretical understanding of the allegory of the sun

Recall Figure 1.3 [Descartes figure] in Sec. 1.2.1, namely,
Figure 1.3 ; [Descartes Figure]: Image of "measurement(=ⓐ+ⓑ)" in dualism

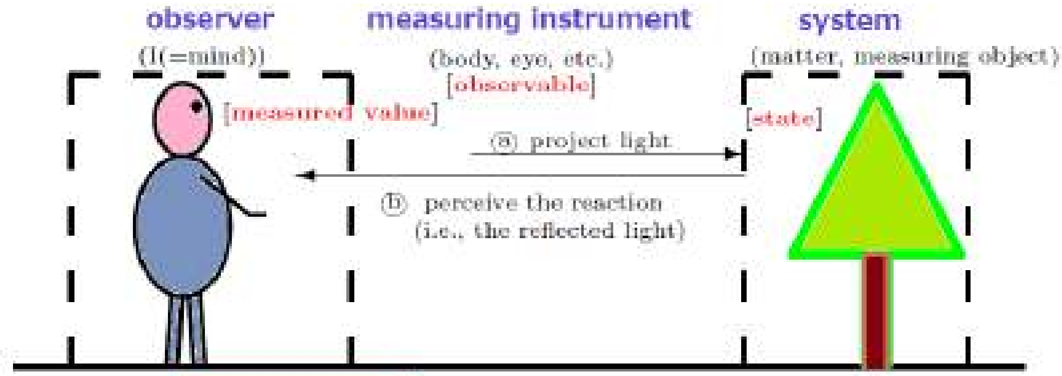
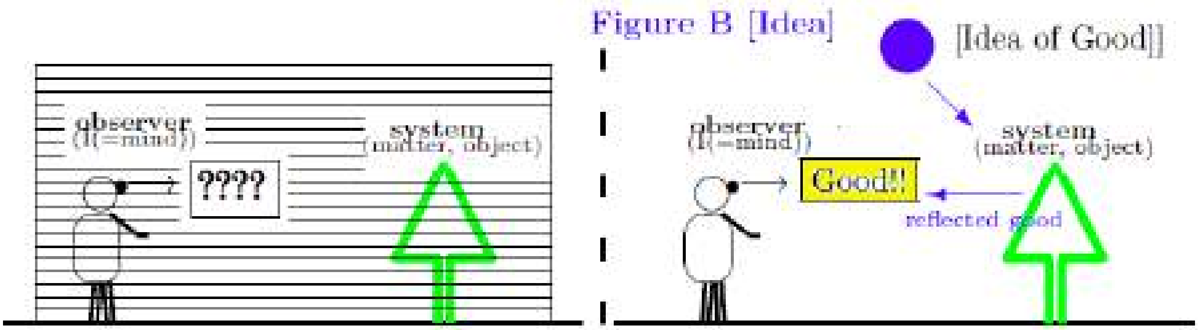


Figure 1.3 Descartes Figure: Image of "measurement(=ⓐ+ⓑ)" in dualism

This is very similar to the "the allogory of the sun such as :



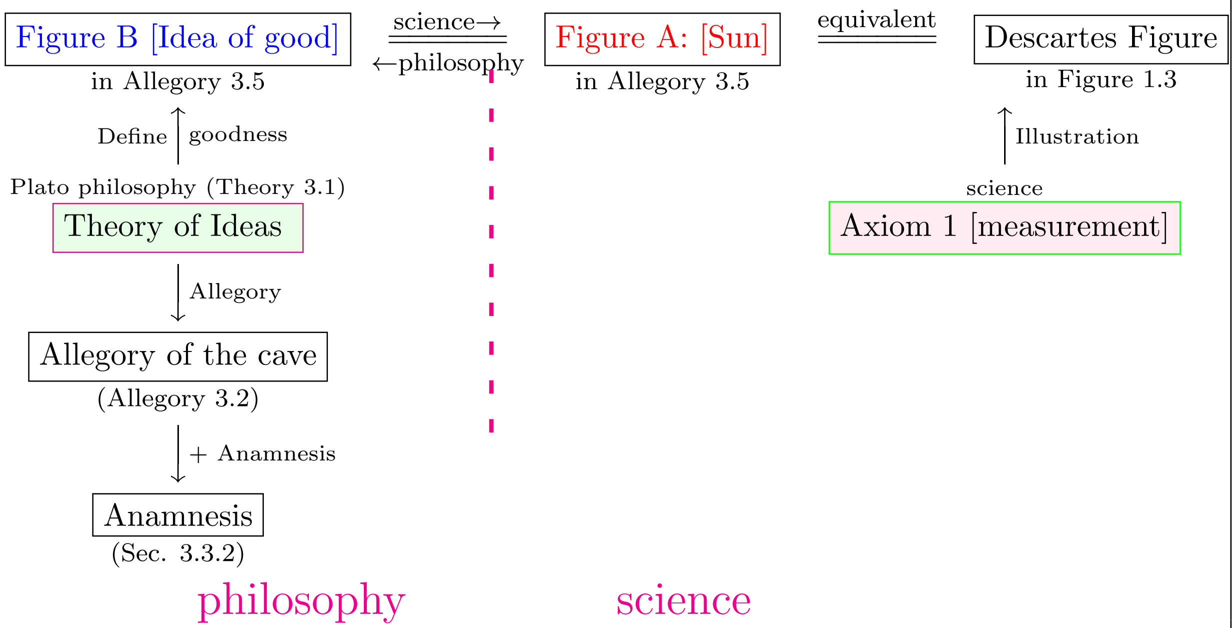
Not only are the figures similar, but it is tempting to assert the following relationship.

Figure B [Idea of good] in Allegory 3.5	science→ ←philosophy	Figure A: [Sun] in Allegory 3.5	equivalent	Descartes Figure in Figure 1.3

For further information, see my homepage

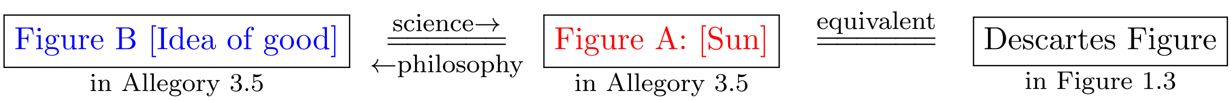
Here, it should be noted that this is essentially the same as the figure [Allegory of the sun].

Summary 3.7. We see:



We consider that

- the understanding of the first line above, namely,



This is equal to the measurement theoretical understanding of Allegory 3.5 [Allegory of the sun]

So, of the three explanations of Idea, "the allegory of the cave," "the theory of Anamnesis," and "the allegory of the sun," I think the one that comes closest to quantum language (= measurement theory) is the allegory of the sun.

♠**Note 3.7.** However, I do not think "The Allegory of the Sun" is a suitable example of Idealism because it is too scientific and easy to understand. Perhaps Plato gave too scientific an explanation in order to make the Idea Theory understandable to his readers. I believe that Plato was not satisfied with the parables in The Allegory of the Sun, so he called The Allegory of the Sun "The Son of Good." In B. Russell's work "History of Western Philosophy," he writes (*cf.* ref. [116])

- *Aristotle's metaphysics, roughly speaking, may be described as Plato diluted by common sense. He is difficult because Plato and common sense do not mix easily.*

If we believe Russell's words, then Idealism and the "the allegory of the sun" cannot easily be mixed. In other words.

- This "Allegory of the Sun" is too scientific and may interfere with a philosophical understanding of Idealism.

However, in Scholastic philosophy, more than a thousand years after Plato, Anselmus was the man who took the Idea theory even more scientifically. This was the beginning of a series of revelations about the scientific aspects of the Ideas. A summary of this is written in Sec. 3.5.

3.4 Plato: The worldview called Idea theory; dualistic idealism

3.4.1 Worldviewism

Review the Platonic worldview (= Idealistic worldview)(*cf.* Sec. 1.1).

(B) Worldview Type

worldview
- (i): realistic worldview (=Aristotelian worldview) (physics, Aristotle philosophy, etc.)
- (ii): idealistic worldview (Plato worldview) (Statistics, Plato, Descartes, Kant , etc.)
- (iii): mathematics(=mathematical thinking) (logical worldview) (Set theory, etc.)[*4]

Above, our interest is in (ii), the idealistic worldview (= Platonic worldview). In other words.

(ii) [Idealistic worldview (=idealism)] is "a way of trying to understand the world through sutra-like statements". We can assume statistics (which can be applied to medicine, economics, engineering, or anything else). Thus, "the worldview comes first and the world comes after". The Idealistic worldview (since it is worded like a sutra) makes no sense to determine right or wrong by experimentation, but "usefulness (usefulness?)" is evaluated by "usefulness". In fact, statistics is very useful, as it is said to be the most powerful science. The idealistic worldview is quite broad, and dualistic idealism is the mainstream of Western philosophy [Plato, Scholasticism, Descartes-Kant's epistemology, analytic philosophy (in the author's opinion)].

the fiction (spell) that "the world is so" [idealistic worldview] —therefore / habitual thinking→ [consideration, discussion] Let's consider so

There are rather non-scientific idealistic worldviews, and in fact, Plato, Schola, Descartes-Kant's epistemology, and analytic philosophy are "rather spurious idealistic worldviews. In Plato's philosophy, Idealism (= fictional worldview) is nothing but a fiction (= parable or allegory).

- **Idealism is nothing more than a quibble (a parable, a fashion) to justify Socrates Plato's ethics.**

Plato's way of speaking philosophy (i.e., "a rather fanciful conceptual worldview") is common to all philosophies of the dualist-idealist lineage, as follows.

[*4] It is impossible to consider mathematics as a kind of worldview, but I will classify it here in the sense of "mathematical thinking." (*cf.* Sec.4.3.4: Set-theoretic Worldview, Logical Worldview)

(A) Plato (the theory of Ideas)⟶Augustinus⟶Thomas Aquinas ⟶Descartes ⟶Kant(epistemology)⟶analytic philosophy

There are, of course, reasons why a "rather frowned upon conceptual worldview" could become the mainstream of Western philosophy as described above. I shall write this below.

3.4.2 The necessity of the worldview

Even if the theory of Ideas is a fairy tale, Plato's idea has a point.

(B) Ethics · morals is dependent on the world (=environment around).

It is a matter of course that there is a difference between the ancient Greek ethics and the modern American ethics. In this sense, strictly speaking, the following (=Plato's way of telling philosophy) is true:

(C) (= (F_2) in Sec. 3.2.1)

the fiction (spell) that "the world is so"
[idealistic worldview]
common sensory soil
$\xrightarrow[\text{habitual thinking}]{\text{therefore}}$
Let's consider so
[Ethics · morals, etc.]
main subject

3.4.2.1 The necessity of idealism (= metaphysical world)

If the (D) is strictly put into practice, this is not philosophy but life consultation. For example,

- If you are really depressed, go to a psychiatrist and ask him to prescribe some medication. Philosophers are not to be trusted.

Thus, philosophers have to assume an unrealistic world (metaphysical world). This is because if the settings can be verified by experiment, mistakes may be pointed out. Therefore,

(D) Metaphysical worldview (i.e., idealism) is desirable

This is the reason to adopt the idealism (i.e., metaphysical world) in Plato's way of telling philosophy

3.4.2.2 The necessity of dualism

Also,

(E) since the goal is ethics and morality, a world that reflects human beings in some way is preferable. In other words, the dualism of "things" and "people" is preferable.

For example,

- We don't usually take moral lessons from monism, such as Newtonian mechanics.

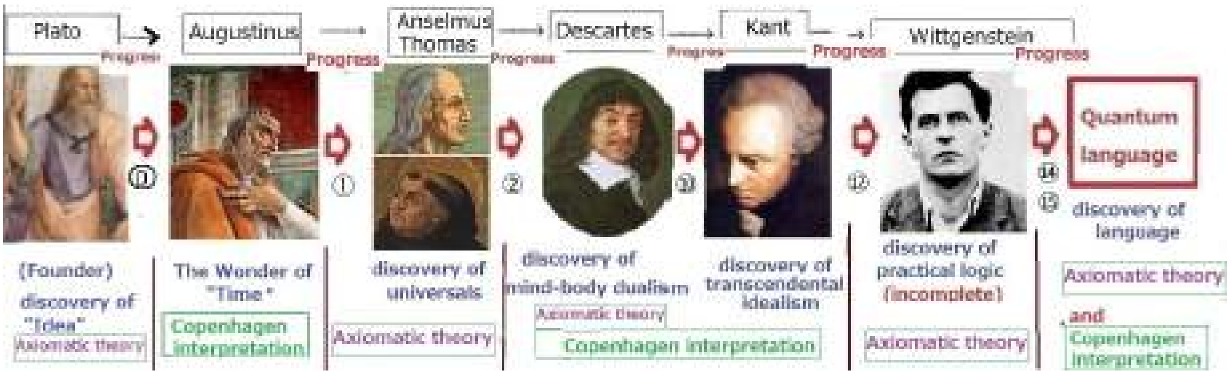
This is the reason to adopt "mind-body dualism" in philosophy.

After all, we conclude that

(F) **in the philosophy of worldview, dualistic idealism is desirable.**

3.5 Appendix: How could Plato's philosophy be the source of Western philosophy?

Let us consider "How could Plato's philosophy be the source of Western philosophy?"



In other words, "How did the above figure come about?" is what we would like to consider. Since this question is to be considered throughout this text, I do not intend to draw a hasty conclusion in this section, but there may be readers who would like to know the conclusion here and now. For such readers, I would like to conclude.

- The word "Idea" just happens to be the best "magic word" in the history of philosophy*5.

This will be explained as follows.

3.5.1 History of development from ideas (Plato) to observables (quantum language)

Descartes' dualistic idealism has the following form:

$$[A](\textbf{mind}) \quad \longleftarrow \underset{\textbf{(medium)}}{[B:\text{body}]} \longrightarrow \quad [C](\textbf{matter})$$

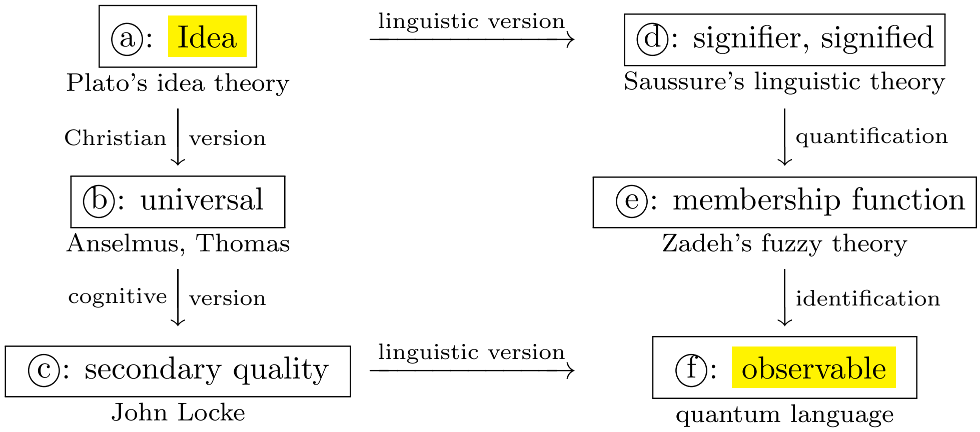
We then consider the position of "Plato's Idea" as shown in the table below.

\	[A](= mind)	[B](Mediating of A and C) (body)	[C](= matter)
Plato (Sun)	actual world	Idea	/ [Idea world]
quantum language	**measured value**	**observable**	**state [system]**

Let's preview here the history of the evolution of "Ideas" to become "Observables." As will be explained throughout this text, the following can be said in summary.

*5 However much of a genius Plato was, I think it was "accidental.

(A)



I will describe it in more detail later, but here I will draw a picture to illustrate the image.

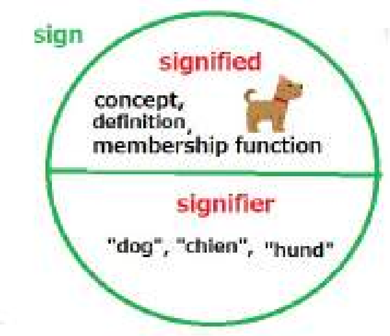
ⓐ Idea
the meter standard of goodness, the meter standard of beauty, , $\cdots$"
$\Longrightarrow$
Device for visualizing goodness, Device for visualizing beauty

---

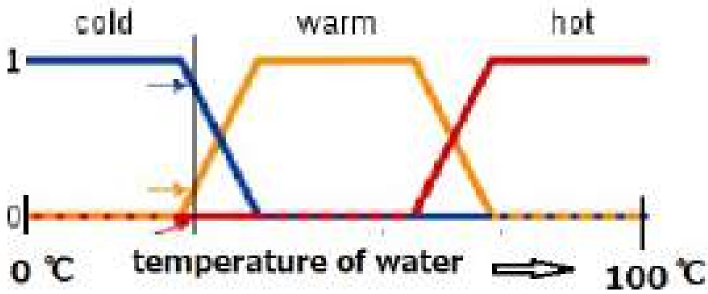
ⓑ universal
mankind, Americans, hero, good people, $\cdots$

ⓒ secondary quality (=sensations of inherent nature (=primary qualities))
"sweet, spicy", "hot, cold", "beautiful, ugly" $\cdots$

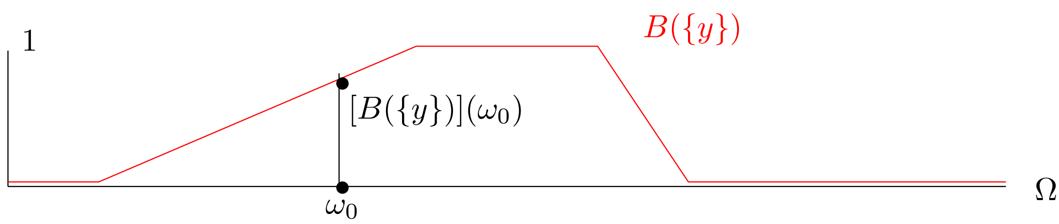
ⓓ Saussure's Signifiant and Signifie



ⓔ fuzzy set (≈ membership function)



ⓕ observable(≈ measuring instrument)
position, momentum, energy··· (in quantum mechanics)



For further information, see my homepage

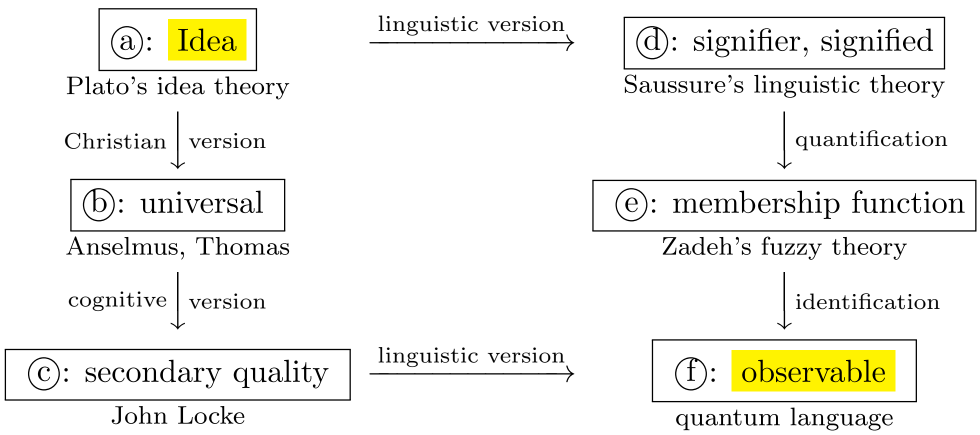
3.5.2 The history of Western philosophy is the history of the pursuit of "What is Idea?"

As mentioned in (A), I think

(B) (=(A)) The history of Western philosophy is the history of the pursuit of "What is Idea?"

That is,

(C) (=(A))



Thus, we may consider the following.

(D) The history of Western philosophy is the history of the pursuit of "What is Idea?"

Even so, if many genius philosophers did not continue to hold the belief that there is a scientific answer to the question "What is an idea?" Therefore, the answer to question (B) is as follows.

(D_1) "Idea" was the basic concept of dualistic idealism
(D_2) As you can see from (C)(=(A)), "idea" has various aspects and is interesting enough to never get tired of.

3.5.2.1 Guardian of christianity

Now, even if (F)(=(F_1)+(F_2)) above is the right answer, it does not mean that the "pursuit of ideas" can continue for more than 2,000 years.

Another reason, I think, is Christianity. Eastern philosophy has continued for 2,500 years in a delicately tense relationship, with various styles of Buddhism, Confucianism, and Taoism mixed up. In the West, on the other hand, Christianity was too strong. Therefore, the main theme (= ethics, morality) was mostly based on "Christianity + Socrates", and various styles were not born. Moreover, it can be said that the position of Plato's philosophy became immovable as a result of Augustine's adoption of Plato's philosophy as the theoretical pillar of Christianity.

Also, according to worldviewism (Plato's philosophical narrative), the introductory part (= the fictional worldview of dualistic idealism) can be changed. Therefore, the progress of Western philosophy (which could not be realized in the East) can be realized in the following way.

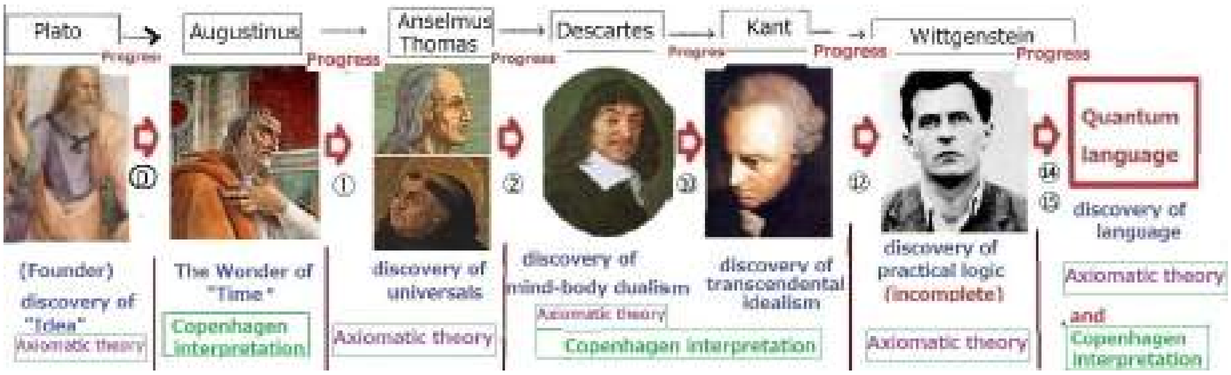
- Plato⟶Augustinus⟶Thomas Aquinas ⟶Descartes⟶Locke⟶...⟶Kant

That is,

By reason of (D_2), western philosophy could be kept fresh

To use an analogy, we think that the effect was like **car model change**. The Platonic way of telling philosophy is almost the mainstream of Western philosophy. This device (i.e., model change) brought about the rise of Western philosophy.

The pursuit of "Idea" is, after all, equivalent to the pursuit of "dualistic idealism. Therefore, in fact, it is not a "mere model change. The following diagram shows this.



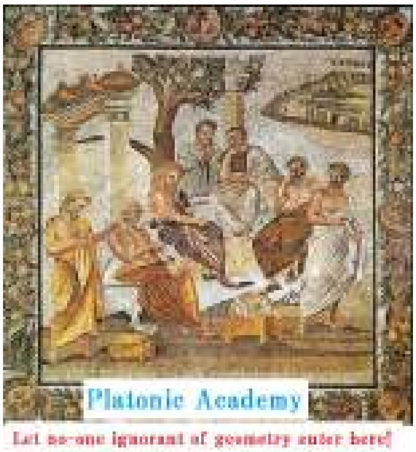
In the case of Eastern philosophy, the founder was always great, and no mechanism was created to pass the baton to his successor.

(E_1) **There were so many different aspects to "Idea" that it was never boring or interesting.**
(E_2) **Western philosophy kept itself fresh by alternating between "axiomatic theories" and the Copenhagen Interpretation.**

3.5.2.2 Thanks to Aristotle (monistic realism), "Idea Theory" became stronger

Compared to Eastern philosophy, a distinctive characteristic of ancient Greek philosophy is its emphasis on "mathematics and logic" represented by Pythagoras.

The entrance gate of the school founded by Plato (the Platonic Academy) reads.
"The person who does not know the geometry should not pass through this gate"



Although Platonic philosophy has nothing to do with mathematics, the signboard was effective enough, as it led Aristotle and other gifted students to pass through the doors of the Platonic Academy.

In fact, I believe that one of the factors that allowed Plato's philosophy (dualistic idealism) to persist and remain in the mainstream of philosophy was the existence of its rival, Aristotle's philosophy (monistic realism). Since monistic realism is connected to Newton (physics), its prosperity has kept it in a firm position until today. In comparison, Plato's philosophy (dualistic idealism) has never given the general public a sense of "usefulness.2

And yet

Aristotle (monistic realism) vs. Plato (dualistic idealism)

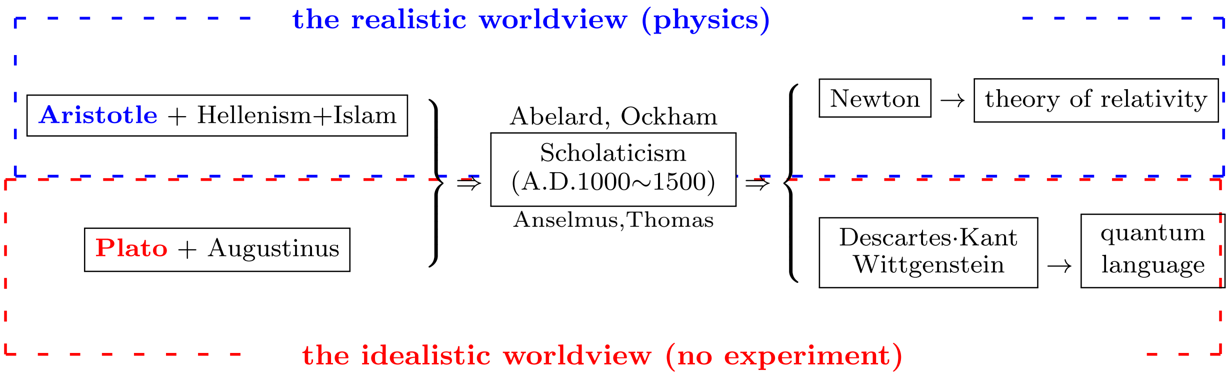
has been the "greatest scientific controversy (see Sec. 4.1.3)" for 2500 years.

While Aristotle's philosophy (monistic realism) was firmly established by Newton, Plato's philosophy (dualistic idealism) continued to boldly challenge Aristotle's philosophy (monistic realism) despite its lack of a solid foundation. By continuing to challenge, dualistic idealism became stronger and stronger. In the East, dualistic idealism became religious because a powerful rival, Newton, did not emerge.

Chapter 4

The Big Three in Greek Philosophy (Aristotle)

Although Aristotle was a disciple of Plato, he did not understand Plato's philosophy. He then proposed an realistic worldview that was quite different from Plato's philosophy. He is called the father of all sciences (≈ the father of the realistic worldview). It is no exaggeration to say that philosophy was started by these two men. That is, we see:

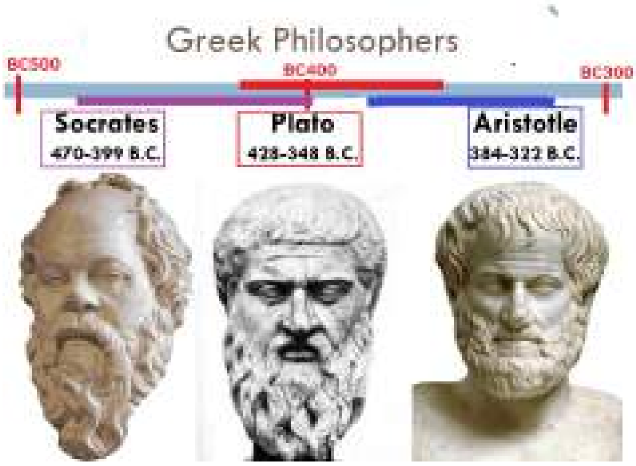


The main theme of this text is the "idealistic worldview," however, it is recommended that the "idealistic worldview" be understood in contrast to the "realistic worldview".

Plato & Aristotle

4.1 Aristotle (BC.384 - BC.322)

4.1.1 Realistic worldview vs. idealistic worldview



Aristotle (BC.384 - BC.322), the student of Plato, is called the father of all sciences ($\approx$ the father of the realistic worldview). He could not accept Plato's theory of Ideas($=\Big[$ asserted fiction $\Big]$). Namely,

- **Philosopher Plato preferred asserted fiction (without experiment) to truth (with experiment)**

On the other hand,

- **Scientist Aristotle preferred truth (with experiment) to asserted fiction (= without experiment)**

♠**Note 4.1.** S. Weinberg (1933 -2021), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said in his book [123] "To explain the world; The discovery of modern science" as follows:

($\sharp$) [in Chapter 3] *I confess that I find Aristotle frequently tedious, in a way that Plato is not, but although often wrong Aristotle is not silly, in the way that Plato sometimes is.*

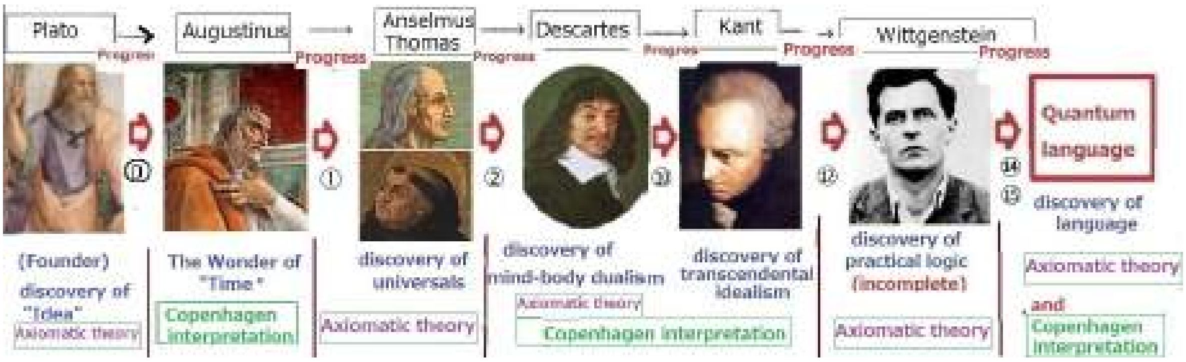
Plato was not aiming for science, that is, Plato's purpose is to support Socrates's ethical philosophy. Therefore, from the scientific point of view, some may feel Plato silly. Namely,

($\flat_1$) a fiction that was asserted (by Plato) over 2000 years ago is somewhat silly, if not tedious.

On the other hand, Aristotle might be aim for science. Thus, from the modern point of view, some may feel Aristotle tedious. Namely,

($\flat_2$) truths discovered (by Aristotle) over 2000 years ago are tedious and often wrong, if not silly

The above ($\flat_1$) and ($\flat_2$) are merely statements of the commonplace. And thus, he may not be saying anything negative. Science progresses, so you'll find the old science boring. But literature does not fade away after more than two thousand years. However, one of our purposes of this text is to show that



Although Aristotle does not appear in this diagram, it is impossible to understand the meaning of this diagram without an understanding of Aristotle.

4.1.2 Edios and Hyle

Aristotle proposed the concepts such as "eidos" and "hyle" as follows.

(A): Edios(Aristotle's Idea) and hyle

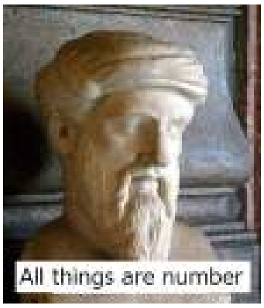
Aristotle said that

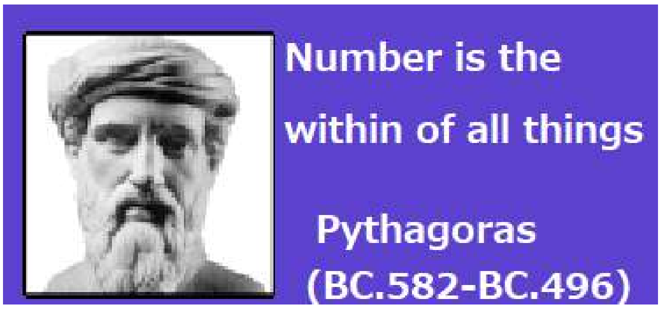
- **Edios** (= Aristotle's Idea = true form) is not in the heaven, but in **hyle** (= matter = particle).

In short, Aristotle simply stated that

"the nature of a thing is in the thing (not in the heavens).

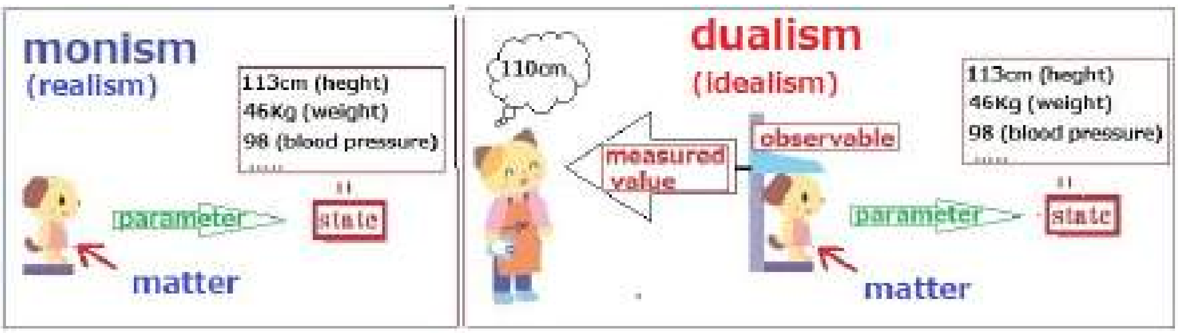
The above is the same as the Pythagorean claim.





I think the reason for this is clear from the figure below left

The final forms of monism and dualism were



Thus, dualism (Plato) is considerably more complex than monism (Aristotle). With this in mind, please read the following.

Summary 4.1. [The key-words of the realistic worldview] The meaning of "hyle" and "edios" in (A) above is difficult to understand. However, note that

- **Aristotle is the founder of the realistic worldview (= physics)**

Aristotle philosophy was completed by Netwonian mechanics. That is,

Thus, we see:

\	[A](= mind)	[B](Mediating of A and C) (body)	[C](= matter)
Aristotle	———	———	hyle [eidos]
Newton	———	———	point mass [state]

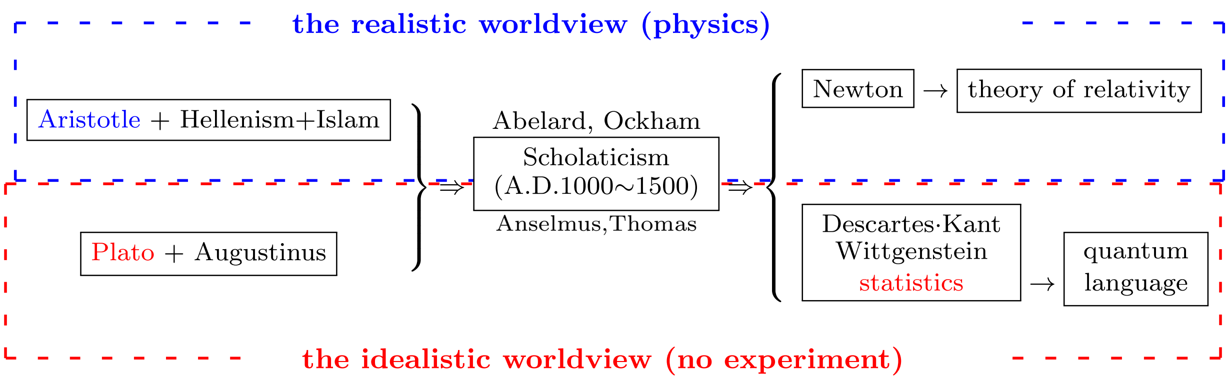
[Note]: The above table should be compared to the following table in Remark 3.3 [i.e. Plato's Idea theory]:

\	[A](= mind)	[B](Mediating of A and C) (body)	[C](= matter)
Plato: Sun Remark 3.6	actual world	Idea	Idea world
quantum language	**measured value**	**observable**	**system [state]**

♠**Note 4.2.** The book : "History of western philosophy" due to B. Russell says that (*cf.* ref. [116]),

- *Aristotle's metaphysics, roughly speaking, may be described as Plato diluted by common sense. He is difficult because Plato and common sense do not mix easily.*

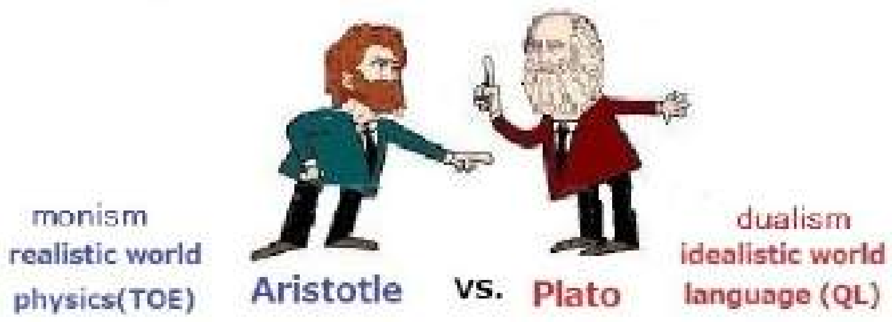
I think this representation is misleading since the two men have very different areas of expertise. Aristotle proposed a monistic realist philosophy that is quite different from the Platonic philosophy of dualistic idealism.



However, modern common sense is also suspect. This is because statistics, the most successful branch of idealism, is generally considered to belong to applied mathematics and is not generally recognized as a type of idealism.
Immediately when Newtonian mechanics was proposed, realism was generally understood. However, I believe that idealism is still not generally understood. In short, scientific

understanding of idealism is as difficult as understanding quantum mechanics.

For "realistic worldview vs, idealistic worldview", see Sec 4.1.3.



4.1.3 Appendix: The history of Aristotle (Monistic realism) vs.Plato (Dualistic idealism)

"Realistic Worldview or Idealistic Worldview? " is one of the greatest debates in the history of philosophy. From our point of view, however, the two are not in a duel, but coexist, as shown in the figure below (Figure 0.1 in Preface).

Figure 0.1 (in Preface) : The location of QL in the history of western philosophy

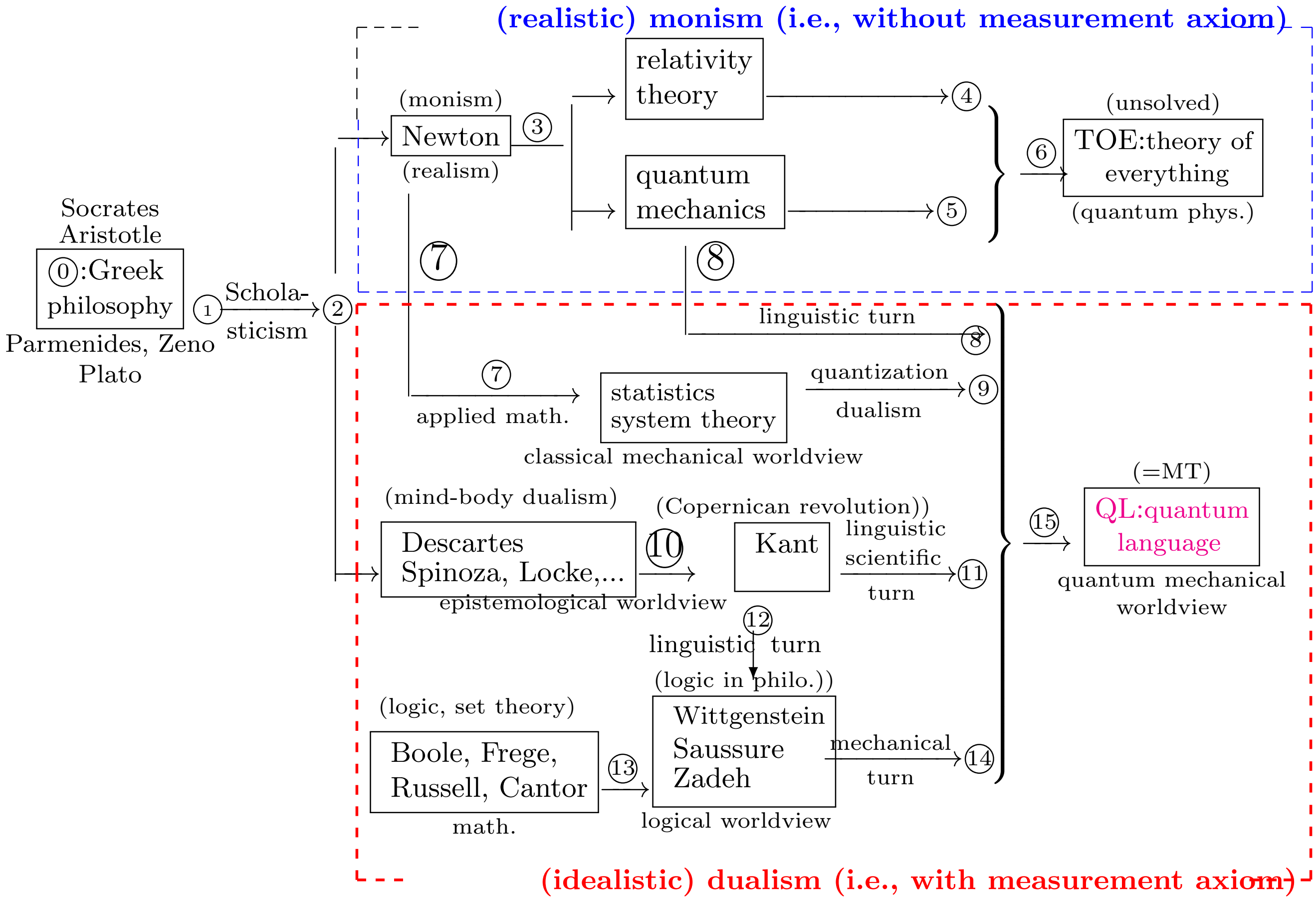


Figure 0.1 (in Preface): The history of the world-descriptions
Philosophy (≈ dualistic idealism) has progressed toward QL
(i.e., ⓪→ ①→ ②→ ⑩→ ⑫→ ⑭→ ⑮)

Assertion 4.2. [Debates: realistic worldview vs. idealistic worldview]

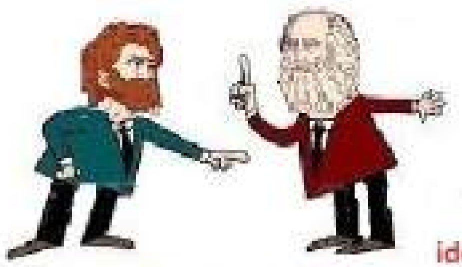


Table 4.1 (=Table 2.1) : realistic worldview (Aristotle) vs. idealistic worldview (Plato)

dispute ╲ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	Ockham	Anselmus
ⓓ: space-time	Newton (Clarke)	Leibniz
ⓔ: quantum theory	Einstein	Bohr
ⓕ:philosophy of science	Carnap	Quine
ⓖ: fuzzy sets	Kalman	Zadeh

♠**Note 4.3.** The following is a brief explanation of the above table, with some caveats. Although the table above is divided by "Realism vs. Idealism," the distinction is not always clear.
ⓐ: this ⓐ was too long ago to know what Parmenides really meant.
ⓔ: Bohr was a physicist and would have considered himself a realist. Thus, the Bohr-Einstein controversy is a controversy within physics. If so, it is a controversy within the TOE of Fig. 1 of the Foreword. Therefore, we will have to wait for the emergence of a genius of Einstein's caliber to resolve it.
ⓕ: As for Carnap and Quine, I only know their conclusions. I did not understand their position or the content of their arguments.
ⓖ: Since Kalman and Zadeh's specialty is dynamical system theory (= statistics), they both belong to idealism in the main textual sense. With the advent of quantum language, I believe that this controversy has been resolved.

ⓐ Heraclitus argued plausibly that "all things are in flux. Parmenides, on the other hand, argued that

- everything is unchanging. There is no movement, no change. Time does not exist. There is only "one," not "many.

and other statements similar to the Copenhagen Interpretation.

ⓑ This was practically the beginning of the "monistic realism (no need for humans) vs. dualistic idealism (need for humans)," and this is where science began.

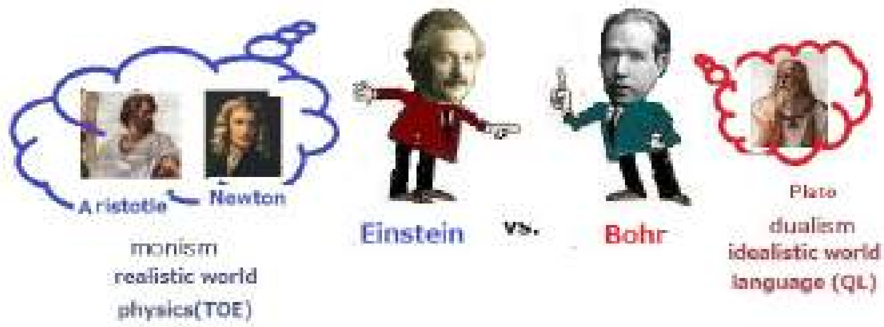
ⓒ The Problem of universals was a 250-year debate between the Aristotelians and the Platonists, with the Platonists gaining the upper hand at first, and then the Aristotelians gradually gaining the upper hand. Although it was not a quantitative scientific debate, the problem of universals shows the high potential of Scholastic philosophy (both Aristotelian and Platonist).

Since "universals" = "observable," from the standpoint of quantum language (Platonic/dualism), we can conclude that "universals" exist.

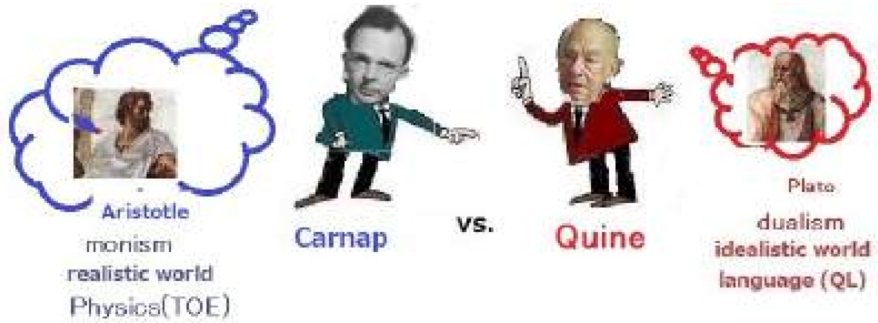
ⓓ The Leibniz-Clarke correspondence (*cf.*) is a dispute between two geniuses, "Newton (and Clarke) vs. Leibniz. Without knowing quantum language, many might judge Newton the winner, but quantum language gives it to Leibniz.

ⓔ The "Bohr-Einstein controversy" in quantum mechanics is the biggest scientific debate of the 20th century. In a nutshell, this is the debate such that "Is quantum mechanics Aristotelian or Platonian? "
In order to settle this issue, I proposed a quantum language. A quantum language, which is a generalized and more powerful modification of quantum mechanics, is clearly not physics. Quantum language, which is a generalized and more pow-

erful modification of quantum mechanics, is clearly not physics. (Bohr-Einstein debates(*cf.* ref. [74])). However, one may argue that this settlement is probably a superficial settlement. However, if there is a "real settlement," only a genius of Einstein's level would be able to achieve it.



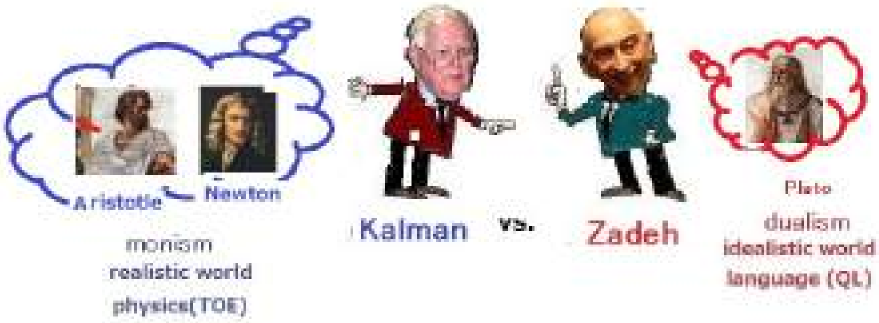
ⓕ The "Carnap-Quine controversy" in the philosophy of science is a controversy over the distinction between analytical and synthetic propositions. In quantum language, "all scientific propositions are synthetic propositions", so we give them to Quine. Because the spirit of Copenhagen interpretation is "if you don't measure, you don't know anything." (*cf.* Sec. 12.3).



ⓖ The "fuzzy set" proposed by Zadeh was overwhelmingly supported by some systems engineers. Zadeh's paper (*cf.* ref. [129], 1965) is the most cited paper of the 20th century. In response, Kalman (the most respected systems engineer of his time and proponent of the Kalman filter) objected on the grounds that "fuzzy sets" did not fit within the framework of dynamical systems theory (classical dynamical worldview).(*cf.* [130]). Now, half a century has passed since then, and the fever of fuzzy sets has cooled down completely, leaving only the impression that Kalman was right after all. However, if we consider that

$$\text{[fuzzy set]}=\text{[universal]}=\text{[observable]}$$

Zadeh's proposal fits within the quantum language (quantum mechanical worldview). Therefore, I think that the engineers who enthusiastically supported "fuzzy sets" at the time had a good point. From the standpoint of quantum language, dynamical systems theory (classical mechanical worldview) is a monistic idealism, and I feel that it is half-baked.


Aristotle
Newton
monism
realistic world
physics(TOE)
Kalman
vs.
Zadeh
Plato
dualism
idealistic world
language (QL)

4.2 Why does the motion happen?

4.2.1 From purpose to causality: Modern science started from the discovery of "causality"

When a certain thing happens, the cause always exists. This is called *causality*(=*causal relation*). You should just recall the next proverb:

Smoke is not located on the place which does not have fire.

However the situation is not so simple as you think. Consider, for example,

- This morning I feel good.

  ($\sharp_1$) Is it because that I slept sound yesterday?

  or

  ($\sharp_2$) Is it because I go to favorite golf from now on?

You will find the difficulty in using the word "causality". In daily conversation, the word "causality" is used in many contexts, mixing up "a cause (past)", "a reason (implication)", and "the purpose and a motive (future)".

As mentioned in Sec. 2.3, the pioneers in the study of movement and change are Hērakleitos and Parmenides:

(A)
- Hērakleitos(BC.540 -BC.480): "Everything flows."
- Parmenides (born around BC. 515): "There is no movement."
  (Zeno's teacher)

I think the reader will have the following question.

- Why are their names still there, even though it was 2,500 years ago?

As I mentioned before, "motion and change" is the most important keyword in science (= "worldview"), that is, I consider:

(B) **[The beginning of World description]**

= [The discovery of movement and change] = { Hērakleitos, Parmenides }

This is why their names are still there.

Thus, from the standpoint of this text (i.e., the scientific standingpoint), we say

(B_2) **Heraclitus and Parmenides are the correct answers to the main theme of ancient Greek philosophy, "What is the origin of all things?"**

(*cf.* Sec.3.1).

However, Aristotle (BC384–BC322) pursued an even more fundamental problem:

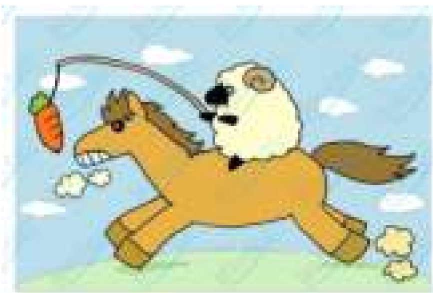
(C) What is the essence of movement and change?

and concluded as follows.

(D):Purpose (Aristotle)

Aristotle asserted that **all the movements had the "purpose".**

- For example, a stone falls because it has the purpose to go downward, and smoke rises because it has the purpose to go upward.
  A heavy stone falls fast because it has a strong purpose of "falling fast".



4.2.1.1 The discovery of science: From purpose to causality

Under the influence of Aristotle, "*Purpose*" had remained as a mainstream idea of "Movement" for a long period of 1500 years or more.

We were freed from the spell of "Purpose", only after Galileo, Bacon, Descartes, and Newton et al. discovered the essence of movement and change lies in "Causality".

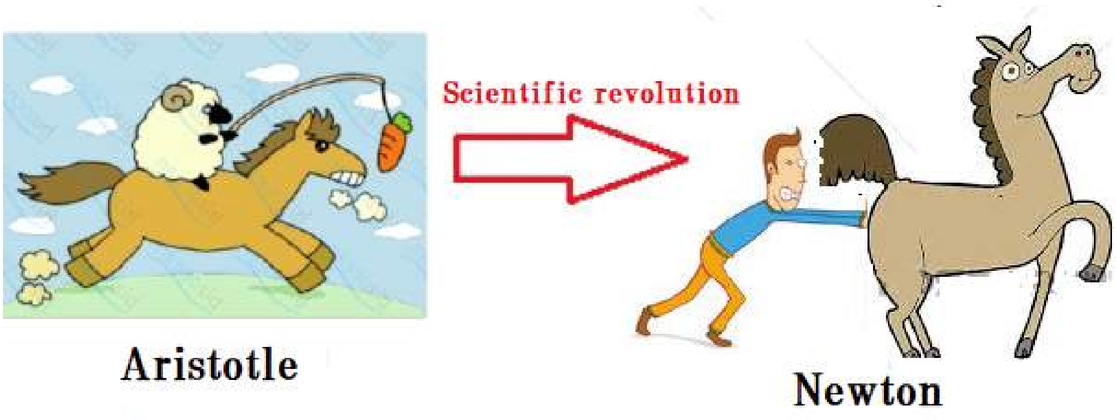
Scientific revolution from "Purpose" to "Causality"

is the greatest paradigm shift in the history of science. It is not an exaggeration even if we call the shift "*birth of modern science*".

(E) the birth of worldview: Movement (Hērakleitos, Parmenides, Zeno, Aristotle)

"purpose" (and no experiment) → Aristotle's spirit :(About 1500 years)

the birth of modern science: Causality (and experiment) (Bacon, Galileo, Descartes, Newton)



I cannot emphasize too much the importance of the discovery of the term: "causality". That is,

(♯) Science is the discipline about phenomena that can be represented by the term "causality". (i.e., "No smoke without fire")

Thus, I consider that the discovery of "causality" is equal to that of science. In the realistic worldview, Newtonian kinetic equation (i.e., the equation of the chain of causality) was final in a sense. However, in the idealistic worldview, the problem "What is causality?" is not solved yet. For the complete answer to the problem, we had wait for the appearance of quantum language (Axiom 2 (causal relation) in Sec.1.2.1, also, see ref. [74]).

Summary 4.3. [Solutions to the causality problem] For example, we see:

(F_1) The causality is represented by Newtonian kinetic equation in Newtonian mechanics
(F_2) The causality is represented by Maxwell's equations in electromagnetism
(F_3) The causality is represented by Schrödinger equation (or equivalently, Heisenberg's kinetic equation) in quantum mechanics
(F_4) The causality is represented by Axiom 2 (in Section 1.2) in quantum language

(Continued to Sec. 10.2: What is causality?).

♠**Note 4.4.** S. Weinberg (1933 -2021), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said in his book [123] "To explain the world; The discovery of modern science" as follows:

- [in Chapter 3]:*We can agree with the classical scholar R. J. Hankinson that "we must not lose sight of the fact that Aristotle was a man of his time and for that time he was extraordinarily perspicacious, acute, and advanced." Nevertheless, there were principles running all through Aristotle's thought that had to be unlearned in the discovery of modern science. For one thing, Aristotle's work was suffused with teleology: things are what they are because of the purpose they serve.*

Recall the above (E). I think that only people after the scientific revolution (17th century) understand the meaning of "science".
Aristotle is also the father of biology. I think the reason Aristotle failed to discover causality is that biology was one of his major research topics. In biology, we tend to think in terms of purpose theory, and causality is very difficult to understand. It is

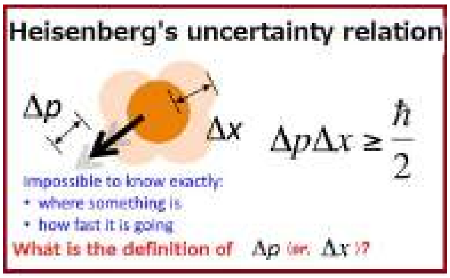
in astronomy and physics that the causal relationship is very easy to see. In addition, causality is relatively easy to experimentally verify. Thus, the scientific revolution was born out of [(i) geodynamics, (ii) causality (Newtonian mechanics), and (iii) the importance of experimentation (i.e., British empiricism)]. That is,

$$\left.\begin{array}{l}\text{(i) Geocentrism}\\ \text{(ii) causality (Newtonian mechanics)}\\ \text{(iii) the importance of experimentation}\end{array}\right\} \Longrightarrow \text{ scientific revolution}$$

Even so, British empiricism (Descartes-Kant epistemology) has become "brain science without experimentation," and the story is not simple.

4.3 Practical logic; Why does logic hold in this world?

♠**Note 4.5.** In 1991, I discovered the mathematical expression of "Heisenberg's uncertainty principle (γ line thought experiment)" (*cf.* ref. [38], or, Sec. 4.3 in ref. [74]). However, I have not been able to invent a practical application for this theory (no one has done it yet).

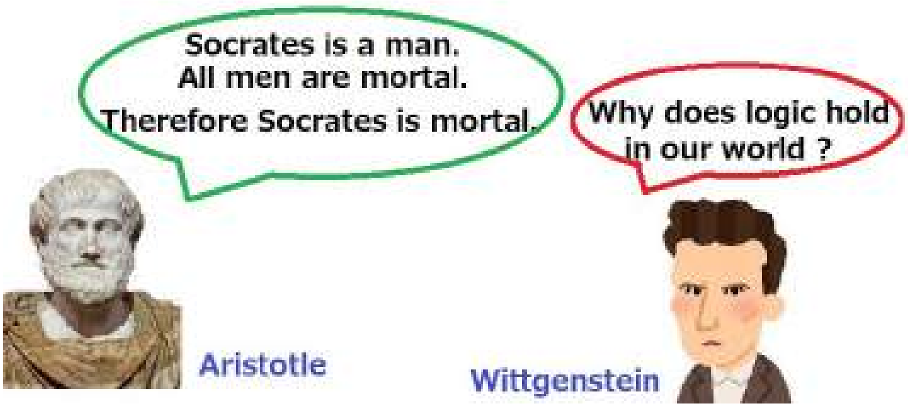


Believing that the reason for the lack of progress in my research was my "immature understanding of the Copenhagen Interpretation", I started a serious research on the Copenhagen Interpretation and chose the following problem (A_0) as my next goal. In answer to (A_0), I obtained the result (*cf.*[41, 42, 43, 44]), which was not well evaluated. I knew I would need to do some pretty extensive theory to get an evaluation. In order to gain recognition, I thought it was necessary to create a rather extensive theory. It is "quantum language" and this is summarized in refs. [74] and [82]. The revised version of [82] is this text. In this sense, problem (A_0) is the starting point of this study.

The following is one of the biggest problems in philosophy.

(A_0) **Why does logic hold in this world?**

We will answer this question in Chapter 12. To answer this we must be prepared for much. This section is one of these preparations.



We consider that two kinds of logic exist such as

(A_1) mathematical logic (= symbolic logic)
(A_2) non-mathematical logic (=practical logic)

Here, mathematical logic (due to Boole, Frege) is elementary and well known. A proposition in mathematical logic is usually considered to be a mathematical proposition. Also, A proposition in practical logic is a non-mathematical proposition, written by ordinary language. For example, the (B_2) below is non-mathematical logic.

4.3.1 Practical logic; Aristotle's syllogism in ordinary language

If the ecology of various animals is observed, it will be clear that the base of language was due to intimidation, solidarity, reproduction. Language was one of the strongest arms for the survival and breeding. Such a time have continued for millions of years. Of course, the biggest events in the "history of language" happened one after the other. For example,

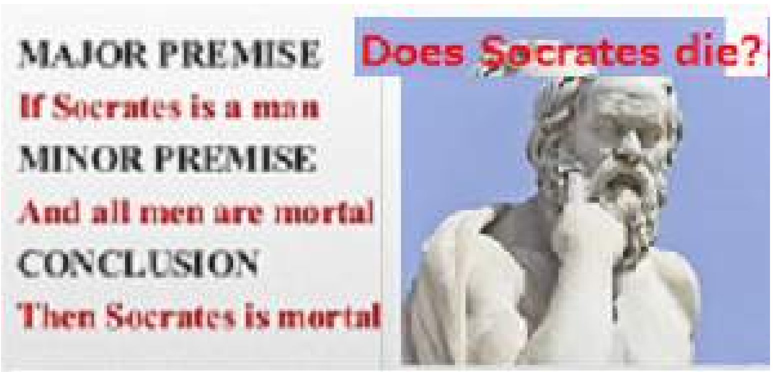
(B_1) "rhythm and song", "logical structure", "quantity concept", "grammar", "tense", etc.

However, it was done gradually by many people, tens of thousands of years ago, and it is not possible to identify the names of the contributors. But the **surprise** that ordinary language had a logical structure is carried on by the term "Aristotelian syllogism." Namely,

Aristotle's syllogism in ordinary language

The following is famous Aristotole's syllogism

**(B_2) Since Socrates is human being,
and human being is mortal,
it follows that Socrates is mortal.**



Although this is quite famous, the reader may have several questions concerning this as follows,

(C_1) Syllogism is essential for mathematical proofs. Therefore, it is natural to assume that Pythagoras already knew syllogism.

(C_2) Also, it is natural to consider that syllogism was frequently used in the debate between Socrates and sophists.

Thus I guess that

(D) The knowledge of syllogism of the time was summarized in Aristotle's book: "Organon", which was compiled by his followers about B.C. 40. And, syllogism was endorsed by Aristotle and remained authoritative for almost 2,000 years.

In fact, Immanuel Kant said that there was nothing else to invent after the work of Aristotle.

♠**Note 4.6.** (i): Here,

($\sharp_1$) Who is the discoverer of the syllogism?

Thalēs and Pythagoras would naturally have known about the syllogism, so it is certainly not Aristotle. It is not possible to identify the names of the discoverer. Similarly,

($\sharp_2$) Who is the discoverer of the motion function method?
($\sharp_3$) Who is the discoverer of probability theory (or, statistics)?

As mentioned in Note 2.16, it is not possible to identify the names of the discoverers. Compared to the clear discoverers of monistic realism (classical mechanics and relativity), the work of dualistic idealism often takes the form of gradual discovery by several. If so,

($\flat$) **it is tempting to formulate ($\sharp_1$)$\sim$($\sharp_3$) in quantum language all at once.**

This was the motivation for my research.

4.3.2 Mathematical logic; due to Boole, Frege

The following "Elementary Logic" is sufficient to read this text.

Postulate 4.4.
[**Symbolic logic; due to G. Boole, G. Frege**]
For any proposition P, the truth function $\phi(P)$ is determined such that

$$\phi(P) = \begin{cases} 1 & (\text{if } P \text{ is true}) \\ 0 & (\text{if } P \text{ is wrong (i.e., not true)}) \end{cases}$$

(E$_1$) Assume that P_1, P_2 are propositions. Then, $P_1 \wedge P_2$, $P_1 \vee P_2$, $\neg P_1$, $P_1 \to P_2$ are propositions.
And it holds that $\phi(P_1 \wedge P_2) = \min\{\phi(P_1), \phi(P_2)\}$, $\phi(P_1 \vee P_2) = \max\{\phi(P_1), \phi(P_2)\}$, $\phi(\neg P) = 1 - \phi(P)$.

where $\wedge$, $\vee$, $\neg$, $\to$ respectively is called "and", "or", "not", "implies". Note that $P_1 \to P_2$ is defined by $\neg P_1 \vee P_2$".

Also, assume that P_θ is a proposition ($\theta \in \Theta \equiv \{1, 2, ..., n\}$), then it holds
(i): $P_1 \wedge P_2 \wedge ... \wedge P_n$ (denoted by $\bigwedge_{\theta \in \Theta} P_\theta$, or $\forall \theta (\in \Theta)[P_\theta]$) is a proposition
(ii): $P_1 \vee P_2 \vee ... \vee P_n$ (denoted by $\bigvee_{\theta \in \Theta} P_\theta$, or $\exists \theta (\in \Theta)[P_\theta]$) is a proposition.
Here, $\phi(P_1 \wedge P_2 \wedge ... \wedge P_n) = \min_{i=1,...,n} \phi(P_i)$, $\phi(P_1 \vee P_2 \vee ... \vee P_n) = \max_{i=1,...,n} \phi(P_i)$.

(E$_2$) The above finite set $\Theta (\equiv \{1, 2, ..., n\})$ can be extended to an infinite set Θ.

[**Predicate logic** (This text does not go deep into predicate logic.)]
(i): $\bigwedge_{\theta \in \Theta} P_\theta$ is denoted by $\forall \theta [[\theta \in \Theta] \to [P_\theta]]$.
(ii): $\exists \theta [Q_\theta]$ is defined by $\neg [\forall \theta [\neg [Q_\theta]]]$.
Therefore,

$$\bigvee_{\theta \in \Theta} P_\theta = \neg [\bigwedge_{\theta \in \Theta} \neg P_\theta] = \neg [\forall \theta [[\theta \in \Theta] \to [\neg P_\theta]] = \exists \theta [[\theta \in \Theta] \wedge [P_\theta]]$$

♠**Note 4.7.** The most important problem in Postulate 4.4 is the "proposition P" in line 3. In the case of mathematical logic, there is no problem because "proposition = mathematical proposition". However, if it is non-mathematical logic (practical logic), there is no definition of "proposition P," which causes a problem. For example,

($\sharp$) Is "I think, therefore I am" a proposition?

I naturally assumed that "What is a proposition?" was written in Wittgenstein's "Tractatus Logico-Philosophicus (widely abbreviated and cited as TLP), and I started reading it, but I remember being disappointed because I did not find the definition of a proposition anywhere in the TLP. But as he writes in Sec. 12.2 (H$_1$), he honestly asks "What is a proposition?" in Philosophical Inquiry (§38 in [126]), he wrote that he was unable to propose the definition of "Proposition". . Thus, I understood TLP as a book that poses the question, "What is a proposition?" In any discipline, not only philosophy, it is often said that "it is more important to pose a problem than to solve it." TLP is a book that raises the question, "Logic is not only about mathematical logic!" and "Someone define a (non-mathematical) proposition!"

4.3.3 Aristotle's syllogism is not a syllogism

Exercise 4.5. The truth table below is well known. Wittgenstein was one of those who contributed to the popularization of the truth table.
(i): The proof of syllogism: $[(p \to q) \wedge (q \to r)] \to (p \to r)$

Truth Table

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$(p \to q) \wedge (q \to r) \to (p \to r)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	0	1	0	1	1	0	1
1	0	0	0	1	0	0	1
0	1	1	1	1	1	1	1
0	1	0	1	0	1	0	1
0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1

Therefore, syllogism: $[(p \to q) \wedge (q \to r)] \to (p \to r)$ always holds.
(ii):Modus ponens: $((p \to q) \wedge p) \to q$

Proof of (ii)

p	q	$p \to q$	$(p \to q) \wedge p$	$(p \to q) \wedge q$	$(p \wedge (p \to q)) \to q$ modus ponens	$((p \to q) \wedge q) \to p$
1	1	1	1	1	1	1
1	0	0	0	0	1	1
0	1	1	0	1	1	0
0	0	1	0	0	1	1

For example, put $p :=$ "it rains", $q :=$ "the ground is wet". The, modus ponens says the followings.

["it rains"$\wedge$["it rains" $\to$ "the ground is wet"]
]$\to$ "the ground is wet".

(iii):[Since Socrates is a human being]$\wedge$[every human being is mortal]$\to$[Socrates is mortal].
Using set theory, we prove it as follows.
Assume that H is the set of all humans and M is the set of all mortals.
. Then,

[Socrates is a human being]$\Leftrightarrow$ [Socrates $\in H$]
[every human being is mortal]$\Leftrightarrow \forall x[x \in H \to x \in M]$

Thus,

$$
\begin{aligned}
&[\text{Socrates} \in H] \wedge \forall x[x \in H \to x \in M] \\
\to &[[\text{Socrates} \in H] \wedge [\text{Socrates} \in H \to \text{Socrates} \in M]]
\end{aligned}
$$

(by Modus ponens)

$$
\to [\text{Socrates} \in M]
$$

That is,

$$
\text{(F)} \quad \Big[\underset{\text{(Socrates is a human being)}}{[\text{Socrates} \in H]} \wedge \underset{\text{(every human being is mortal)}}{\forall x[x \in H \to x \in M]} \Big] \to \underset{\text{(Socrates is mortal)}}{[\text{Socrates} \in M]}
$$

Thus,

- The above (F) says: Aristotle's syllogism:

  [Socrates is a human being]∧[every human being is mortal]→[Socrates is mortal]

  is not a syllogism but a variant of modus ponens.

□

♠**Note 4.8.** It seems that the term "three-stage argument" is sometimes used in multiple senses, but here we will assume that it means (i) above. Aristotle's syllogism (iii)" is often cited as a typical example of syllogism, but this usage should be used with caution because it may cause confusion. In the next section, I will use "Aristotle's syllogism (iii)" as an example of "the danger of logic in everyday language."

4.3.4 Set-theoretical worldview (= logical worldview)

4.3.4.1 Set-theoretical worldview

Aristotle's syllogism (Socrates' death) in Sec. 4.3.3 such that

(G_1) **Since Socrates is a human being, and every human being is mortal, then Socrates is mortal**

is very instructive. Translating this into the language of set theory, we get the following

$$(G_2)\ (=(F))\ \Big[\underset{\text{(Socrates is a human being)}}{[\text{Socrates} \in H]} \wedge \underset{\text{(every human being is mortal)}}{\forall x[x \in H \to x \in M]}\Big] \to \underset{\text{(Socrates is mortal)}}{[\text{Socrates} \in M]}$$

Thus, what appeared to be a syllogism in (G_1) was actually a variant of modus ponens in (G_1). Thus, the "logic" of everyday language is insufficient,

(G_3) The meaning of "logic" becomes clear only after (G_1), in which the everyday language (G_1) is described under the set-theoretic worldview.

Thus, the set-theoretic worldview (= the style of describing phenomena in the language of set theory (= the language using "symbol $\in$" and symbolic logic)) is a very, very good worldview.

Now, let us consider the following.

Exercise 4.6. Is Descartes' cogito proposition "I think therefore I am" a proposition?
[Proof?] Define T as the set of all thinking organisms. Let E be the set of all that exist. Then, it is clear that $T \subseteq E$. Put I="I".

$$\text{Then, we see that } I \in T \text{ and } T \subseteq E \text{, therefore, } I \in E$$

However, I do not think that the above argument is related to the cogito proposition "I think therefore I am". Thus, the set-theoretic worldview may impotent on the cogito proposition. □

4.3.4.2 From a Questioning of the Set-theoretic Worldview to a Quantum Mechanical Worldview

Even so, there is room to consider whether a set-theoretic worldview is the best way. For example, the set-theoretic worldview cannot answer the following questions

(H_1) In the first place, is there such a scientific concept as "the set of all humans"? (This is a problem called "Hempel's raven Problem"(*cf.* Sec. 12.7). It was Hempel who first questioned the set-theoretic worldview.
(H_2) Is the first proposition of philosophy "I think, therefore I am" a proposition?
(H_3) Is there a relationship between the set-theoretic worldview and Descartes-Kant's epistemology?
(H_4) Does the syllogism hold in quantum systems?
(H_5) Isn't symbolic logic useful only for mathematics (=set theory)?

Thus, various problems emerge. The most important of these problems is the following,

(I) **Why does logic work in this world?**

Wittgenstein tried to solve this problem in his philosophical book TPL (i.e., ref. [125]: "Tractatus Logico-Philosophicus", but he was theoretically unsuccessful. However,

(J) In philosophy, it is sometimes more important to pose a problem (I) than to solve it

In this sense, TPL is a good book.

Our interest in this text is the following quantum mechanical worldview.

We then claim the following.

(K_1) quantum language is the language of science

And in [74], we already showed that

(K_2) Statistics is one aspect of quantum language.

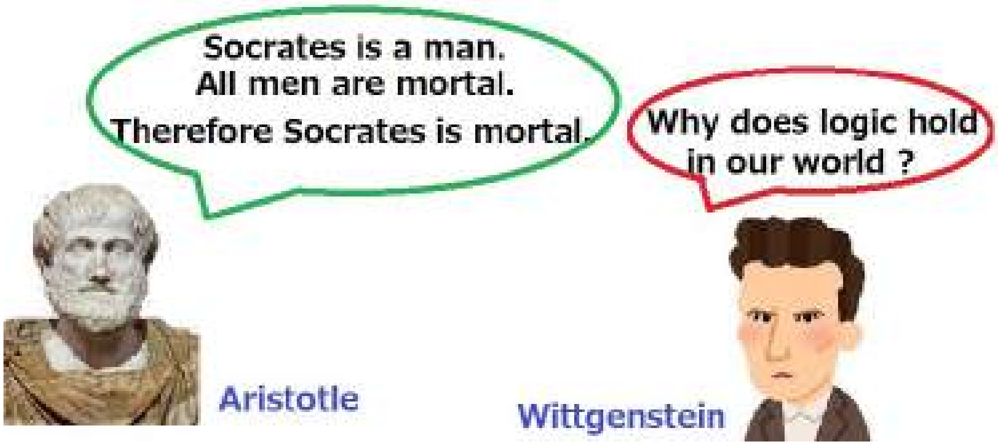
Therefore, to answer the question "Why does logic work in this world? we can show the following.

(K_3) By defining "propositions" in the quantum language, we show that the logic is valid.

This will be discussed in Chap.12.

After all, Figure 0.1 in Preface asserts the following.

(K_4) **Statistics, quantum mechanics, scholastic philosophy, Cartesian-Kantian philosophy, and analytic philosophy are each an aspect of quantum language (or an immature state of quantum mechanics).**



4.3.5 Appendix: Syllogism does not always hold in quantum systems?

We have the following theorems:

Theorem 4.7. (L_1) Syllogism does not necessarily hold in quantum systems
(L_2) Syllogism always holds in both classical and quantum systems

////

Since (L_1) and (L_2) have different premises, (L_1) and (L_2) are not contradictory (*cf.* Remark 12.9 later). The proof of (L_2) is given in Corollary 12.8.

Proof 4.8. The proof of Theorem 4.7 (G_1) due to the following:

- [79] Ishikawa, S. (2020) Wittgenstein's picture theory in the quantum mechanical worldview Journal of quantum information science, Vol. 10, No.4, 104-125, DOI:10.4236/jqis.2020.104007
(https://www.scirp.org/journal/paperabs.aspx?paperid=106233)

(M) : [Syllogism does not hold in quantum systems]:
i.e. the following does not always hold in quantum language:
- if $P_1 \longrightarrow P_2$ and $P_2 \longrightarrow P_3$, then it holds $P_1 \longrightarrow P_3$.

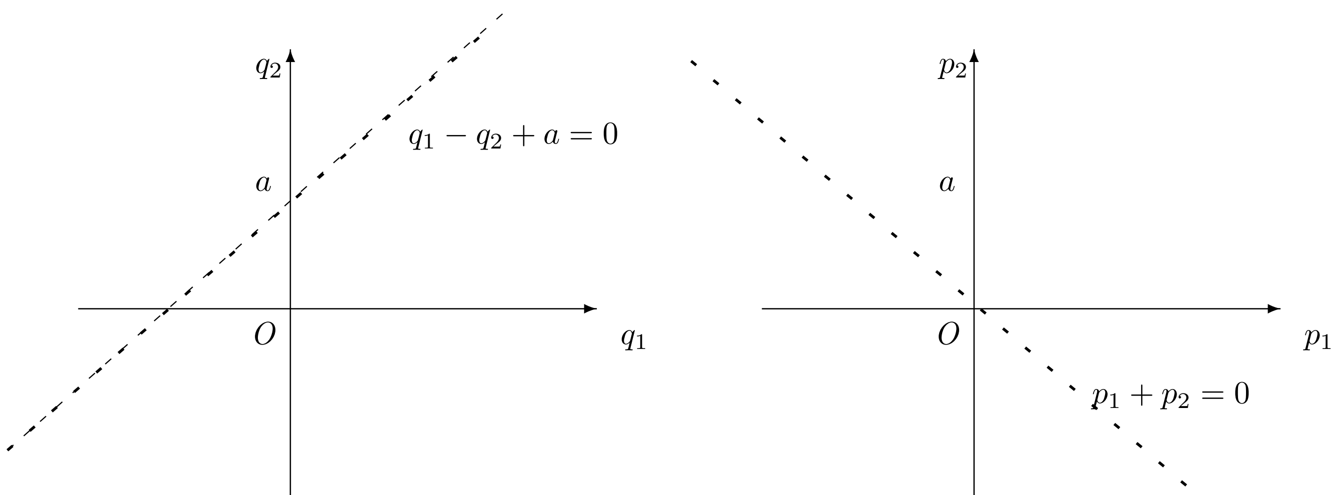
Let us prove it as follows. A quantum two particles system S is formulated in a tensor Hilbert space $H = H_1 \otimes H_1 = L^2(\mathbb{R}_{q_1}) \otimes L^2(\mathbb{R}_{q_2}) = L^2(\mathbb{R}^2_{(q_1,q_2)})$. The state u_0 ($\in H = H_1 \otimes H_1 = L^2(\mathbb{R}^2_{(q_1,q_2)})$) $\Big($ or precisely, $\rho_0 = |u_0\rangle\langle u_0|\Big)$of the system S is assumed to be

$$u_0(q_1, q_2) = \sqrt{\frac{1}{2\pi\epsilon\sigma}} e^{-\frac{1}{8\epsilon^2}(q_1-q_2+a)^2 - \frac{1}{8\sigma^2}(q_1+q_2)^2} \tag{4.1}$$

where $a \neq 0$, a positive number ϵ is sufficiently small, and a positive number σ is sufficiently large. Thus, we see that

$$|\widehat{u_0}(p_1, q_2)| = |\sqrt{\frac{1}{2\pi\epsilon\sigma}} e^{-\frac{1}{8\sigma^2}(p_1-p_2)^2 - \frac{1}{8\epsilon^2}(p_1+p_2)^2}| \tag{4.2}$$

where $\widehat{u_0}$ is the Fourier transform of u_0



For each $k = 1, 2,$ define the self-adjoint operators $Q_k : L^2(\mathbb{R}^2_{(q_1,q_2)}) \to L^2(\mathbb{R}^2_{(q_1,q_2)})$ and $P_k : L^2(\mathbb{R}^2_{(q_1,q_2)}) \to L^2(\mathbb{R}^2_{(q_1,q_2)})$ by

$$\begin{aligned} Q_1 &= q_1, \qquad P_1 = \frac{\hbar\partial}{i\partial q_1} \\ Q_2 &= q_2, \qquad P_2 = \frac{\hbar\partial}{i\partial q_2} \end{aligned} \tag{4.3}$$

($\sharp_1$) Let $\mathsf{O}_1 = (\mathbb{R}^3, \mathcal{B}_{\mathbb{R}^3}, F_1)$ be the observable representation of the self-adjoint operator $(Q_1 \otimes P_2) \times (I \otimes P_2)$. And consider the measurement $\mathsf{M}_{B(H)}(\mathsf{O}_1 = (\mathbb{R}^3, \mathcal{B}_{\mathbb{R}^3}, F_1), S_{[|u_0\rangle\langle u_0|]})$. Assume that the measured value $(q_1^0, p_2^0, p_2^0) (\in \mathbb{R}^3)$. That is,

$$\underset{\text{(the position of } A_1\text{, the momentum of } A_2)}{(q_1^0, p_2^0)} \Longrightarrow \underset{\text{the momentum of } A_2}{p_2^0}$$

($\sharp_2$) Let $\mathsf{O}_2 = (\mathbb{R}^2, \mathcal{B}_{\mathbb{R}^2}, F_2)$ be the observable representation of $(I \otimes P_2) \times (P_1 \otimes I)$. And consider the measurement $\mathsf{M}_{B(H)}(\mathsf{O}_2 = (\mathbb{R}^2, \mathcal{B}_{\mathbb{R}^2}, F_2), S_{[|u_0\rangle\langle u_0|]})$. Assume that the measured value $(p_2^0, -p_2^0) (\in \mathbb{R}^3)$. That is,

$$\underset{\text{the momentum of } A_2}{p_2^0} \Longrightarrow \underset{\text{the momentum of } A_1}{-p_2^0}$$

($\sharp_3$) Therefore, if syllogism holds, we may conclude that

$$\underset{\text{(the position of } A_1\text{, the momentum of } A_2)}{(q_1^0, p_2^0)} \Longrightarrow \underset{\text{the momentum of } A_1}{-p_2^0}$$

$$\Big(\text{ that is, the momentum of } A_1 \text{ is equal to } -p_2^0\Big)$$

But, the above argument (particularly, "syllogism") is not true. That is because

($\sharp_4$) $(Q_1 \otimes P_2) \times (I \otimes P_2)$ and $(I \otimes P_2) \times (P_1 \otimes I)$ (therefore, O_1 and O_2) do not commute, and thus, the simultaneous observable does not exist.
Thus, we can not test the ($\sharp_3$) experimentally.

Remark 4.9. Some arguments differ from the above. In Corollary 12.8 (= Remark 12.9) later, we present the different result, i.e.

($\sharp$) syllogism always holds in classical and quantum systems.

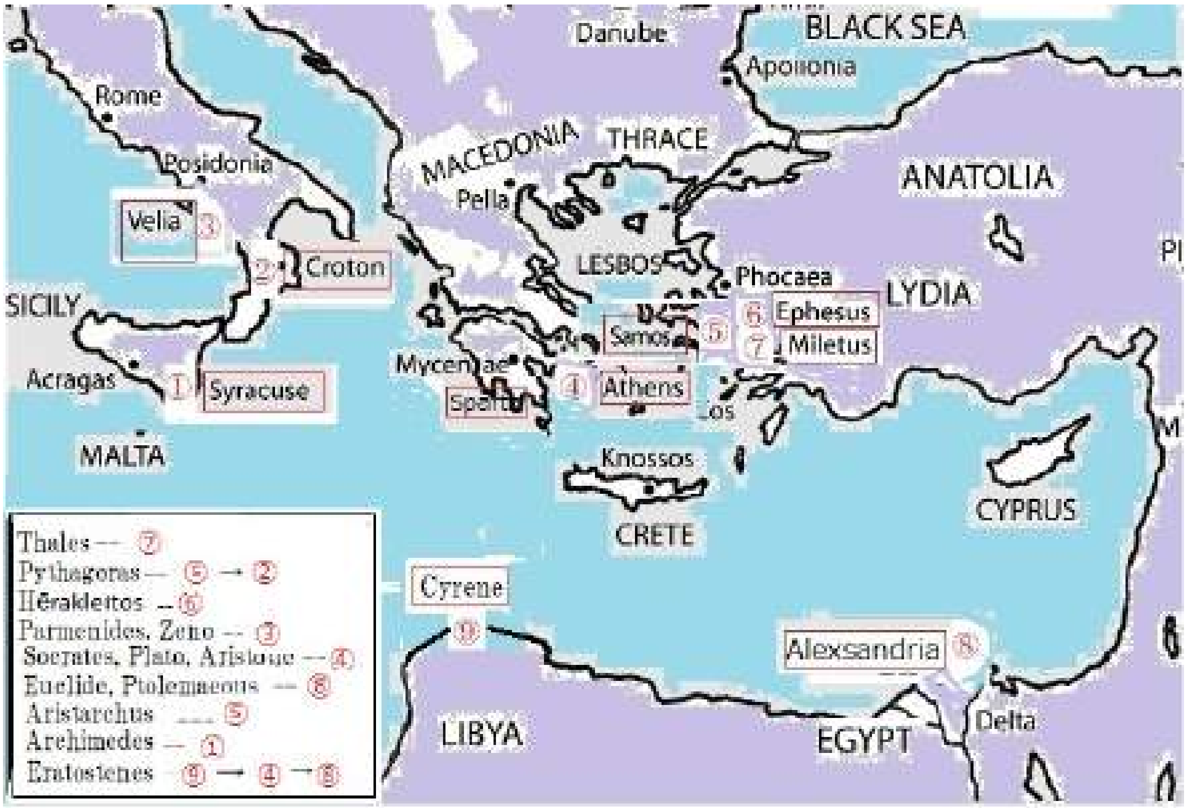
which is more formal. the above ($\sharp$) does not contradict the result of this section, since the definitions of "proposition" are different.

Chapter 5

Around Alexandria; Hellenistic period

Wisdom of pyramid building for thousands of years was accumulated by Egypt (Alexandria). Bright people studied in Egypt from each place of the Mediterranean Sea coast to learn it. For example,

Euclid$\cdots$geometry	Aristarchus$\cdots$Heliocentrism
Archimedes$\cdots$buoyancy, lever	Eratosthenes$\cdots$the measurement of the earth
Ptolemaios$\cdots$Geocentrism	



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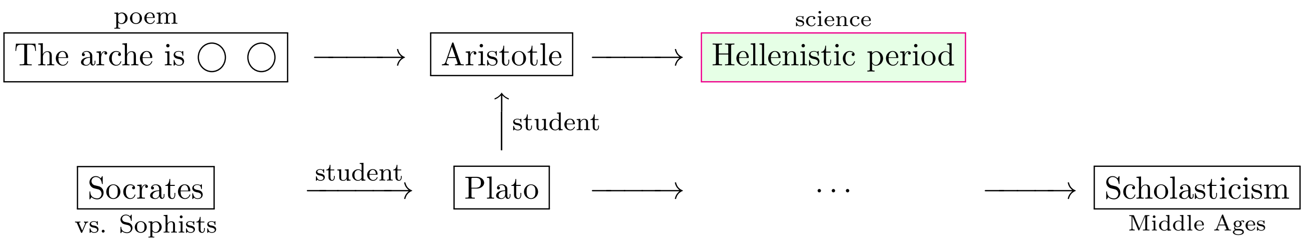
5.1 Around Alexandria; Hellenistic period

Influenced by the tradition of pyramid construction engineering, the studies of the Alexandrian school are solid and scientific. Put differently, it could be said that there was no philosophical appeal that transcended mathematical logic. The Alexandrians must have known of Plato's work. However, they had little influence from Plato. I guess that

- they didn't think that Plato's philosophy would survive more than 2,000 years later

Hellenistic period is located as follows:

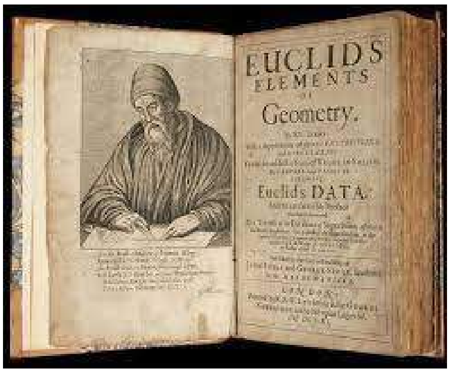


Many Greek philosophers were not willing to put their theories to work. But, in Hellenistic period, Practical research was respected. In Section 5.3, I will explain Heliocentrism of Aristarchus a little in detail as the preparation of Chap. 7, in which we say that

- "Geocentrism vs. Heliocentrism" is a metaphysical problem that cannot be put on black-and-white in the experiment.

5.2 Euclid(BC.330 - BC.275)

5.2.1 Euclid geometry - Parallel postulate

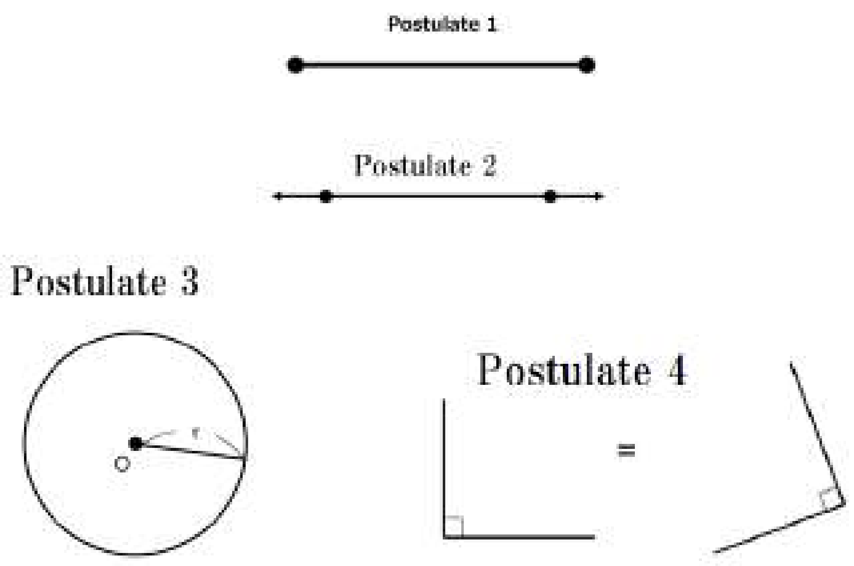


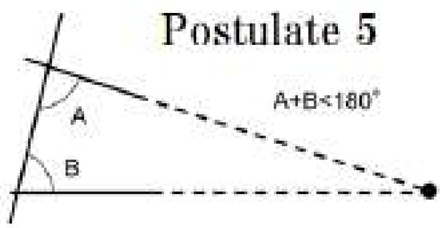
Three great pyramids in the Egyptian Giza desert (deceased person is Khufu, Khafre, Menkaure) erecting time of is the 2500 BC. Since then more than 2,000 years later, Euclid (BC.330 - BC.275) was born. Euclid is referred to as the "father of geometry" who was active in Alexandria (the mouth of the Nile). His book "Elements" is one of the most influential works in human cultural history. It has been estimated to be second only to the Bible in the number of editions published since the first printing in 1482 AD. In "elements" (Volume 1), 23 definitions, 5 postulates, and 9 axioms are listed at the beginning, and 48 theorems (such as Pythagoras' theorem) are proved on the basis of them. In short, what Euclid did was

(A_1) After determining the obvious (?) axioms (or axioms), complex theorems (e.g. Pythagorean theorem) were proved or deduced from them.

This can be seen as a mathematical version of [worldviewism (*cf.* Sec.1.1)].
Here, five postukates are illustrated as follows.

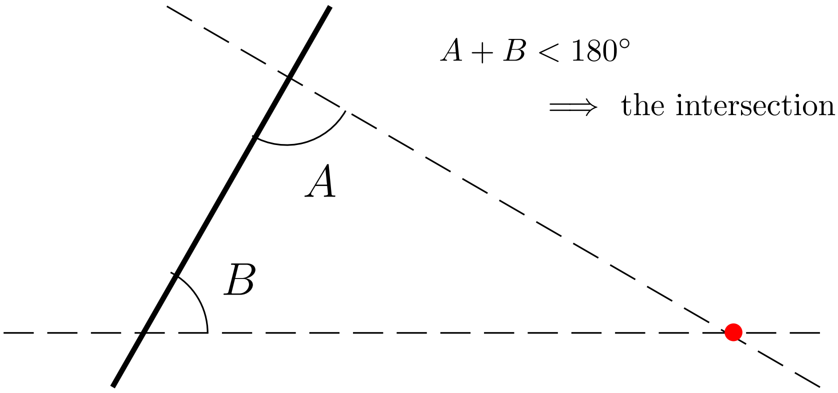
Euclide geomety: Postulates 1∼ 5





This is explained below. We focus on the postulate 5 (i.e., the parallel postulate).

- If a line segment intersects two lines on the same side whose sum of two interior angles is less than two right angles, then extending the two lines infinitely, the sum of the angles ($= \alpha + \beta$) intersects the line on the side whose sum of the angles is less than two right angles.



Therefore,

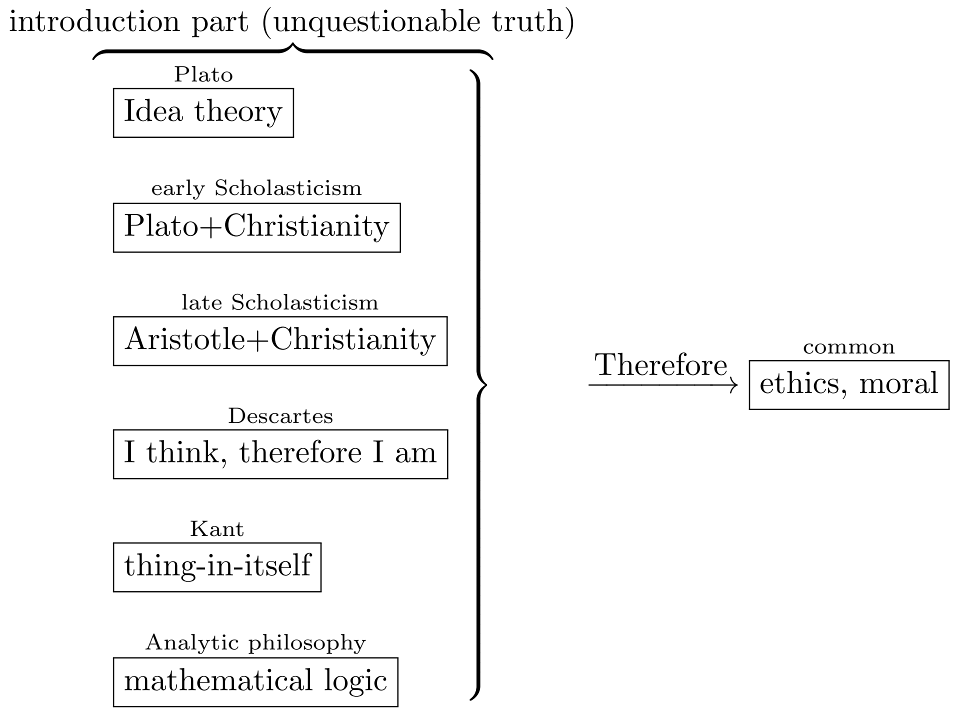
(A$_2$) Euclid was a mathematician who proposed the axiomatization of geometry, considered the parallel postulate, and intuited that the notion of "obviousness" is non-trivial.

Despite Euclid's meticulous care, the following false beliefs were formed by "Elements".

(B) **It is best to start from the obvious.**

Many philosophers (Descartes, Spinoza, etc.) have fallen prey to this false belief. It is already well known that Descartes' cogito proposition "I think, therefore I am" is far from self-evident. In other words, nothing is self-evident (i.e., there is no unquestionable truth).

However, many philosophers have emulated the "Elements" as follows.



It should be noted here that the worldviews on the left side above are all propositions and theories that are impossible to deny. In short, they are all starting from "pseudo-scientific propositions" in the sense of Popper's disprovability.

♠**Note 5.1.** (i): In "Elements", geometry is not only written but also algebra. For example, it is shown that prime numbers are infinite. The proof is as follows.

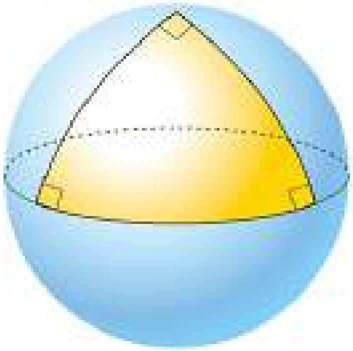
($\sharp$) Assume that the set of prime numbers is finite, that is, $\{2, 3, 5, 7, ..., n\}$. Put

$$N = (2 \times 3 \times 5 \times 7 \times ... \times n) + 1$$

Then, N is a prime number or it can be divided by the larger prime number than n. In each case, it contradicts the assumption that n is the largest prime number. □

(ii): Euclid's axiomatic system is now known not to provide a complete logical foundation for the modern world, and now seems to have only historical value. Birkhoff's axiomatic system is a well-known modern version, but there are many others, of which the author is not familiar.

5.2.2 non-Euclidean revolution



Discovery of non-Euclid geometry (due to Gauss(1777 - 1855), etc.) defeated the wrong belief (B) and asserted

(C) **Start from "productive" than "self-evident"!**

that is, "all is well that ends well". In this text, the (C) is called the non-Euclidean revolution, that is,

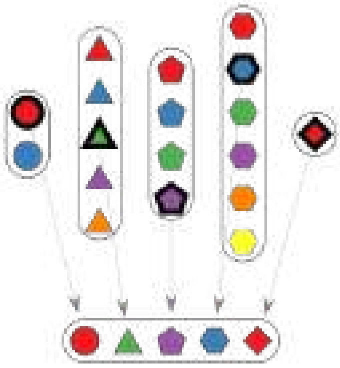
(D) non-Euclidean revolution $\Big[$(B): self-evident $\xrightarrow[\text{non-Euclidean revolution}]{}$ (C): productive$\Big]$

It can't be said that the non-Euclidean revolution is still generally also recognized sufficiently in today. There is no successful theory which starts from "self-evident things". For example, Newtonian mechanics, the theory of relativity, quantum mechanics, etc. do not start from "self-evident things". Paradoxically saying, we see that

(E) The question: "What is 'self-evidence'?" is not self-evident.

Axiom of choice of mathematics is not self-evident, where axiom of choice is as follows.

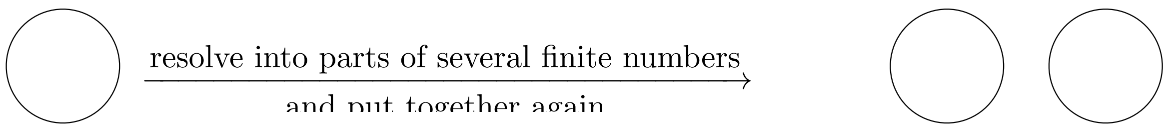
- Given any set X of pairwise disjoint non-empty sets, there exists at least one set C that contains exactly one element in common with each of the sets in X.
  (For example, consider a set $X = \{\{a, b\}, \{c, d, e\}, \{g\}, \{h, i, j, k\}\}$. Then, we can construct a set $C = \{a, c, g, j\}$)

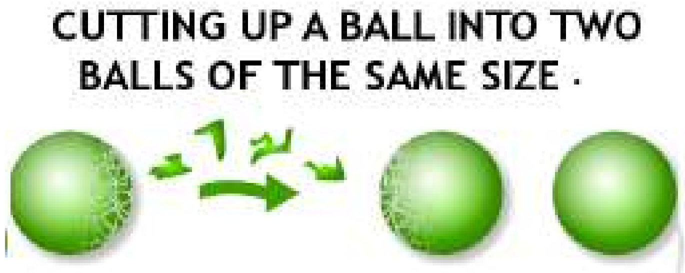


This is not self-evident (i.e., trivial). For instance, Banach-Tarski theorem says that

(F) If we adopt axiom of choice, we must admit the following

- A ball B is resolved into parts of several finite numbers, and we assume that it's put together again. Then, we can get the same two balls which are also the same as the ball B.





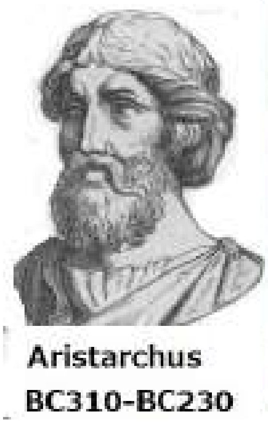
Then, we want to doubt axiom of choice, but a description of the mathematics largely decreases when I do not accept axiom of choice. Hence, we usually accept axiom of choice.

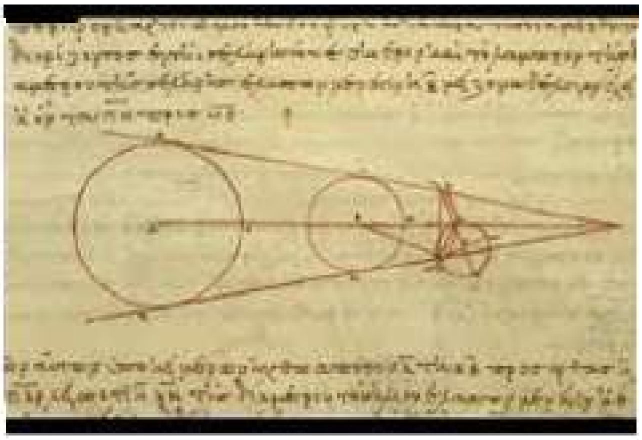
♠**Note 5.2.** There was also tradition of pyramid construction, and Egypt was an advanced country of mathematics. Pythagoras and Archimedes also learned geometry in Egypt. Then Alexandria was an academic city as there was Alexandria library having 700,000 collection of books. After Euclid, we know that

- Eratosthenes (BC.275 - BC.194) : He was determined to 46250km the whole circumference of the earth. Cf. Sec. 5.5.
- Cleopatra(BC.69 - BC.30): The most beautiful woman in human history.
- Ptolemaios (AD.83 - 168): Geocentrism

5.3 Aristarchus (BC.310 - BC.230)

5.3.1 the diameter of the moon: the diameter of the sun





Aristarchus (BC.310 - BC.230) was an ancient Greek astronomer and mathematician who presented Heliocentrism. He calculated as follows.

Proposition 5.1.

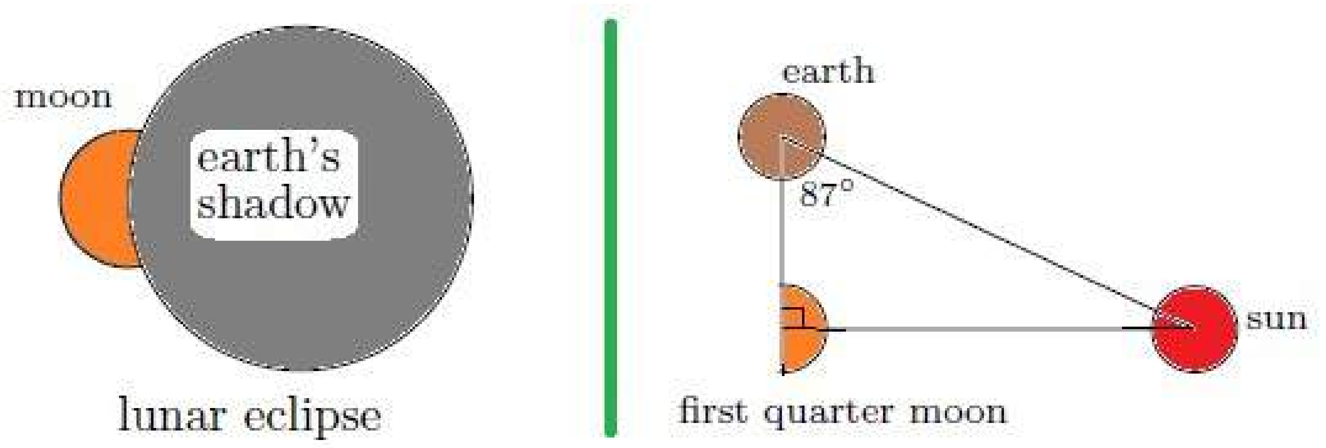
(A_1) the diameter of the moon: the diameter of the earth ≈1:3 (Recent result says that 1 : 3.669), where $a : b = c : d$ means $a/b = c/d$.
(A_2) the diameter of the moon: the diameter of the sun≈1:19
(A_3) Thus, the diameter of the earth : the diameter of the sun ≈1:6.333 (Recent result says that 1 : 109)
(A_4) Since each volume is proportional to [diameter]^3, the sun is much larger than the earth.

The answer to (A_1) Look at the lower left figure (lunar eclipse). Since the sun is very far, it suffices to consider that

$$\text{the diameter of the earth} \approx \text{the diameter of the earth's shadow}$$

Hence, measuring by eye, we see (A_1).
The answer to (A_2): Look at the lower left figure (the first quarter moon). Note that $\cos 87° \approx 1/19$. And using the fact that the sun and the moon are seen as the same size, we can calculate:

$$\frac{\text{the diameter of the moon}}{\text{the diameter of the sun}} = \frac{\text{the distance between the moon and the earth}}{\text{the distance between the sun and the earth}} = \cos 87° \approx \frac{1}{19}$$



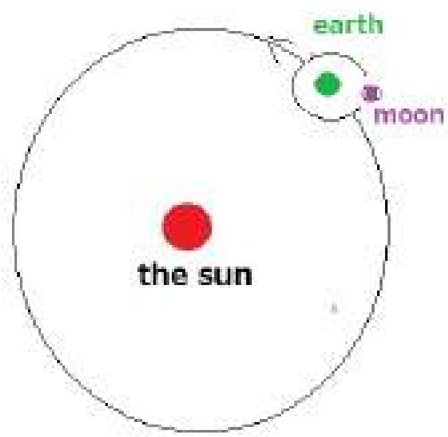
5.3.2 Ancient Heliocentrism

Aristarchus considered as follows:

(B_1) The sun is overwhelmingly larger than the Earth. If so, it is wrong that the big sun goes around the small earth. It is sure that the small earth goes around the big sun.

That is,

(B_2) Aristarchus proposed Heliocentrism



His argument is almost complete since the difference between "the volume" and "the mass" is trivial.
Therefore, I agree that

Aristarchus was the first proponent of the heliocentric theory

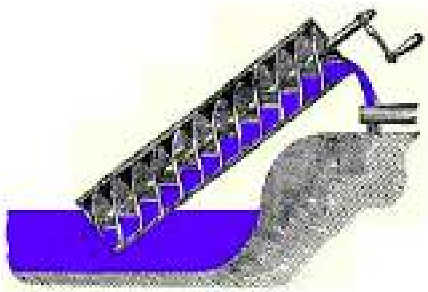
Next problem is as follows.

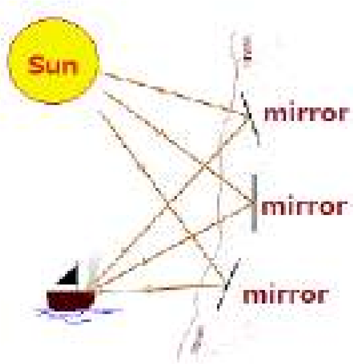
(C) measuring the diameter of the earth

This was solved by Eratosthenes (*cf.* Sec.5.5).

5.4 Archimedes (BC.287 - BC.212)

Archimedes was born in Syracuse on the island of Sicily in the Mediterranean. Archimedes studied in Alexandria that was a center of the study and engaged in the study of "Elements" with pupils of Euclid afterwards. He returned to Syracuse later and spent life in Syracuse.





5.4.1 Buoyancy (Archimedes' principle)

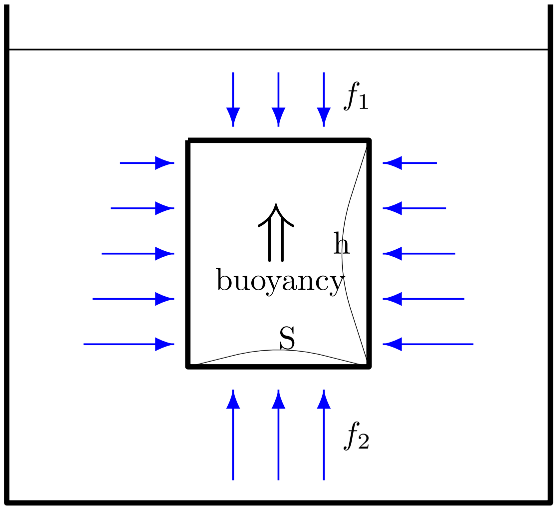
Archimedes' principle on buoyancy is as follows.

(A) Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object. If some want to avoid the term "force", then

$$\begin{aligned}&\big[\text{the weight of the matter in water}\big]\\=&\big[\text{the weight of the matter}\big]-[\text{buoyancy}]/g\end{aligned}\tag{5.1}$$

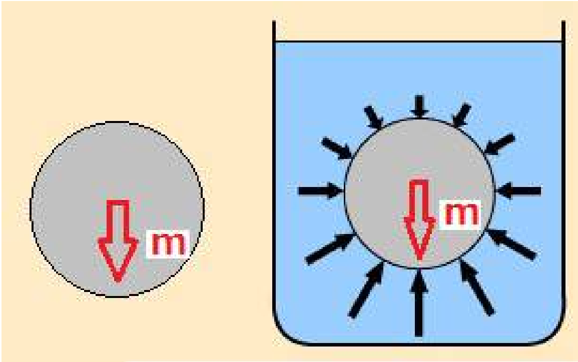
where (g: gravitational constant)

$$\begin{aligned}[\text{buoyancy}]=&\ \big[\text{Sum of the water pressure from the bottom of the object}\big]\\&-\big[\text{Sum of the water pressure from the top of the object}\big]\\&=(f_1+f_2)S=(|f_2|-|f_1|)S=hSg=Vg\quad(\text{the density of water is }\rho(=1).)\end{aligned}$$

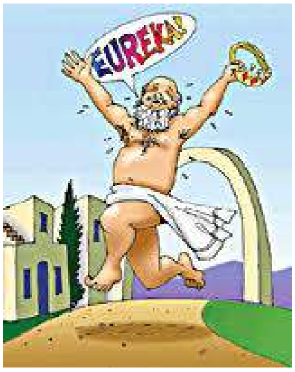


S: bottom area, h: height
$V=hS$; volume
f_k: water pressure, $k=1,2$

For further information, see my homepage



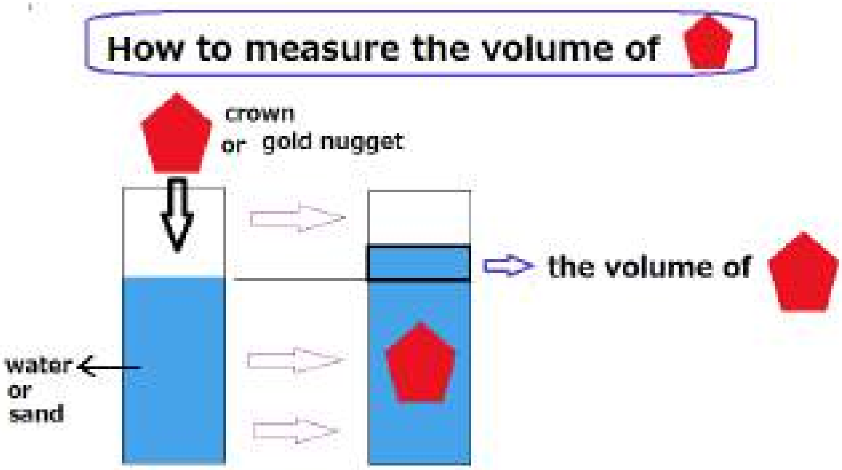
♠**Note 5.3.** A famous anecdote of the golden crown is the delicate anecdote that there is not connected with Archimedes' principle. In like there is a relationship, I try to write this in what follows.



- The King of Syracuse asked Archimedes "Can you check whether silver is not mixed by the crown without breaking the crown". Archimedes notices next answer (♯) during bathing: while shouting with joy too much "Heureka!"(="I have found it!"), was running around the streets naked without even wearing clothes.

He found:

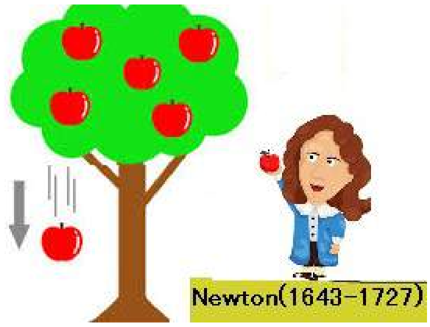
(♯) Prepare a gold nugget of the same weight as the crown. Next, determine the volume of the gold nugget and the volume of the crown as shown in the figure below. This allows us to determine the specific gravity of the gold nugget and the crown. This way, we do not need to use Archimedes' principle of buoyancy.

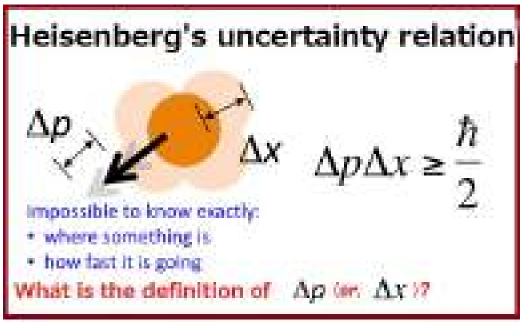


This way, Archimedes' principle of buoyancy would not have to be used. However, there is no way that Archimedes would jump out of the bath and run around the city because of something as simple as this. It would be more natural to assume that Archimedes knew the above before he took a bath and discovered the "Archimedes' principle of buoyancy" when he took a bath.

♠**Note 5.4.** For each great discovery, an anecdote (or, a catchphrase, stage effect) is left as follows.

($\sharp_1$) Archimedes$\cdots\cdots$golden crown, heureka! (*cf.* Sec.5.4)
($\sharp_2$) Galileo$\cdots\cdots$Leaning Tower of Pisa, "And Yet It Moves" (*cf.* Sec.7.3.4)
($\sharp_3$) Newton$\cdots\cdots$ ①:Newton's apple, "Geocentrism vs. Heliocentrism"(*cf.* Note 7.8)
($\sharp_4$) Descartes$\cdots\cdots$ ①:fly on the ceiling (*cf.* Note 8.2), ②: I think, therefore I am, (*cf.* Sec.8.2)
($\sharp_5$) Kant$\cdots\cdots$clock (*cf.* Note 10.2), dogmatic slumber (*cf.* Note 10.5)
($\sharp_6$) Wittgenstein$\cdots\cdots$primary school teacher, Gardener, Guardian: Russell (*cf.* Sec.12.1.1)
($\sharp_7$) Einstein$\cdots\cdots$Elevator
($\sharp_8$) quantum mechanics$\cdots\cdots$Heisenberg's uncertainty principle (*cf.* ref. [38]).





I have the following opinion about ($\sharp_8$):

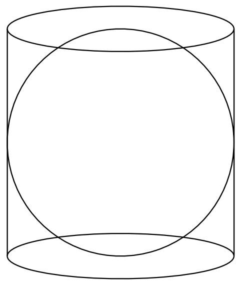
($\flat$) There is a big gap between Heisenberg's uncertainty relation (refs. [28, 38]) and Kennard=Robertson uncertainty relation (refs [90, 111]). Therefore, just as the "Newton's apple" was used as a symbol for Newton mechanics, "Heisenberg's uncertainty relation" was used as a symbol for quantum mechanics (*cf.* Note 4.1 of ref. [74]).

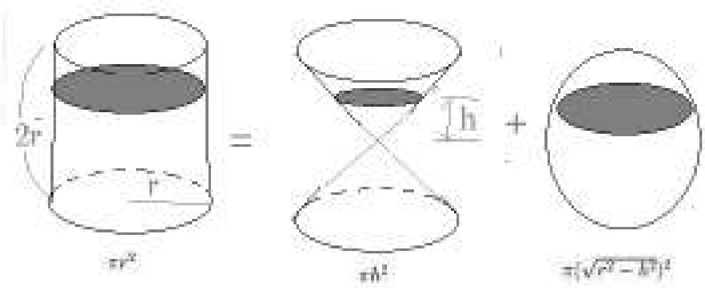
5.4.2 The tomb of Archimedes

Consider the ball B of radius r. Archimedes showed the followings:

$$\text{The volume of the ball } B = \frac{4\pi r^3}{3}, \qquad \text{The surface area of the ball } B = 4\pi r^2$$

If you are a mathematical genius, you may find the proof by seeing the lower illustration ('the cylinder which is circumscribed to a ball' called "the tomb of Archimedes"). If you are not genius, you can calculate it by using the differential and integral calculus.





Areas of gray areas of the same height

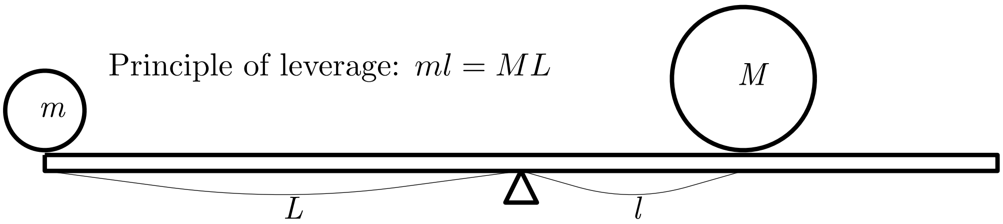
Thus,

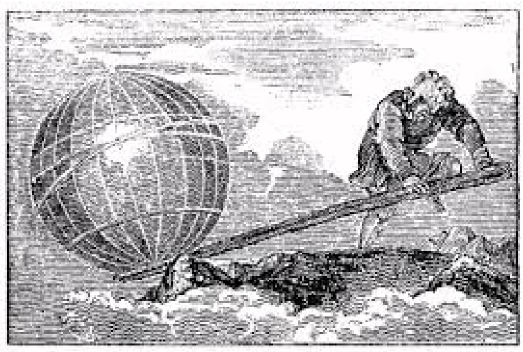
$$
\begin{aligned}
\text{"Volume of Sphere"} &= \text{"Volume of Cylinder"} - 2 \times \text{"Volume of Cone"} \\
&= 2\pi r^3 - 2 \times \frac{\pi r^3}{3} = \frac{4\pi r^3}{3}
\end{aligned}
$$

□

5.4.3 Principle of leverage

Archimedes found" principle of a lever" and did more various invention with a lever.





Archimedes said "Give me a lever long enough and a fulcrum on which to place it, and I shall move the world". In spite that he referred Aristarchus' Heliocentrism in his book: "The Sand Reckoner", he supported Aristotle's Geocentric model. However, Archimedes, found " principle of a lever", have to restate Aristarchus' Heliocentrism((B_1) in Sec.5.3) as

(B) Because the sun is so much bigger than the earth. The center of gravity of the combined Earth and Sun is extremely close to the Sun. Hence, **the Sun and the Earth revolve around the gravity of both the earth and the Sun.**

If Archimedes said so, science history would be history which is completely different from now.

♠**Note 5.5.** Archimedes' arguments were so clear that even elementary school students could understand them, and he did not say ambiguous and unintelligible things like philosophy (Plato, etc.). This clarity is a factor in Archimedes' popularity. As I have mentioned before, his words pierce our hearts,

- Heureka!"(="I have found it!")
- Give me a lever long enough and a fulcrum on which to place it, and I shall move the world

and so on. And the "last words" was

- Do not disturb my circles!

The city of Syracuse, where Archimedes lived, was a battleground between Carthage (Hannibal the General) and Rome. The Roman army knew that Archimedes was a famous scientist, so they instructed him not to do any harm. However, when Archimedes was thinking about writing a figure on the sand, he was almost taken away by the Roman soldiers, who refused to do so, saying "Do not disturb my circles! And thus he was killed. It can be said that he was the "greatest star of the ancient scientists" until the end of his life.

♠**Note 5.6.** Note that

- Archimedes did not speak ambiguous things like Plato's philosophy.

Therefore the work of Archimedes is quantitative, clear and easy to understand. Since political power could interpret the vague philosophy conveniently, philosophy could influence to maintain harmony with religion or politics. In fact, philosophy survived in the middle ages as a maid of theology. On the other hand, Archimedes' work was almost forgotten.

5.5 Eratosthenes (BC.275 - BC.194)

BC276-BC194

Sieve of Eratosthenes

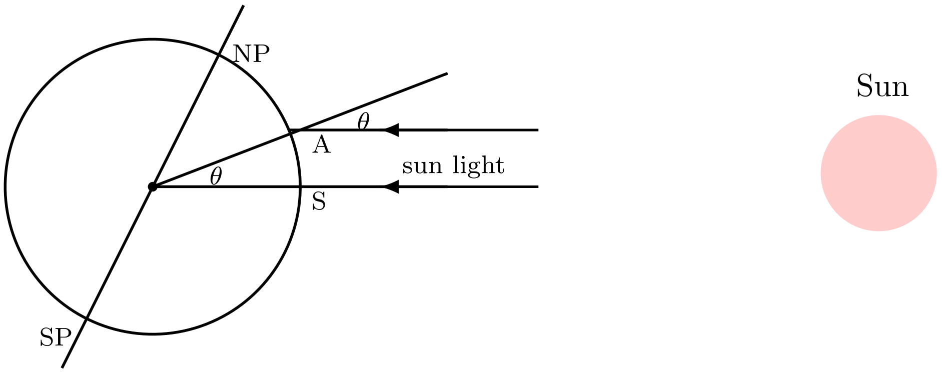
5.5.1 The biggest ancient observer

Pythagoras believed that the earth must be a beautiful shape and believed that the earth was a sphere. Aristotle deduced that the lunar eclipse was the shadow of the Earth and believed that the Earth was a sphere. When you look at the ocean in the distance, it looks like an arc, so there were probably people who believed that the earth was a sphere since long ago. However, if we were to mention the two certain discoverers, it would be the scientist Eratosthenes (BC.275 - BC.194) and the explorer Magellan (AD.1480 - AD.1521).

Aswan High Dam

- Syene (=Aswan) is on the tropic of cancer, thus, the sun is seen in right above at noon on the summer solstice.
- Syene (=Aswan) is located just south of Alexandria. The distance $=AS=$925km.

Eratosthenes measured the whole circumference of the earth as follows.



NP:North pole, SP:South pole, A:Alexandria, S:Syene(=Aswan)
Hence,

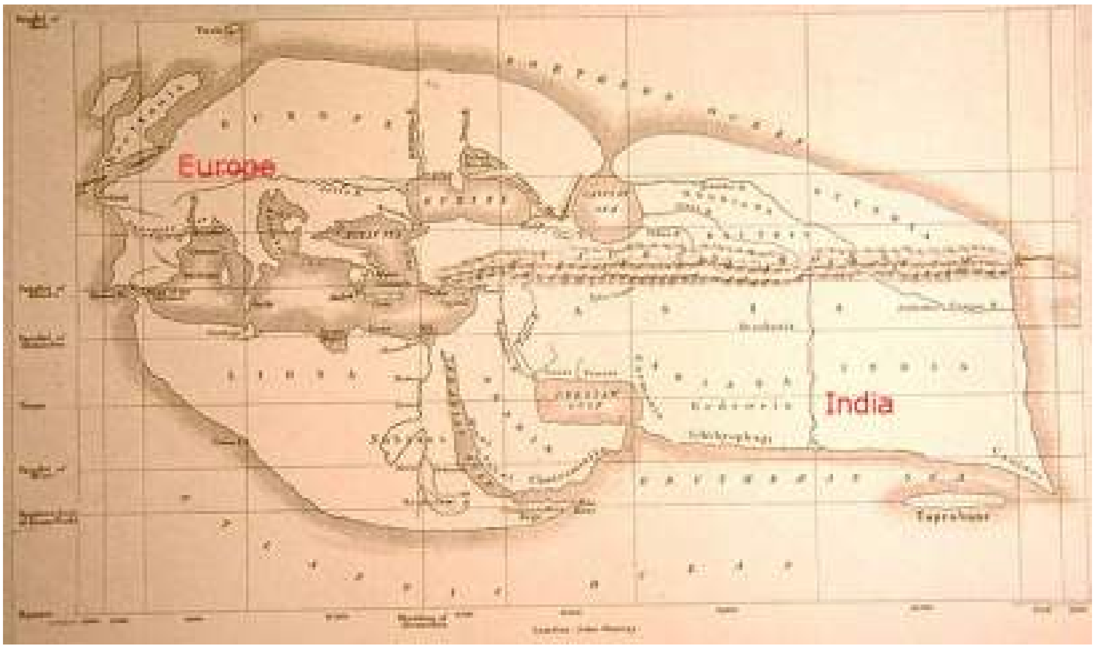
$$\text{the whole circumference of the earth} = 2 \times 3.14 \times [\text{the radius of the Earth}] = 360AS/\theta$$
$$=360 \times 925/7.2 = 46250km$$

As the recent result:40009km, it may be surprising.

♠**Note 5.7.** Since Aristarchus discovered

$$\big[\text{the diameter of the moon}\ \big] : \big[\text{the diameter of the earth}\ \big] : \big[\text{the diameter of the sun}\ \big]$$
$$= \ 1 : 3 : 19$$

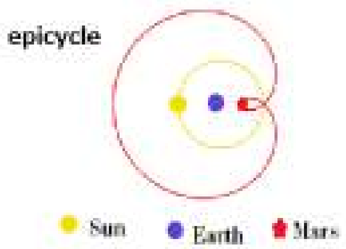
then, by Eratosthenes's result, we know that
$\big[$the diameter of the moon $\big]$, $\big[$the diameter of the earth $\big]$, $\big[$the diameter of the sun $\big]$.

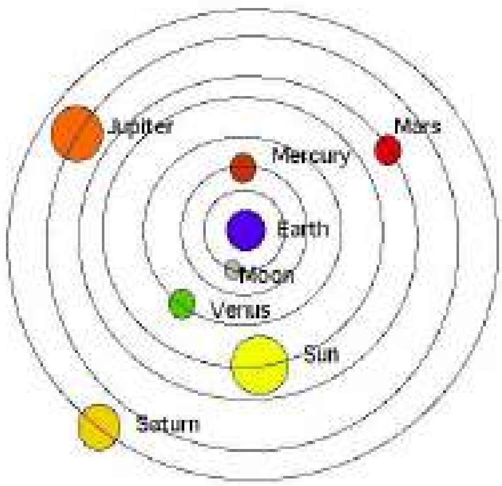


5.6 Claudius Ptolemaios (AD.83 - AD.168)

5.6.1 A culmination of Ancient Astronomy

Ptolemaic Dynasty is ruined by the death of Cleopatra, Rome became the heyday of the Five Good Emperors era. At this time, Ptolemaios (AD.83 - 168) played an active part in Alexandria. In his book "Almagest", he adopted Aristotle's Geocentrism (i.e., the sun goes around the earth). Ptolemaios explained the retrogression seen at a planet in Mars such as Mars revolves around the earth while drawing a small circle as "epicycle".





Ptolemaios compiled the latest theory in those days and concluded the Geocentrism (= Ptolemaic system) under the enormous measured data.

(A) Ptolemaios followed Aristotle, Archimedes, etc.

And it is sure

(B) **Ptolemaios is a top-notch researchers.**

He was the scientist who gave the most importance to observation among the ancient scientists.

Although, approximately 1500 years later (at Galileo's trial (1633)), his Ptolemaic system was replaced by the Copernican system, he was surely one of scientists who thought observation and experiment as important most.

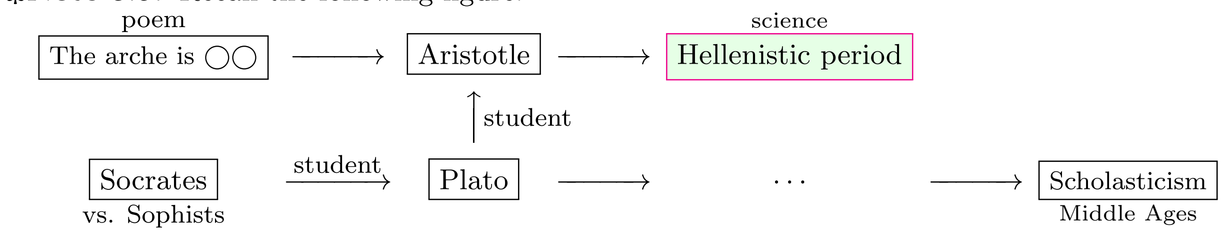
♠**Note 5.8.** S. Weinberg (1933 -2021), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said of his book [123] "To explain the world; The discovery of modern science" as follows:

- [in Chapter 8]: *In one respect the work on this theory described in the Almagest is strikingly modern in its methods. Mathematical models are proposed for planetary motions containing various free numerical parameters, which are then found by constraining the predictions of the models to agree with observation.*

Decades ago, when I saw the planetarium when I was a kid, a commentator explained that "Ptolemaios asserted a foolish Geocentrism, and this was corrected by Copernicus."

However, when I went to the planetarium recently, a commentator praised Ptolemaios, saying, "His epicycle model is groundbreaking at the time." It was great to hear a similar opinion to Weinberg's at a planetarium in the Far East island nation.

♠**Note 5.9.** Recall the following figure:



That is, Aristotle bridged the gap between this pseudoscience (≈ the arche is ◯◯) and the base for the foundation that would become science. Archimedes is the discoverer of "principle of buoyancy", which belongs the realistic worldview. Ptolemaic system is based on the realistic motion function method.

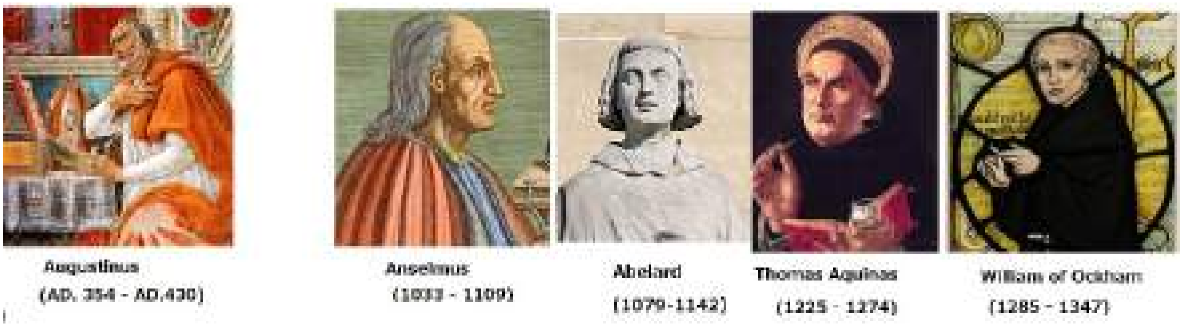
Chapter 6

The Middle Ages

It is often said that

(★) the Middle Ages is characterized as "the time of the thought stop for about 1500 years" Thus, it is called "Philosophy is a maidservant of theology"

However, I think that this is due to our modern low philosophical background.
Thus the (★) is completely wrong. Of all the philosophers to date, I believe it is Anselmus, Thomas Aquinas (and Descartes) who have understood Platonic philosophy the best.



In this chapter, we discuss:

($\sharp_1$) Augustinus(354 - 430): Christianity became the state religion of the Roman Empire. His time theory (i.e., subjective time theory) should be reconsidered in the Copenhagen interpretation.

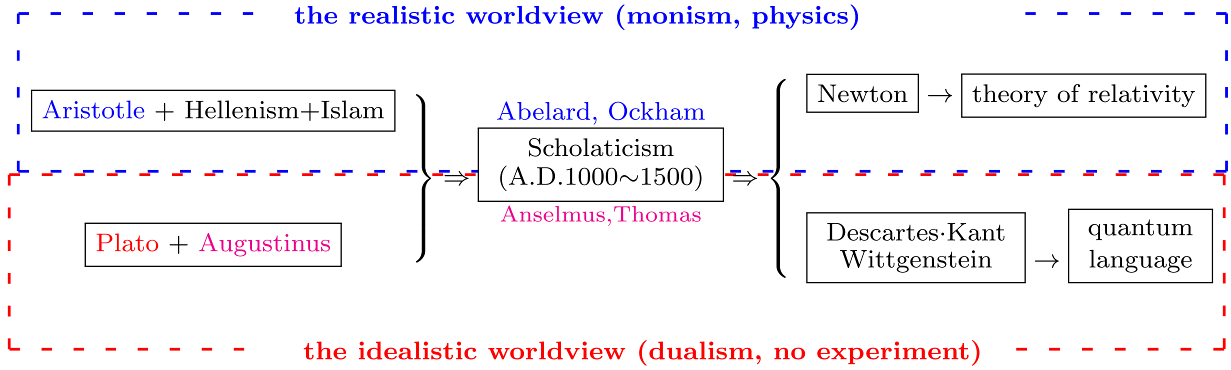
($\sharp_2$) Anselmus(1033 - 1110): the father of Scholasticism, Arguments for the existence of God, "Realismus (i.e., dualism)" in Problem of universals

($\sharp_3$) Abelard, (1079 - 1142): Argued for monism due to Aristotle's influence

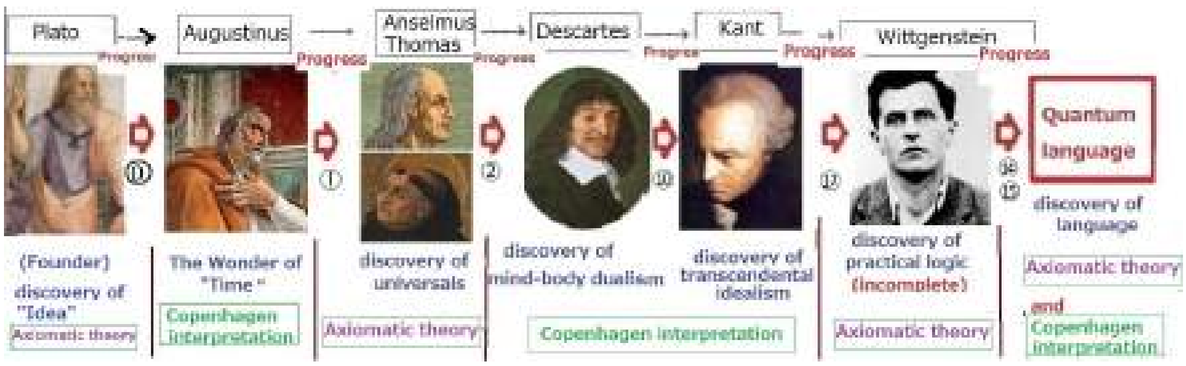
($\sharp_4$) Thomas Aquinas(1225 - 1274): Completion of the Scholasticism (Summa Theologica): He deepen Anselmus's theory in dualism despite Aristotle's influence

($\sharp_5$) Ockham(1285 - 1347): Ockham's razor, "Nominalismus (i.e., monism)" in Problem of universals

And therefore, I assert that they are located as shown in the next figure.



Abelard and Ockham were followers of Aristotle's philosophy. Notable above is [Anselmus, Thomas]. They adopted Aristotle's method to deepen Plato's philosophy. In the history of Western philosophy (from Plato to analytic philosophy), these two are the only two who have attempted to take the theory of ideas head-on and rationally. Spinoza, Leibniz, and Wittgenstein also tried and failed. I think they (Anselmus, Thomas) are too geniuses to be properly appreciated.



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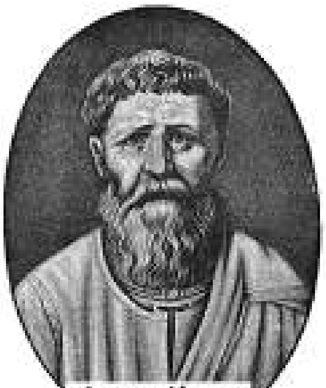
6.1 Augustinus(AD. 354 - AD.430)

6.1.1 The benefactor who saved "Idea theory"


Augustinus
(AD.354-430)


Saint Augustine in His Study
by Botticelli (1480)

One of the largest events in the Western history is

AD.380: Christianity became the state religion of the Roman Empire

A wonder of Western philosophy(≈Plato philosophy) is:

(A) Western philosophy met with a dying crisis many times. Each time, Western philosophy was rescued by a hand of someone's help (such as a hand of help of a god).

Augustinus (354-430), the greatest Catholic priest, adopted Plato's Idea theory to reinforce Christianity such as

(B)

Plato	:	real world	═	Idea	═	Idea world
↓		↓		↓		↓
Augustinus	:	city of this world	═	church	═	City of God

Here, it should be noted that "church" is a bridge between "city of this world" and "City of God". Augustinus' theory is too visible since this is a theory for Christians, written in a way that everyone can understand. However, Augustinus' dualism is rather religious (i.e., good and evil dualism), not scientific. All proceeded as Augustinus' plan.

♠**Note 6.1.** If we emulate Remark 3.6 and forcefully apply the three key words of dualism to Augustine, we get the following

dualism \ three key-words	[A](= mind)	[B](Mediating of A and C) (body)	[C](= matter)
③: Plato: sun Remark 3.6	actual world	Idea (Sun)	Idea world
Augustinus; Christianity	earthly city	church	the city of God
quantum language	**[measured value]**	**[observable]**	**system [state]**

This idea of "church" alone cannot be said to have advanced Idea theory, but in the sense that it saved Idea theory from dying out, it should be noted as follows.

(♯) Plato $\xrightarrow[\text{regress}]{}$ Augustinus $\xrightarrow[\text{progress}]{}$ $\cdots$ $\xrightarrow[\text{progress}]{}$ Quantum language

Also, as stated in the next section, Augustinus is closely related to the Copenhagen interpretation. Thus I assert (♯).

6.1.2* "Confessions" by Augustinus: Only the present exists

This section is written in the following reference:

(C) [76]:Ishikawa, S: Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language Open Journal of philosophy, Vol. 8, No.5, 466-480, 2018, DOI: 10.4236/ojpp.2018.85032 (https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862)

[Revised version] (https://philpapers.org/rec/ISHLCB)
(http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf)

- [85] S. Ishikawa, K. Kikuchi, (2021) *Quantum Fuzzy Logic and Time, Journal of Applied mathematics and physics,* Vo.9 No.11 2021, 2609-2622
(https://www.scirp.org/journal/paperinformation.aspx?paperid=112972)

We want to know:

① How should we live?
② How is the world made?

Augustine (354-430), the greatest Catholic priest, used Plato's philosophy as the "God's intellect = Idea" and armed Christianity. And everything proceeded according to Augustine's plan and intention. What we ordinary people want to know most is, how should we live? Christian Fathers, as God's spokespersons, preached this to the people as the teachings of Christ. Therefore, Christian fathers, like God, had to be able to answer any questions immediately. Among them, the question that puzzled them were, "how is the world made?", "if this world was made by God, what was it like before God made it?" etc. Bible says:

(D_1) This world was made by God.

If so, people may have a question:

(D_2) How about before God made it?

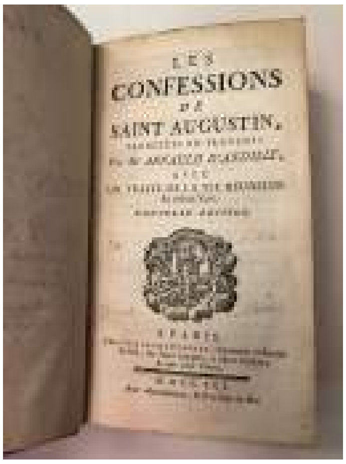
However, if (D_1) is believed, then the priest would respond as follows.

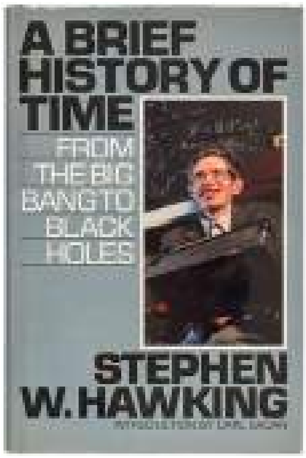
(D_3) Time was also made at the same time as the world.

Therefore,

(D_4) The sentence "before God made it" is nonsense.

If we are told by fathers of Christianity so, We are convinced, "I see, I understand." It should be noted that people want such a short story, and not scientific arguments. That is, note that it is not an understanding of the world for the sake of truth, but an understanding of the world for the sake of deepening our faith.

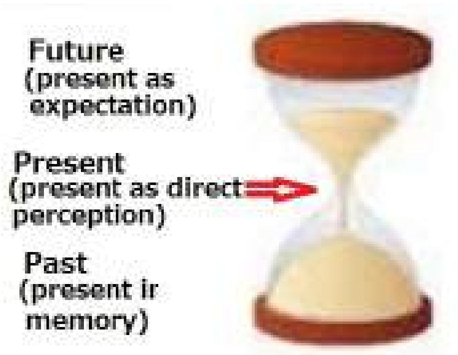




Augustinus asserted the following in his book "Confessions".

(E):Augustinus' theory of time

(E) **Only present exists, and neither future nor past exist.**



It is well known that St. Augustinus said that

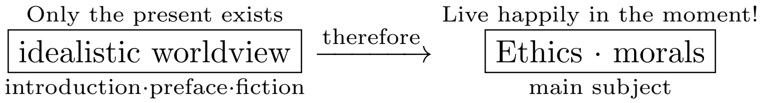
(F) *the past does not exist because of its being already gone, that the future does not exist because of its not coming yet, and that the present really exists.*

Here, consider

(G) "Only the present exists"

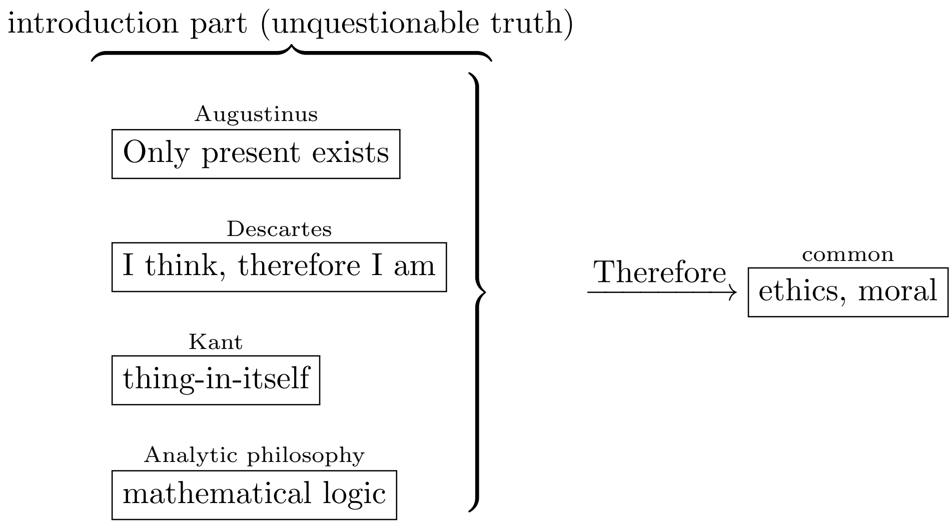
Note that this proposition (G) is related to "tense". Here, according to the Copenhagen interpretation (E_2) in Sec. 1.3.3, there is no tense in quantum languages. Therefore, this (G) is not a statement of a quantum language. Therefore, this (G) is not scientific, i.e., there is no experiment to verify (or deny) it. This means that (G) is a very convenient statement for religious people.

This is the beginning of subjective time. This "time" cannot be used in science, but can be used in philosophy as follows.



The Copenhagen Interpretation is "a rule that draws a line between scientific and pseudoscientific propositions," but it does not mean that pseudoscientific propositions are useless. For example, the pseudoscientific proposition "only the present exists" is a proposition that cannot be denied by experiment (it does not have Popper's disprovability). If so, "Only the present exists" could be used as an "absolute truth" in non-scientific fields.

Rather, the use of "pseudoscientific absolute truth" as a catchphrase is a staple of Western philosophy, as shown below. For example



For further information, see my homepage

6.1.3 "Subjective time" is a magic word which excites our delusion

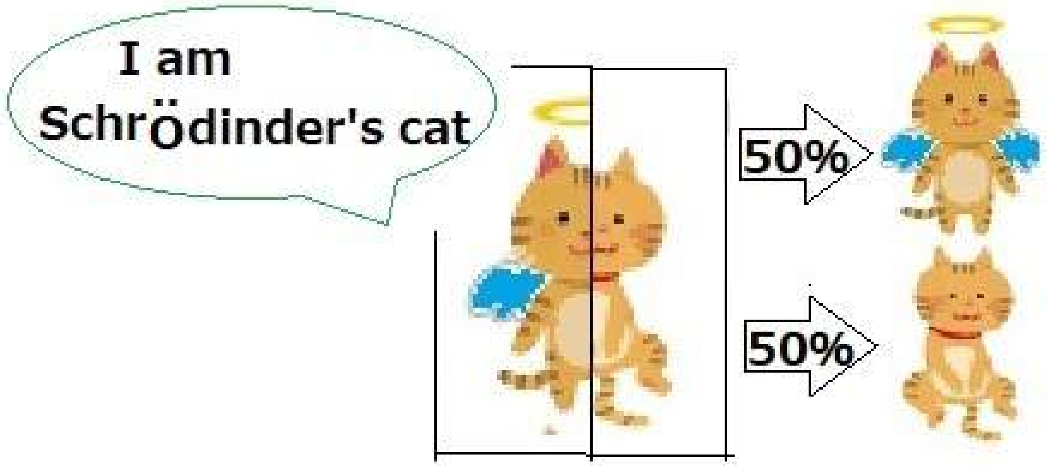
Augustine's problems such as "subjective time," "tense," and "observer's time" did not enter into the realm of science, but they continued to attract the interest of philosophers. For example, Bergson, a philosopher of "subjective time", tried to challenge Einstein of "theory of relativity" to an argument. But he was rejected by Einstein, saying "I don't know the time of philosophers". Even now, some are still misled by this "observer's time". In quantum mechanics, for example, observer's time is often assumed. For example, some researchers may accept "So-called Copenhagen interpretation" such as

- at the moment when an observer measures it, a wave function collapses.

In order to explain "At the moment when observer measured it", von Neumann made a non-scientific word "abstract ego", and said

- "At the moment when observer measured it" is "at the moment when a signal reach abstract ego"

which is of course prohibited by the linguistic Copenhagen interpretation (*cf.* (E_2) in Sec.1.3.3 earlier).



For the quantum linguistic understanding of "wave function collapse", see:

ref. [65] S. Ishikawa, *Linguistic interpretation of quantum mechanics; Projection Postulate,* JQIS, Vol. 5, No.4, 150-155, 2015, DOI: 10.4236/jqis.2015.54017 (http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=62464)

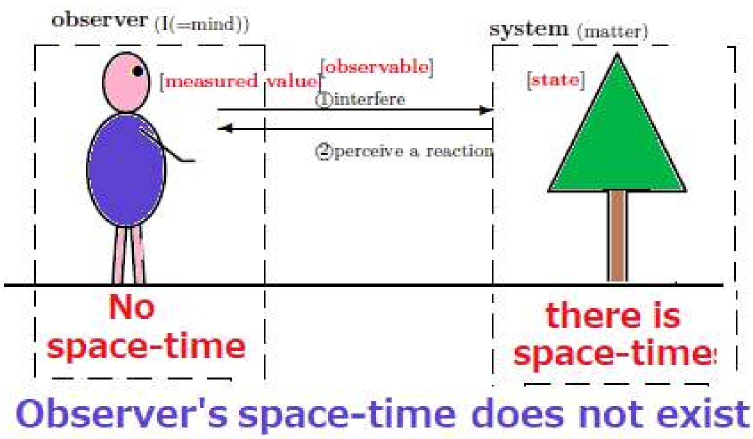
6.1.3.1 Summing up

I think Augustine knew that word play was a useful tool for the spread of Christianity. And I think Augustine also knew that subjective time is a treasure trove of wordplay. Many philosophers were interested in "subjective time". For example,

($\sharp_1$) McTaggart's paradox: "
($\sharp_2$) Russell's "Five-minute hypothesis"
($\sharp_3$) Bergson's subjective time
etc.

However, the linguistic Copenhagen interpretation says that

(H) Observer's space-time does not exist. That is, the subjective space-time (= observer's space-time) is non-sense



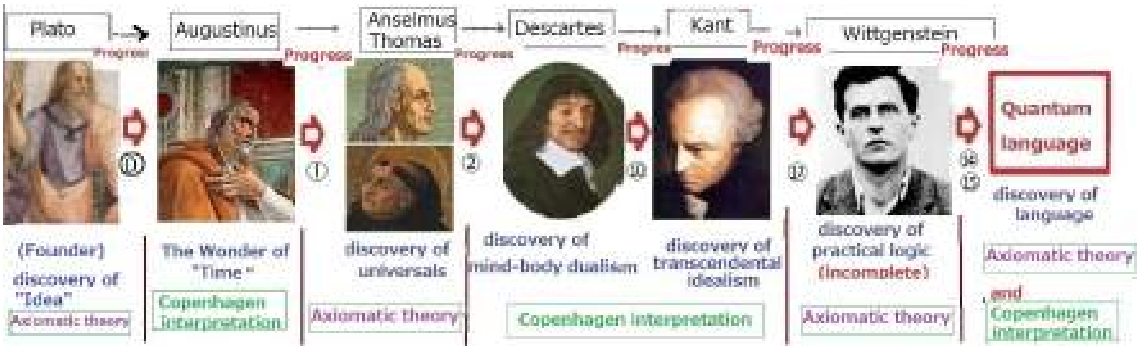
Therefore, QL does not bother with "subjective time.

However, it was Augustine who first noticed the affinity between Platonic philosophy and the Copenhagen interpretation.

♠**Note 6.2.** "What is the subjective time?" This is a problem of brain science. It is sure that cats and dogs etc. have clock gene or biological clock, thus they surely feel the subjective time. This is a scientific problem. Also, measuring the time with a clock is also a measurement. However, when you measure time with your brain clock, it is not a measurement. A measurement that only you can make is not a measurement. In science, "I", "now" and "here" are forbidden. Thus, 'Now I am here' is not a scientific proposition (i.e., a proposition in quantum language). Enjoyment of wordplay is an important part of a successful philosophy, e.g., "I know I know nothing", "Only present exists", "I think, therefore I am", etc. However, it should be noted that these are not statements in quantum language, i.e. these violate the linguistic Copenhagen interpretation.

♠**Note 6.3.** After all, I think Augustinus's subjective time theory deepened our understanding of the Copenhagen interpretation. .Therefore, we insist as follows.

$$\boxed{\text{Plato}} \xrightarrow[\text{progress}]{①} \underset{\text{linguistic Copenhagen interpretation}}{\overset{\text{subjective time}}{\boxed{\text{Augustinus}}}}$$



The key question is, "To whom do they tell their philosophy? ".

Augustine··· believers, general public

Anselmus, Thomas ··· theological controversy, thus, professional priests

Descartes,...,Kant··· by Gutenberg's typography, the general public

Wittgenstein - - - - ··· professional philosophers and general public[*1]

In short, when it is aimed at the laypersons, the Copenhagen Interpretation is chosen. For professionals, "Axiomatic theory" tends to be chosen.

[*1] The part of the book that tried to be written to professional philosophers fell apart, but it had a strong message and helped popularize analytic philosophy.

6.2 Scholasticism –Bamboo (=Aristotle) cannot be grafted to a tree (=Plato) –

6.2.1 Aristotle's philosophy spread to the Islamic world

Plato's philosophy survived with the backing of Christianity (Augustine). The philosophy of Aristotle spread to Islam. I don't know the details of the reason why,

(A) Plato's philosophy spread to the Christian world and Aristotle's philosophy to the Muslim world.

Probably, there were various conflicts in Christianity, and the winners stayed in Rome and supported Plato's philosophy. The losers were driven to the Muslim world, and Aristotle's philosophy must have spread to the Muslim world by such a process. Eastern Islam was centered in Baghdad which was famous on the Arabian Nights. Western Islamic culture developed around Cordoba in the Andalusian region of southern Spain and became the largest city in the world in the 10th century. At that time, the Islamic world learned a lot of wisdom from the books of ancient Greeks and Romans and developed its own thought and technology. The Islamic culture was at the forefront of the world under Aristotle's philosophy.

♠**Note 6.4.** In this text we adopt the story such as (A). Actually, it may not be such a simple story.

6.2.2 Crusade expedition and Inflow of Islamic culture

In the era of crusade expedition (1096 - 1270), the Western countries were in a downturn. Such public opinion had been drifting.

- The achievements of the Crusades do not rise by Plato's way. Thus, let's study Aristotle at the tip of the Islamic culture!

I think it is true that

- in every age and every place in the world, the human resources required are

the humanities in peacetime, and the sciences in wartime.

As a byproduct of the pilgrimage to the Holy Land of Jerusalem and the crusade to recapture it, interaction with Islamic culture was facilitated. Aristotle's philosophy flowed into the West, merged with Plato's philosophy, and settled in as Scholastic philosophy. That is, Scholasticism was born. As the typical persons of Scholasticism, we list up as follows.

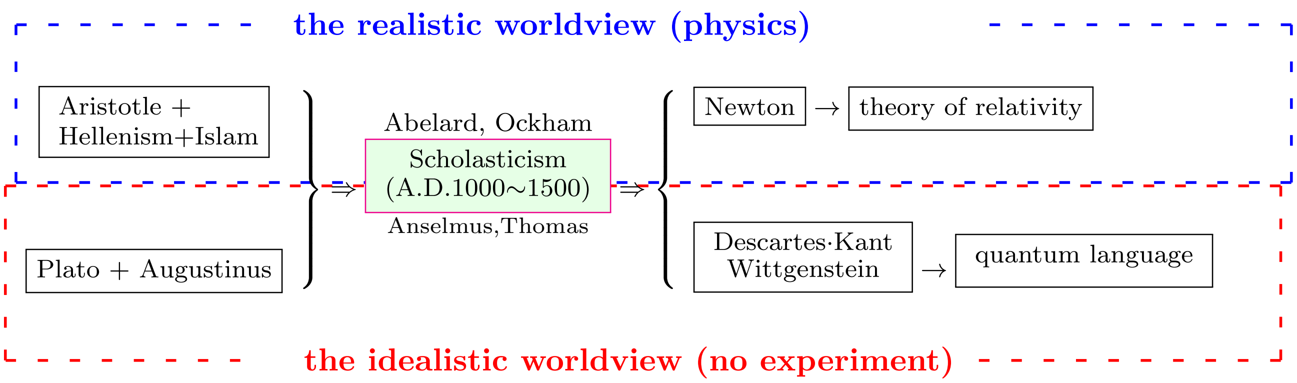
(B_1) Anselmus (1033 - 1109) "The father of Scholasticism", "Realismus(≈dualistic idealism)"
(B_2) Abelard (1079-1142) "Nominalismus(≈monistic realism)"
(B_3) Thomas Aquinas (1225 - 1274) "Realismus", "Summa Theologica", Greatest theologian in Scholasticism
(B_4) Ockham (1285 - 1347) "Ockham's razor", "Nominalismus"

After all,

- in the beginning, Plato's dualistic idealism was the most popular, but gradually Aristotle's influence increased. And it has become like a product of the fusion of Platonic and Aristotelian philosophies.

Of course, it is impossible to succeed this "fusion". That is because Plato philosophy and Aristotle philosophy are "oil (dualistic idealism) and water (monistic realism)", and these are different categories (cf. Figure 0.1 [the history of worldview] in Preface). However, in this text, we prepare the story such as

- In the process of fusing Platonic and Aristotelian philosophy, Idealism was deepened by Anselmus, Thomas Aquinas, etc.



6.3 The discovery of zero

6.3.1 Positional notation (= the discovery of zero): Arabic numerals

As mentioned in the previous section,

(A) Plato was introduced to the Christian world and Aristotle to the Muslim world.

Although many people must have suffered miserably during the Crusade expedition, the merits for Christian culture is that Aristotle philosophy and the positional notation flowed into Europe from Islam.

♠**Note 6.5.** The phrase "the discovery of zero" can mean many things. In this text, "discovery of zero" = " positional notation ".

The positional notation is how to write numbers to learn in an elementary school. That is,

- The next number after 9 is written as 10

For example,

$$+5040302, \quad -15, \quad +39.045, \quad -81.5, \quad +0.3333\cdots$$
$$-1000, \quad +0.009876, \quad +3.1415\cdots, \quad 0,$$

and so on. That is, By 13 symbols "0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, −, . (radix point)", we can express all real numbers by the positional notation . Hence, we may say

- the discovery of the positional notation (= Arabic numerals) = the discovery of real numbers.

(the radix point was discovered in Europa of 16 century AD.) Of course, the discovery of zero is

(B) the discovery of how to use zero called the positional notation

For further information, see my homepage

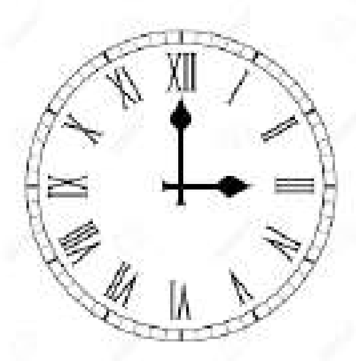
6.3.2 Arabic numerals and Roman numerals

Roman numerals are often used on the clock face such as

$$1=\text{I},\ 2=\text{II},\ 3=\text{III},\ 4=\text{IV},\ 5=\text{V},...,10=\text{X},\ 11=\text{XI},$$

However, it is too hard to represent large numbers such as

$$495 = \text{CDXCV},\ 1888 = \text{MDCCCLXXXVIII},\ 3999 = \text{MMMCMXCIX}$$



6.3.3 The explosion of mathematics

European mathematics originally had the high potential of Euclidean geometry. With the introduction of the positional notation (the discovery of zeros), computation became easier. The word "ALGEBRA (algebra)" is originally an Arabic word. The formula for the solution of the quadratic equation: $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

was also understood by the Arabic mathematician al-Khwarizmi (about 790 - about 850) Mathematician Gauss(1777 - 1855) said

(C) "If genius Archimedes invented the positional notation , I am certain that the mathematics must have progressed drastically."

The positional notation triggered the "math explosion" such that

(D) Solution of algebraic equations, complex numbers, the function concept, betting of problem (probability), analytic geometry (Descartes coordinates), calculus, differential equations, linear algebra, number theory, etc.

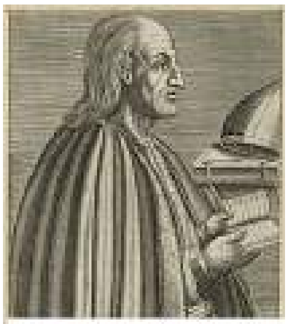
♠**Note 6.6.** There may be several opinions about the three big discoveries of mathematics. We think as follows.

① the discovery of the plane (geometry)
② the discovery of zero (positional notation)
③ the discovery of sets

Of course, it is needless to say that the biggest discovery is "⓪:the discovery of natural numbers". And "④:the invention of computer" would be almost certain.

6.4 Anselmus; the problem of universals and the proof of the existence of God

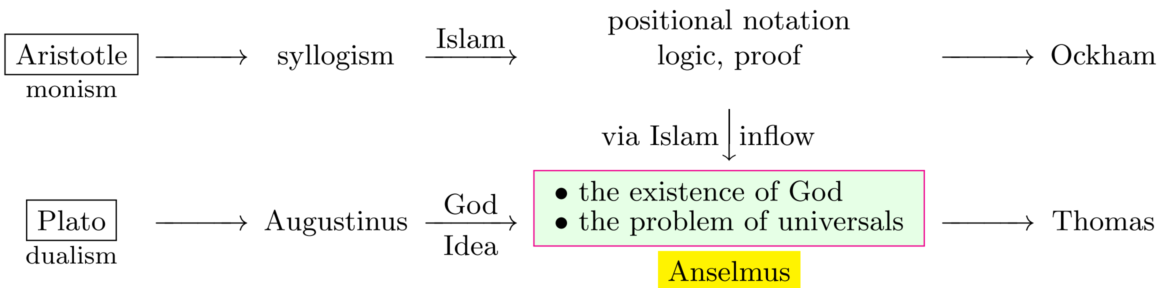
Anselmus is the founder of Scholastic philosophy, famous for "the problem of universals" and his "Proof of the Existence of God".



Anselmus (1033-1109)

6.4.1 Aristotelian Philosophy's Inflow from Islam

The figure below shows the location of Anselmus.



As can be seen from above, Anselmus is located at the confluence of the "Platonic-Augustinian line of faith" and the "Aristotelian line of reason via Islam." Thus, Anselmus' next two jobs make sense.

$$
(\mathrm{A}) \begin{cases} (\mathrm{A}_1)\text{: Considering God rationally} & \longrightarrow \text{the existence of God} \\ (\mathrm{A}_2)\text{: Considering Idea theory rationally} & \longrightarrow \text{the problem of universals} \end{cases}
$$

Now anyone can see that (A_1) failed. Opinions may differ on (A_2), but this text considers (A_2) to be the greatest advance in Plato's philosophy. If it were not for the work of Anselmus, the Idea Theory would never have gone beyond the realm of a fairy tale.

6.4.2 Review: the worldviewism

In fact, the two issues in (A) are closely related to worldviewism. Let's review worldviewism here .

(B): Worldviewism (*cf.* Sec.1.1)

The worldviewism is as follows.

(B) $\underset{\text{premise}}{\overset{\text{world is so}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{subject}}{\overset{\text{conclusion}}{\boxed{\text{discussions, calculation, logic, properties}}}}$

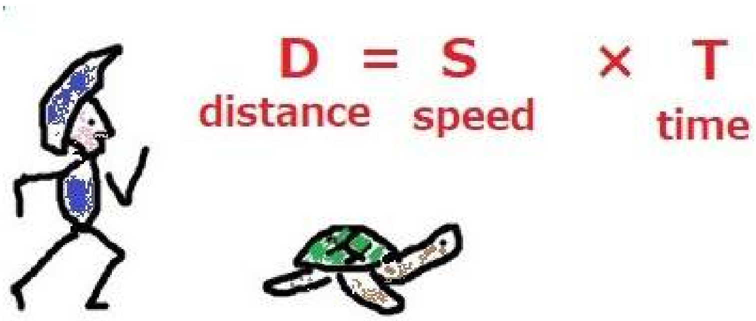
That is,

(B_1) Worldviewism is the spirit of "Let's start with worldview!"

To put it more clearly.

(B_2) Without a worldview, there's nothing to say.

We have seen many paradoxes (e.g., Zeno's paradox, etc.) that arise from ignoring the worldview.



In this text, we have mainly discussed the quantum mechanical worldview (= quantum language), but a worldview based on relativity is also acceptable, and we do not deny that the worldview may change depending on the problem to be solved.

6.4.3 The proof of the existence of God

Anselmus is the founder of Scholastic philosophy, famous for his "Proof of the Existence of God". His proof is nothing more than a play on words that we use in our daily lives. Its premises are not clearly stated (i.e., "under which worldview is the proof?" is not explicitly stated). Also, the meaning of "existence" is not clear. Therefore, there is no need to read the proofs seriously.

Proof 6.1. Anselmus: the proof of God's Existence

ⓐ: God is a being than which none greater can be imagined . That is, the greatest possible being that can be imagined.
ⓑ: It is obvious that God exists as an idea in the mind.
ⓒ: A being that exists as an idea in the mind and in reality is greater than a being that exists only as an idea in the mind.
ⓓ: Thus, if God exists only as an idea in the mind, then we can imagine something that is greater than God. But we cannot imagine something that is greater than God.
ⓔ: Therefore, God exists in reality.

□

My opinion on this proof is as follows.

(C) **Before examining the above proofs in detail, it is necessary to ascertain "under what worldview is it being discussed? As we will soon see, this is not an argument from a certain worldview. Thus, ⓐ ∼ ⓔ is not worth verifying. This is not a proof but a play on words.**

♠**Note 6.7.** (i):The "proof of God's existence" has been challenged by many philosophers. However, we believe that the only credible proof is the following.

(♯) What is described in the Bible is the Christian worldview. And the Bible says that God exists. Therefore, God exists . ([Existence] = [keyword]. *cf.*Note 1.2) □

(ii):All scientists are interested in God. What are the neural circuits of the brain regarding faith? What is subjective time? (= What about the biological clock?) These are one of the most interesting questions in neuroscience.

6.4.4 Overview of the problem of universals; see next section for details.

The problem of universals is the following controversy.

Problem of universals

(D) It is clear that individuals such as Jack and Betty exist. So the question is, "Does the human race exist?"
Realism claims that it "exists", and Anselm is the representative.
Nominalism asserts that "it does not exist," and Occam is the representative.

Anyone would ask, "Why did the great priests spend 250 years in such a play on words?" But if this is a profound question, we must marvel at the level of scholastic philosophy.

The problem of universals is the greatest theme not only in Schola philosophy but also in Western philosophy. This shows the high potential of Scholastic philosophy (i.e., the depth of its understanding of Plato's philosophy). In fact. The following ⓐ∼ⓖ are all problems of the same kind as the universal argument.

Table 6.1 (=Table 4.1) : monistic realism (Aristotle) vs. dualistic idealism (Plato)

dispute ╲ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	"Nominalismus"(Ockham)	"Realismus"(Anselmus)
ⓓ: space-time	Newton (Clarke)	Leibniz
ⓔ: quantum theory	Einstein	Bohr
ⓕ:philosophy of science	Carnap	Quine
ⓖ: fuzzy sets	Kalman	Zadeh

For the precise argument, see Table 4.1.

ⓒ The Problem of universals was a 250-year debate between the Aristotelians and the Platonists The Problem of universals was a debate between the Aristotelians and the Platonists in Scholasticism, with the Platonists gaining the upper hand at first, and then the Aristotelians gradually gaining the upper hand. Although it was not a quantitative scientific debate, the problem of universals universal shows the high philosophical competence of Scholastic philosophy (both Aristotelian and Platonist).

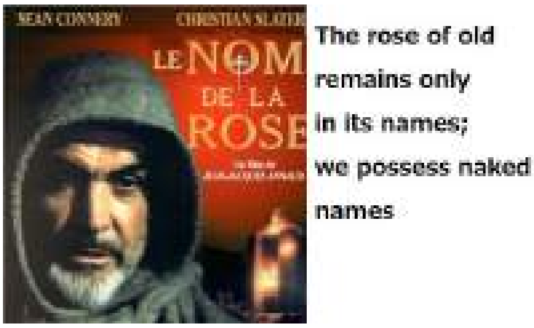
This will be discussed in the following section (Sec. 6.5).

6.5 Scholasticism; Problem of universals

6.5.1 What is the problem of universals?

This section is due to the following paper:

- ref.[83],S. Ishikawa, (2022) *The problem of universals from the scientific point of view: Thomas Aquinas should be more appreciated* Open Journal of Philosophy, Vol. 12, No. 1, 86-104, (https://www.scirp.org/journal/paperinformation.aspx?paperid=115252)



In the figure below, let us review again the historical position of Anselmus.

Aristotle (monism) ⟶ syllogism $\xrightarrow{\text{Islam}}$ positional notation logic, proof ⟶ Ockham

via Islam ↓ inflow

Plato (dualism) ⟶ Augustinus $\xrightarrow[\text{Idea}]{\text{God}}$ [• the existence of God • the problem of universals] (Anselmus) ⟶ Thomas

As can be seen from above, Anselmus is located at the confluence of the "Platonic-Augustinian line of faith" and the "Aristotelian line of reason via Islam." Thus, Anselmus' next two jobs make sense.

$$
(A) \begin{cases} (A_1)\text{: Considering God rationally} & \longrightarrow \text{the existence of God} \\ \\ (A_2)\text{: Considering Idea theory rationally} & \longrightarrow \text{the problem of universals} \end{cases}
$$

Now anyone can see that (A_1) failed. The (A_2) is a controversy that arises when Plato's Idea Theory enters the realm of science, and readers will be surprised at Anselmus' deep knowledge of Plato's philosophy.

(B): What is "Problem of universals"?

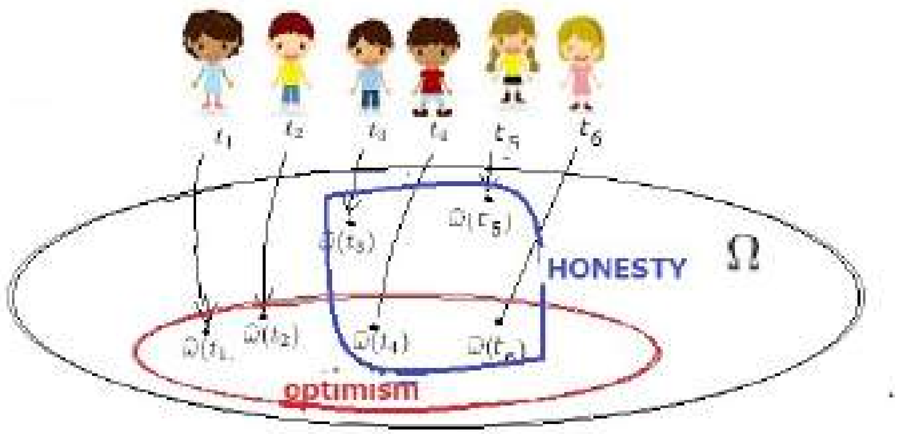
Problem 6.2. "Problem of universals" is as follows.

(♯) It is certain that Mx. Smith, Mx. White, Mx. Brown, etc. exist as matters (i.e., individuals). Then, we have the following problem:

Do "universals" (i.e., "Honesty", "Optimism", "Kindness", "Female", "Intelligence",etc.) exist?

If "Yes", then, "Realismus". If "No", then, "Nominalismus".

The following figure will promote the reader's understanding



Many readers may ask the following question?

(B′) Why did the great Fathers argue so fervently on such a matter of wordplay?

When studying the problem of universals scientifically, we must first answer two questions.

(B_1) Under what worldview should we discuss the problem of universals ?
(B_2) What is "universal"?

In fact, however, "universal" is a concept that can be clearly understood for the first time under a scientific worldview.

Therefore, in the next section, review the scientific worldviews and then answer questions (B_1) and (B_2)

6.5.2 Review of Sec 1.4 (Appendix: The mathematical foundations of quantum language)

6.5.2.1 Axioms 1 and 2

Now we review Axioms 1 and 2 of QL in Sec. 1.4 as follows.

Axiom 1 [State, Measurement]

•[General (i.e., quantum and classical) case]:

(C_1) [State]: With any *system* S, a C^*-algebra $\mathcal{A}(\subseteq B(H))$ can be associated. A *state* of the system S is represented by an element $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$.

(C_2) [Measurement]: An *observable* is represented by an observable $\mathsf{O} = (X, \mathcal{P}(X), G)$ in $\mathcal{A}$. And, the *measurement of the observable* O *for the system* S *with the state* ρ is denoted by $\mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]})$ (or more precisely, $\mathsf{M}_{\mathcal{A}}(\mathsf{O} = (X, \mathcal{P}(X), G), S_{[\rho]})$). An observer can obtain a measured value x $(\in X)$ by the measurement $\mathsf{M}(\mathsf{O}, S_{[\rho]})$. The probability that a measured value x $(\in X)$ obtained by the measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O} = (X, \mathcal{P}(X), G), S_{[\rho]})$ is given by $\rho(G(\{x\}))(\equiv {}_{\mathcal{A}^*}\langle \rho, G(\{x\})\rangle_{\mathcal{A}})$.

•[Classical case, i.e. the case that $\mathcal{A} = C(\Omega)$]: The above (C_1) and (C_2) can be summarized for the classical system as follows

(C_1^c) [Classical state]: A classical state of the system S is represented by an element $\omega(\in \Omega)$.

(C_2^c) [Classical measurement]: When an observer take a measurement of an observable $\mathsf{O} = (X, \mathcal{P}(X), G)$ for a system with a state $\omega(\in \Omega)$, the probability that a measured value x $(\in X)$ is obtained is given by $[G(\{x\})](\omega)$.

Note that it suffices to identify "observable" with "measuring instrument".

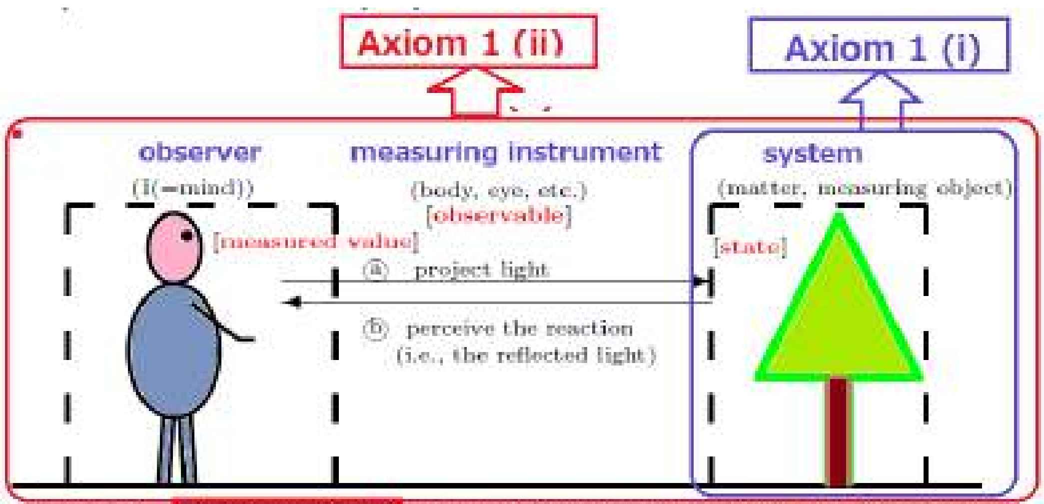


Figure 1.2. Descartes Figure: Image of "measurement(=ⓐ+ⓑ)" in dualism

(D): Axiom 2 [Causality]

Let $(\mathbb{T} \equiv \{t_0, t_1, ..., t_n\}, \leq)$ be a (finite) tree-like semi-ordered set.
(D_1): [General (i.e., quantum and classical) case]
Causality is represented by a causal relation $\{ \Phi_{t_2,t_1} : \mathcal{A}_{t_2} \to \mathcal{A}_{t_1},\ t_1, t_2 \in \mathbb{T}\ (t_1 \leq t_2) \}$.

(D_2): [Classical case]
Classical causality is represented by a classical causal relation $\{ \phi_{t_2,t_1} : \Omega_{t_1} \to \Omega_{t_2},\ t_1, t_2 \in \mathbb{T}\ (t_1 \leq t_2) \}$.

6.5.2.2 Three worldviews

Here we focus on three worldviews

Remark 6.3. (= Remark 1.11)

(E):quantum language (=QL)

QL = [state, measurement] (Axiom 1; (i) and (ii)) + [causality] (Axiom 2) + [Copenhagen interpretation] (how to use Axioms 1 and 2)

(F): classical quantum language (= classical QL)

classical QL = [state, measurement] (Axiom 1[Classical case (i), (ii)]) + [causality] (Axiom 2[Classical case]) + [Copenhagen interpretation] (how to use Axioms 1 and 2)

Also, we define "abstract classical mechanics" as follows.

(G): abstract classical mechanics

abstract classical mechanics = [state] (Axiom 1 [Classical case] (i)) + [causality] (Axiom 2 [Classical case])

6.5.3 Scientific definition of "universal"

As mentioned in the previous section, "universal" is a concept that can be clearly understood for the first time under a scientific worldview. And, when studying the problem of universals scientifically, we must first answer two questions.

- (=(B_1)) Under what worldview should we discuss the problem of universals ?
- (=(B_2)) What is "universal"?

Then, we can assert the definition:

(H): Definition of "universal" in scientific worldview

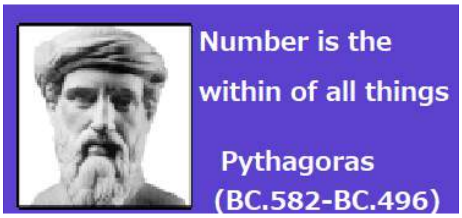
Definition 6.4. [Universal]
Assume a certain scientific worldview (worldview expressed in mathematics).

(H_1) The axiom system of the assumed world view has several keywords. Among those keywords, those with mathematical expressions (= parameter expressions) are called "universal".

[Note]: For non-scientific worldviews (worldviews not expressed in mathematics), "universal" cannot be defined. However, I dare say that "universal" = "essential keywords". This is because it is an axiom that quantitatively expresses the relationship between several essential keywords.

♠**Note 6.8.** Thus, to use Pythagorean language, we can express it as follows

($\sharp_1$) Number is the within of all universals.



6.5.4 Examples: universals in Classical QL and Abstract Classical Mechanics

6.5.4.1 Example: universals in Classical QL (or, QL)

Explained in classical QL (same for QL), but for simplicity I will explain in classical QL.
(I): The classical QL is formulated as (F)

$$\boxed{\boxed{\text{classical QL}}} = \underset{\text{(Axiom 1[Classical case (i), (ii)])}}{\boxed{\begin{matrix}\text{state}\\\text{measurement}\end{matrix}}} + \underset{\text{(Axiom 2[Classical case])}}{\boxed{\text{causality}}} + \underset{\text{(how to use Axioms 1 and 2)}}{\boxed{\text{Copenhagen interpretation}}}$$

Here, the key-words in Axiom 1 [Classical case (i), (ii)] and Axiom 2 [Classical case] are as follows.

(J) "state", "system", "observable", "measurement", "measured value", "probability", "causality"

Among the above, those that have mathematical expressions are as follows.

(K) "state" $(\omega \in \Omega)$,
"observable" $(\mathsf{O} = (X, \mathcal{P}(X), G))$
"measured value" $(x \in X)$,
"causality" $(\{ \phi_{t_2,t_1} : \Omega_{t_1} \to \Omega_{t_2})$

(Here, "measurement" $\mathsf{M}_{C(\Omega)}(\mathsf{O} = (X, \mathcal{P}(X), G), S_{[\omega]}))$ and "probability" $[G(\{x\})](\omega)$ are not simple, thus, the two are omitted.)

Therefore, the universals in classical QL are as follows.

- **"state", "observable", "measured value", "causality"**

6.5.4.2 Example: universals in abstract classical mechanics

The same is true for Newtonian mechanics, but for the sake of simplicity I will explain it in abstract classical mechanics.
(L): abstract classical mechanics is formulated as follows (= (G)):

$$\boxed{\boxed{\text{abstract classical mechanics}}} = \underset{\text{(Axiom 1 [Classical case] (i))}}{\boxed{\text{state}}} + \underset{\text{(Axiom 2 [Classical case])}}{\boxed{\text{causality}}}$$

Therefore, key-words are as follows:

"state", "causality"

Among the above, those that have mathematical expressions are as follows.

"state" $(\omega \in \Omega)$,
"causality" $(\{ \phi_{t_2,t_1} : \Omega_{t_1} \to \Omega_{t_2})$

Therefore, abstract classiical mechanics has the following universals:

- **"state", "causality"**

6.5.5 Detailed discussion: Problem of Universals in science

[[I]: The problem of universals in classical QL]:
Let Ω be the compact state space, in which every human's state is assumed to be represented. Let $m_H : \Omega \to [0,1]$ [resp. $m_O : \Omega \to [0,1]$] be the membership function of "Honesty" [resp. "Optimism"] (see Figure below) (*cf.* ref. [129], 1965).

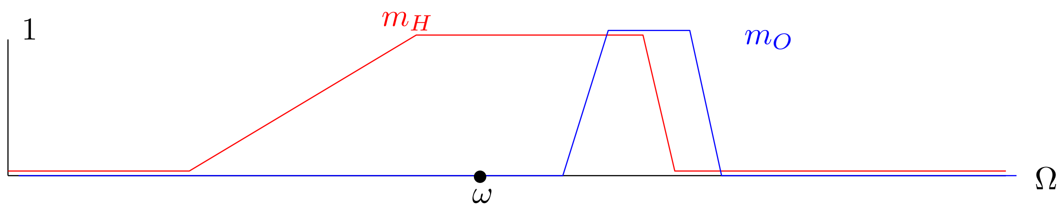


Figure 6.1: [Membership functions m_H (resp. m_O) of "Honesty" (resp. "Optimism")]

Here, assume that the value $m_H(\omega)$ $(\forall \omega \in \Omega)$ is determined as follows:

(M) [The probabilistic interpretation of QL]: Suppose a group of 100 respondents are asked the question "Is this person (with the state $\omega \in \Omega$) honest or not? and $100 m_H(\omega)$ of the respondents answered "yes". Then, we can define the membership function $m_H : \Omega \to [0,1]$ concerning "Honesty". Similarly, $m_O : \Omega \to [0,1]$ is defined.

Further, define the observables $\mathsf{O}_H = (\{y,n\}, 2^{\{y,n\}}, G_H)$ and $\mathsf{O}_J = (\{y,n\}, 2^{\{y,n\}}, G_O)$ (where "y"="yes", "n"="no") such that

$$[G_H(\{y\})](\omega) = m_H(\omega), \quad [G_H(\{n\})](\omega) = 1 - m_H(\omega)$$
$$[G_o(\{y\})](\omega) = m_o(\omega), \quad [G_o(\{n\})](\omega) = 1 - m_o(\omega)$$

for all $\omega \in \Omega$.
And thus, we have the following identifications:

$$\mathsf{O}_H \approx m_H, \qquad \mathsf{O}_O \approx m_O$$

Therefore, we see that

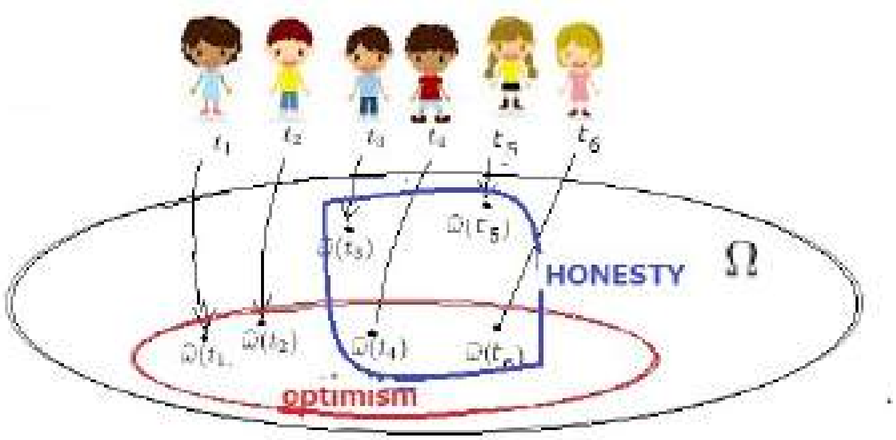
(N) the probability that a measured value y $(\in X = \{y,n\})$ obtained by the measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}_H = (\{y,n\}, \mathcal{P}(\{y,n\}), G_H), S_{[\omega]})$ is given by $[G_H(\{y\})](\omega) (= m_H(\omega))$.

That is, choosing one at random from a group of 100 people, you ask to him/her the question "Is this person (with state ω) honest or not?. Then he/she surely answers, with the probability $m_H(\omega)$, "This person is honest".

Thus, in the framework of Axioms 0 and 1, we can say that the measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}_H = (\{y,n\}, \mathcal{P}(\{y,n\}), G_H), S_{[\omega]})$ is meaningful, that is, three important key-words (i.e., three universals) concerning "Honesty" etc. exist as follows.

(O$_1$) measured values $\{y, n\}$,
(O$_2$) observable O_H (equivalently, membership function $m_H : \Omega \to [0,1]$),
(O$_3$) state ω

This is our conclusion in scientific dualism (i.e., QL).
Now, the original universal was as shown in the figure below.



If so,

- observableO_H is called **"primary universal"** or simply, **"universal"**

Also,

- state $\omega(\in \Omega)$ is called **"secondary universal"**
- measured value (i.e., the element of $\{y, n\}$) is called **"tertiary universal"**

[[II]:Universal in Abstract classical mechanics]
Now, in the case of abstract classical mechanics,

(P) state $\omega(\in \Omega)$

is meaningful ($\approx$ exists). Therefore, as in the case of [I], we see

- state $\omega(\in \Omega)$ is **"secondary universal"**.

For example, assume that some say that

(Q) A person with the state ω is honest (i.e. $m_H(\omega) = 1$)"

However, abstract classical mechanics does not have an observable that represents "honesty (i.e., primary universal)". Thus, the statement (Q) is only sound.
[[III]: Summary]

Summary 6.5. Putting the above together, we get the following

/	tertiary universal	primary universal or simply, universal	secondary universal
monism Newtonian mechanics abstract classical mechanics	/	/	state
dualism quantum mechanics QL, classical QL	measured value	observable	state

♠**Memo 6.1.** Instead of Summary 6.5, there is a reason to define as follows.

(♭) "state", "observed value" and "measured value" are respectively "primary universal" and "secondary universal" "tertiary universal"

In fact, in ref. [82], this (♭) was adopted. However, in this text, Summary 6.5 is adopted because it is close to the spirit of Anselmus and Thomas Aquinas.

Remark 6.6. It is generally believed that the problem of universals is difficult to understand. The reason for this is that the problem of universals can only be fully understood within the context of scientific dualism. And, this is because the discovery of scientific dualism (that is, the discovery of quantum language (or quantum mechanics)) is only recent. It is amazing to see the genius of Anselmus, who took Plato's ideas one step further in the direction of science.

6.5.6 Primary universals in incomplete worldview

Now we have the following problem:

(R) Can we discuss the problem of universals in the case that the worldview is non-scientific?

It is a matter of course that in an incomplete worldview, only an incomplete argument can be made. However, we think that

(S) If we can assume that

$$\boxed{\text{incomplete worldview (S)}} \xrightarrow[\text{progress}]{} \boxed{\text{Newtonian mechanics}}$$

then, the incomplete worldview (S) does not have the (primary) universal.

(T) If we can assume that

$$\boxed{\text{incomplete worldview (T)}} \xrightarrow[\text{progress}]{} \boxed{\text{Quantum language}}$$

then, the incomplete worldview (T) have the (primary) universal.

If so, we see that following progress of "(primary) universal" in the history of philosophy:

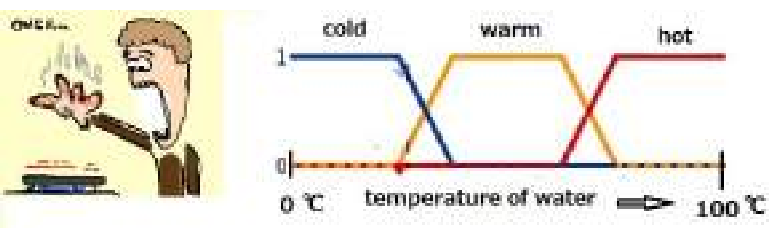
(U) The progress of (primary) universals in history of philosophy

Idea Plato's idea theory	$\xrightarrow{\text{linguistic version}}$	signifier, signified Saussure's linguistic theory
↓ Christian version		↓ quantification
(primary) universal Anselmus, Thomas		membership function Zadeh's fuzzy theory
↓ cognitive version		↓ identification
secondary quality John Locke	$\xrightarrow{\text{linguistic version}}$	observable quantum language

♠**Note 6.9.** The arguments in (S) and (T) above are valid for the history of philosophy (U), but generally inadequate. For example. we think

(♯) $\boxed{\text{statistics}} \xrightarrow[\text{progress}]{} \boxed{\text{Quantum language}}$ (*cf.* see Note 1.3)

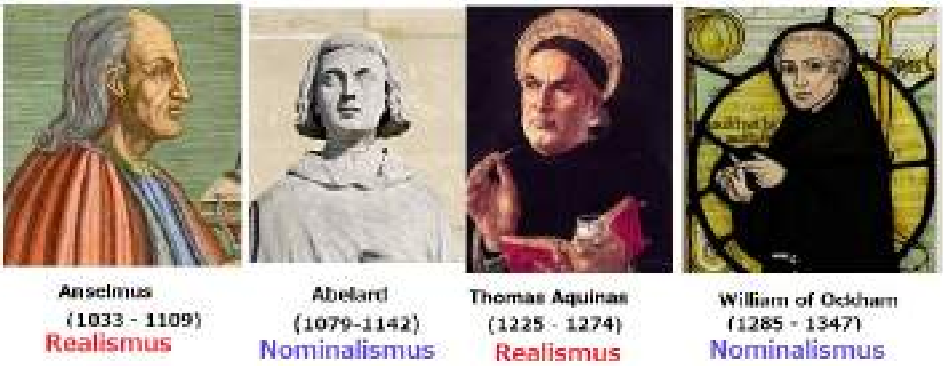
But, I don't want to say that statistics has "universals (= primary universals)".



When I was in high school, I studied Locke's "secondary quality". The simplicity of "secondary quality" left a lasting impression on me, but I never dreamed that this was a central concept in philosophy. However, the fact that it was mentioned in high school textbooks must mean that ordinary philosophers knew that it was a central concept in philosophy.

6.6 The problem of universals in history of western philosophy

6.6.1 Problem of universals in Scholasticism; Anselmus, Thomas Aquinas, etc.



6.6.1.1 Anselmus (1033-1109); dualism, Realismus

First, Anselmus of England, the father of Scholasticism, argued that

(A) universals are real and exist before individuals,

and preached realism (i.e., conceptual realism, in this text it is also called dualistic idealism in spite of the risk of confusing you). We appreciate (A) from a scientific point of view. Because Plato is fairy tale and Augustine is too religious.

6.6.1.2 Abelard (1079-1142) ; monism, Nominalismus

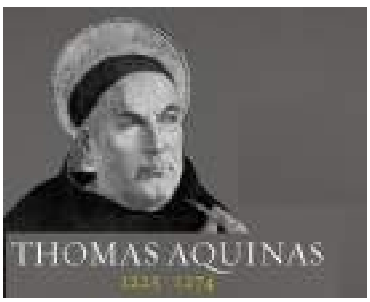
Since "eidos" is usually regarded as "Aristotle's Idea", it is natural to consider that

(B_1) universals are in individuals

Therefore, it is reasonable that Roscellinus (1050-1125), Abelard (1079-1142) and others proposed an argument known as nominalism argued in what follows:

(B_2) universals (in the sense of Anselmus) are merely nominal terms or "sound"

6.6.1.3 Thomas Aquinas (1225-1274); dualism, Realismus



The Catholic priest: Thomas Aquinas (1225 - 1274) wrote "Summa Theologica" as the summing-up of Scholasticism. He was the most important at the intermediate time of Scholasticism (or, Problem of universals), that is,

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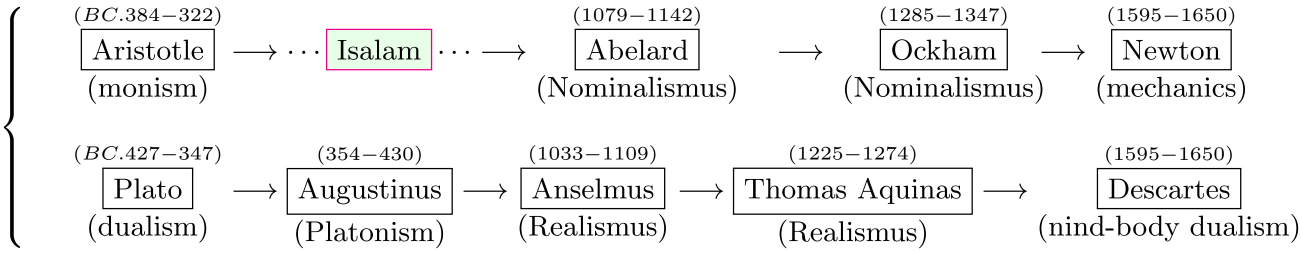
(B_3)



He developed Realismus (due to Anselmus) as follows.

It is usually said that Thomas Aquinas (1225-1274) proposed to integrate conceptual realism (= dualistic idealism) and nominalism (= monistic realism) such as

Theory of Heaven(by Thomas)

(C_1) :universals exist'after individuals' in the human intellect .
[universalia post rem] as actual world
QL: (i.e., measured value is sensed by the brain (i.e., mind)).

(C_2) :universals exist 'before individuals' in the divine intellect, .
[universalia ante res] as Plato's Idea
QL: (i.e., observable (or, measuring instrument) could be in the heavens, like the North Star (*cf.* Note 1.7))

(C_3) :universals exist 'in individuals' in the world, .
[universalia in rebus] as Aristotle's eidos
QL: (i.e., state exists in matter)

♠**Note 6.10.** In Sec. 8.3.3, I introduce Descartes mind-body dualism, which is also called "Theory of Earth" in this text.

I would like to think, however, that Thomas pursued true dualism and not a Platonic/Aristotelian compromise.
I think that

(D) The closest theories to quantum language in the history of philosophy are Thomas's [(C_1)∼(C_3)] above and Descartes problem (Sec. 8.3.2).

6.6.1.4 Ockham (1285-1347); Monism, Nominalismus

In the 14th century, however, William Ockham and others revived nominalism (≈ monistic worldview), which led to the emergence of modern thought that abolished conceptual thinking and sought truth through observation and experimentation.

Nominalism was very helpful in enlightening Aristotelian philosophy. As I've said many times so far, I think that monism and dualism are not mutually exclusive, but coexist such as the above (B_3).

Under the trend that the correct answer to the problem of universals is Newtonian mechanics, nominalism has somehow become dominant, and this continues to this day. On the other hand, because no theory succeeding Thomas has emerged, the philosophical meaning of the problem of universals has remained unclear until the present day.

Table 6.2 : Summing up: universals in worldview (monism and dualism)

/	/	A(= mind) [tertiary universal(A)]	B(= body; medium) [(primary) universal (B)]	C(= matter) [secondary universal(C)]
monism	Aristotle	———	———	hyle [eidos]
	Abelard (1079-1142)	———	———	individual [universal]
	Ockham (1285-1347)	———	———	individual [universal]
	Newton (complete form)	———	———	particle [state]
dualism	Plato (Allegory of the sun)	actual world	Idea	idea world
	Augustinus (354-430)	city of this world	church	City of God
	Anselmus (1033-1109)	[/]	[universal]	individual [/]
	Thomas Aquinas (1225-1274)	human intellect [universale post rem]	divine intellect [universale ante rem]	individual [universal in re]
	Descartes (1596-1650)	mind	body	matter
	Quantum language	observer [measured value]	measuring instrument [observable]	matter [state]

I think philosophy is to answer "What is a dualism?", or equivalently, "What is Plato's Idea?"

In what follows, the reader will see that the meaning of "universal" is clear only in Newtonian mechanics (= scientific monism) and quantum language (= scientific dualism), and that in all other cases the meaning of "universal" is ambiguous.

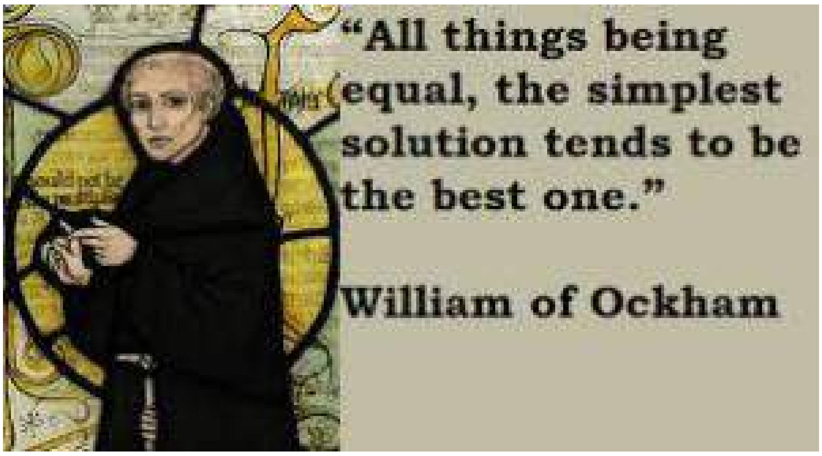
6.7 Ockham's razor and Plato's beard

William of Ockham (1285 - 1347), a Scholastic philosopher or theologian born in Ockham in England, who revived nominalism (≈ monistic worldview), which led to the emergence of modern thought that abolished conceptual thinking and sought truth through observation and experimentation. Further he is known as an advocate of Ockham's razor(=the law of parsimony) in philosophy and science.

(A):Ockham's razor(=the law of parsimony)

Ockham's razor is as follows:

(A) Shave unnecessary assumptions with a razor!
(A') All things being equal, the simplest solution tends to be the best one.



However, this may be a self-evident truth. For example,

(B) Assume that you were a student of Plato and Plato asked you

- "The sun goes around the earth? or the earth goes around the sun?"

Then, which did you answer to Plato?

Probably, you, by Ockham's razor, answer that the sun goes around the earth. In fact Aristotle did so. Ockham's razor is dependent on the environment around. If the surrounding environment changes, so will the way of thinking. Thus I have a question:

(C) Is there a case as which Ockham's razor is functioning effectively (besides the mathematical theorems) ?

We see that

(D_1) Relying on Ockham's razor, Ptolemy claimed the theory of celestial motion from the observational data of the time.

And

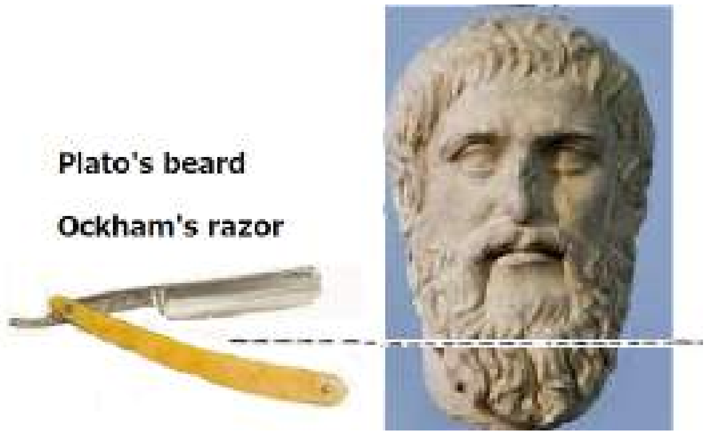
(D_2) relying on Ockham's razor, Galileo claimed for a geodynamic theory from the observational data of the time.

Ockham's Razor is an explanation added later and thus, powerless. But then again...

(E) Is there a case (other than mathematics) in which "Ockham's Razor" is working effectively?

This makes me skeptical of Ockham's razor. Even in the Middle Ages, it is difficult to imagine that people would have been impressed by such a statement if it had been made without any background. The question is, "In what kind of background and context did Ockham say it? I think so. The author is a layman and does not know much about it. Ockham, however, is the leading edge of the Aristotelian sect. Therefore, I agree the following opinion:

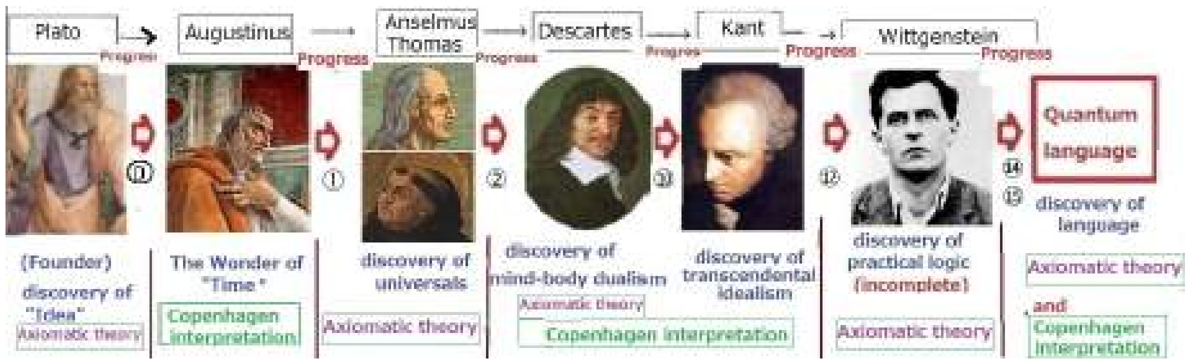
(F) What Ockham wanted to shave with a razor is Plato's beard (= the theory of Ideas)



It makes sense, then. However, it would be too short-sighted to think that it must be better to eliminate such hypotheses as the existence of "something ideal in the heavens. Plato must have thought that the idea theory was a fairy tale. It would be immature and foolish to cut off the fairy tale. As mentioned above, Ockham's razor is dependent on the environment around. Under Newtonian mechanics, Idea theory, purpose (= final cause), Geocentric model must be cut off. And thus, we think that

(G) under QL, ideationalism need not be truncated

Because, I would like to argue the following story:



6.7.1 Statistics or quantum language: which will Occam's razor cut off?How sharp is Ockham's razor?

Figure 0.1 (Preface) is restated as follows.

Figure 0.1: The location of QL in the history of western philosophy

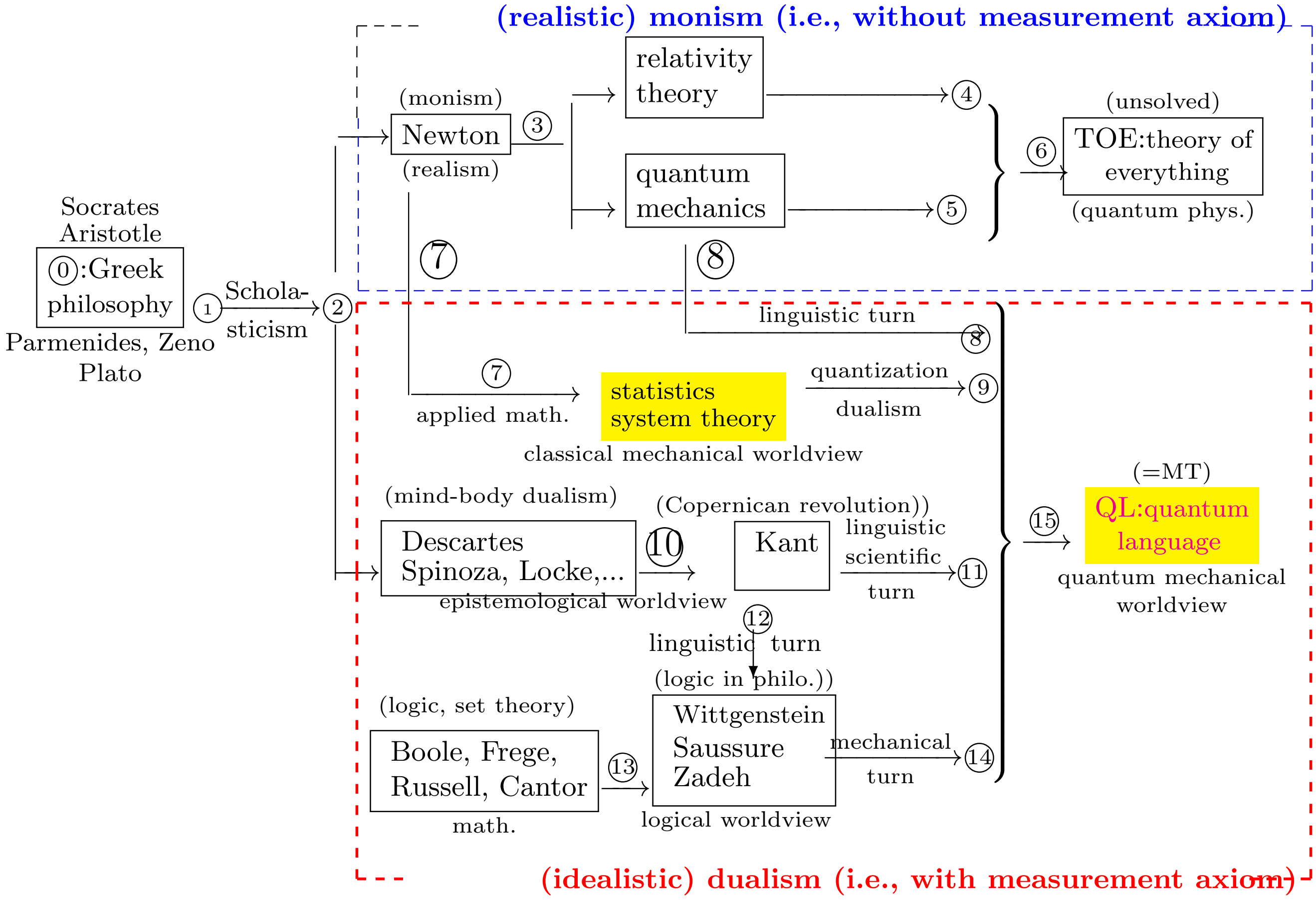
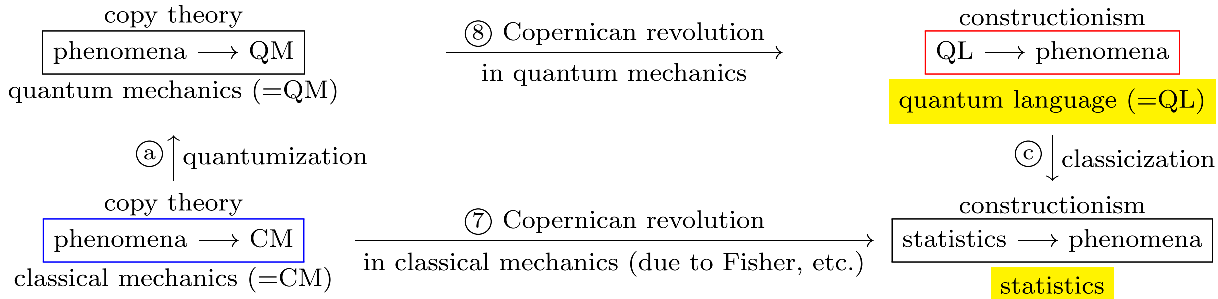


Figure 0.1 (in Preface): The history of the world-descriptions
Philosophy (≈ dualistic idealism) has progressed toward QL
(i.e., (0)→ (1)→ (2)→ (10)→ (12)→ (14)→ (15))

As will be discussed in Sec. 11.7, the Copernican revolution [(7) and (8)](i.e., from realism to idealism) from Newtonian mechanics and quantum mechanics respectively produce statistics and quantum mechanics.



However, I believe that the true form of statistics is the "simplification (or, classicization

ⓒ in the above Figure) of quantum language".

Scope of application of statistics (=dynamical system theory) $\subset$ Scope of application of QL

Why doesn't Occam's razor work?

Chapter 7

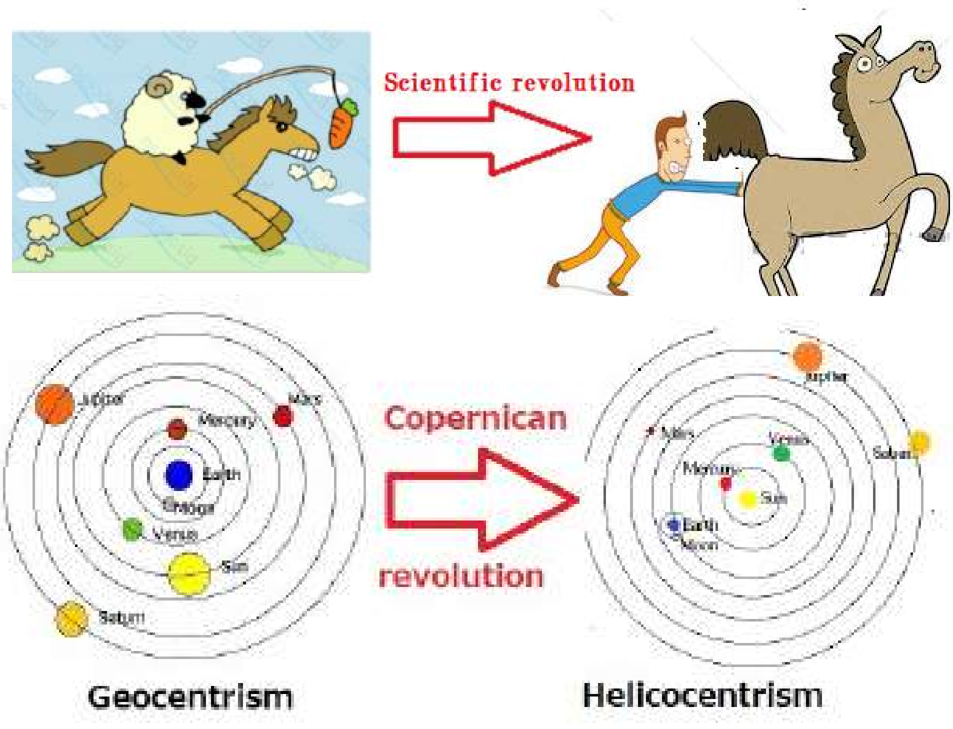
Early modern – Scientific revolution: From Geocentrism to Heliocentrism

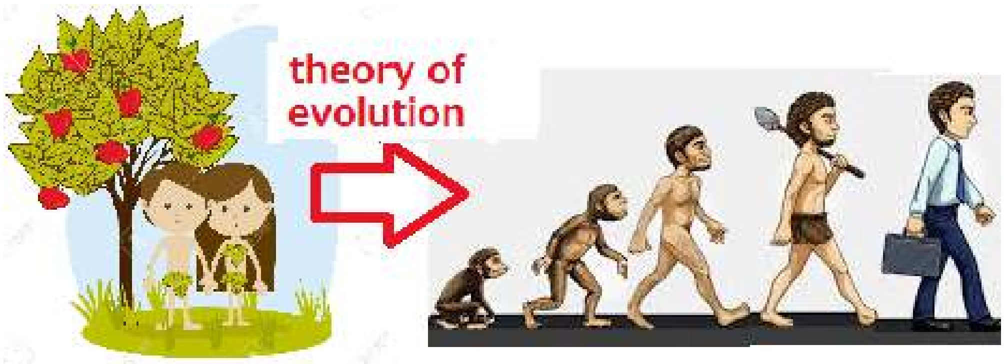
We assume that the three greatest paradigm shifts are as follows

($\sharp_1$) Aristotelian worldview (purpose) $\longrightarrow$ Newtonian worldview (causal relation)
($\sharp_2$) Ptolemaic system: Geocentrism $\longrightarrow$ Copernican system: Heliocentrism
($\sharp_3$) Christianity: Adam and Eve $\longrightarrow$ Darwin: evolution theory

In this chapter, we are concerned with ($\sharp_1$) and ($\sharp_2$), and conclude that

- ($\sharp_2$) is a metaphysical dispute, which cannot be made clear by experiments. And it was clarified by ($\sharp_1$). In this sense, ($\sharp_2$) is included in ($\sharp_1$).
- The ($\sharp_3$) is peculiar to the West.




theory of
evolution

7.1 Paradigm shift

Eastern Roman Empire was made to be ruined by Ottoman Turkey in 1453.

- 1453:The Eastern Roman Empire extinction (Constantinople surrender)

The influence on Christ cultural area of this great event is immeasurable.
Traffic of "Silk Road" became inconvenient. And thus,

Age of Discovery had begun

Also, engineers, artists, cultural people, etc. (of Eastern Roman Empire) had flowed into Western Europe as refugees. And hence,

Renaissance rose suddenly.

The time series is as follows.

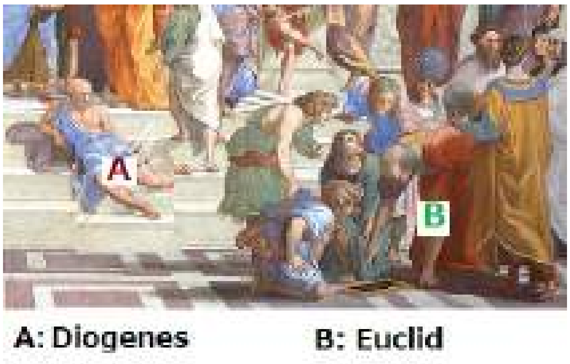
Table 7.1. Scientific revolution; Chronological table

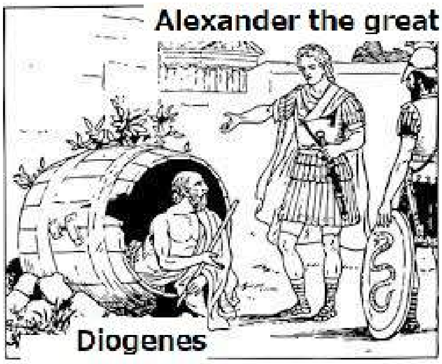
Before Galileo: The era of observation and experiment

- 1450: Gutenberg's printing press — inventor
- 1453:The Eastern Roman Empire extinction (Constantinople surrender)
- 1492: Columbus, discovery of the American Continental — navigator
- 1498: Vasco da Gama, discovery of the sea route to India — navigator
- 1500s: Leonardo da Vinci, "Mona Lisa's smile" — artist
- 1510: Copernicus, Heliocentrism — scientist
- 1510: Raffaello, "The School of Athens", Admiration to ancient Greece — artist
- 1517: Luther, Protestant Reformation — religionist
- 1519 - 20: Magellan, the first circumnavigation of the Earth
- 1540s: Michelangelo, "The Last Judgment" — artist
- 1600: G. Bruno, who supported heliocentrism along with Galileo, was burned at the stake by the Vatican — philosopher
- 1609∼1619: Kepler's laws of planetary motion — scientist
- 1610: Galileo, A telescope was made and moons of Jupiter were found — scientist
- 1620: F. Bacon, "knowledge is power", the father of British Empiricism — illuminator
- 1633 Galileo's trial "And yet it moves" — scientist


"The school of Athens" (Raffaello : 1510) Longing for ancient Greece


A: Alexander the Great
B: Socrates
C: Plato
D: Aristotle
Part of
"The School of Athens"
Artist Raphael
Year 1509–1511
Pythagoras (BC.582 - BC.496)
Parmenides (BC.515 - ?)
Herakleitos (BC.540 - BC.480)





After Galileo: The era of thought

Table 7.2.

- **1637: "Discourse on the Method", Rene Descartes (1596 - 1650), the father of modern philosophy, Cogito proposition** — philosopher
- 1670: Pascal, "Pensèes" — enlightener
- 1685 - 1750: Bach — artist
- **1687: Newton, "Principia"** — scientist
- 1688: Glorious Revolution
- 1690: John Locke, the father of British Empiricism, "An Essay Concerning Human Understanding", tabula-rasa, the secondary quality — philosopher
- around 1700: Jakob Bernoulli, the law of large numbers — mathematician
- 1703: Leibniz, "New Essays on Human Understanding" — philosopher
- 1715 - 16: Leibniz-Clarke correspondence (*cf.* Sec. 9.2.2)
- 1739: Hume, "A Treatise of Human Nature" — philosopher
- **1781: Kant, "Critique of Pure Reason"** — philosopher
- 1788: Lagrange, Mécanique analytique — scientists
- 1789: Lavoisier, Elementary Treatise on Chemistry — scientists
- 1799-1825: Laplace, Traité de mécanique céleste — scientists

♠**Note 7.1.** The law of large numbers, discovered by J. Bernoulli(1654 - 1705), is as follows.

(♯) Suppose a fair coin (with a probability of heads equal to 1/2) is tossed N times. Then it holds that

$$\lim_{N\to\infty} \frac{\text{Number of times the coin comes up Heads}}{N} = 1/2$$

$$\lim_{N\to\infty} \frac{\text{Number of times the coin comes up Tails}}{N} = 1/2$$

the law of large numbers
J. Bernoulli (1654-1705)

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I think that Bernoulli's achievement equals Galileo's achievement. That is,

$$\begin{cases} \text{Scientific pioneer in the realistic worldview} \cdots \text{Galileo} \\ \text{Scientific pioneer in the idealistic worldview} \cdots \text{J. Bernoulli} \end{cases}$$

It is difficult to identify the founder of the probability theory to one person. But, I think that J. Bernoulli is one of the most important founders (e.g., P.S. Laplace (1749-1827), A. Kolmogorov (1903-1987), etc.).

7.2 Francis Bacon (1561 - 1626): The father of empiricism, Inductive reasoning


7.2.1 How to create science: The exclusion of idols (=prejudice, preconception)

F. Bacon has been called the father of empiricism. He was the greatest enlightener of "scientific revolution" as follows.

- the birth of worldview **Movement** (Hērakleitos, Parmenides, Zeno, Aristotle, Ptolemaios) —"purpose" (and no experiment) / Aristotle's spirit :(About 1500 years)→ the birth of modern science **Causality (and experiment)** (Galileo, Bacon, Descartes, Newton)

In 1620, he proposed "how to create science" (called inductive reasoning, or induction principle) in his book "Novum organum".

(A):Induction principle (by bad idols), how to create science

His proposal is as follows.

(A) ①:Exclusion of bad idols
⟶②:data, collection by observation and experiment⟶③:scientific theory, principle

Let us explain this in what follows.

① : Firstly, we have to exclude bad idols (=prejudice, preconception) Here, idols is as follows.
Idols of the Tribe: prejudice due to sense organs
Idols of the Cave: prejudice due to custom, the education
Idols of the Market: prejudice due to language
Idols of the Theatre: prejudice due to thought, theory
② : Next, we have to collect data by observation, experiments,
③ : Lastly, find the essence from the data, and build science theory.

Here, "②+③" is called "induction".

Recall that Greek philosophy starts from "principle", e.g., "the arche (= the first principle of all things) is ◯◯", or, Euclid advocated geometric axiomatization, and a lot of theorems were derived from Euclid axioms, that is,

$$(\mathrm{B}_1)\ \text{Mathematics:}\qquad \text{Euclid axioms} \xrightarrow{\text{deduction}} \text{theorems}$$

On the other hand, Bacon emphasized the importance of observation (or experiments), that is,

$$(\mathrm{B}_2)\ \text{Science:}\qquad \text{data, collections} \xrightarrow{\text{induction}} \text{principle}$$

which is the scientific method proposed by Bacon (who was called the father of empiricism).

7.2.1.1 Isaac Newton (the exclusion of bad idols) (1642-1727)

Newton said:

"I frame no hypotheses"

And he practiced Bacon's induction principle, and proposed Newtonian mechanics as follows.

(C): ①exclude bad idols (i.e., Aristotle's purpose, Geocentrism) ⟶ ②Data collection (due to Tycho Brahe, Kepler, Galileo) ⟶ ③Science theory (Newtonian mechanics)

♠**Note 7.2.** This may be said on the philosophy side. The next section (good idols) must be added.

7.2.1.2 Isaac Newton (good idols) (1642-1727)

Bacon's induction principle is not simple. there is another way (by good idols) such as

(D): Induction principle (by good idols)

Induction principle (by good idols) is as follows.

(D) ①believe good idols ⟶ ②Data collection ⟶ ③Science theory

Newton said:

"I frame no hypotheses"

And he practiced Bacon's induction principle (good idols), and proposed Newtonian mechanics as follows.

(E): ①believe good idols (i.e., Causal relation) ⟶ ②Data collection (due to Tycho Brahe, Kepler, Galileo) ⟶ ③Science theory (Newtonian mechanics)

♠**Note 7.3.** Although ironically,

(♯) Bacon, who proposed the exclusion of idols, was also one of discoverers of "good idols" called "causal relation".

If so, what Bacon wanted to say really may be

$$\big[\text{ bad idols }\big] = \big[\text{ dogmatism in Scholasticism (or, Aristotle's purpose)}\big]$$

♠**Note 7.4.** S. Weinberg (1933 -2021), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said in his book [123] "To explain the world; The discovery of modern science" as follows:

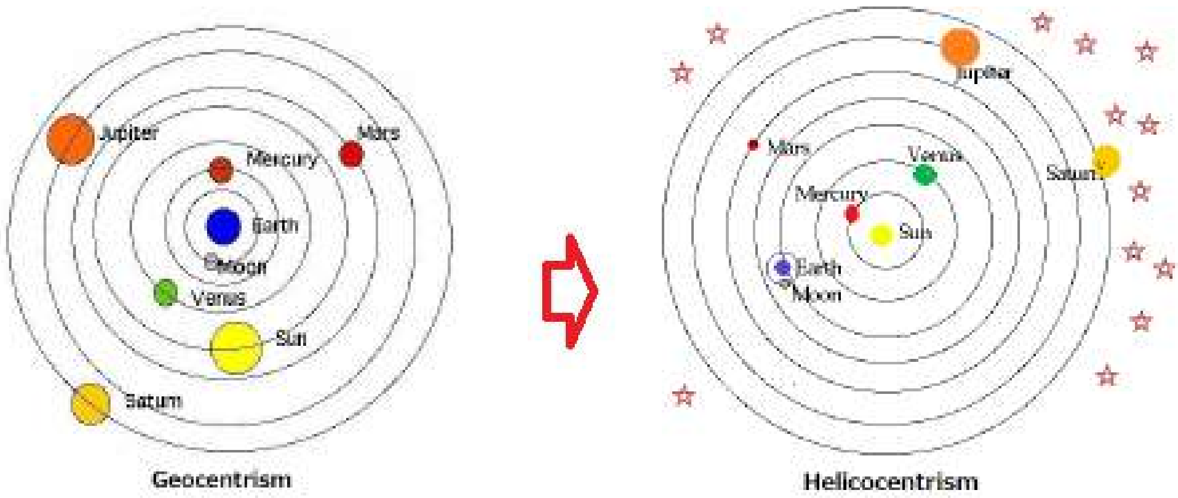
($\sharp_1$) [in Chapter 13]: *They (i.e., Bacon and Descartes) are, in my opinion, the two individuals whose importance in the scientific revolution is most* **overrated***. .... Scientists in the seventeenth and eighteenth centuries would invoke Bacon as a counterweight to Plato and Aristotle, ... It is not clear to me that anyone's scientific work was actually changed for the better by Bacon's writing. Galileo did not need Bacon to tell him to do experiments, and neither I think did Boyle or Newton.*

From the pure scientific point of view, I almost agree to the above opinion. However, it should be noted that the scientific revolution was not only achieved by scientists, but by the collective power of philosophers, enlighteners, astronomers, mathematicians, adventurers, artists, educators, politicians, religionists, and the general public. The difficult and important thing is to get public support for the scientific revolution. I think Bacon did this work well. He enlightened the method to create science (i.e., the importance of causality and experiment) to the general public, not the scientists. As I will show in the next chapter, I consider as follows:

($\sharp_2$) the greatest task of modern philosophy (Descartes, Kant, analytic philosophy) is to enlighten the general public about science.

Even so, I respect Dr. Weinberg's honest opinion.

7.3 From Geocentrism to Heliocentrism



7.3.1 What is "Geocentrism vs. Heliocentrism"?

As mentioned in Chapter 5,

- Heliocentrism due to Aristarchus (BC.310 - BC.230) is based on the arguments:

  The big sun cannot go around the small earth.

  I think his Heliocentrism to have reached the scientific level. (*cf.* Sec. 5.3).
- Geocentrism due to Ptolemaios (AD.83 - AD.168) can explain the motion on planets by epicycle theory. Thus, I also think his Geocentrism to have reached the scientific level at the time. (*cf.* Sec. 5.6).

However, Heliocentrism due to Copernicus (1473 - 1543) might be controversial whether it had reached the scientific level. For example, there is an opinion that

- At the time, Europe is in the cold period, the masses were hungry for "the sun". The public was hungry for the sun central principle. Therefore, there is a foundation that allows the germination of Heliocentrism.

In spite of the above, I want to assert that Copernicus is a great scientists. The reason I think so is written in the following Note 7.5.

♠**Note 7.5.** S. Weinberg (1933 -2021), An American physicist, won a Nobel Prize of physics in 1979, said in his book [123] "To explain the world; The discovery of modern science" as follows:

(♯) [Chapter 11] *Copernicus could not claim in the Commentariolus that his scheme fitted observation better than that of Ptolemy. For one thing, it didn't. Indeed, it couldn't, since for the most part Copernicus based his theory on data he inferred from Ptolemy's Almagest, rather than on his own observations.* **Instead of appealing to new observations, Copernicus pointed out a number of his theory's advantages**.

It was very interesting to me that Copernicus, the flag bearer of the scientific revolution, valued beauty over observation. In fact, my conclusion in section 7.4 of this text,

”Heliocentrism vs. Geocentrism,” is also ”Heliocentrism was chosen because it is more beautiful.” I’ve grown to love Copernicus. Thus I would like to regard him as the great scientist.

The following question is the main theme of this chapter.

(A): What is ”Heliocentrism vs. Geocentrism”?

Now,

(A_1) Note that motion is relative. Thus, if the earth is assumed to be at center, the sun goes around the earth (i.e., Geocentrism). Also, if the sun is assumed to be at center, the earth goes around the sun (i.e., Heliocentrism). Hence,

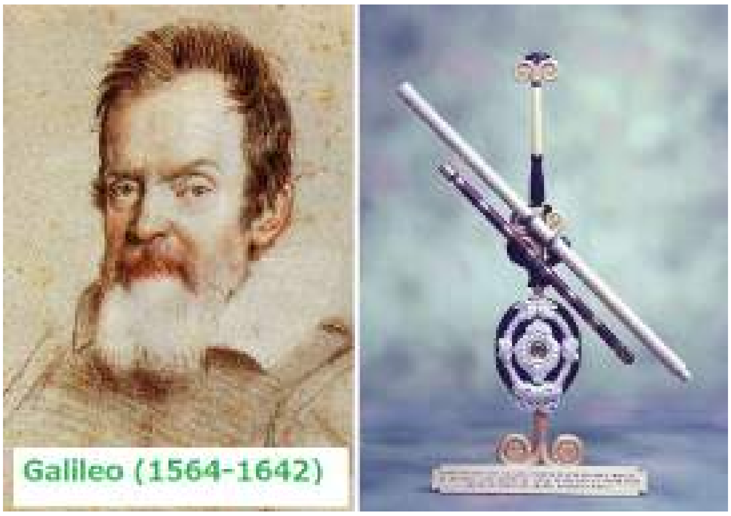
The difference between Heliocentrism and Geocentrism may be only a difference of how to take the coordinate system.

Or,erael

(A_2) Under what worldview should ”Heliocentrism vs. Geocentrism” be considered?

7.3.2 Somehow ”from Geocentrism to Heliocentrism”

In what follows, I will arrange the history of ”Heliocentrism vs. Geocentrism”.



- 1510: Copernicus, Heliocentrism in the Commentariolus
- 1600: G. Bruno, who supported heliocentrism along with Galileo, was burned at the stake by the Vatican

Analyzing the enormous data obtained by Tycho Brahe’s steady astronomical observation, Kepler found the following laws:

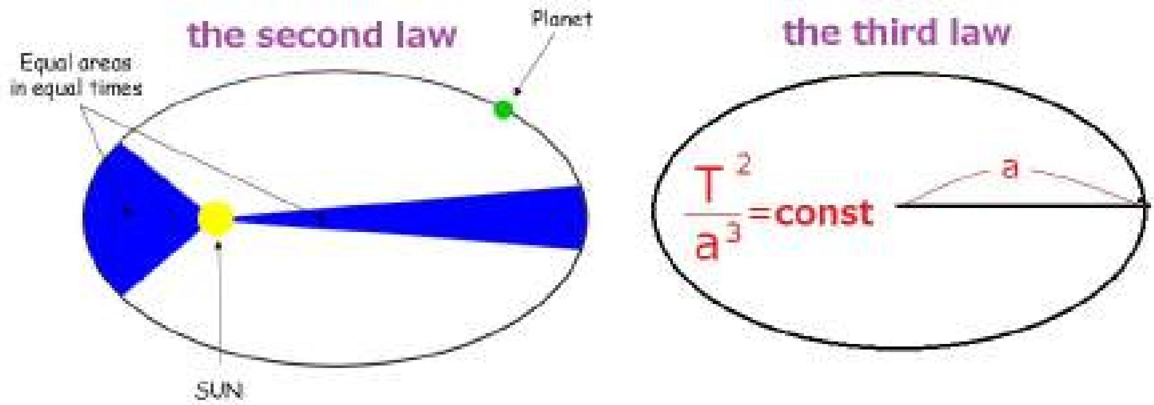
- Kepler’s laws of planetary motion:
  1609:

($\sharp_1$) The first law of elliptical orbits,

($\sharp_2$) The second law of areal velocity,

1619:

($\sharp_3$) The third law of Periods:



And

1610: Galileo found the moons of the Jupiter by his telescope of the self-made

And further,

1633: Galileo said "And yet, it moves" in the Trial of Galileo

♠**Note 7.6.** In the next year of 1642 when Galileo died, Isaac Newton was born in the British country.

1687: Newton "Principia"

In this way, we think:

(B) Somehow the air "to Heliocentrism" has been formed.

Still, I am worried about this problem (A)"What is 'Geocentrism vs. Heliocentrism'?" That is,

(C_1) Did both Galileo and the church understand the essence of 'Geocentrism vs. Heliocentrism'?

which is equivalent to

(C_2) In order to win the definitive victory, what should they (Galileo or the church) have done?

In order to answer to this question, we first have to clarify the meaning of "Heliocentrism vs. Geocentrism".

7.3.3 "Geocentrism vs. Heliocentrism" is the problem of the worldview

As mentioned in the previous section, how to decide "Geocentrism vs. Heliocentrism" is somewhat difficult. That is because

(D) Thus, if the earth is assumed to be at center, the sun goes around the earth (i.e., Heliocentrism). Also, if the sun is assumed to be at center, the earth goes around the sun (i.e., Geocentrism). Hence, The difference between Heliocentrism and Geocentrism is only a difference of how to take the coordinate system.

In the same sense, we say that

(E) **No matter how much there are exact observation data, we cannot decide "Geocentrism or Heliocentrism"**

In the famous trial of Galileo, he said

"And Yet It Moves"

However, I wonder if Galileo knew the (E)?

♠**Note 7.7.** No matter how much there are exact observation data, we cannot decide "Geocentrism vs. Heliocentrism" we have to need the worldview. Namely,

($\sharp_1$) it is a matter of course that there is no science without measurement

However, we believe that

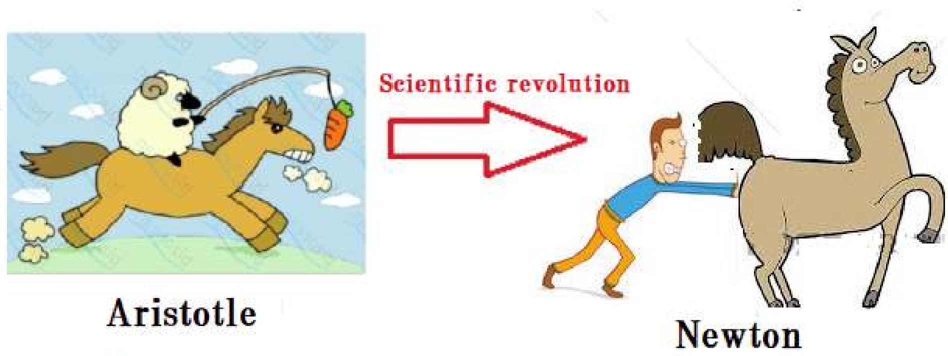
($\sharp_2$) there is no science without worldview

Thus, as seen later, we cannot decide "Geocentrism vs. Heliocentrism" without worldview

7.3.4 The Galileo legend; Leaning Tower of Pisa, Trial of Galileo

The worldview of Aristotle has kept its position for 1500 years such as (*cf.* Sec. 4.2.1):

- the birth of worldview: Movement (Hērakleitos, Parmenides, Zeno, Aristotle, Ptolemaios) —"purpose" (and no experiment)→ Aristotle's spirit :(About 1500 years) the birth of modern science: Causality (and experiment) (Galileo, Bacon, Descartes, Newton)



Thus, Aristotle's worldview is not a so bad worldview.

But, this worldview was a little inconvenient to organize the data, obtained by technological innovation (e.g., telescope, navigation, etc.). The history of the increase of the inconvenience is as follow:

$$[\text{Copernicus}] \Rightarrow [\text{Kepler}] \Rightarrow [\text{Galileo}]$$

However, these are not sufficient to decide "Geocentrism vs. Heliocentrism". It is a matter of course that there were excellent persons in the church. And they might think:

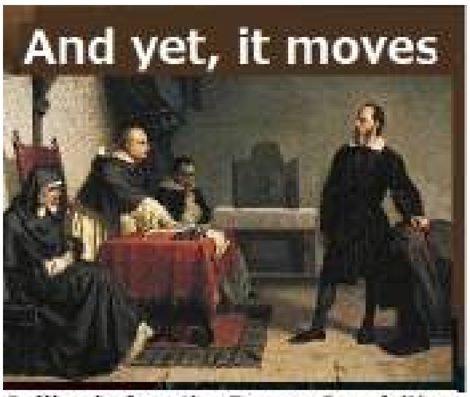
- if they insisted that motion is relative, they did not lose the dispute, at least, they could make "Geocentrism vs. Heliocentrism" endless dispute.

Galileo legend Galileo (1564 - 1642) was an active leader of the overthrow of the worldview of Aristotle, and his targets were the following (F_1) and (F_2):

(F_1) Ptolemaic Geocentrism
(F_2) Aristotelian purpose such as "Heavy objects fall faster"

Concerning the two, We have two episodes called "Galileo legend" as follows.

For (F_1), "And Yet it moves" in trial of Galileo
For (F_2), Leaning Tower of Pisa



Galileo before the Roman Inquisition
(by Cristiano Banti, 1857)

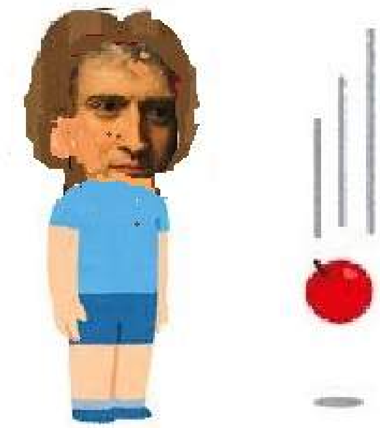
Thus, Galileo might think it was a matter of measurement data, but the church might think it was a matter of worldview. Therefore,

- at the time (1633) of the trial of Galileo, the Church was convinced that it would not lose its argument with Galileo and, at worst, could bring it into an endless dispute.

An endless dispute implies the win of the church. The church is not so stupid.

No way, the church did not think that Newton would appear

No one would have predicted the appearance of Newton.



After all, Galileo was the active leader of the overthrow of the worldview of Aristotle, but he could not propose the new worldview. In that sense, the Galileo legend is just the beginning of Newton's appearance

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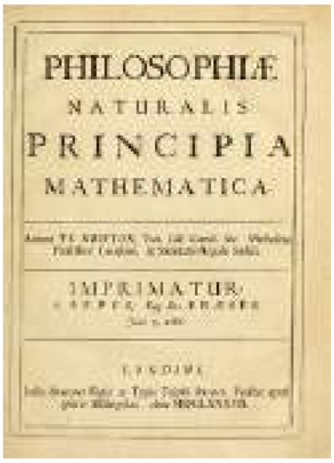
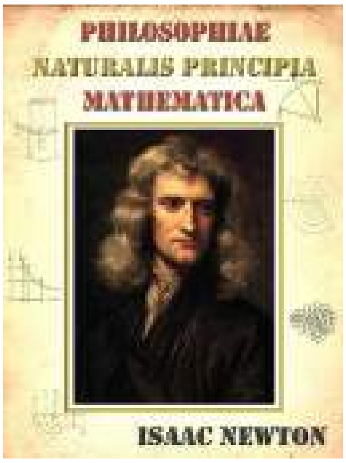
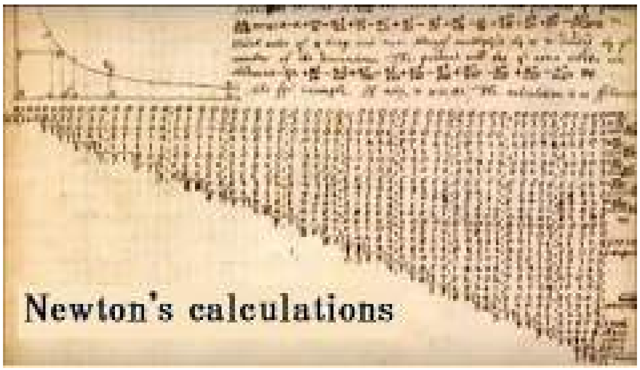
7.4 Principia; Newtonian worldview

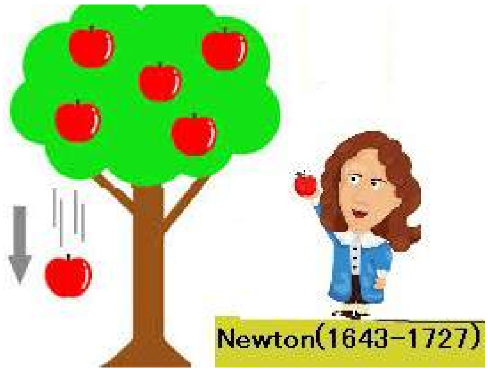
7.4.1 Principia (1687)





"PhilosophiæNaturalis Principia Mathematica" (in short, "Principia") , written by Newton (1687), is the most famous and important book in science. Three laws of Kepler were derived from three laws of dynamics and the law of universal gravitation. Principia was written based on elementary geometry and not the differential and integral calculus. Why did Newton (= advocate of differential and integral calculus) not write Principia based on differential and integral calculus? Although there may be several opinions for this question, The work (based on differential and integral calculus) was succeeded by Leibniz, J. Bernoulli, Euler, d'Alembert, Lagrange and Laplace, etc. and was completed.





7.4.2* What is "Geocentrism vs. Heliocentrism"? After all, the worldviewism

The following biggest paradigm shift in the history of science is as follows.

(A) Motion [Motion function method: (Parmenides, Zeno, Aristotle, Ptolemaios) *cf.* Sec. 2.4]

$\xrightarrow[\text{paradigm shift}]{}$

Causal relation [Kinetic differential equation method: (Newton)]

That is, we see:

(B): Paradigm shift [Newtonian mechanical worldview]

Now,

(B) Aristotle's worldview (purpose) $\xrightarrow[\text{paradigm shift}]{}$ Newtonian worldview (causal relation)

Here, Newtonian worldview means Newtonian mechanics, that is,

Newton's kinetic equation (i.e., the chain of causal relations)
+ the law of universal gravitation

Recall that the main theme of this text is the worldviewism(*cf.* Sec. 1.1). As mentioned frequently up to this point,

(C_1) The argument in ordinary language (or, in the motion function method (*cf.* Sec.2.3.4)) is fuzzy, and thus, "Geocentrism vs. Heliocentrism" cannot be decided. Thus, we need a new worldview.

In Principia, Newton proposed Newtonian mechanics (i.e., Newtonian worldview) and showed that

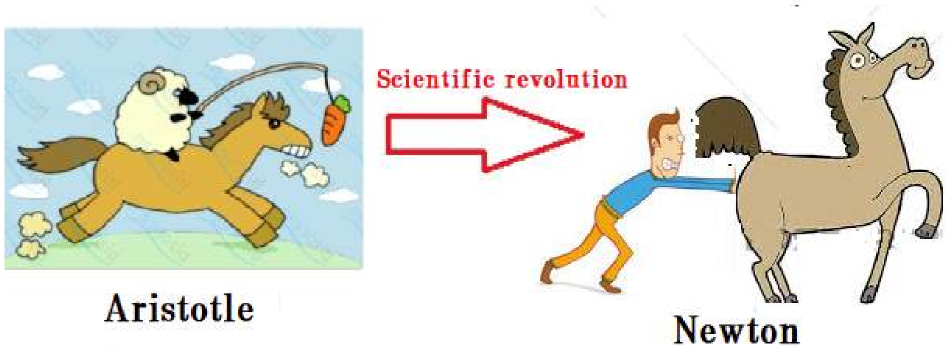
(C_2) **When the motion of the sun and the planets is studied, the calculation becomes easy under the assumption that the planets go around the sun.**

Therefore, even the definitions "center" and "go around" depend on the worldview. After all, we conclude that

(D) "Geocentrism or Heliocentrism" is not an issue that can be settled, no matter how accurate the observations are. That is, "Geocentrism vs. Heliocentrism" is not the problem of measurements, but the problem of the worldview.

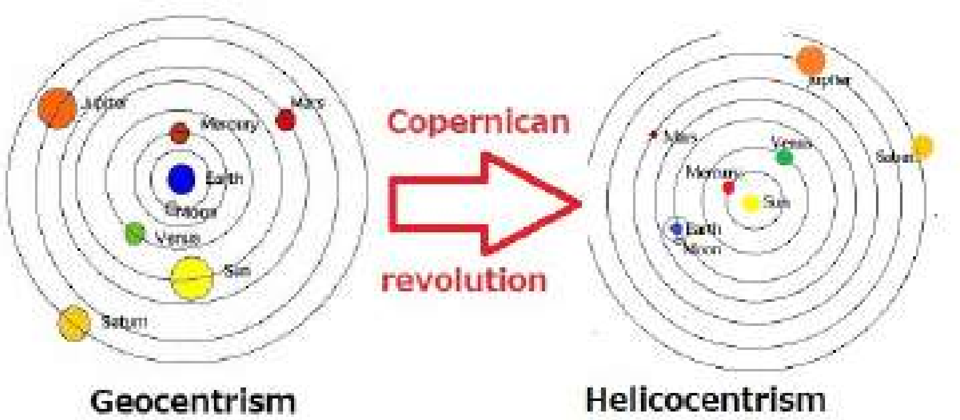
♠**Note 7.8.** (i): There are many ways to understand the "Geocentrism vs. Heliocentrism". Since the main theme of this text is "Nothing can be said without a worldview," I wrote a fiction based on this theme.
(ii): Next is said to be the three major discoveries of modern science

($\sharp_1$) Aristotelian worldview (purpose) $\longrightarrow$ Newtonian worldview (causal relation)

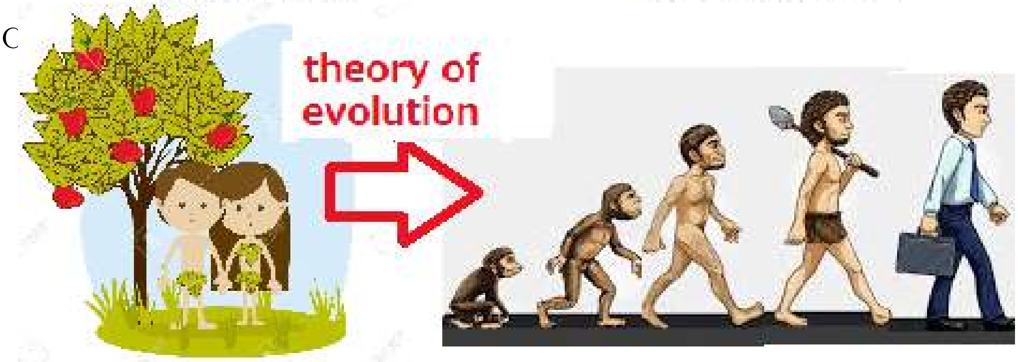


For further information, see my homepage

($\sharp_2$) Ptolemaic system: Geocentrism $\longrightarrow$ Copernican system: Heliocentrism



($\sharp_3$) C



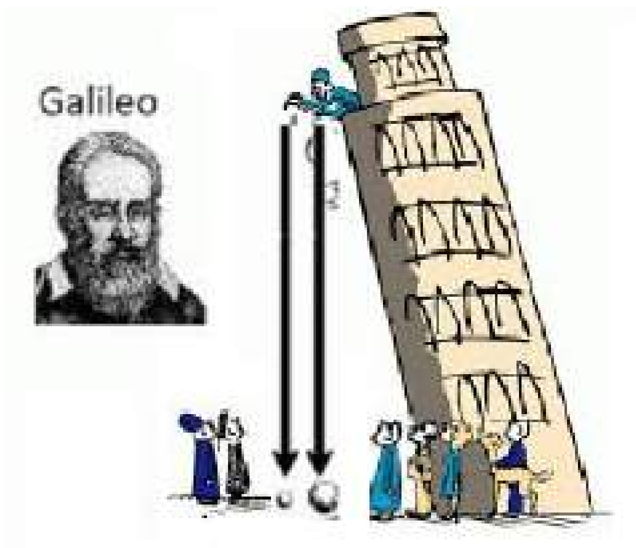
However, it should be noted that ($\sharp_2$) is a consequence of ($\sharp_1$). There may be a reason to consider that ($\sharp_2$) is a great episode of the birth of ($\sharp_1$).

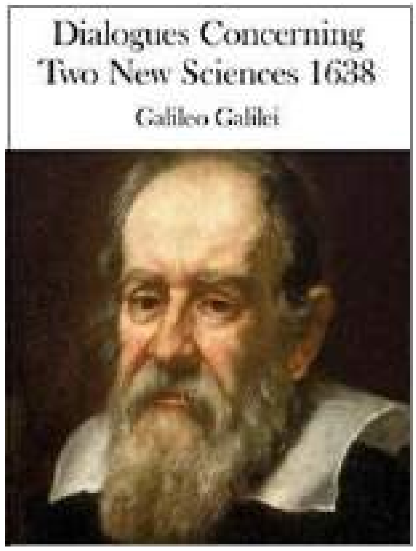
7.5 Appendix; About "Dialogues concerning two new sciences" by Galileo Galilei

The following issues are discussed

- If you drop a heavy iron ball A and a light iron ball B at the same time, which one will fall to the ground faster?

The only way to settle this issue would be to conduct an experiment. In fact, Galileo conducted this experiment on the Leaning Tower of Pisa, and proved that "two iron balls fall at the same time". This is the famous "Leaning Tower of Pisa legend". So far, so good.





But, in "Dialogues concerning two new sciences", Galileo must have wanted to say something like this:

- It is not necessary to experiment to know that two iron balls fall at the same time. You don't have to go to the Leaning Tower of Pisa to conclude that "two iron balls fall at the same time" if you think about it while lying in bed in your own home.

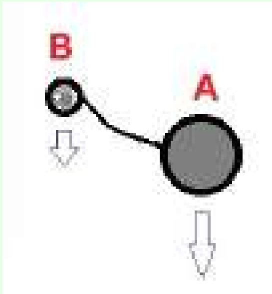
I will explain Galileo's idea in the following.

(A):"Dialogues concerning two new sciences" by Galileo Galilei ref. [24]

Here, assume that

(A_1) The heavier iron ball A falls faster than the lighter ball B.

If so, at what speed will the iron ball [A+B], which is made up of a heavy iron ball A and a light iron ball B connected by a string, fall?



Common sense tells me that

(A_2) The iron ball [A+B] will fall at a speed halfway between the heavy iron ball A and the light iron ball B

However, this contradicts (A_1). Since the iron ball [A+B] is heavier than the heavier iron ball A, according to the assumption (A_1), the iron ball [A+B] should fall faster than the heavier iron ball A. The assumption (A_1) is wrong because it is strange that the falling speed increases only by connecting with a string. Assuming that the lighter iron ball falls faster than the heavier one at (A_1) is a contradiction as well. Therefore,

(A_3) without experiment (and without Newtonian mechanics) , we can conclude a heavy iron ball and a light iron ball will fall at the same time.

If it is true, was the "Leaning Tower of Pisa" experiment unnecessary?

The above discussion seems both strange and obvious. I would like the reader to consider this exercise.

♠**Note 7.9.** For convenience, consider two kinds of propositions such as

(B_1) analytic propositions: propositions grounded in meanings, independent of matters of fact.

(B_2) synthetic propositions: propositions grounded in fact.

If so, we consider:

(C_1) Is the proposition "A heavy ball and a light ball fall at the same speed" an analytic proposition or a synthetic proposition?

(C_2) Does (A_3) imply that the problem (C_1) is a analytic proposition without Newtonina mechanics?

I don't have a definitive answer to this question, but I can say the following in terms of quantum language:

(D) all proposition in QL are synthetic (Sec. 12.3).

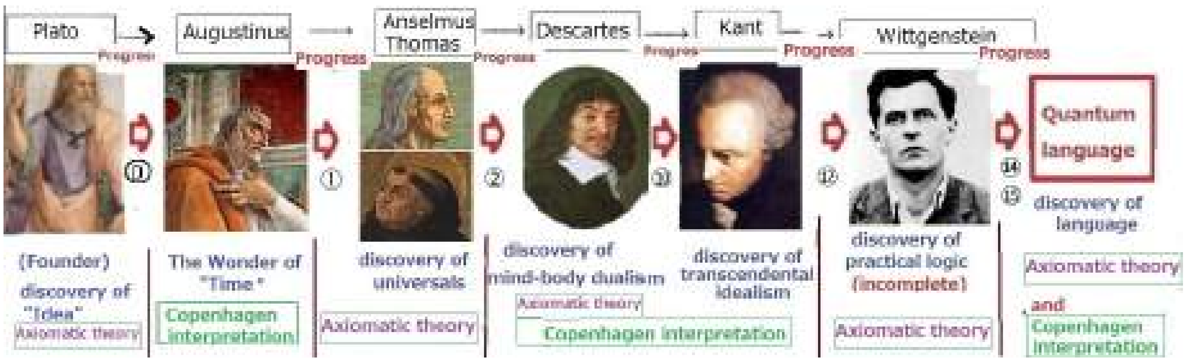
In addition, the problem (A) raises the following issue

(E) There may be a worldview of common sense (or the worldview called common sense). (Sec. 12.2.3: Wittgenstein's paradox)

Chapter 8

Modern philosophy: Continental rationalism: Descartes, Spinoza, Leibniz

The genealogy of dualistic idealism is as follows

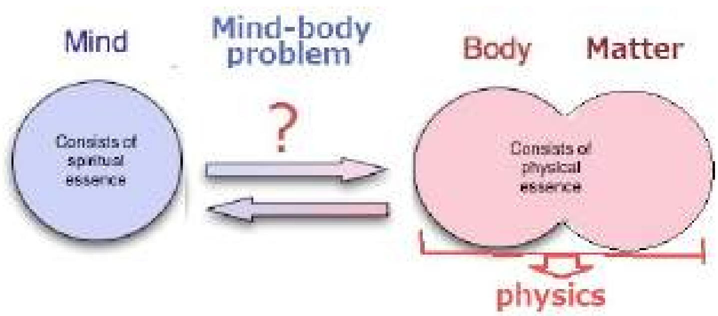


Scholastic philosophy was "philosophy for the priests." Descartes opened the door to modern philosophy by proposing a "philosophy for the general public. He raised two problems: the mind-body problem and the subjectivity problem (the so-called Descartes problem (=Cartesian problem)).

Descartes problem = mind-body problem and subjectivity problem

Here,

($\sharp_1$) mind-body problem: How are the mind and body connected?
($\sharp_2$) subjectivity problem: Is the world I see the same world you see?



In summary:

($\sharp_3$)

(classical) Descartes problem (Establish mind-body dualism!)	mind-body problem (Continental rationalism)	⋯How are the mind and body related?
	subjecyive problem (British empiricism)	⋯Is the world I see the same world you see?

Some readers may ask, "Why is the Cartesian problem important?" or "Is it a philosophical problem?". Therefore, to state the conclusion first, the Cartesian problem is the same as the following "(modern) Cartesian problem".

($\sharp_4$)

(modern) Descartes problem (Establish mind-body dualism!) (Establish QL!)	mind-body problem (Continental rationalism)	⋯ Find Axioms including the key-words (mind, body, matter)
	subjectivity problem (British Empiricism)	⋯Find the interpretation concerning this Axioms

"Why are ($\sharp_3$) and ($\sharp_4$) the same?" will be explained in this section.

Modern philosophy (Descartes-Kantian philosophy) enjoyed 300 years of prosperity. I think the reason is as follows.

($\flat_1$) The Cartesian problem ($\sharp_3$) was presented using plain language [($\sharp_1$) and ($\sharp_2$)] that the general public could understand

($\flat_2$) As a matter of consequence, Plato's Idea is a magic word such as

$$[\text{Idea}] \xrightarrow[\text{progress}]{} [\text{universal}] \xrightarrow[\text{progress}]{} [\text{body}]$$

$$\xrightarrow[\text{progress}]{} [\text{secondary quality}] \xrightarrow[\text{progress}]{} [\text{observable}]$$

($\flat_3$) The general public expected Descartes-Kant philosophy to be a counterweight (i.e., rival) to Newtonian mechanics

The often stated opinion that "epistemology could have been done without" is an opinion worth listening to, but we firmly disagree with it.

In this chapter, I will explain Descartes philosophy (i.e., mind-body dualism) and present my solution to Descartes problem. In addition, I will discuss continental rationalism (i.e., Spinoza and Leibniz).

8.1 Popularization of philosophy; Descartes=Kant philosophy

8.1.1 Background of Descartes=Kant philosophy

The historical background of Descartes=Kant philosophy is as follows.

Table 8.1. **Scientific revolution; Chronological table**

Historical Background of "Discourse on the Method": R. Descartes

- Scholastic philosophy, Thomas Aquinas (1225 - 1274), Ockham (1285 - 1347)), The Black Death pandemic that struck Europe between 1347 and 1351 was the worst in history, claiming the lives of one-third of the European population.
- 1450: Gutenberg's printing press — inventor
- 1453:The Eastern Roman Empire extinction (Constantinople surrender)
- 1492: Columbus, discovery of the American Continental — navigator
- 1498: Vasco da Gama, discovery of the sea route to India — navigator
- 1500s: Leonardo da Vinci, "Mona Lisa's smile" — artist
- 1510: Copernicus, Heliocentrism — scientist
- 1510: Raffaello, "The School of Athens", Admiration to ancient Greece — artist
- 1517: Luther, Protestant Reformation — religionist
- 1519 - 20: Magellan, the first circumnavigation of the Earth
- 1540s: Michelangelo, "The Last Judgment" — artist
- 1600: G. Bruno, who supported heliocentrism along with Galileo, was burned at the stake by the Vatican — philosopher
- 1609∼1619: Kepler's laws of planetary motion — scientist
- 1610: Galileo, A telescope was made and moons of Jupiter were found — scientist
- 1620: F. Bacon, "knowledge is power",The father of British Empiricism — illuminator
- 1633 Galileo's trial "And yet it moves" — scientist
- **1637: "Discourse on the Method", Rene Descartes (1596 - 1650)**, the father of modern philosophy, Cogito proposition — philosopher
- 1670: Pascal, "Pensèes" — enlightener
- 1677: Spinoza, "Ethica" — enlightener
- 1685 - 1750: Bach — artist
- **1687: Newton, "Principia"** — scientist
- 1688: Glorious Revolution
- 1690: John Locke, the father of British Empiricism, "An Essay Concerning Human Understanding", tabula-rasa, the secondary quality — philosopher
- around 1700: Jakob Bernoulli, the law of large numbers — mathematician
- 1703: Leibniz, "New Essays on Human Understanding" — philosopher
- 1715 - 16: Leibniz-Clarke correspondence (*cf.* Sec. 9.2.2)
- 1739: Hume, "A Treatise of Human Nature" — philosopher
- **1781: Kant, "Critique of Pure Reason"** — philosopher
- 1788: Lagrange, Mécanique analytique — scientists
- 1789: Lavoisier, Elementary Treatise on Chemistry — scientists
- 1799-1825: Laplace, Traité de mécanique céleste — scientists

8.1.2 Philosophy for the general public (≈ literary philosophy≈ science without formulas≈ Copenhagen interpretation)

The persons Scholastic philosophers talk about philosophy are priests. However, in modern ages (i.e., since the invention of the Gutenberg printing press), the persons philosophers talk about philosophy are the general public. In this sense, modern philosophy may be a kind of literature or enlightenment philosophy. Descartes was the first to talk about philosophy to the general public.

Descartes's "Discourse on the Method" was the first philosophical book written in French, whereas philosophical works at the time were usually written in Latin. It was written in plain language for the general public. Thus I think that

(A_1) Descartes-Kant philosophy was a philosophy for the general public

That is, I think that Descartes-Kant philosophy is a type of enlightenment philosophy. As can be seen in Table 8.1, at the beginning of the scientific revolution, Descartes and Kant philosophy was born to educate the general public on how to deal with science.

On the other hand, in Chap. 6, I asserted the following.

(A_2) Scholastic philosophy was a philosophy for professionals.

As shown throughout this text, the difference between "philosophy for the general public" and "philosophy for professionals" often caused confusion in Western philosophy. For example, the philosophy of science was originally supposed to be a "philosophy for professionals (i.e., scientists)," but historically it has been presented as a "philosophy for the general public." Because of this, the philosophy of science was not accepted by scientists. (See ref. [78], in which "philosophy of science for scientists" is written.)

It is well known that quantum mechanics is formulated as follows.

			axiomatic theory		empirical theory
(B)	quantum mechanics (dualism)	=	Axiom 1 (measurement) Axiom 2 (causality)	+	Copenhagen interpretation
			math style		narrative style

That is, there are two main components: the axioms (laws) and the Copenhagen Interpretation (a manual on how to use the axioms).

Therefore, if Descartes-Kant philosophy and quantum mechanics are related, and if Descartes-Kant philosophy is philosophy for the general public, then,

- Descartes-Kant philosophy must resemble the Copenhagen interpretation (since Axioms 1 and 2 are too mathematical).

Also, recall that

			axiomatic theory		empirical theory
(C)	Newtonian mechanics (monism)	=	Axiom (causality) Newtonian equation	+	No interpretation (or obvious interpretaion)
			math style		narrative style

Thus, Newtonian mechanics has no "interpretation" (or if it does, it is an obvious interpretation). This suggests that a philosophy founded on Newtonian mechanics is not very promising. In fact, nominalism (by Abelard, Ockham, and others) has not produced a fruitful philosophy. Statistics (= dynamical system theory), which is a mathematical generalization of Newtonian mechanics, has been regarded as a kind of applied mathematics rather than philosophy. In other words, a "theory without interpretation," such as (C) above, is hardly a philosophy for the general public. That is,

(D) Monism is hardly a philosophy.

The above can be summarized as follows.

(E)
- Philosophy for Professionals · Axiomatic theory
  - The problem of universals, Continental Rationalism, Statistics, Philosophy of Science (Hempel, etc.), QL (Axiomatic theory)
  - (A great scientific success has been statistics, but it is not generally regarded as a philosophy since it does not belong to dualism)
- Philosophy for the Masses · Empirical theory
  - Descartes-Kant Epistemology, Philosophy of Science (Popper, etc.) Philosophy of Mind, QL (Copenhagen Interpretation)
  - (Gaining the support of the masses tends to be considered a success)

The caveat above is that the difference between professionals and amateurs is not the level of research. One can think of it as "professional=mathematics required (especially Axioms

1 and 2)" and "amateur=explanation of rules that cannot be expressed in mathematical formulas (Copenhagen Interpretation)". For example, although not mentioned much in this text, various examples discussed in the philosophy of mind (e.g., the qualia problem and the brain in a vat (Sec. 9.4)) are very useful for forming Copenhagen interpretations.

♠**Note 8.1.** Many readers will ask the following question.

($\sharp_1$) While quantum mechanics has an "interpretation (i.e., the Copenhagen interpretation)," classical mechanics and the theory of relativity do not have "interpretations". Why is this?

($\sharp_2$) If the Copenhagen Interpretation is a "manual on how to use axioms", is there such a thing as a complete manual? (*cf.* Sec. 12.2.3 Wittgenstein's paradox).

These questions are one of the motivations for my interest in the study of the Copenhagen interpretation. I do not yet know the exact answer to these questions, but so far I believe that

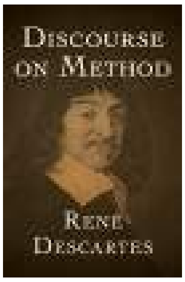
($\flat_1$) All theories, except mathematics, possess some "interpretation" of some sort. Thus, both classical mechanics and relativity have "interpretation". However, since they are "common sense and self-evident interpretation," there was no particular inconvenience in not being aware of the "interpretation".

($\flat_2$) I believe that there is no perfect interpretation. I believe that all theories, not only quantum mechanics (except mathematics), cannot be perfect (*cf.* Sec. 12.2.3 Wittgenstein's paradox).

I don't quite understand the "Wittgenstein paradox", but this paradox is related to the above.

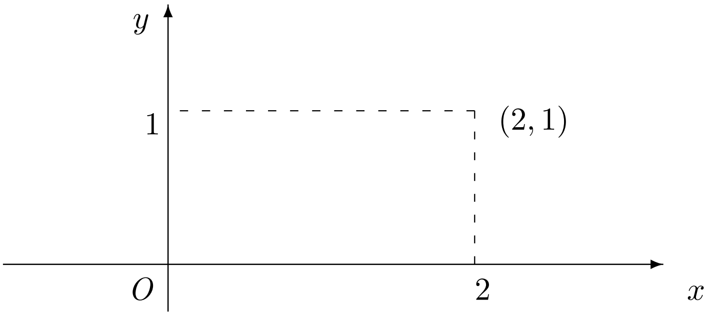
8.2 The cogito proposition "I think, therefore I am" is a catchphrase of modern philosophy

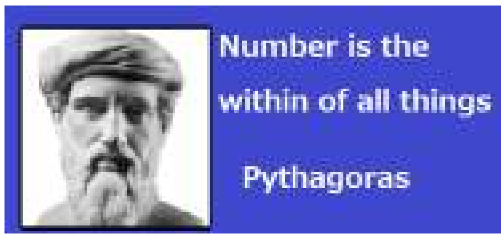
8.2.1 Discourse on the Method (1637)



Descartes(1596-1650) was a French philosopher, mathematician, and scientist. He is also widely regarded as one of the founders of modern philosophy.

♠**Note 8.2.** Descartes' well-known mathematical achievement is the discovery of Cartesian coordinates or analytic geometry. There is even a plausible legend that Descartes, who was sleeping in his bed, came up with the system in order to track the position of a fly moving around the ceiling. However, Cartesian coordinates have nothing to do with Cartesian philosophy.





[Cartesian coordinates; parameterization]

Let us study the most famous book in philosophy: "Discourse on the Method (1637)" by René Descartes, which is the abbreviation of "The method of rightly conducting one's reason and of seeking truth in the sciences". Probably after Descartes read Bacon's "Novum organum", he decided "Start from the unquestionable truth". And he found the most famous philosophical proposition (= cogito proposition):

"I think, therefore I am"

That is, Descartes think:

I think that 'I think, therefore I am'

In spite that this is a most doubtful proposition (and thus, it belongs to pseudoscience and not in quantum language), Descartes believed that it was the unquestionable truth.

And further, bearing in mind Euclid's success in geometry, Descartes was convinced that

(A) Every statement derived from the cogito proposition are absolutely trusted (despite the fact that there is nothing that can be derived from the cogito proposition)

This is just "The method of rightly conducting one's reason and of seeking truth in the sciences". As I have said many times, Descartes issued his philosophy to the general public, and I believe he used forced logic as a technique to communicate his philosophy.

Summing up, we see:

Proclaim 8.2. The first principle (= cogito proposition) in philosophy
Descartes doubted everything. And he arrived in the cogito proposition which has no doubted room. That is, he arrived in

(B_1) "I think, therefore I am."

And, he proclaimed that the cogito proposition (B_1) is the first principle in philosophy, and asserted:

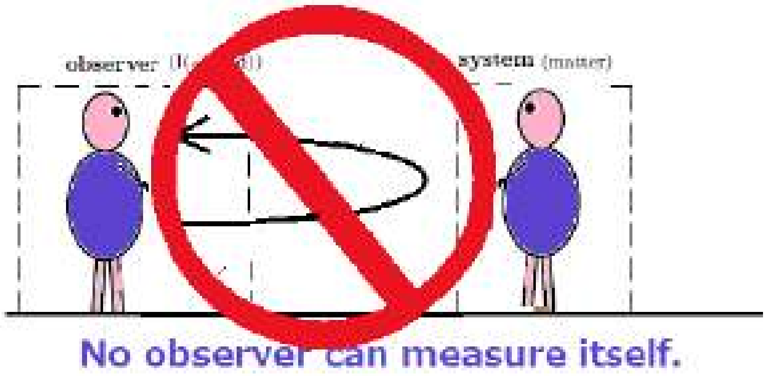
(B_2) "Everything deduced from the cogito proposition is absolutely correct."

♠**Memo 8.1.** This was enough because Descartes aimed for a philosophy for the general public. If Descartes aimed at a philosophy for philosophers, he had to start with the question "What is a proposition?" (See Chap. 11). And since the cogito proposition is not a (scientific) proposition, (B_1) and (B_2) are absurd. However, as we will see in the next section (Sec. 8.3), a miracle occurs. It is not clear whether all of this is Descartes' calculation or not, but I think that Descartes was convinced that "mind-body dualism" was the way forward for philosophy.

8.2.2* Regrettably, the cogito proposition is not a (scientific) proposition.

What is described in this section has been mentioned many times in the references [46]~[76].

Recall the linguistic Copenhagen interpretation:"No observer can measure himself."



We will show two (scientific or non-scientific) interpretations of the cogito proposition: "I think, therefore I am".

Lemma 8.3. **[(i): Scientific interpretation of the cogito proposition]:**
Let start from the scientific interpretation of the cogito proposition.
For the sake of convenience, put "I"="Tom". Thus,

- "I think, therefore I am"= "Tom thinks, therefore Tom is"

If this is a scientific proposition, it must be experimentally verifiable. However, it is easy. That is because it is usual that

(C_1) a doctor says "Tom's brainwaves are normal, so he's alive."

which is of course a statement in quantum language, since

"observer = doctor", "matter = Tom".

However, the interpretation (C_1) is a trivial one, and this would not be Descartes' intention.

Proposition 8.4. **[(ii): Non-scientific interpretation (i.e., Descartes' intention) of the cogito proposition]:**
Similarly, put "I"="Tom". Put

(♭) "I think, therefore I am"= "Tom thinks, therefore Tom is"

Thus, we consider the situation (which is the same as Descartes's intension) :

(C_2) Tom thinks "Tom thinks, therefore Tom is"

which means that

"observer = Tom", "matter = Tom",

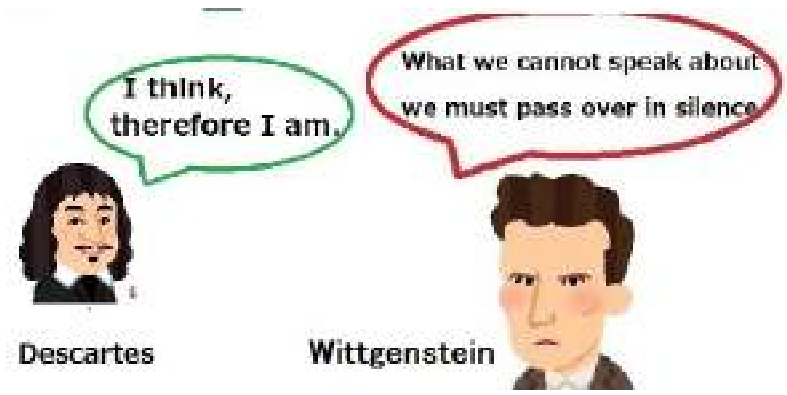
That is, this violates the linguistic Copenhagen interpretation: "No observer can measure himself." Therefore, the (C_2) is not in quantum language, that is, it is just a play on words.
Thus, the (♭) is a non-scientific proposition (i.e., an experimentally unverifiable proposition), but this play on words had captured the interest of philosophy enthusiasts. That is,

This play on words opened the door to modern philosophy.

♠**Note 8.3.** As concluded in chapter 12, "I think, therefore I am" is not a proposition (precisely, a proposition in QL). in spite that it is called "the first proposition of philosophy". Using Wittgenstein's words, we says that

$$
\begin{aligned}
&\text{"I think, therefore I am"}\\
=&\text{"non-proposition"} = \text{"what we cannot speak about"}
\end{aligned}
$$

Therefore, Wittgenstein's famous words (i.e., "*What we cannot speak about we must pass over in silence*") may have been invoked in the Descartes-Kant philosophy.



The catchphrase "I think, therefore I am" may be unscientific, but it does not mean that Descartes' philosophy is unscientific. On the contrary, Descartes' philosophy is a great achievement (comparable to Newtonian mechanics in physics (monism)).

Descartes opened the door to modern philosophy.

We will discuss this below.

8.3 Descartes' genius strategy

8.3.1 From the cogito proposition to mind-body dualism

The most important key-word in Descartes' philosophy is "I". However, Descartes thought that

> Nobody pays attention even if Descartes appeals for the importance of "I" aloud.

Thus,

(A) Descartes used the advertising slogan (i.e., the cogito proposition): "I think, therefore I am"

The cogito proposition is nonsensical, but it is very impressive. Thus, this proposition could be a perfect advertising slogan.

[cogito proposition] $\xrightarrow[\text{derivation}]{\text{non-scientific}}$ [the existence of "I"] $\xrightarrow[\text{Note 1.2}]{\text{"exist"=keyword}}$ ["I" as keyword]

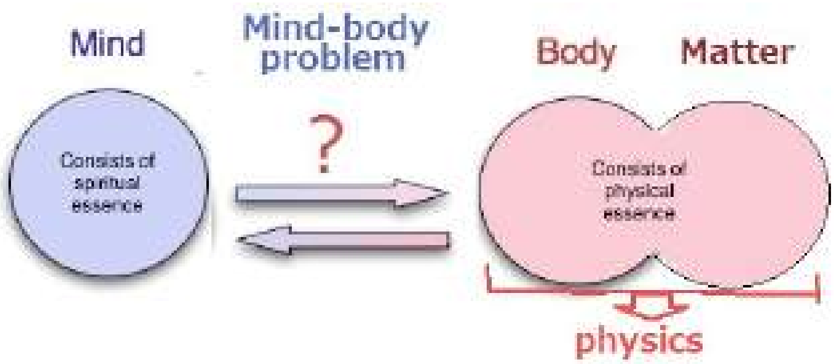
Thus, by the cogito proposition, what Descartes wanted to say was

(B) "I" is the most important key-word in Descartes philosophy.

His strategy succeeded wonderfully. If "I" is accepted, the existence of "matter" (which is perceived by "I") is accepted. And further, the medium of "I" and "matter" is automatically accepted as "body (= sensory organ)". Therefore, the key-words of Descartes philosophy (= mind-body dualism = mind-body·matter dualism) is

(C)

$\underbrace{\boxed{\text{"I"(="mind", "brain", "observer")}}}_{\text{spiritual dynamics}}, \underbrace{\boxed{\text{"body"(="sensory organ")}}, \boxed{\text{"matter"}}}_{\text{physical dynamics}}$



in this sense, we usually assume that "mind-body dualism" = "mind-body·matter dualism" ="mind-matter dualism".

Again, it should be noted that this is not a consequence of the cogito proposition. That is the cogito proposition is the reason added afterwards. The cogito proposition is nothing more than a catchphrase. This is the greatest catchphrase in human history.

Hence, I think

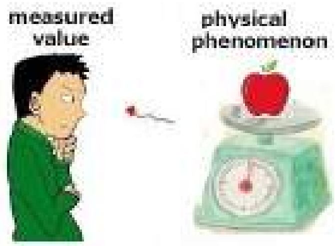
- "cogito proposition" does not matter as long as Descartes can insist on "mind-body dualism".

♠**Note 8.4.** (i): Descartes derived "existence of I" from "cogito proposition" as the first proposition of philosophy, and further derived

- "existence of things", "existence of body", "existence of causality" and "existence of God".

It can only be said that it is a "miracle of Western philosophy" that such an unreasonable story led to quantum language.
(ii): The next gap is quite big.
[I] $\longrightarrow$ [observer], [body] $\longrightarrow$ [sensory organ] [mind-body dualism] $\longrightarrow$ [epistemology]
This may be due primarily to John Locke (in Sec 9.1)
(iii): Note that

- For example, assume that an apple is put on food scale. We are not sure if he is trying to weigh an apple or make sure the food scale isn't broken (see Figure below). Thus, the difference between "body" and "matter" is arbitrary. This is determined by the observer. Both "body" and "matter" obey the laws of physics. However, the laws governing "mind"(i,e., spiritual dynamics) are assumed to be unknown.



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Principle 8.5. The principle of Descartes' philosophy is

(D) **Think of everything in terms of mind-body dualism!**

In other words, Descartes proposed a worldview of mind-body dualism that takes the three key words **"mind, body, and matter"** as its starting point.

This text claims that "the scientific realization of Descartes mind-body dualism is quantum language.

(♯) [quantum language (dualism)] = [axiomatic theory: Axiom 1 (measurement) Axiom 2 (causality); math style] + [empirical theory: Copenhagen interpretation; narrative style]

Axiom 1 [measurement]

"I" (="mind", "brain", "observer"), "body" (="sensory organ"), "matter"

spiritual dynamics — physical dynamics (Axiom 2 [causality])

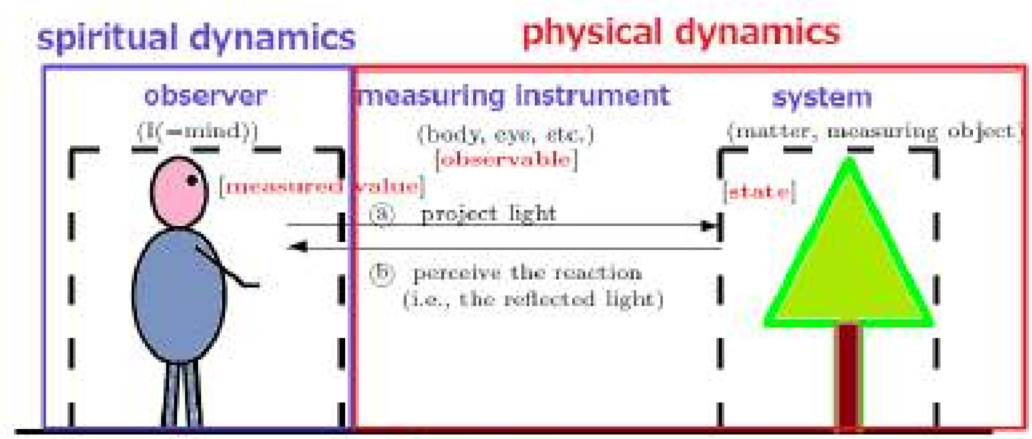


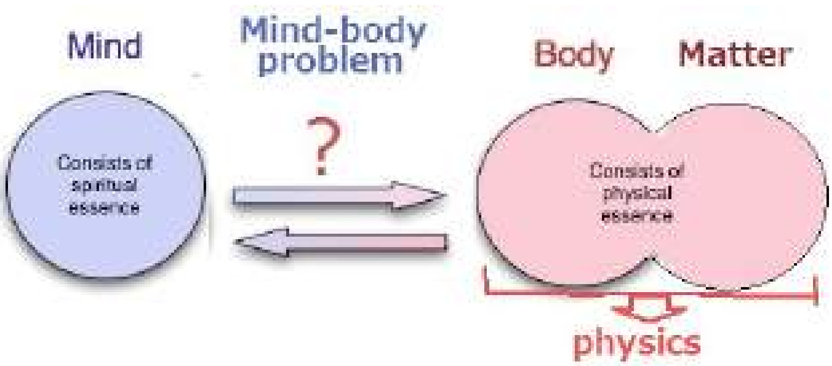
Figure 1.3. Descartes Figure: Image of "measurement(=ⓐ+ⓑ)" in dualism

8.3.2 Descartes problem (= mind-body problem + subjectivity problem

And, Descartes proposed the following two problems;

Problem 8.6. [Descartes problem (mind-body problem, subjectivity problem)]
Raising the following two issues is Descartes's greatest task in philosophy.

(E_1) **mind-body problem:**
How are "mind" and "body" connected?



(E_2) **subjectivity problem (= qualia problem):**
Is the world I perceive the same as the world you perceive?
Or more precisely,
How do we use mind-body dualism to solve the subjectivity problem (and problems mentioned in List (D_1) in Preface)?

In summary, the following is a summary.

(E_3)

(classical) Descartes problem (Establish mind-body dualism!)
- mind-body problem (Continental rationalism) – How are the mind and body related?
- subjective problem (British empiricism) – Is the world I see the same world you see?

♠**Note 8.5.** The above two problems have been touted as the two most difficult problems in modern philosophy. Without raising Descartes' issue, the prosperity of modern philosophy might not have been realized. In fact, Kant, Husserl, and others challenged these issues, and their results were supported by the general public. Although their approach is rather pseudo-psychological (i.e., pseudo-scientific), they are respected as great philosophers. However, no scientist thinks that their achievements have "guaranteed the objectivity of natural science." (However, as will be mentioned in Chap. 10, Kant's transcendental idealism is the gold standard of modern philosophy.)

No scientist would interpret the (original) Descartes problem in the literal sense. Of course, another way to describe the Descartes problem is as follows.

(E_4) Descartes problem is as follows.

(♯) Establish mind-body dualism so that the mind-body and subjectivity problems can be solved!

Answer 8.7. **[The solutions of the above Descartes' problems]:** Now, if we agree with (E_4), then Descartes problem has already been solved (in terms of this text) and we have

(E_5)

(modern) Descartes problem (Establish mind-body dualism!) (Establish QL!)	mind-body problem (Continental rationalism)	···	Find Axioms including the key-words (mind, body, matter)
	subjectivity problem (British Empiricism)	···	Propose the interpretation concerning this Axioms

In short, with the discovery of the "quantum language" the (modern) Descartes problem (E_5) is solved. A few additional explanations are provided below.
That is,

(F_1) **mind-body problem:**
[How do mind, body, and matter relate?]
This answer has already been answered in Axiom 1. Because
($♯_1$) Axiom 1 explains the quantitative relation among measured value(≈mind), observable(≈body), state(≈matter)
Also, see (♭) in Principle 8.5). The mind-body problem is said to be the biggest problem in philosophy, and although it may be persistent, Sec. 12.8 will also discuss it in detail.

(F_2) **subjectivity problem (= qualia problem):**
[Is the world as I perceive it the same as the world as you perceive it?]
This problem is called the Qualia problem, but it is not the only one, and our goal was to create a "Copenhagen Interpretation" so that the various unsolved problems described in the "Foreword to (D_1)" could be solved all at once.
($♯_2$) Our goal was to "create a Copenhagen interpretation," and we answered this in Sec. 1.3 and throughout this text.

Therefore, from the quantum language standpoint, (F_2) has already been solved.

Putting together (F_1) and (F_2), we see;

		axiomatic theory		empirical theory
quantum language (dualism)	=	Axiom 1 (measurement) Axiom 2 (causality)	+	linguistic Copenhagen interpretation
		math style; **mind-body problem**		narrative style; **subjectivity problem**

Thus, we say that

The solution to the Cartesian problem was made possible by quantum language

Throughout this text, the above answers will be explained precisely.

♠**Note 8.6.** (i): I believe that no matter how lofty a theory may be, it is meaningless if it does not solve a variety of problems. What I have done is to rewrite the "Copenhagen Interpretation" to solve the problems mentioned in List (D_1) in Preface.
(ii): Many philosophers may not be immediately convinced because our approach is not traditional. However, I am sure that our approach is truly philosophical and scientific. That is, I think that philosophers should not regard Problem 8.6 as the problems in psychology and brain science. I believe that Kant, Husserl, and others have become immersed in pseudoscience. Even so, their considerations have facilitated our understanding of the "Copenhagen Interpretation."
(iii):

As mentioned in Note 1.6, in Mermin's book [101], he proposed the excellent and interesting explanation of "Copenhagen interpretation" as follows.

($\sharp_1$) If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be **"Shut up and calculate"**

($\sharp_2$) Stop being bothered!

If it were a quantum mechanics course at a university, these ($\sharp_1$) and ($\sharp_2$) would be appropriate advice. However, in quantum language, I think [The linguistic Copenhagen interpretation (E_0)-(E_7) in Sec. 1.3] is necessary. I believe that the linguistic Copenhagen interpretation [(E_0),... ,(E_7)] is the "true Copenhagen Interpretation".

8.3.3 Two Geniuses: Thomas (Theory of Heaven) to Descartes (Theory of Earth)

As noted in Sec. 6.6.1, Thomas Aquinas asserted "Theory of Heaven" as follows.

Theory of Heaven(by Thomas)

(C_1) :universals exist'after individuals' in the human intellect .
[universalia post rem] as actual world
QL: (i.e., measured value is sensed by the brain (i.e., mind)).

(C_2) :universals exist 'before individuals' in the divine intellect, .
[universalia ante res] as Plato's Idea
QL: (i.e., observable (or, measuring instrument) could be in the heavens, like the North Star (*cf.* Note 1.7))

(C_3) :universals exist 'in individuals' in the world, .
[universalia in rebus] as Aristotle's eidos
QL: (i.e., state exists in matter)

It was Descartes who rewrote this to "Theory of Earth".

Theory of Earth (by Descartes)

Solve the following Cartesian problem in mind-body dualism, which consists of three key words:**mind, body, and matter**!

(E_3) in Sec. 8.3.2

(classical) Descartes problem (Establish mind-body dualism!)
- mind-body problem (Continental rationalism) – How are the mind and body related?
- subjective problem (British empiricism) – Is the world I see the same world you see?

In the same sense:

(E_5) in Sec. 8.3.2

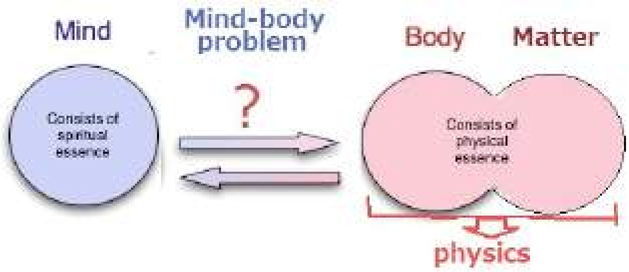
(modern) Descartes problem (Establish mind-body dualism!) (Establish QL!)
- mind-body problem (Continental rationalism) ··· Find Axioms including the key-words (mind, body, matter)
- subjectivity problem (British Empiricism) ··· Propose the interpretation concerning this Axioms

These two might be greater than Kant!

8.4 Continental Rationalism; Descartes, Spinoza, Leibniz

Inspired by Descartes's mind-body dualism, several philosophers (Spinoza, Leibniz, etc.) sought to further develop mind-body dualism. Their philosophy was called Continental Rationalism. Therefore, they devoted themselves to the mind-body problem:

(A) How are "mind" and "body" connected?



That is,

(B) They wanted to find a principle for the relationship between "mind" and "body (matter)"

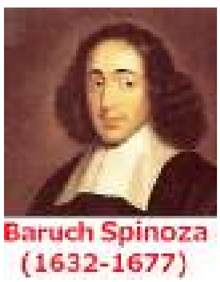
just as Newton discovered a principle for the motion of "matter".

However, as one can easily surmise, their challenge was not successful as a science.

A scientific failure does not mean a philosophical failure, but from the standpoint of this text, we cannot devote a page to it.

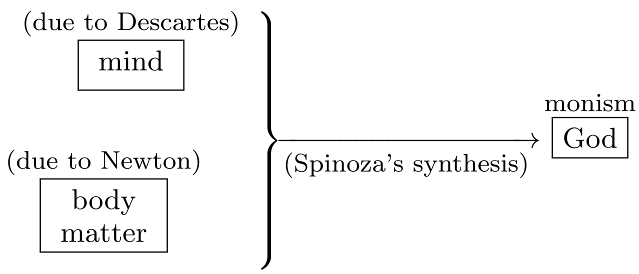
However, their attempt to break out of the shell of dualism left a mark on the history of Western philosophy, so I will mention a few things.

8.4.1 Spinoza: Pantheism



Baruch Spinoza (1632-1677) was a Dutch philosopher, who was one of the foremost exponents of 17th-century Rationalism. In his work "Ethica", he claimed to have improved on Descartes' mind-body dualism as follows.

(C)



That is, Spinoza combined the two entities ("mind" and "body/matter") into one entity called "God". Namely, he proposed "God monism" (i.e., pantheism).

Since the interest of this text is the "scientific part of Western philosophy," it must be an unfavorable assessment for Spinoza.

However, the spirit with which he challenged the great attempt to unify the "mind-body problem," "God," and "ethics" was favorably received by many philosophical enthusiasts.

♠**Note 8.7.** This text claimed the following

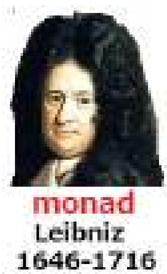
($\sharp_1$) The logic is determined by the worldview. For example, mathematics has a mathematical logic, quantum language has a quantum language logic (*cf.* Sec. 12.2).

Now, if it is a philosophy that is more akin to literature or religion, "Is it logical or not?" is not often pursued. However, since "Ethica" was written by Spinoza with the intention of "logical reflection," the question of "logical or not? is often debated. And it is often pointed out that it is "absurdly logical" from a modern point of view. I think this point is probably right on the mark. And I think the main way to read "Ethica" is to enjoy its "absurd logic" from a literary point of view. Of course, this kind of literary enjoyment is acceptable in philosophy. However, from the standpoint of this text, I would like to consider the following possibilities.

($\sharp_2$) Pantheism has the logic of pantheism.

In other words, it would be interesting if we could discover a "logic of pantheism" different from the "logic of mathematics" or the "logic of quantum language". Quantum language is a quantification and verbalization of Cartesian philosophy (*cf.* Answer 8.7), so pantheism (and monads) may also be quantifiable and verbalizable. Descartes philosophy and pantheism are equally dubious from a scientific point of view as monads are from a scientific point of view. And yet, since quantum language emerged from Cartesian philosophy, I expect something scientific to emerge from both pantheism and monads. I hope that a rival to quantum language will emerge.

8.4.2 Leibniz: Monadology



Gottfried Wilhelm Leibniz (1646-1716) was one of the great thinkers of the seventeenth and eighteenth centuries and is known as the last "universal genius". He made deep and important contributions to the fields of metaphysics, epistemology, logic, philosophy of religion, as well as mathematics, physics, geology, jurisprudence, and history. In particular, he discovered Calculus at the same time as Newton.

Leibniz claimed to have improved on Descartes' mind-body dualism as follows.

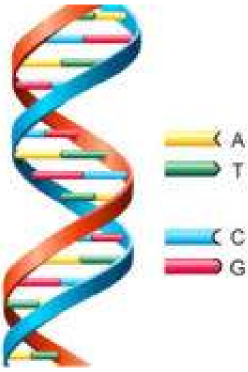
(D) The true elements that make up the world are the monads, which are mutually independent, indivisible entities, and the source of all vital activity. Every monad is different from each other.

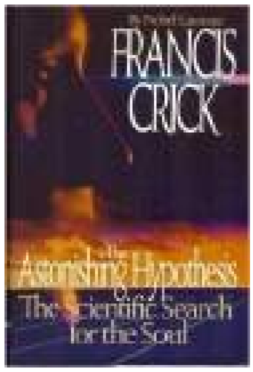
In this sense, it is pluralism. I believe that the success of Newtonian mechanics inspired

Leibniz. I think Leibniz aimed at the summit of both the material and spiritual worlds. The genius Leibniz must have thought so. However, since the interest of this text is "the scientific part of Western philosophy as seen from modern science (quantum language)", I have to give Leibniz an unfavorable evaluation. An evaluation from the perspective of science 100 years later might be the opposite of my opinion.

However, Leibniz's space-time theory in the Leibniz-Clarke correspondence (in Sec. 9.2) is one of the most important achievements of the Copenhagen interpretation, and his genius is on full display.

♠**Note 8.8.** In the book "The astonishing hypothesis" (by F. (the most noted for being a co-discoverer of the structure of the DNA molecule in 1953 with James Watson)), Dr. Crick said that





($\sharp_1$) *You, your joys and your sorrows, your memories and your ambitions, your sense of personal identity and free will, are in fact no more than the behavior of a vast assembly of nerve cells and their associated molecules.*

That is, he believed the monistic realism such as

($\sharp_2$) the movement of the human spirit is also a kind of physical phenomenon.

I agree to his opinion. And I believe that with the development of brain science, even consciousness can be understood in the future. However, it should be noted that Crick's claim and my claim (H) are not contradictory. Because Crick's interest is brain science, on the other hand, our interest is philosophy (more precisely, the language of science). For further discussion, see Sec. 12.8.

Chapter 9

Modern philosophy: British empiricism: Locke, Leibnitz, Berkeley, Hume

Kant proposed a "Kantian synthesis"[*1] to answer the following question.

($\sharp_1$) Why are there two schools of thought, Continental rationalism and British empiricism?

($\sharp_2$) Are these two schools of thought to be merged?

However, "Kantian synthesis" is pseudoscientific and cannot be adopted from the standpoint of this text. Our assertion is "QL synthesis (integrated by quantum language)" as follows.[*2]

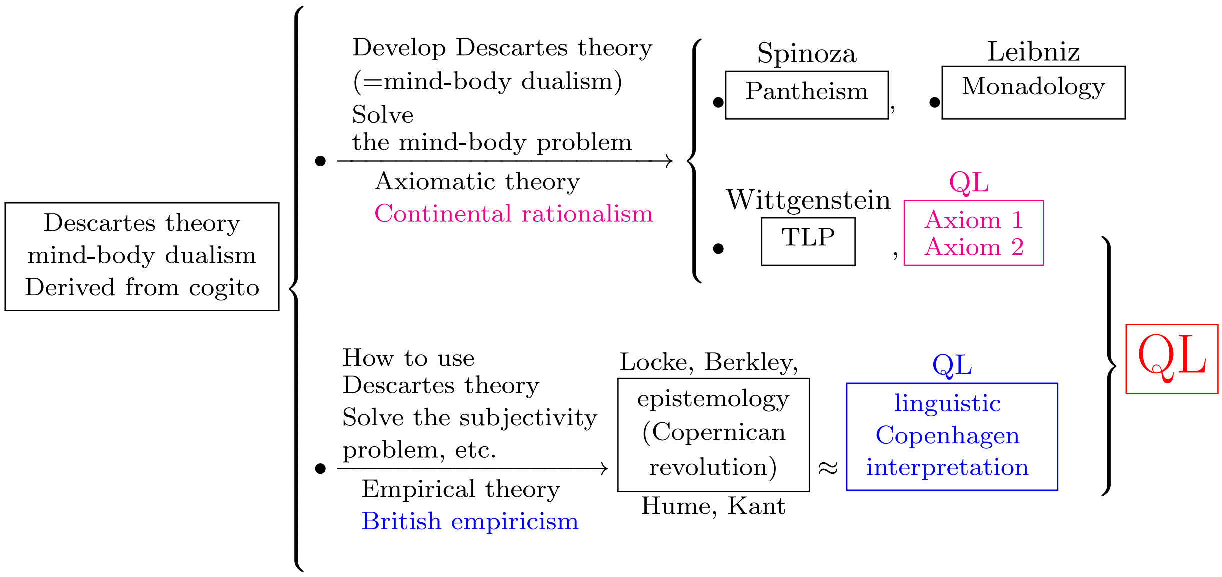


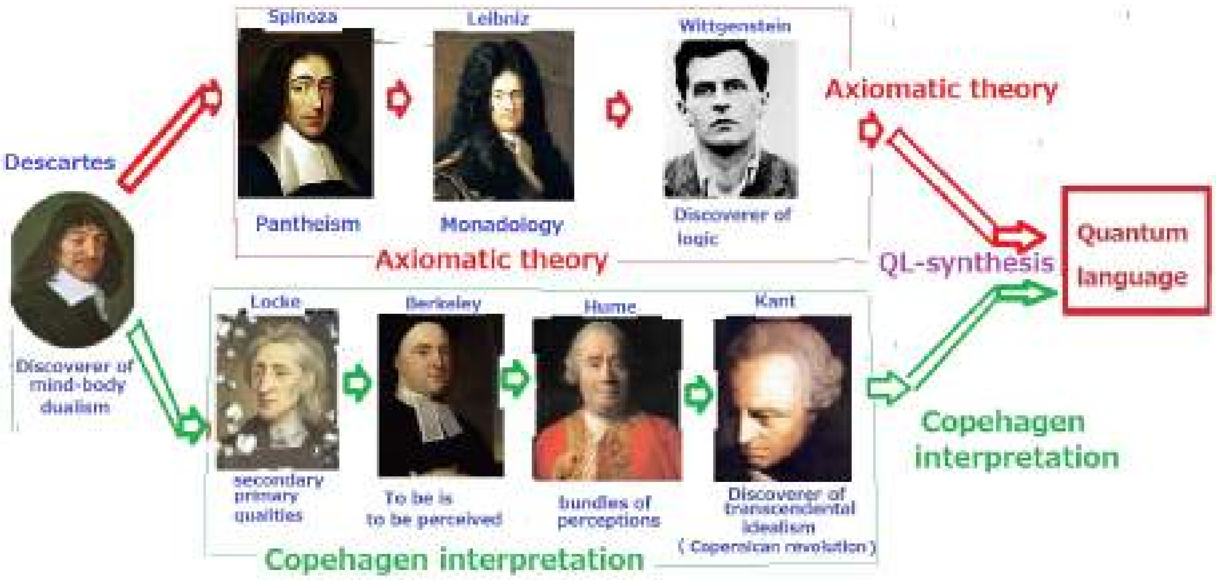
Figure 9.1: QL synthesis

The main purpose of chapters 9 and 10 is to explain this.

[*1] See Sec. 9.1.3

[*2] This was, for the first time, proposed in ref. [84]: S. Ishikawa, (2022) *Empiricism, Rationalism, and Kantian synthesis in quantum linguistic point of view* OJPP, Vol. 12, No. 2, 182-198, (`https://www.scirp.org/journal/paperinformation.aspx?paperid=117114`)

In the previous chapter, we concentrated on Continental Rationalism (= Descartes, Spinoza, Leibniz), especially Descartes' mind-body dualism. Then, I showed that the Cartesian problem (i.e., the mind-body problem and the subjectivity problem) can be solved if we assume that Descartes' mind-body dualism evolves and moves toward quantum language. We also showed that the works of Spinoza and Leibniz are not scientifically successful (i.e., their theories do not resemble Axiom 1 (measurement) at all).
In this and the next chapters, we will discuss British empiricism (Locke, Berkeley, Hume, and Kant) and show that their theories (Descartes-Kant epistemology) were scientifically successful and had a great influence on the Copenhagen interpretation of quantum language. The following figure summarizes the results.

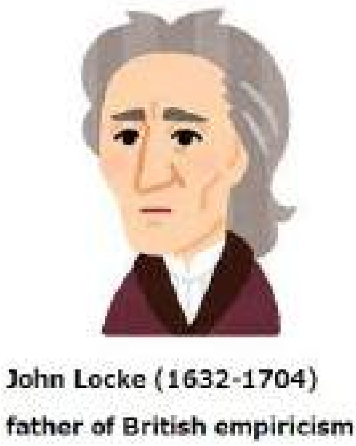


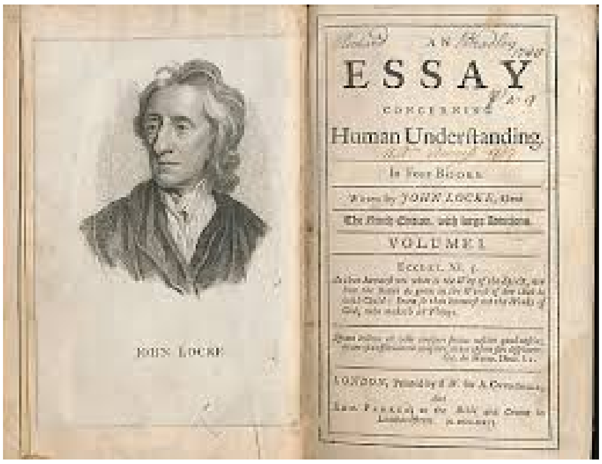
(However, attempts at Axiomatic theory were too difficult at the time.)

9.1 Locke (1632 - 1704): The father of British Empiricism

9.1.1 "An Essay Concerning Human Understanding" by Locke (1689)

In England, there has been a tradition of empiricism and inductive reasoning since F. Bacon (founder of empiricism, 1561-1626). John Locke (1632 - 1704) attempted to view Descartes' work (mind-body dualism) empirically and inductively.





John Locke was called the father of British empiricism and systematized empirical epistemology in his main book, An Essay Concerning Humanity.

(A) :"An Essay Concerning Human Understanding" by Locke (1689)

Locke is the heir to Descartes philosophy. He clarified the most difficult to understand "body" in Descartes mind-body dualism. Since Isaac Newton published his "**Principia**(= Mathematical Principles of Natural Philosophy (1686))", John Locke humbly wrote in his "Essay Concerning Human Understanding (1689)" that.

(B) *... and in an age that produces such masters as the great Huygenius and the incomparable Mr. Newton, with some others of that strain, it is ambition enough to be employed as an under-labourer in clearing the ground a little, ...*

♠**Note 9.1.** I think that "the incomparable Mr. Newton" is the most important key-word in the whole of Descartes-Kant philosophy (i.e., in British empiricism as well as continental rationalism). I think almost all philosophers (or, the general public) at the time wanted someone to tell them that philosophy is as great as Newtonian mechanics (or, more honestly, philosophy is greater than Newtonian mechanics). And it was Kant who said it. It is through this process that the general public learned to deal with science. In this sense, Cartesian Kantian philosophy is an Enlightenment philosophy that communicates to the general public the coming of the scientific revolution. Thus,

(C_1) even if Descartes-Kant philosophy is a pseudoscience about cognition, its importance remains constant.

(C_2) also note that the scientific revolution that began 500 years ago is still ongoing. (*cf.* Note 3.4).

With the advent of Newtonian mechanics, the role of the scientific part of philosophy threatened to disappear. For example, I think the nominalists of Scholastic philosophy (Ockham's successors) could have been more influential than Descartes. However, while there may have been a rivalry of "dualism" against "monism," British empiricism (Locke, Berkeley, Hume, Kant) found a field that, in hindsight, corresponds to the Copenhagen interpretation of quantum mechanics. This insight is so great that no amount of praise can be too much. I think it is the "sense of balance" of the people of that time. And thus, I think

(C_3) the emergence of a monistic giant, Newtonian mechanics, resulted in the strengthening of dualistic idealism and the flourishing of modern philosophy

9.1.2 Tabula·Rasa (British empiricism) vs Nativism (Continental rationalism)

The main purpose in Chap. 9(=this chapter)a and Chap. 10 is as follows.

(A) To assert the superiority of " quantum linguistic synthesis (= QL-synthesis)" over "Kant's synthesis."

Therefore, in this section, we will give an outline of "Kant's synthesis" and " QL-synthesis.

9.1.2.1 Kantian synthesis

Philosophy enthusiasts like the story that Descartes' philosophy developed in England and on the Continent as empiricism and rationalism, respectively, and that Kant united the two about a hundred years later. Let me explain this theory as follows.

The Flower of Modern Philosophy (Kantian Synthesis)

Descartes' work gave rise to two philosophies: British empiricism and continental rationalism.

British empiricism[**"tabula rasa(= blank paper)**]
"An Essay Concerning Human Understanding (by Locke, 1690)

(B) He eliminated the possibility of innate knowledge before experience. Human being is born as the blank state ("tabula rasa") . In other words, the "brain circuit" is initially a blank slate, but concepts (i.e., complex brain circuits) are created through seeing and hearing various things. (Locke, Berkeley, Hume, $\cdots$)

Continental rationalism[**nativism**]
"New Essays on Human Understanding" (by Leibniz, 1703) says that

(C) Nativism: Human "brain circuits" already contain ideas and thoughts when we are born. (Spinoza, Leibniz$\cdots$)

That is, the opposition structure:

British empiricism (tabula·rasa) **vs.** **Continental rationalism (nativism)**

was born.

Then, about 100 years later, after many twists and turns, Kant came up with [Critique of Pure Reason: 1781];

(D) to synthesize "tabula·rasa and nativism"

Kant, however, has taken both sides of the issue into account, and concluded that

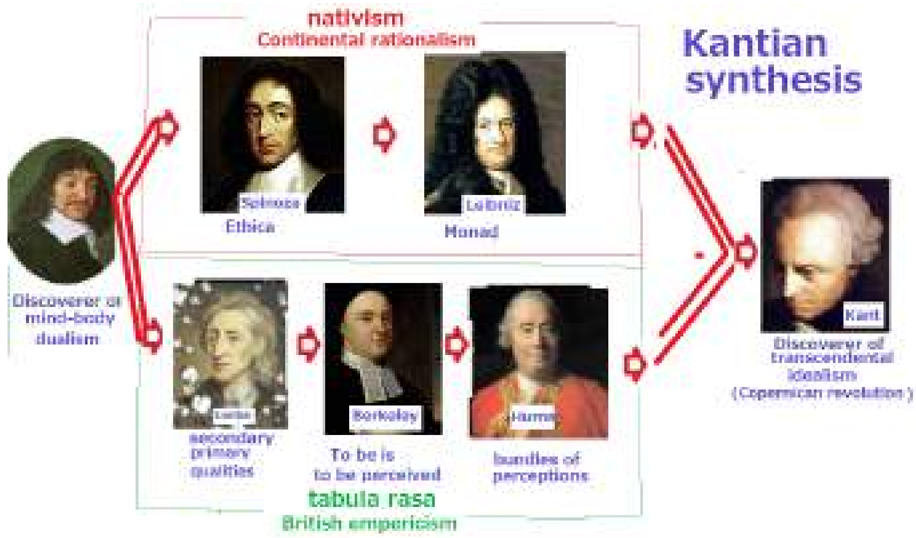
- while innate qualities are important (innate theory), experience, training, and education are also important (empiricism).

This is illustrated in the following figure.

(E) Kantian synthesis

mind-body problem
Descartes (the father of modern phil.
subjectivity problem
→
- Deduction (due to Descartes)
  Continental Rationalism (nativism)
  [Spinoza,Leibniz]
- Induction (due to Bacon)
  British Empiricism (tabula rasa)
  [Locke, Berkeley, Hume]

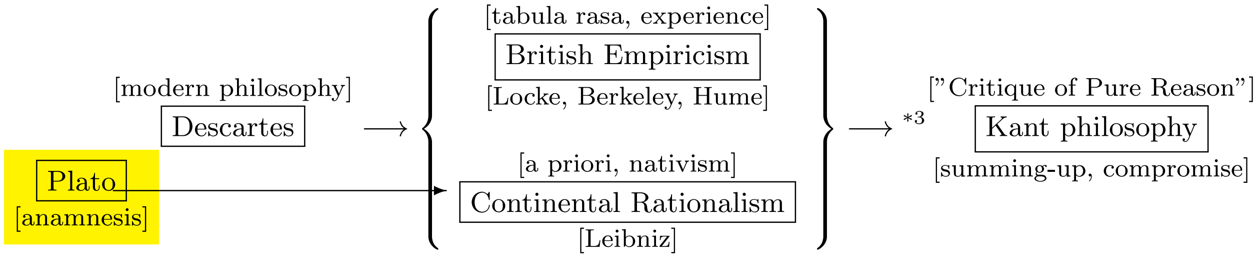
→ (synthesis) Kant



♠**Note 9.2.** Recall the theory of Plato's anamnesis (Sec.3.3). Let's recall:
Plato presented the concept of "recollection" (=anamnesis) as a mediator between the real world and the ideal world. i.e.

- We saw the Idea before we were born. But when we are born we have forgotten it. Therefore, to know an Idea is to recall it. In other words, to learn is nothing but to recall (=anamnesis).

In the history of Western philosophy, the theory of anamnesis is placed as follows.



I believe that "tabula rasa vs. nativism" is based on the tradition of Plato's philosophy. It is a pseudo-scientific argument in , and I think that sticking to it risks losing sight of the essence of modern philosophy. Therefore, I think Kant's synthesis needs to be reconsidered.

9.1.2.2 QL-synthesis

I think the above "Synthesis of Kant" is too pseudoscientific. Write my opinion in the note below.

♠**Note 9.3.** Thinking in terms of "language" instead of "recognition"

($\sharp_1$) "Everyday language" is similar to tabla-rasa. When a baby is born, it does not know any ordinary language at all (= tabula rasa). The baby acquires everyday language through repeated trial and error.

($\sharp_2$) Mathematics is similar to Continental Rationalism. All mathematical theorems can be proved (in principle) if only the axioms of set theory are determined.

($\sharp_3$) The quantum language is similar to the compromise between ($\sharp_1$) and ($\sharp_2$) above as follows.

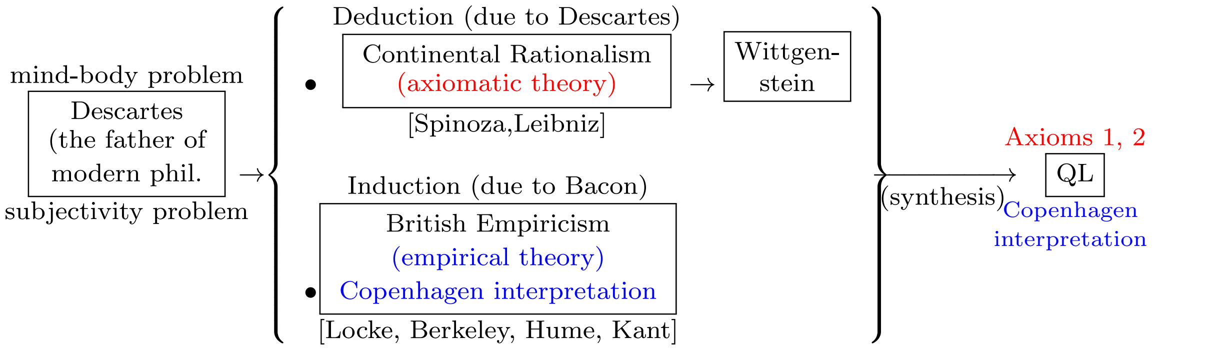
[Quantum language] = [rationalism: Axiom 1 (measurement) / Axiom 2 (causality)] + [(empiricism): linguistic Copenhagen interpretation]

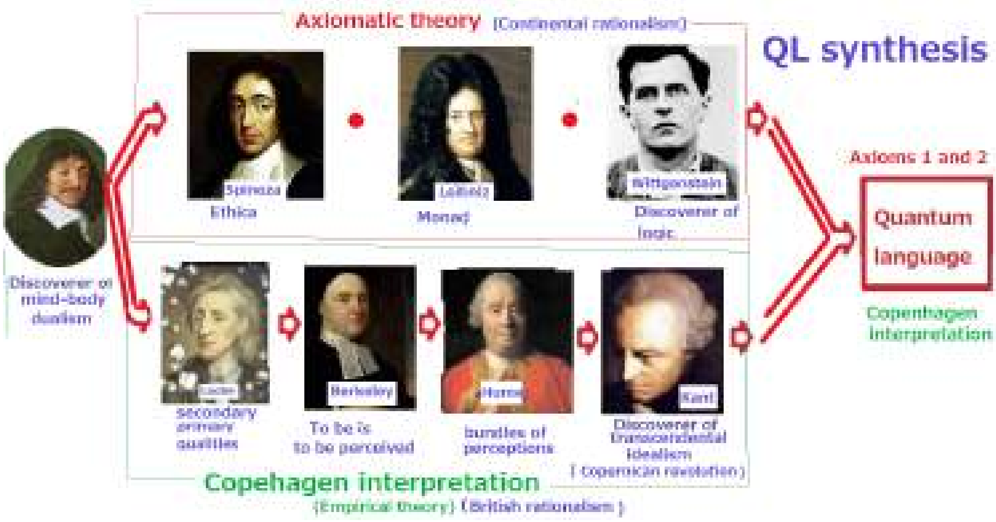
////

Taking this Note 9.3 as a hint, I consider the following. The main purpose of this text (chapters 8, 9, and 10) is to propose a rewriting of the history of modern Western philosophy (i.e., from [(E): Kantian synthesis] to [(F): QL synthesis]).

(F) QL-synthesis)

This will be explained in chapters 8, 9, and 10, but only a figure is given here.





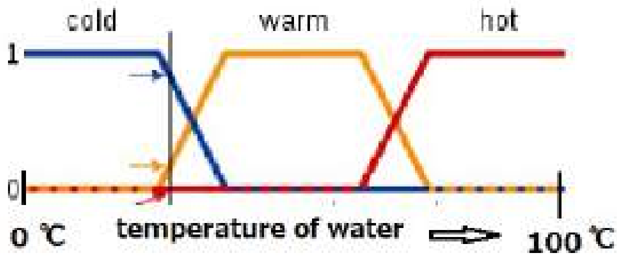
(However, attempts at Axiomatic theory were too difficult at the time.)

9.1.3 Secondary quality (and primary quality)

According to Locke, we see

(G_1) **primary quality** (i.e., inherent nature (=primary quality)) $\cdots$ weight, temperature, length, etc.

(G_2) **secondary quality** (i.e., quantities perceived by sensory organs)$\cdots$ sweet, red, hot, salty, etc.



In terms of quantum language, we say:

primary quality$\Longrightarrow$state,
secondary quality$\Longrightarrow$observable

Here,

(H) Locke represents the most important concept in dualistic idealism as the term "secondary quality". The terms such as Idea, body, etc. may not be epistemological. However,

- "secondary quality" is a word we can understand. Hence, Locke's achievement should be honored.

Again, note that "secondary quality" is a word that forms the foundation of mind-body dualism.

Of course, it was Locke who steered Cartesian philosophy in the direction of "recognition and measurement". Thus, he argues the following.

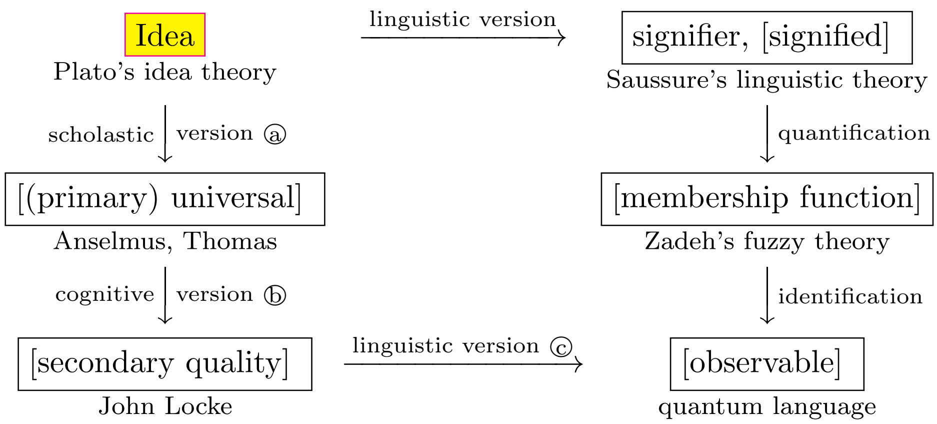
(Idea) Plato $\xrightarrow[\text{model-change}]{\text{progress}}$ [(primary) universal] Anselmus, Thomas $\xrightarrow[\text{model-change}]{\text{progress}}$ (body) Descartes

$\xrightarrow[\text{model-change}]{\text{progress}}$ (sensory organ), [secondary quality] Locke (epistemology) $\xrightarrow[\text{model-change}]{\text{progress}}$ (measuring instrument), [observable] quantum language

British empiricist philosophers spoke philosophy to ordinary philosophy enthusiasts. Thus, they always spoke philosophy in plain language. The term "secondary quality" can be understood by anyone.

Corollary 9.1. (Cf. (W) in Sec. 6.5.4) Of these three ("measured value," "observed value," and "state"), "observable" is the most difficult to understand. So, the history of "observable" can be summarized as follows.

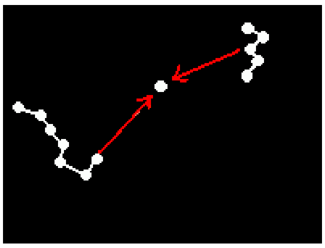
(I)

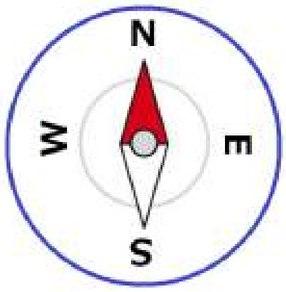


(The history from Plato Idea to observables)

Through scholastic philosophy ⓐ, Idea became the scientific concept of "universal. After ⓑ by Locke, Idea became a concept that everyone can understand [secondary properties]. After that, it only becomes a [computable form] called an "observable" by ⓒ.

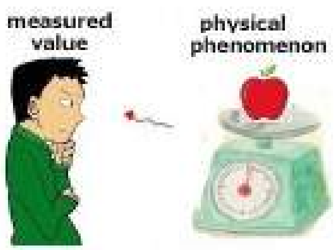
♠**Note 9.4.** (i):The concept of "measuring instrument" (≈ body) is not easy. For example, telescopes, glasses and eyes are a type of measuring instrument. A directional magnet is, of course, a measuring instrument. If so, then the polar star may be also regarded as a type of measuring instrument.





(ii): By the way, Merleau-Ponty (1908 - 1961) might think in the following manner.

- I shake hands with my right hand and the left hand. In this case, if I regard the right hand as the measuring instrument, I feel the existence of my left hand. On the contrary, if I regard the left hand as the measuring instrument, I feel the existence of my right hand.



(iii): I have mentioned this before, but I will write it again. For example, assume that an apple is put on food scale. We are not sure if he is trying to weigh an apple or make sure the food scale isn't broken. Thus, the difference between "body" and "matter" is arbitrary. This is determined by the observer. Both "body" and "matter" obey the laws of physics. However, the laws governing "mind" are assumed to be unknown. Maybe there is no such thing.

(v): As readers are already aware:

- the magic word "Idea" is what made Western philosophy flourish!

A.N.Whitehead (1861 - 1947) said as follows.

(♯) **Western philosophy is characterized as a series of footnotes to Plato**

In short, the word "Idea" is a profound word with many aspects, as described in (G), and it has kept philosophy enthusiasts occupied for 2,500 years.

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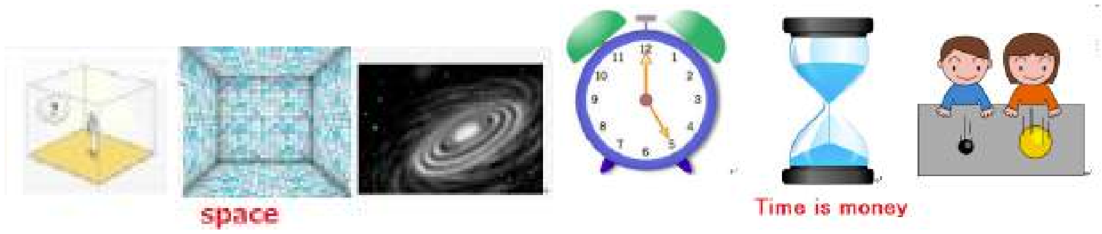
9.2* Leibniz-Clarke Correspondence: What is space-time?

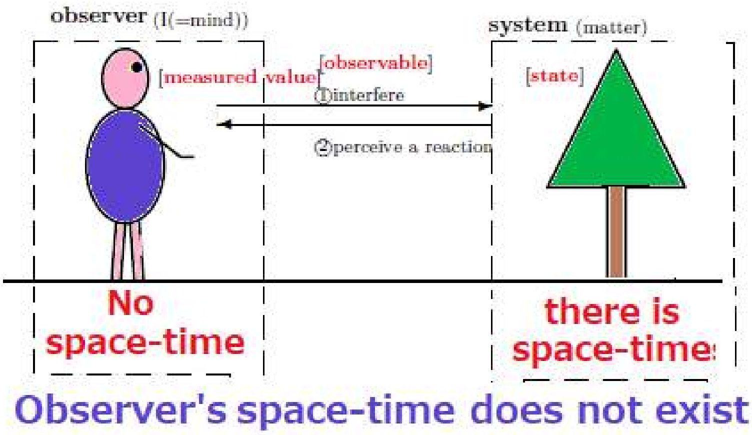
The contents of this section are published in the followings:

- ref. [76]: S. Ishikawa; *Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language* Open Journal of philosophy, Vol. 8, No.5, 466-480, 2018, DOI: 10.4236/ojpp.2018.85032 (`https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862`)
- ref. [77]; S. Ishikawa; *Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language*; [Revised version] ; Keio Research report; 2018; KSTS/RR-18/001, 1-15 (`https://philpapers.org/rec/ISHLCB`) (`http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf`)
- [85] S. Ishikawa, K. Kikuchi, (2021) *Quantum Fuzzy Logic and Time, Journal of Applied mathematics and physics,* Vo.9 No.11 2021, 2609-2622 (`https://www.scirp.org/journal/paperinformation.aspx?paperid=112972`)



The problems ("What is space?" and "What is time?") are the most important in modern science as well as the traditional philosophies. In this section, we give the quantum linguistic answer to these problems. As seen later, our answer is similar to Leibniz's relationalism concerning space-time. In this sense, we consider that Leibniz is one of the discoverers of the linguistic Copenhagen interpretation. No one could understand Leibniz's monad in science (*cf.* Sec. 8.4), but everyone can understand his excellent "space-time theory" in science.

9.2.1 “What is space?” and “What is time?”



9.2.1.1 Space in quantum language

(How to describe “space” in quantum language)

(*Cf.* the C^*-algebraic formulation in Sec 1.4) is used)

In what follows, let us explain “space” in measurement theory (= quantum language). For example, consider the simplest case, that is,

$$\text{(A)} \qquad \text{“space”} = \mathbb{R}_q (\text{one dimensional space})$$

Since classical system and quantum system must be considered, we see

$$\text{(B)} \begin{cases} (\text{B}_1)\text{: a classical particle in the one dimensional space } \mathbb{R}_q \\ \\ (\text{B}_2)\text{: a quantum particle in the one dimensional space } \mathbb{R}_q \end{cases}$$

In the classical case, we start from the following state:

$$(q, p) = (\text{“position”}, \text{“momentum”}) \in \mathbb{R}_q \times \mathbb{R}_p$$

Thus, we have the classical basic structure:

$$(\text{C}_1) \qquad [C_0(\mathbb{R}_q \times \mathbb{R}_p)^{*4} \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p)]$$

Also, concerning quantum system, we have the quantum basic structure:

$$(\text{C}_2) \qquad [\mathcal{C}(L^2(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q)]$$

Summing up, we have the basic structure

$$\text{(C)} \ [\mathcal{A} \subseteq \overline{\mathcal{A}} \subseteq B(H)] \begin{cases} (\text{C}_1)\text{: classical } [C_0(\mathbb{R}_q \times \mathbb{R}_p) \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p)] \\ \\ (\text{C}_2)\text{: quantum } [\mathcal{C}(L^2(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q)] \end{cases}$$

Since we always start from a basic structure in quantum language, we consider that

*4 See Note 1.11

How to describe "space" in quantum language

$\Leftrightarrow$ How to describe [(A):space] by [(C):basic structure]

This is done in the following steps.

Assertion 9.2. [The linguistic Copenhagen interpretation concerning "space"]
How to describe "space" in quantum language

(D$_1$) Begin with the basic structure:

$$[\mathcal{A} \subseteq B(H)]$$

(D$_2$) Next, consider a certain commutative C^*-algebra $\mathcal{A}_0(= C_0(\Omega))$ such that

$$\mathcal{A}_0 \subseteq B(H)$$

(D$_3$) Lastly, the spectrum Ω ($\approx \mathfrak{S}^p(\mathcal{A}_0^*)$) is used to represent "space".

Therefore, in quantum language, we see

- space is a kind of state of a "thing".

[Note]: Therefore,

(D$_4$) Space can be thought of as a kind of state space (a space representing the state of "things").
Therefore, if there is no "thing", there is no basic structure,
and therefore there is no space.

For example,

(E$_1$) in the classical case (C$_1$):

$$[C_0(\mathbb{R}_q \times \mathbb{R}_p) \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p))]$$

we have the commutative $C_0(\mathbb{R}_q)$ such that

$$C_0(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p))$$

And thus, we get the space $\mathbb{R}_q$ as mentioned in (A)

(E$_2$) in the quantum case (C$_2$):

$$[\mathcal{C}(L^2(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q))]$$

we have the commutative $C_0(\mathbb{R}_q)$ such that

$$C_0(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q))$$

And thus, we get the space $\mathbb{R}_q$ as mentioned in (A)

9.2.1.2 Time in quantum language: How to describe time in quantum language?

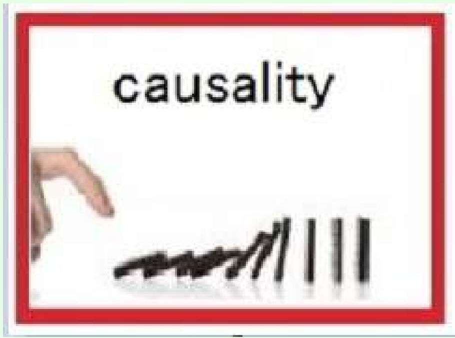
Axiom 2 (i.e., causality) should be restated (For details, see ref. [74]).

Axiom 2 [Causality] in the C^*-algebraic formulation

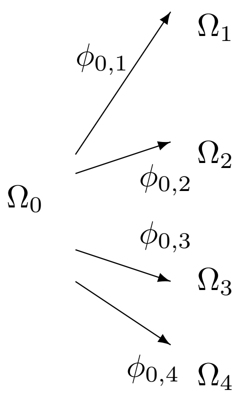
•[General case (i.e., quantum case ($\mathcal{A}_{t_i} = B_c(H_{t_i})$) and classical case ($\mathcal{A}_{t_i} = C(\Omega_{t_i})$))]:
Assume $[t_1 \le t_2]$.
($\sharp_1$): Causality is represented by a causal operator $\Phi_{t_1,t_2} : \mathcal{A}_{t_2} \to \mathcal{A}_{t_1}$
In the same sense,
($\sharp_2$): Causality is represented by a dual causal operator $\Phi^*_{t_1,t_2} : \mathcal{A}^*_{t_1} \to \mathcal{A}^*_{t_2}$

•[Classical case ($\mathcal{A}_{t_i} = C(\Omega_{t_i})$)]
Assume $[t_1 \le t_2]$:

($\flat_1$): Classical causality is represented by a causal operator $\Phi_{t_1,t_2} : C(\Omega_{t_2}) \to C(\Omega_{t_1})$
In the same sense,
($\flat_2$): Classical causality is represented by a dual causal operator $\Phi^*_{t_1,t_2} : C(\Omega_{t_1})^* \to C(\Omega_{t_2})^*$
Also,
($\flat_3$): Classical causality is represented by a continuous map $\{ \phi_{t_1,t_2} : \Omega_{t_1} \to \Omega_{t_2}, (t_1 \le t_2) \}$



♠**Note 9.5.** In the above, $t_1, t_2, ...$ does not always imply time. For example, when formulating regression analysis in quantum language, use the parallel structure shown in the lower left figure (*cf.* ref. [74]). If it is semi-ordered, it is acceptable.



$$C(\Omega_0) \xleftarrow{\Phi_{0,1}} C(\Omega_1) \xleftarrow{\Phi_{1,2}} C(\Omega_2) \xleftarrow{\Phi_{2,3}} C(\Omega_3)$$

Assertion 9.3. [Copenhagen Interpretation of "Time"]
How to describe "time" in quantum language

Let $[\mathcal{A} \subseteq B(H)]$ be a C^*-basic structure. Let T be the real line $\mathbb{R}$. for any $t_1, t_2 \in T (t_1 \le t_2)$, define a causal operator $\Phi_{t_1,t_2} : \mathcal{A} \to \mathcal{A}$ such that

$$\Phi_{t_1,t_2}\Phi_{t_2,t_3} = \Phi_{t_1,t_3} \qquad (\forall t_1, \forall t_2, \forall t_3 \in T \text{ such that } t_1 \le t_2 \le t_3) \tag{9.1}$$

Also, let $\Phi^*_{t_1,t_2} : \mathcal{A}^* \to \mathcal{A}^*$ be the dual causal operator of $\Phi_{t_1,t_2} : \mathcal{A} \to \mathcal{A}$. Let $\rho_0 (\in \mathfrak{S}(\mathcal{A}^*)$ be an initial state. Then, the time evolution is repretented by $\Phi^*_{0,t}\rho_0$ $(0 \le t < \infty)$.
[Note]

(F) In the above, time T is a parameter that to describe the change in state of the "thing"
Therefore, without "things", there is no basic structure, and time is meaningless.

♠**Note 9.6.** According to the Copenhagen interpretation, the thing (state) does not move. the above (F) formally states that

(F') time T is a parameter used to describe the change of an observable

The Schrödinger picture (change of state) is more intuitive than the Heisenberg picture (change of observables), so I wrote it as above.

For example, consider the following specifics. Note that $T(\in T)$ is treated as a parameter, and without the "thing", there is no basic structure and time T is meaningless.

[I]: For quantum systems(the case of C^*-basic structure:$[B_c(H) \subseteq B(H)]$): Consider the Schrödinger equation with Hamiltonian $\widehat{\mathcal{H}}$ as follows:

$$i\hbar \frac{d}{dt}|\psi_t\rangle = \widehat{\mathcal{H}}|\psi_t\rangle \quad (\text{ in } H) \quad (t \in T, \psi \in H, ||\psi||_H = 1)$$

Clearly the solution is equal to $\exp[\frac{\widehat{\mathcal{H}}t}{i\hbar}]|\psi_0\rangle$. Using the dual causal operator $\Phi^*_{t_2,t_1} : \mathcal{A}^*_{t_1} \to \mathcal{A}^*_{t_2}$ (i.e., $\mathfrak{S}^p(B_c(H)^* \to \mathfrak{S}^p(B_c(H)^*$), the causality is written by

$$\Phi^*_{t,0}(|\psi_0\rangle\langle\psi_0) = \exp[\frac{\widehat{\mathcal{H}}t}{i\hbar}]|\psi_0\rangle\langle\psi_0| \exp[\frac{\widehat{\mathcal{H}}t}{i\hbar}]$$

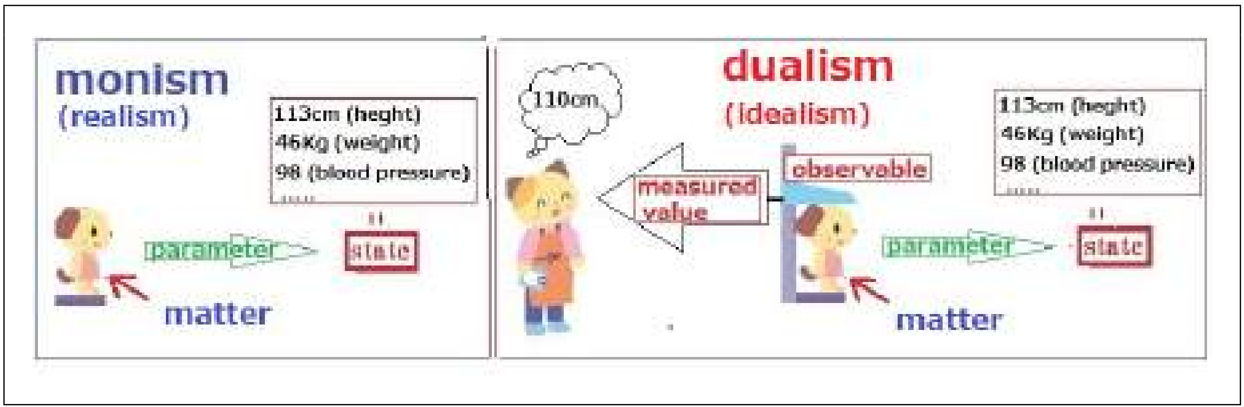
[II]: For classical case(C^*-basic structure:$[C(\Omega), B(H)]$): Consider the dynamical system $\{ \phi_{t_1,t_2} : \Omega \to \Omega, (t_1, t_2 \in T$ such that $t_1 \leq t_2) \}$, which is equivalento Newtonian equation. Clearly, we see

$$\phi_{t_3,t_2}\phi_{t_2.t_1} = \phi_{t_3,t_1} \quad (0 \leq t_1 \leq t_2 \leq t_3 < \infty)$$

Assume the initial state is $\omega_0(\in \Omega)$. Then we see

$$\omega_t = \phi_{0,t}(\omega_0) \quad (t \geq 0)$$

9.2.2 Leibniz-Clarke Correspondence



Read the following while looking at the above figure.

The above argument urges us to recall Leibniz-Clarke Correspondence (1715–1716: *cf.* [1]), which is important to know both Leibniz's and Clarke's (=Newton's) ideas concerning space and time.

(G) [The realistic space-time]
Newton's absolutism says that the space-time should be regarded as a receptacle of a "thing." Therefore, even if "thing" does not exits, the space-time exists.

On the other hand,

(H) [The metaphysical space-time]
Leibniz's relationalism says that

(H_1) Space is a kind of state of "thing".

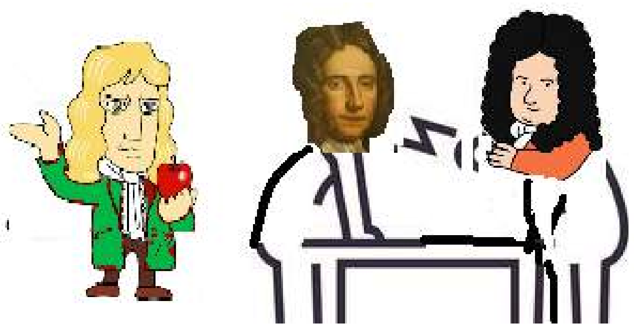
(H_2) Time T is also a kind of state space $\mathbb{R}$ (or, $\mathbb{Z}$), which represents an order of occurring in succession which changes one after another.

Thus,

(I) Without "things", there would be no space-time. (*cf.* (D_4) to (F_4))

Therefore, I regard this correspondence as

Newton ($\approx$ Clarke) (realistic view) $\overset{\longleftrightarrow}{\text{vs.}}$ Leibniz (linguistic and idealistic view)



which should be compared to

$$\underset{\text{(realistic view)}}{\boxed{\text{Aristotle}}} \quad \underset{\text{vs.}}{\longleftrightarrow} \quad \underset{\text{(linguistic and idealistic view)}}{\boxed{\text{Plato}}}$$

(also, recall Note 4.4).

In the end, it is the difference between existential and linguistic worldviews that can be summarized as follows

$$\begin{cases} ① : \underset{\text{(realistic worldview)}}{\text{Newton, Clarke}} \cdots \underset{\text{“What is space-time?”}}{\overset{\text{(space-time in physics)}}{\boxed{\text{realistic space-time}}}} \quad \text{(successors: Einstein, etc.)} \\ \\ ② : \underset{\text{(linguistic worldview)}}{\text{Leibniz}} \cdots \underset{\text{“How should space-time be represented?”}}{\overset{\text{(space-time in measurement theory)}}{\boxed{\text{linguistic space-time}}}} \quad \text{(i.e., spectrum, tree)} \end{cases}$$

Therefore, both Clarke (≈Newton) and Leibniz are correct, but the decisive difference is as follows.

(J_1) Newton created the language of Newtonian mechanics, under which he discussed space-time.

(J_2) Leibniz discussed space-time without (quantum) language, only by his genius intuition.

Because of this difference, Leibniz's theory of space-time relationalism has been difficult to understand until today.

I would also ask you to review the following table again

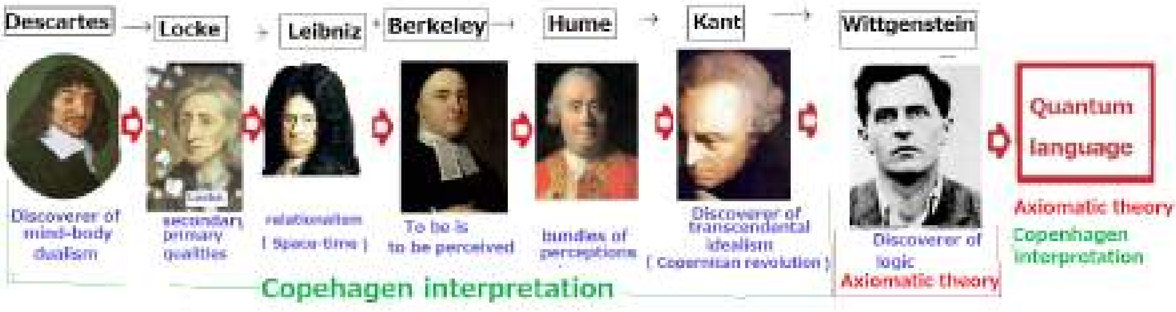
Table 9.1 (=Table 4.1) : realistic worldview (Aristotle) vs. idealistic worldview (Plato)

dispute ╲ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	Ockham	Anselmus
ⓓ: space-time	Newton (Clarke)	Leibniz
ⓔ: quantum theory	Einstein	Bohr
ⓕ:philosophy of science	Carnap	Quine
ⓖ: fuzzy sets	Kalman	Zadeh

For the precise argument, see Table 4.1.

♠**Note 9.7.** Leibniz is an undisputed genius. I think Leibniz is the only one who can discuss space-time on equal terms with Newton. Leibniz space-time theory is almost the same as the space-time theory in the Copenhagen interpretation. Thus, Leibniz was one of the leading contributors to the creation of the Copenhagen interpretation. Therefore, I would like to consider that Leibniz also belongs to British empiricism.

I cannot say anything about this because I do not understand Leibniz's monadology. However, I would have hoped that the genius Leibniz would have discussed space-time under the worldview of monadology and put this up against Newton.



9.2.3 Appendix: motion function method (in classical system)

The " motion function method (Sec. 2.3)" discussed in the Zeno's paradox is supplemented here with its fundamentals.

9.2.3.1 Schrödinger and Heisenberg pictures are equivalent in the classical system

According to Leibniz, "time" is just a "parameter" that can be conveniently created. Let's introduce "Parallel time" and "Series time*5.

Consider a classical dynamical system (Ω, ϕ_{t_1,t_2}). Here, Ω is compact space, $t_1, t_2 \in T = [0,1]$ such that $0 \le t_1 \le t_2 \le 1$, a map $\phi_{t_1,t_2}(\cdot) : \Omega \to \Omega$ is bi-continuous and satisfies the following condition:

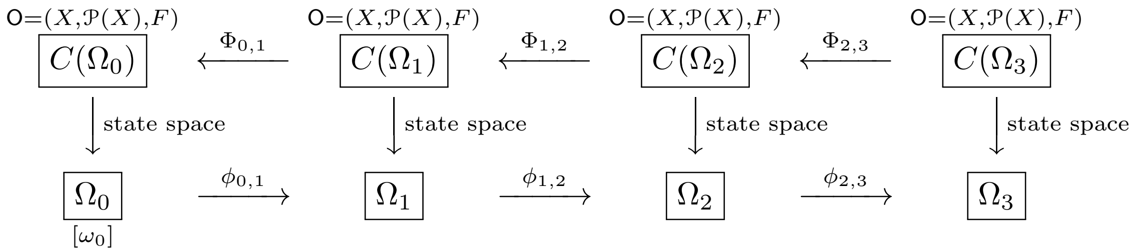
($\sharp_1$) $\lim_{t_2 \to t_1} \phi_{t_1,t_2}(\omega) = \omega \quad (\omega \in \Omega)$
($\sharp_2$) $[\phi_{t_2,t_3} \circ \phi_{t_1,t_2}](\omega) = \phi_{t_2,t_3}(\phi_{t_1,t_2}(\omega)) = \phi_{t_1,t_3}(\omega) \quad (\omega \in \Omega, 0 \le t_1 \le t_2 \le t_3 \le 1)$

As mentioned before

(K) there exists a homomorphism $\Phi_{t_1,t_2} : C(\Omega) \to C(\Omega)$ such that

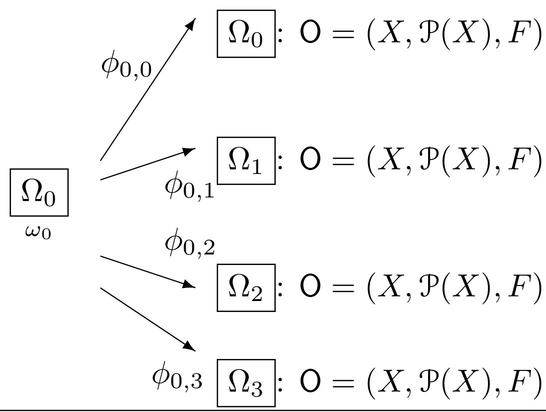
$$[\Phi_{t_1,t_2}(g_{t_2})](\omega_{t_1}) = g_{t_2}(\phi_{t_1,t_2}(\omega_{t_1})) \qquad (\forall \omega_{t_1} \in \Omega, \forall g_{t_2} \in C(\Omega)),$$

Consider the following time series (i.e., the case that $N = 3$, $\Omega_i = \Omega$, $i = 0,1,2,3$):
(L)



[(i): Schrödinger pictures (a state moves) :Parallel time)] of (L):

Figure (the case that $N = 3$; $\Omega = \Omega_i$, $i = 0,1,2,3$)



$\phi_{0,0}$: identity map
$\phi_{0,2} = \phi_{1,2} \circ \phi_{0,1}$
$\phi_{0,3} = \phi_{2,3} \circ \phi_{1,2} \circ \phi_{0,1}$

---

*5 See [85]; Parallel time represents the time lapse of a dice throw or the law of large numbers, etc.

Assume that the state $\omega_0(\in \Omega)$ at time $t_0(=0)$ evolves in time to become $\phi_{0,t_k}(\omega_0)$ $(k=0,1,...,N)$ as follows:

($\sharp_3$) state $\phi_{0,t_0}(\omega_0)=\omega_0$ at time $t_0=0/n(=0)$
state $\phi_{0,t_1}(\omega_0)$ at time $t_1=1/n$
state $\phi_{0,t_2}(\omega_0)$ at time $t_2=2/n$
...
state $\phi_{0,t_k}(\omega_0)$ at time $t_k=k/n$
...
state $\phi_{0,t_n}(\omega_0)$ at time $t_n=n/n=1$

And assume:

(M) At each time $t_0(=0), t_1(=1/n), ..., t_k(=k/n), ..., t_n(=1)$, measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}=(X,\mathcal{P}(X),F), S_{[\phi_{0,t_k}(\omega_0)]})$ is taken.

That is, putting $T_n=\{t_0(=0), t_1(=1/n), \cdots, t_n(=1)\}$, we take the tensor exact measurement:

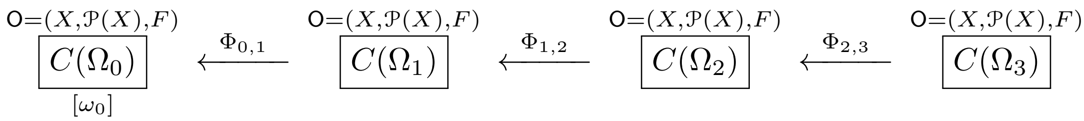
$$\bigotimes_{t_k\in T_n} \mathsf{M}_{C(\Omega)}(\mathsf{O}=(X,\mathcal{P}(X),F), S_{[(\phi_{0,t_k}(\omega_0))]})$$
$$=\mathsf{M}_{C(\Omega^{T_n})}(\bigotimes_{t_k\in T_n}\mathsf{O}_{E_\Omega}=(\Omega^{T_n},\mathcal{B}(\Omega^{T_n}),\bigotimes_{t_k\in T_n}F), S_{[(\phi_{0,t_k}(\omega_0))_{t_k\in T_n}]})$$

Then, we see that, for any $\Xi_k\subseteq X$ $(k=1,2,...,n)$,

(N) the probability that the measued value belongs to $\times_{i=0}^{k}\Xi_k$ is given by $\times_{k=0}^{n}[F(\Xi_k)](\phi_{0,t_k}(\omega_0))$

[(ii): Heisenberg picture (observable moves: (Series time)]:

Figure (the case that $N=3$; $\Omega=\Omega_i,\ \ i=0,1,2,3$)



As mentioned in the above, assume that the state $\omega_0(\in\Omega)$ at time $t_0(=0)$, and $T_n=\{t_0(=0), t_1(=1/n), \cdots, t_{n-1}(=(n-1)/n), t_n(=1)\}$.

Assume that, at each $t_0(=0), t_1(=1/n), \cdots, t_{n-1}(=(n-1)/n), t_n(=1)$, an observable $\mathsf{O}=(X,\mathcal{P}(X),F)$ is set.

($\flat_4$) the observable $\mathsf{O}(=(X,\mathcal{P}(X),F))$ at time $t_n(=1)$ is identified with the observable $\Phi_{t_{n-1},t_n}\mathsf{O}(=(X,\mathcal{P}(X),\Phi_{t_{n-1},t_n}F))$ at time t_{n-1}. At time t_{n-1}, we originally have an observable O, and the product of this O and $\Phi_{t_{n-1},t_n}\mathsf{O}$ gives the observable at time t_{n-1}:

$$\mathsf{O}\times(\Phi_{t_{n-1},t_n}\mathsf{O})\quad (=(X^2,\mathcal{P}(X^2),\widehat{F}_{n-1}))$$

Similarly, the observable it time t_{n-2} is represented by

$$\mathsf{O}\times(\Phi_{t_{n-2},t_{n-1}}(\mathsf{O}\times(\Phi_{t_{n-1},t_n}\mathsf{O})))\quad (=(X^3,\mathcal{P}(X^3),\widehat{F}_{n-2}))$$

Further, the observable at time t_{n-3} is represented by,

$$\mathsf{O} \times (\Phi_{t_{n-3},t_{n-2}}(\mathsf{O} \times (\Phi_{t_{n-2},t_{n-1}}(\mathsf{O} \times (\Phi_{t_{n-1},t_n}\mathsf{O})))) \quad \big(= (X^4, \mathcal{P}(X^4), \widehat{F}_{n-3})\big)$$

Iteratively, after all, the observable at time t_0 is represented by,

$$\begin{aligned}\widehat{\mathsf{O}}_{t_0} &= \mathsf{O} \times (\Phi_{t_0,t_1}(\cdots(\mathsf{O} \times (\Phi_{t_{n-3},t_{n-2}}(\mathsf{O} \times (\Phi_{t_{n-2},t_{n-1}}(\mathsf{O} \times (\Phi_{t_{n-1},t_n}\mathsf{O})))))\cdots)) \\ &= (X^{n+1}, \mathcal{P}(X^{n+1}), \widehat{F}_0)\end{aligned}$$

Thus, we get the measurement $\mathsf{M}_{C(\Omega)}(\widehat{\mathsf{O}}_{t_0}, S_{[\omega_0]})$ at time $t = 0$. Therefore, putting $\Xi_k \subseteq X$ $(k = 1, 2, ..., n)$, we see that

(O) the probability that its measured value belongs to $\times_{i=1}^{k}\Xi_k$ is given by $[\widehat{F}_0(\Xi_0 \times \Xi_1 \times \cdots \times \Xi_n)](\omega_0)$

Here, we see

$$\begin{aligned}&[\widehat{F}_0(\Xi_0 \times \Xi_1 \times \cdots \times \Xi_n)](\omega_0) \\ =&[F(\Xi_0)](\omega_0) \times \Phi_{t_{n-1},t_{n-2}}[\widehat{F}_1(\Xi_1 \times \cdots \times \Xi_n)](\omega_0) \\ =&[F(\Xi_0)](\omega_0) \times [\widehat{F}_1(\Xi_1 \times \cdots \times \Xi_n)](\omega_1) \\ \cdots& \\ =&\bigtimes_{k=0}^{n}[F(\Xi_k)](\phi_{0,t_k}(\omega_0))\end{aligned}$$

Here, note that (N)=(O) holds. Thus, we can conclude tha

(P) Schrödinger and Heisenberg pictures are equivalent in the classical system

9.2.3.2 Derivation of the motion function method from (classical) quantum language

In classical system, we see that the Schrödinger picture (N) and Heisenberg picture (O) are equivalent, we use Schrödinger picture (N) from here. Put $T = [0, 1]$. Consider the case that $\mathsf{O} = (X, \mathcal{P}(X), F)$ is the precise observable O_E (*cf* Sec. 1.4.5), namely, $\mathsf{O} = (X, \mathcal{P}(X), F) = \mathsf{O}_E = (\Omega, \mathcal{B}(\Omega), E_\omega)$. That is, we consider the case that $X = \Omega, F = E_\omega$, namely,

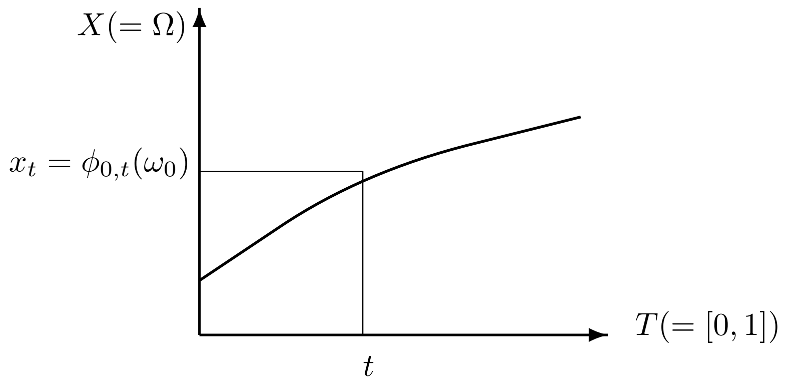
$$\text{precise measurement } \mathsf{M}_{C(\Omega)}(\mathsf{O}_E = (\Omega, \mathcal{B}(\Omega), E_\omega), S_{[\phi_{0,t}(\omega_0)]}) \quad (t \in T = [0, 1])$$

And further, consider the tensor product precise measurement

$$\begin{aligned}&\bigotimes_{t\in T}\mathsf{M}_{C(\Omega)}(\mathsf{O}_E = (\Omega, \mathcal{B}(\Omega), E_\Omega), S_{[\phi_{0,t}(\omega_0)]}) \\ =&\mathsf{M}_{C(\Omega^T)}(\bigotimes_{t\in T}\mathsf{O}_E = (\Omega^T, \mathcal{B}(\Omega^T), \bigotimes_{t\in T}E_\Omega), S_{[(\phi_{0,t}(\omega_0))_{t\in T}]})\end{aligned}$$

Proposition 1.20 says:

(Q) (=(F$_2$) in Proposition 1.20): When the tensor product precise measurement $\mathsf{M}_{C(\Omega^T)}(\bigotimes_{t \in T} \mathsf{O}_{E_\Omega} = (\Omega^T, \mathcal{B}(\Omega^T), E_{\Omega^T}),\ S_{[(\phi_{0,t}(\omega_{\omega_0})_{t \in T}]})$ is take, the probability that the measured value $(x_t)_{t \in T} (\in \Omega^T)$ belongs to any open set which includes $(\omega_t)_{t \in T} (\in \Omega^T)$ is 1. In the same sense, the measured value $(x_t)_{t \in T} (\in \Omega^T)$ is surely equal to $(\phi_{0,t}(\omega_0))_{t \in T}$

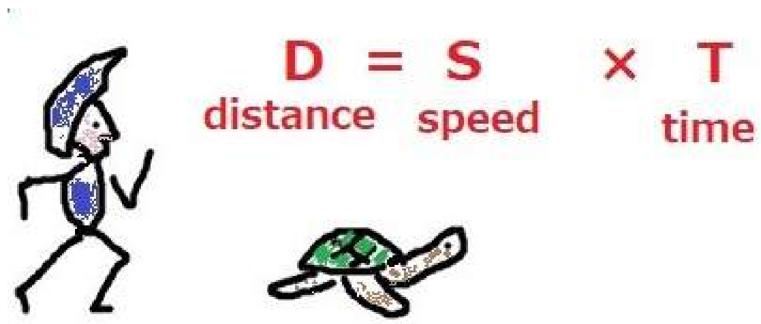


In general, define the position map $P' : \Omega(= X) \to X'$ such that

$$\Omega(= X) \ni \text{[state]=[position, momentum]} \overset{P'}{\mapsto} \text{[position]}(= X')$$

Then, the motion function $m : T \to X'$ can be written as follows.

$$m(t) = P'(\phi_{0,t}(\omega_0)) \qquad (\forall t \in T)$$



9.3 Subjective idealism: Berkeley, "To be is to be perceived"

9.3.1 Priest: Berkeley

Berkeley (1685 -1753) is famous as follows.

(A_1) Berkeley is a priest, and he interpreted Locke's primary quality as the state of things that come from a supernatural power such as a god. Thus his philosophy is called subjective idealism.

Let's check the next table to see where Berkley is located.

Table 9.2: The key-words of mind-body dualism

mind-body dualism	A(= mind) [tertiary universal]	B(body:between A and B [(primary) universal]	C(= matter) [secondary universal]
Plato (Allegory of the sun)	actual world	Idea	idea world
Thomas (1225-1274)	human intellect [tertiary universal]	divine intellect [(primary) universal]	individual [secondary universal]
Descartes (1596-1650)	I, mind, brain	body	matter
Locke (1632-1704)	mind	body (≈ sensory organ) [secondary quality]	matter [primary quality]
Berkeley (1685 -1753)	mind	body (≈ sensory organ) [secondary quality]	God
Kant (1724-1804)	phenomenon	cognition	thing-in-itself
quantum language	**observer [measured value]**	**measuring instrument [observable]**	**system [state]**

If we were to evaluate this Table description alone, we would have to say the following.

$$\boxed{\text{Locke}} \xrightarrow[\text{regress}]{} \begin{cases} \boxed{\text{Berkeley}} \\ \boxed{\text{Kant}} \end{cases}$$

Berkley was too extreme and Kant said too many pseudo-scientific nitpicks. Of course, they hit an inverted home run.

Berkeley was an important figure in the following sense.

(A$_2$) Berkeley indicated that Newton's definition of differentiation "$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$" is not complete

(A$_3$) He said

- To be is to be perceived,
- If a tree falls in a forest and no one is around to hear it, does it make a sound?

which represented the essential spirit of the mind-body dualism

Summing up, Berkeley was always the standpoint of anti-Newton (= anti-realism).

- If we think that modern philosophy (from Descartes to Kant) has significance as a buffer zone of Christianity with Newtonian mechanics, we can conclude that Berkeley was honest.

♠**Note 9.8.** The mathematical definition of $\lim \frac{0}{0}$ (i.e., (ϵ, δ)-definition of limit) was more important than Newton thought, and it was discovered one hundred and tens of years later (by Cauchy (1789-1857), Weierstrass (1815-1897), etc.).

9.3.2 (A_3): To be is to be perceived

Berkeley is too extreme and many may disagree next:

$$(\sharp) \qquad \boxed{\text{Locke}} \xrightarrow[\text{progress}]{} \boxed{\text{Berkeley}}$$

However, his saying "To be is to be perceived" is the essence of the Copenhagen interpretation. Thus, I agree to the (♯).

Consider the following saying:

(B_1) There is no science without measurement
(≈ [To be is to be perceived])

Everyone may believe that this saying (B_1) is absolutely true. In fact, the importance of "measurement" is emphasized as follows (*cf.* Sec. 1.1).

$$\underset{\text{(=quantum language)}}{\boxed{\text{measurement theory}}} = \underset{\text{[Axiom 1]}}{\boxed{\text{measurement}}} + \underset{\text{[Axiom 2]}}{\boxed{\text{causal relation}}} + \underset{\text{[the manual to use Axioms 1 and 2]}}{\boxed{\text{linguistic Copenhagen interpretation}}} \tag{9.1}$$

But, it is Genius Newton (and Einstein) that neglect this absolute truth (B_1). In fact, Newtonian mechanics is formulated as follows.

$$\boxed{\text{Newtonian mechanics}} = \boxed{\text{No measurement}} + \underset{\text{[Newtonian kinetic equation]}}{\boxed{\text{causal relation}}} + \underset{\text{[the manual to use Newtonian mechanics]}}{\boxed{\text{No interpretation}}} \tag{9.2}$$

Here, note that Newton removed "measurement" from (9.2) in spite of the maxim that there is no science without measurement. Perhaps he intuited that adding "measurement" would be tricky. If so, the insightfulness of Newton is surprising. A genius isn't confused by "the absolute maxim (B_1)". The following is my fiction:

(B_2) "Exclusion of measurement" is the conclusion reached by the deep consideration of Newton. However, Berkeley, an excellent controversialist of anti-Newton, considered that the exclusion was a weak point of Newtonian mechanics. And he said

To be is to be perceived

This is the golden rule of anti-Newtonianism (i.e., anti-physicalism, idealism). The opposing structure: [Newton vs. Berkeley] continues to [Einstein vs. Bohr] as mentioned in next section.

♠**Note 9.9.** "Why does Newtonian mechanics (9.2) have no" interpretation "? I don't know the answer. Perhaps, the way to interpret Newtonian mechanics is so simple that I don't bother to mention it. I believe that there is always an "interpretation" of any non-mathematical theory (*cf.* ref. [84]). I also believe that this is related to the "Wittgenstein paradox" (*cf.* Sec. 12.2.3).

I think that a theory that contains something that cannot be explained in one word requires "interpretation". And I think that is the " idea (or observable quantity, or measurement)". This thinking explains the perpetuation of Western philosophy, with Plato as its source.

9.3.3 "Einstein-Tagore Meeting" and "Bohr-Einstein debates"

Concerning "realistic worldview vs. idealistic worldview", Einstein-Tagore (poet, thinker in India) meeting in 1930 is famous, in which they asserted as

- Tagore: *Truth is always limited by human perception.*
- Einstein: *Truth is independent of our consciousness, For instance, if nobody is in this house, yet that table remains where it is*[*6].

In the above, Tagore's assertion is similar to Berkeley's "To be is to be perceived", which belongs to the situation of dualistic idealism(=idealistic worldview).

On the other hand, Einstein's saying:

(C_1) if nobody is in this house, yet that table remains where it is (= Does the moon disappear when I'm not looking at it?)

is the same as

(C_2) Truth is independent of us (= realistic worldview)

Thus, Einstein and Newton are similar, in the sense that

Truth is independent of human being (i.e., physics holds without measurement)

Therefore, it should be noted that the formula (9.2) is significant. In this text, we are not concerned with Bohr-Einstein debates in quantum mechanics (in order to solve this problem, I proposed quantum language), (*cf.* ref. [74])). However, Bohr-Einstein debates is similar to the above. Thus, summing up, we see:

[*6] Einstein often said this kind of statement at various places, for example, "*Does the moon disappear when I'm not looking at it?*"

Table 9.3 realistic worldview vs. idealistic worldview

Realistic worldview [monism, realism, no measurement]	Idealistic worldview [dualism, idealism, measurement]
Newton	Berkeley
Newton (and Clarke)	Leibniz
Einstein	Tagore
Einstein	Bohr

Concerning Bohr-Einstein debates, The impression that Einstein lost now has been left, but the author does not think so (*cf.* ref. [74]).

♠**Note 9.10.** Omitting "Newton vs. Berkeley" and "Einstein vs. Tagore" in the above table, I repeatedly mention the following table (*cf.* Assertion 4.2):

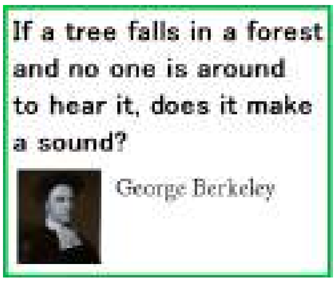
Table 9.4 realistic worldview vs. idealistic worldview (*cf.* Table 4.1 in Sec. 4.1.3)

dispute ╲ [R] vs. [L]	Realistic worldview (monism, no measurement)	Idealistic worldview (dualism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	"Nominalismus"(Ockham)	"Realismus"(Anselmus)
ⓓ: space-time	Newton	Leibniz
ⓔ: quantum theory	Einstein	Bohr
ⓕ:philosophy of science	Carnap	Quine

For the precise argument, see Table 4.1.

ⓐ is my fiction, ⓒ is a confusion (Aristotle vs. Plato). ⓓ is the Leibniz=Clarke correspondence (*cf.* Sec. 9.2.2), ⓔ is Bohr-Einstein debates. Quantum language is proposed as one of answers to Bohr-Einstein debates(*cf.* ref. [74]). Quantum language is proposed as one of answers to Bohr-Einstein debates(*cf.* ref. [74]). ⓕ: Quine understood the spirit of the linguistic Copenhagen interpretation (i.e., "If you don't measure it, you don't know anything") in the Carnap=Quine debate (*cf.* Sec. 12.3).

♠**Note 9.11.** In Japan, I learned the dualistic proposition such as "If a tree falls in a forest and no one is around to hear it, does it make a sound?" in a Zen dialogue (i.e., a question-and-answer exchange between Zen priests and their followers). Zen is one school of Buddhism. In modern Japan, most people may think that Zen monologue is a kind of wordplay.



9.3.4 Bohr-Einstein debates: Do the laws of physics require measurement?

For Bohr-Einstein debates, I discussed in ref. [74]. However, I would like to write something in this section. Almost people agree to the following maxim:

- There is no science without measurement

However, genius Newton neglected this maxim, and proposed Newtonian mechanics as follow: Newtonian mechanics and quantum mechanics are formulated as follows:

$$(\sharp_1)\quad \boxed{\text{Newtonian mechanics}} = \text{Nothing} + \underset{\text{(Newtonian equation)}}{\boxed{\text{causality}}}$$

On the other hand, quantum mechanics is formulated as follows.

$$(\sharp_2)\quad \underset{}{\overset{\text{(=quantum language)}}{\boxed{\text{measurement theory}}}} = \overset{\text{[Axiom 1]}}{\boxed{\text{measurement}}} + \overset{\text{[Axiom 2]}}{\boxed{\text{causality}}} + \underset{\text{[the manual to use Axioms 1 and 2]}}{\text{linguistic Copenhagen interpretation}}$$

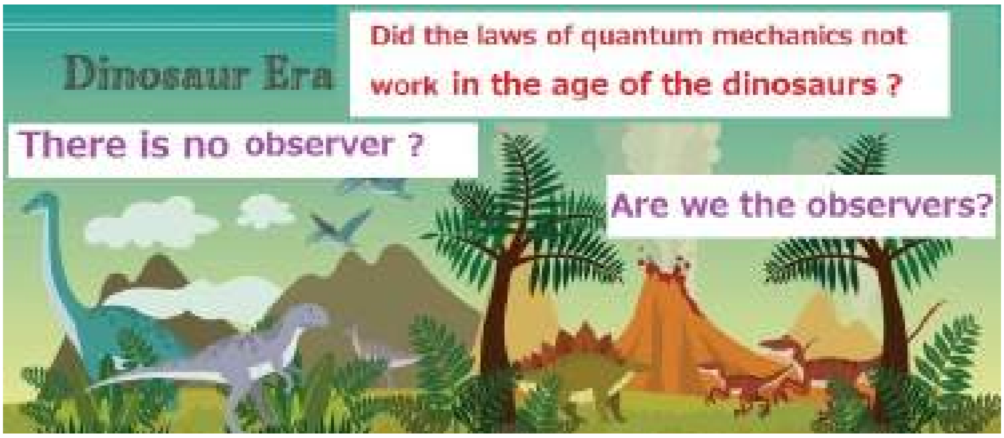
N. Bohr, the leader of the Copenhagen school, agreed to the ($\sharp_2$), on the other hand, A. Einstein asserted that measurement is not needed for physics since he believed in

($\sharp_3$) *The moon is there whether one looks at it or not.*

in Einstein and Tagore's conversation. So far, many experimental results support Bohr. However, if Einstein says the following ($\sharp_4$), everyone has no choice but to shut up.

($\sharp_4$) Then, did the laws of physics not work before the birth of mankind (e.g., the age of the dinosaurs)?

Thus, I think that Bohr-Einstein debates is not settled yet. All we can do is to wait for an Einsteinian genius to emerge.


Dinosaur Era
Did the laws of quantum mechanics not work in the age of the dinosaurs ?
There is no observer ?
Are we the observers?

9.4 Qualia problem and Brain in a vat argument

This section was written with reference to the following.

- [76]:Ishikawa, S: Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language Open Journal of philosophy, Vol. 8, No.5, 466-480, 2018, DOI: 10.4236/ojpp.2018.85032 (https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862)

  [Revised version] (https://philpapers.org/rec/ISHLCB)
  (http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf)
- [85] S. Ishikawa, K. Kikuchi, (2021) *Quantum Fuzzy Logic and Time, Journal of Applied mathematics and physics,* Vo.9 No.11 2021, 2609-2622 (https://www.scirp.org/journal/paperinformation.aspx?paperid=112972)

9.4.1 * The problem of qualia (= the subjectivity problem)

Jack and Betty were looking at the pink flowers. Betty had the following question (i.e., qualia problem) .

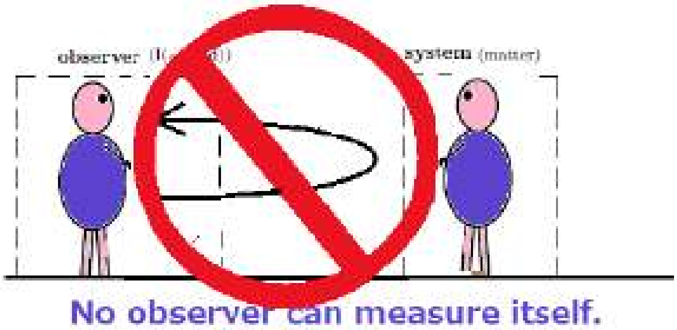
($\sharp$) Is the "pink" that I felt the same as the "pink" that Jack felt?

This question can be answered in the following way.
[(A): Scientific answer (To measure is to believe)]:
With Jack and Betty as test subjects, you (i.e., scientist) can perform various tests (color-blindness test, EEG measurement, electroretinogram, etc.). And if no difference is found in any of the tests, we can conclude that each "pink" felt by Jack and Betty is the same.
[(B): Non-scientific argument (To think is not to measure)]:
However, Betty may say:

($\flat_1$) I like the "pink" so much that I has pink walls in my room too. Jack loves blue and always wears blue. Therefore, I find "pink" to be more beautiful than Jack.

However, the scientist may say that

($\flat_2$) According to the linguistic Copenhagen interpretation (*cf.* (E_1) in Sec. 1.2.1), you (= Betty) are not qualified to be an observer. Even if you are qualified to be an observer, you should perform the same tests as in [(A):Scientific answer] for Jack and yourself as test subjects, that is, various tests (colorblindness test, EEG measurement, electroretinogram, etc.). However, if so, it is the same as the situation [(A):Scientific answer].



Then, Betty says:

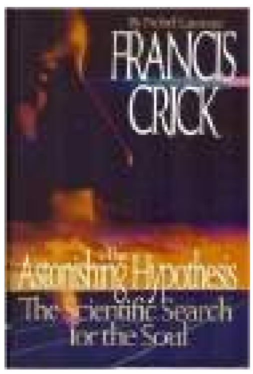
($\flat_3$) OK. I (=Betty) can understand. In short, the measurements I can only make against myself are not scientific measurements. Or, a measurement that only I can make is not a measurement. Is it OK?

Then, the scientist says to Betty:

($\flat_4$) That's right.

□

♠**Note 9.12.** In the book "The astonishing hypothesis" (by F. Crick (the most noted for being a co-discoverer of the structure of the DNA molecule in 1953 with James Watson)), Dr. Crick said that



($\sharp_1$) *You, your joys and your sorrows, your memories and your ambitions, your sense of personal identity and free will, are in fact no more than the behavior of a vast assembly of nerve cells and their associated molecules.*

That is, he believed the monistic realism such as

($\sharp_2$) the movement of the human spirit is also a kind of physical phenomenon.

I agree to his opinion. And I believe that with the development of brain science, even consciousness can be measured in the future. However, no matter how much brain science develops, I believe that there is no solution to the qualia problem other than the one described above.

9.4.2* Brain in a vat argument

Let us introduce a famous problem "Brain in a vat" due to H. Putnam (*cf.* ref. [108]) as follows:

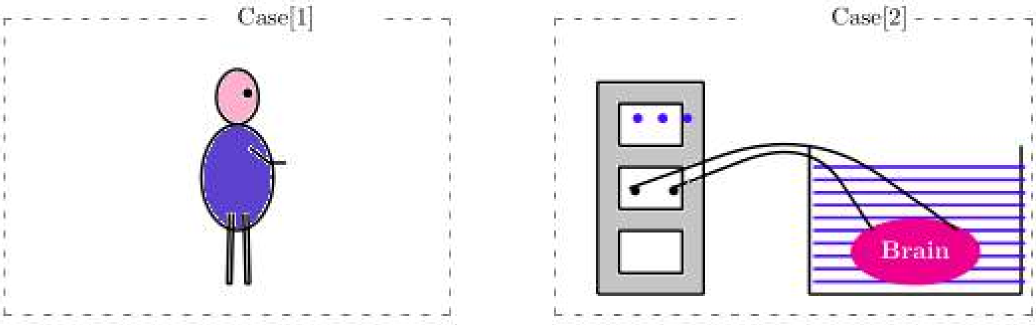


Figure 9.1: [Brain in a vat]

There is a possibility of the following.

(C_1) a mad scientist has removed your brain, and placed it into a vat of liquid to keep it alive and active. The scientist has also connected your brain to a powerful computer, which sends neurological signals to the brain in the way the brain normally receives them. Thus, the computer is able to send your the data to you brain to fool you into believing that you are still walking around a forest.

Here, you may say as follows.

(C_2) Am I a brain in a tank? Or can you confirm that I am not?

And you may say

(C_3) "I cannot decide if I am a brain vat or not".
That is, "I cannot decide if I am in Case[1] or Case[2]?

Therefore, you may say:

(C_4) "I cannot know if I have a limb or not."

Then, we have the following problem:

Problem 9.4. [The problem concerning "Brain in a vat"]

- Are the above arguments correct?

[Ansewr]:
One might think that (C_3) and (C_4) are obviously correct since (C_1) is assumed, but this is not the case from the quantum language standpoint. Because.you ask someone, "Am I walking through the woods?" or "Do I have a limb?" Then, they reply, "yes," so you can be sure that you are walking through the woods and have a limb. In short.

- You just have to trust the measurement results.

After all, we can believe in the following "spirit of quantum language (Copenhagen interpretation)".

- To be is to be perceived

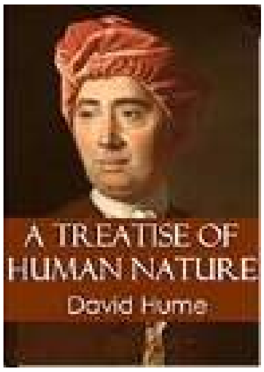
□

♠**Note 9.13.** I believe that "the above discussion has solved the 'brain in a vat,'" but one might disagree with this. In fact, some of my friends are not convinced by the above explanation. If you are one of them, I would like you to try the following problem.

($\sharp$) What rules should be added to the Copenhagen Interpretation (Sec. 1.3.3) to put an end to the "brain in a vat"? The question is: What rules should be added to the Copenhagen interpretation (Sec. 1.3.3)?

As I have said many times, the author believes that there is no perfect Copenhagen Interpretation. What we can do is to make the Copenhagen interpretation as close to perfect as possible. The most interesting part of my position is to collect problems that cannot be solved by any means, and to sort out the laws that they have in common.

9.5 Hume; skeptic who didn't measure, "A Treatise of Human Nature"



9.5.1 Hume's straying[Brain pseudo-science (≈Less than brain science)]; Hume's wordplay

Descartes takes the "Cogito proposition [I think therefore I am]" as the first proposition of philosophy, and from this he derives the "existence of I". And further, he asserted that he can derive the followings:

(A) "Existence of Matter", "Existence of Body", "Existence of Causality", ..., "Existence of God"

Given modern common sense, the cogito proposition is not a scientific proposition, and everything Descartes claimed to have derived from it is suspect.

However, these were just Descartes' tricks and what Descartes really wanted to say was to propose a mind-body dualism (consisting of "mind", "body" and "matter").

But Descartes philosophy is a philosophy that risks entering into pseudoscience. In fact, Hume fell into the zone of pseudoscience.

In "A Treatise of Human Nature" (1739), Hume doubted (A) above. Probably, Hume assumed something like the "brain in a vat" described in the previous section.

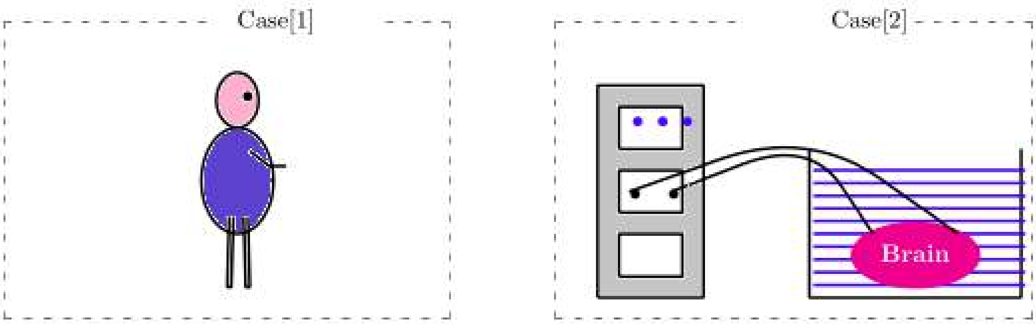
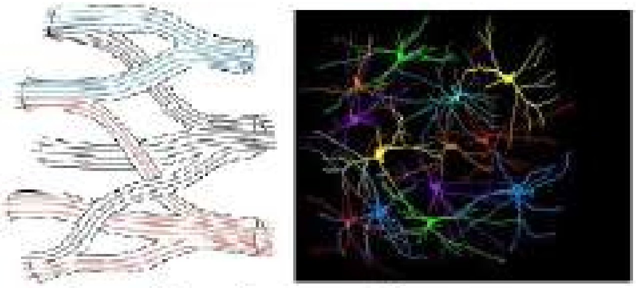


Figure 9.1 in Sec. 9.4.2: [Brain in a vat]

And he concluded that

(B) the existence of "bundle of perceptions (= brain circuit)" is certain.
- [matter], [body], [causality] etc. are kinds of states of brain circuit



bundle of perceptions ?

It is said that Hume's philosophy is the achievement of British empiricism (i.e., no further progress can be expected).

I think the reason they say so is as follows.

(C_1) Once we fall into the trap of the "aquarium brain," we cannot easily get out of it. As shown in the previous section, the "brain in a vat" can be solved only when the meaning of "measurement" is clarified.

(C_2) The study of perceptual bundles (= brain circuits) is a matter of brain science, which is impossible in Hume's time.

In either of these two cases, Hume was 300 years too early.

If so, I am forced to conclude as follows:

$$\boxed{\text{Descartes}} \xrightarrow[\text{progress}]{} \boxed{\text{Locke}} \xrightarrow{\text{regress}} \boxed{\text{Hume}}$$

This is because Descartes and Locke found three key words [mind, body, matter], which are the essence of mind-body dualism, but Hume placed [body, matter] under [mind].

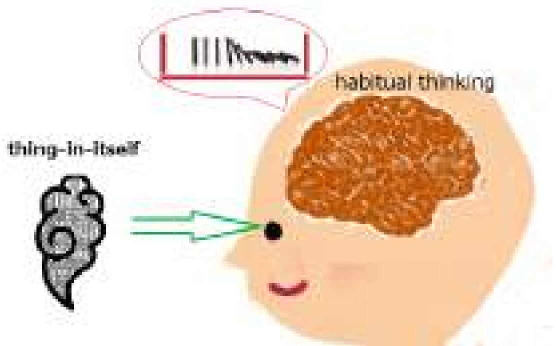
However, Hume is revived by Kant (see Kant: Copernican revolution in the next chapter). There, we see that

$$\boxed{\text{Descartes}} \xrightarrow[\text{progress}]{} \boxed{\text{Locke}} \xrightarrow[\text{progress}]{} \boxed{\text{Hume}} \xrightarrow[\text{progress}]{} \boxed{\text{Kant}}$$

9.5.2 Hume; The causality problem

It is a matter of course that the understanding of "causal relation" is the most important theme in worldview. In Newtonian mechanics, the causality is represented by Newtonian kinetic equation. In Descartes=Kant philosophy, the representation of "causal relation" is as follows.

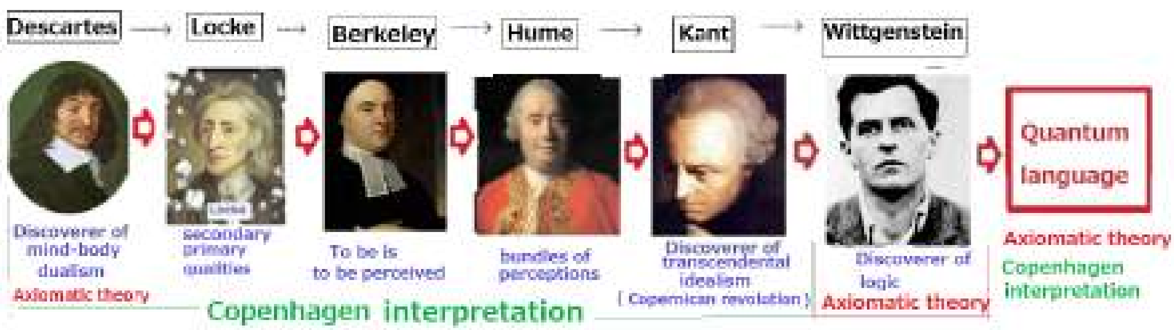
($\sharp_1$) **[Cognitive causality]:** David Hume, Immanuel Kant, etc. thought as follows. :
We cannot say that "causality" actually exists in the world, or that it does not exist in the world. And when we think that "something" in the world is "causality", we should just believe that the it has "causality". Hume argues that scientific wisdom is a product of "habitual thinking", not objective truth.



Most readers may regard this as "a kind of rhetoric", however, several readers may be convinced in "Now that you say that, it may be so." Surely, since you are looking through the prejudice "causality", you may look such. This is Kant's famous "Copernican revolution" (i.e., "Kant was awakened from his dogmatic slumber by Hume's idea and came up with the Copernican revolution", this will be discussed in Sec. 10.2 [What is causality?]), that is,

"cognition constitutes the world."

which is considered that the cognition circuit of causality is installed in the brain, and when it is stimulated by "something" and reacts, "there is causal relationship." Probably, many readers doubt about the substantial influence which this ($\sharp$) had on the science after it. However, Hume served as a bridge between Locke and Kant. Thus we think that

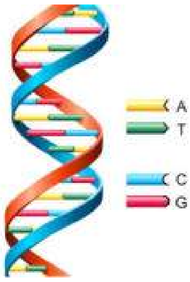


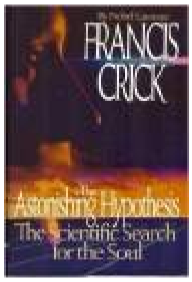
This will be discussed again in the next chapter (*cf.* Sec. 10.2 What is causality?).

♠**Note 9.14.** (i): Note that

$$\text{habitual thinking (in Hume philosophy)} \underset{\text{correspondence}}{\longleftrightarrow} \text{Copenhagen interpretation (in QL)}$$

(ii): In the book "The astonishing hypothesis" (by F. Crick (the most noted for being a co-discoverer of the structure of the DNA molecule in 1953 with James Watson)),





Dr. Crick said that

($\sharp_1$) *You, your joys and your sorrows, your memories and your ambitions, your sense of personal identity and free will, are in fact no more than the behavior of a vast assembly of nerve cells and their associated molecules.*

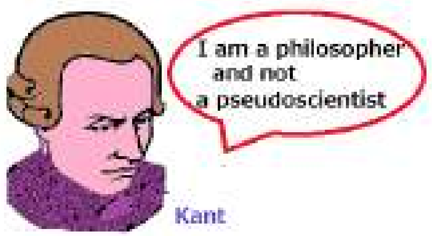
That is, he (as well as we) believed the monistic realism such as

($\sharp_2$) the movement of the human spirit is also a kind of physical phenomenon.

However, Crick did not reject the mind-body dualism. Because I think that

($\sharp_3$) British empiricism and brain science have nothing to do with each other.

which is also what Kant emphasized (though Hume may have confused pseudoscience with philosophy).



Throughout this text, I argue that

($\sharp_4$) it is the "Copenhagen interpretation (i.e., how to use Axioms 1 and 2)", not the "brain science", that is related to British empiricism ($\approx$ Descartes=Kant epistemology).

9.6 Hume's problem of induction in the quantum mechanical worldview

This section was written with reference to the following.

- [78]:Ishikawa, S: Philosophy of science for scientists; The probabilistic interpretation of science Journal of quantum information science, Vol. 9, No.3, 140-154, DOI: 10.4236/jqis.2019.93007 (https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

9.6.1 If discussions are not based on a certain worldview, the discussions will turn into chit-chat, and will not be scientific

For example, consider the following inferences (= inductive inferences):

(A_1) Until now, the sun has always risen in the east. So, tomorrow the sun will rise in the east again.
(A_2) When a coin is thrown ten times, we get (H,H,T,H,H,H,H,H,H,T) (where H="head", T="tail"). Thus, we infer that "H" will be obtained with probability 8/10 by the next coin-tossing
(A_3) White birds are swimming in a row. I can't see the last bird very well, but it must be a white bird.

Here, our current problem (= Hume's problem of induction) is

(B) Can we justify the above enumerative induction (i.e., inductive reasoning)?

If this is a problem of science, this is one of the most important problems in science. Now, the position of this text is always the same

(C) If discussions are not based on a certain worldview, the discussions will turn into chit-chat, and will not be scientific

For example, when I discussed "Zeno's paradox (*cf.* Sec.2.4), I explained this spirit (C) to the point of endless exposition. Therefore, what we should do is

(D) Within a quantum language (i,e., under a quantum mechanical worldview), justifying inductive inferences such as (A_1)$\sim$(A_3) above

♠**Note 9.15.** It is important to doubt a certain worldview, but you need to be careful about how you doubt it. For example, some may be interested in the justifications of the followings:

($\sharp_1$) Newtonian mechanics has been right to this day. So, will the Newtonian mechanics be right tomorrow?

($\sharp_2$) Quantum language have been very useful to this day. So, will quantum language be useful tomorrow?

The justification problem of these may be called Hume's problem of induction, though David Hume suspected the justification of induction. It cannot be said that these are meaningless. In fact, the following ($\sharp_3$) was exactly Einstein's interest.

($\sharp_3$) Newtonian mechanics has been right to this day. However, Is Newtonian mechanics correct when particles move at very high speeds?

However, Einstein was praised for advocating the theory of relativity. He would not have been evaluated if he had only doubted as in ($\sharp_3$). It is important to doubt, but if there is no result, it is a word-play. In the end, we consider that there are two kinds of "inductive inference" as follows.

($\flat_1$) Inductive reasoning under a certain worldview

($\flat_2$) Inductive reasoning for the validity of a certain worldview

Our interest is in ($\flat_1$), that is,

The Problem of Hume's Induction in the Framework of Quantum Language

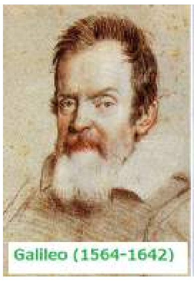
You are free to be skeptical about ($\flat_2$), but it will be accepted as a science only when it produces results like Einstein.

9.6.2 The solution of Hume's problem of induction; Without a worldview, nothing can be said

If Hume's induction problem has a scientific aspect, then there must be scientists who have clarified this scientific aspect without knowing Hume's induction problem. Thus, I think it is natural to expect that the scientific part of Hume's problem of induction is almost identical to already well-known scientific results. That is, I consider Hume's problem of induction to be essentially equivalent to the law of large numbers.

♠**Note 9.16.** In Note 7.1, we say that Bernoulli's achievement (i.e., the discovery of the law of large numbers) equals Galileo's achievement. That is, I think that Bernoulli's achievement equals Galileo's achievement. That is,

$$\begin{cases} \text{Scientific pioneer in the realistic worldview} \cdots \text{Galileo} \\ \text{Scientific pioneer in the idealistic worldview} \cdots \text{J. Bernoulli} \end{cases}$$



Galileo (1564-1642)

the law of large numbers
J. Bernoulli (1654-1705)

The reason for this is that Galileo-Newtonian mechanics can predict the future in serial time, and Bernoulli's law of large numbers has the power to predict the future in parallel time. (*cf.* Sec. 9.2.3, or ref. [85]). Furthermore, in an essential sense, I think

Only these two theories have the power to predict the future.

Hume's problem of inductive inference is a problem of predicting the future (in parallel time), I think that Hume's problem of inductive inference and Bernoulli's law of large numbers are closely related.

In this section, we show that the validity of Hume's problem of induction is easily resolved in the quantum mechanical worldview. First, "Hume's principle of the uniformity of nature" is the following assumption

(E) **[Hume's principle of the uniformity of nature]** Events that occur normally in this world do not occur completely randomly, but there is some kind of order. If it is confirmed that the same phenomenon occurs over and over again under similar conditions, the same phenomenon will occur again under similar conditions.

Of particular importance in (E) above is the part about "Events that occur normally in this world". Therefore, I think that "the problem of Hume's induction" is as follows.

(F) Quantum language is a world view that describes events that occur normally in this world, so we can justify inductive reasoning by defining the "uniformity principle of nature" under quantum language.

The theme of this text is the quantum mechanical worldview, and I assert:

[To do science]=[To speak in quantum language]

Thus, we assume a quantum mechanical worldview as a worldview that describes the events that normally occur in this world. Therefore, we must start from Definition 9.5 [The uniformity principle of nature] in quantum language.

Definition 9.5. [The uniformity principle of nature] Quantum system is also acceptable, but for simplicity, the C^*-basic structure of the classical system is used. Consider $[C(\Omega) \subseteq B(L^2(\Omega,\nu))]$. A family of measurements $\{\mathsf{M}_{C(\Omega)}(\mathsf{O}_i := (X, \mathcal{P}(X), F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ is said to be satisfied with the condition "the uniformity principle of nature", if it satisfies that

($\sharp$) there exists a probability space $(X, \mathcal{P}(X), \mu)$ that satisfies

$$[F_i(\Xi)](\omega_i) = \mu(\Xi) \qquad \forall \Xi \in \mathcal{P}(X), \forall i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$$

////

Under the above definition, we assert the following theorem (essentially the same as the law of large numbers).

Theorem 9.6. [Inductive inference(= inductive reasoning); (The solution of Hume's problem of induction] Consider a classical C^*-basic structure $[C(\Omega) \subseteq B(L^2(\Omega,\nu))]$. Assume that A family of measurements$\{\mathsf{M}_{C(\Omega)}(\mathsf{O}_i := (X, \mathcal{P}(X), F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ satisfies the uniformity principle of natur Let $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., x_N) \in \times_{i=-n}^{N} X$ be a measured value of the parallel measurement $\bigotimes_{i=-n}^{N} \mathsf{M}_{C(\Omega)}$ $(\mathsf{O}_i := (X, \mathcal{P}(X), F_i),\ S_{[\omega_i]})$. Then, it holds that

$$\frac{\sharp\{k \mid x_k \in \Xi, k = -n, -n+1, ..., -1\}}{n} \approx \mu(\Xi)(= [F_i(\Xi)](\omega_i)) \qquad (9.3)$$
$$\big(\Xi \in \mathcal{P}(X), i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\big)$$

where n is assumed to be a sufficiently large number. Also, $\sharp[\Theta]$ is the number of the elements of a set Θ.

Proof: The proof is simple, but just to be sure, it is described below.
. Let $\Xi_i \in \mathcal{P}(X)$ $(i = -n, -n+1, ..., -1, 0, 1, ..., N)$. According to Axiom 1 [measurement] (in Section 1.2.1), we say

($\sharp$) the probability that a measured value $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., x_N)$ of a parallel measurement $\bigotimes_{i=-n}^{N} \mathsf{M}_{C(\Omega)}$ $(\mathsf{O}_i := (X, \mathcal{P}(X), F_i),\ S_{[\omega_i]})$ belongs to $\times_{i=-n}^{N} \Xi_i$ is equal to $\times_{i=-n}^{N}[F_i(\Xi_i)](\omega_i) = \times_{i=-n}^{N} \mu(\Xi_i)$

Therefor, a sequence $\{x_i\}_{i=-n}^{N}$ can be regarded as a sequence which is independently selected from the probability space $(X, \mathcal{P}(X), \mu)$. Therefore, by the law of large numbers, we get (9.3). □

□

After all, we may say

inductive inference

$\approx$ the quantum linguistic representation of the law of large numbers

♠**Note 9.17.** (i): For the idea that a parallel measurement $\bigotimes_{i=-n}^{N} \mathsf{M}_{C(\Omega)}$ $(\mathsf{O}_i := (X, \mathcal{P}(X), F_i),\ S_{[\omega_i]})$ can be regarded as the time series of the family of measurements $\{\mathsf{M}_{C(\Omega)}(\mathsf{O}_i := (X, \mathcal{P}(X), F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$, see ref. [85].
(ii): The reader may wonder why philosophers have failed to formulate inductive reasoning? Because they did not first declare a worldview and then formulate induction under that worldview. We cannot say anything without a worldview (such as quantum language). Again, note that

- No formulation is possible without assuming a worldview.
- If you don't assume a worldview, it can only be a chat

9.6.3 Examples (Coin toss, Emerald)

Example 9.7. [Coin tossing]. [Coin tossing]. Let us discuss the unfair coin tossing as the most understandable example of Theorem 9.6 [Inductive reasoning]. Consider a basic structure $[C(\Omega) \subseteq B(L^2(\Omega, \nu))]$. Here, Ω is the state space, $\omega (\in \Omega)$ is a state of "how to throw a coin". Let $\{\omega_i\}_{i=-n}^{N}$ be a sequence in Ω, where ω_i is the state of i-th coin tossing $(i = -n, -n+1, ..., 0, 1, 2, 3, ..., N)$. Let $\mathsf{O} = (X, 2^X, F)$ be an observable in $C(\Omega)$ such that

$$
\begin{aligned}
& X = \{H, T\}, \text{(where } H\text{: head, } U\text{: tail)} , \\
& [F(\{H\})](\omega_i) = \mu(\{H\}) = 2/3, \quad [F(\{T\})](\omega_i) = \mu(\{T\}) = 1/3 \\
& \qquad (\forall i = -n, -n+1, ..., -1, 0, 1, 2, ..., N)
\end{aligned} \tag{9.4}
$$

Therefore, a family of measurements$\{\mathsf{M}_{C(\Omega)}(\mathsf{O} := (X, 2^X, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ satisfies "the uniformity principle of nature".

Let $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., x_N) \in \times_{i=-n}^{N} X$ be a measured value of the parallel measuremment $\bigotimes_{i=-n}^{N} \mathsf{M}_{C(\Omega)}$ $(\mathsf{O} := (X, 2^X, F), S_{[\omega_i]})$.

Theorem 9.6 [Inductive reasoning] says that it is natural to assume that, for sufficiently large n,

$$
(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0) = (\underbrace{\text{T H H T H H H T T ..... T H H}}_{n+1})
$$

where

$$
\begin{cases} \sharp[\{i = -n, -n+1, ..., 0 \mid x_i = H\}]/(n+1) \approx 2/3 \\ \sharp[\{i = -n, -n+1, ..., 0 \mid x_i = F\}]/(n+1) \approx 1/3 \end{cases} \tag{9.5}
$$

Therefore,

$$
\begin{cases} x_i = H & \text{(probability } 2/3) \\ x_i = T & \text{(probability } 1/3) \end{cases} \qquad (i = 1, 2, ..., N) \tag{9.6}
$$

Here, it should be noted that

($\sharp$) Even without knowing the condition (9.4)(i.e., $\mu(\{H\}) = 2/3$), we can know the future (9.6) from the past data(9.5). future (9.6). However, if we have no information about the future, we cannot estimate the future. Note that we assume that we know the information that both the past and the future will have the same skewness.

////

Example 9.8. [Emerald]. In order to prepare for the next section, we will divide the discussion into two parts: "[I]: Solutions without quantum language" and "[II]: Solutions with quantum language". All of these are too easy, but they are exercises in preparation for the next section.

[I]: Solution without quantum language

Suppose there is an emerald mine M_1. Assume the following.

(G) Assume that the probability that an emerald excavated at mine M_1 is green is $p (0 \ll p \leq 1)$. (However, the specific value of p is unknown.)

Unearth emeralds in the mine M_1.

Consider times $-n, -n+1, ..., -1, 0(= t_0), 1, 2, ..., N$. For simplicity, put $n = 10$, $N = 9$. For example, the result of the emerald excavation is "green at time -10 (=Y= Y_{gr})" and "not green at time -9 (=N= N_{gr})", ... Suppose that the following table is obtained.

Table 9.5: Mine M_1; time; -10, -9, -8,...,-1

time	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
'Green', or 'not Green'	Y	N	Y	Y	Y	Y	N	Y	Y	Y

Then, from Table 9.5 we can infer,

$$p \approx 0.8$$

If so, we can infer, by the hypothesis (G), that at time $t = 0, 1, 2, ..., 9, ...$, the color of the emerald that will be found in mine M_1 is as follows.

$$\begin{cases} \text{Green probability } p & \approx 8/10 \\ \text{Non-green probability } (1-p) & \approx 2/10 \end{cases}$$

Thus, it is not surprising if we obtain the following table.

Table 9.6: Mine M_1; time; 0,1,...,9

time	0	1	2	3	4	5	6	7	8	9
'Green', or 'not Green'	Y	Y	Y	N	Y	Y	Y	Y	N	Y

[II]: Solution that uses quantum language

As above, consider assumption (G) for the excavation of emeralds in the mine M_1.

(G′) (=(G)) Assume that the probability that an emerald excavated at mine M_1 is green is $p (0 \leq p \leq 1)$. (However, the specific value of p is unknown.)

Let us translate the above meaning into quantum language. Consider a C^*-basic structure$[C(\Omega) \subseteq B(L^2(\Omega, \nu))]$. a parallel measurement $\bigotimes_{i=-n}^{N} \mathsf{M}_{C(\Omega)}$ $(\mathsf{O}_i := (X, \mathcal{P}(X), F_i),\ S_{[\omega_i]})$. Here,

$$\Omega = \{\omega \mid \omega \text{ is a state of an emerald}\},$$

$$\Omega_{M_1} = \{\omega \in \Omega \mid \omega \text{ is a state of an emerald in mine } M_1\}$$

Let $\omega_i (\in \Omega_{M_1})$ be an state of an emerald (discovered in mine M_1) at time $i(= -n, -n+1, ..., 0, 1, 2, ..., N)$. Also, the observable $\mathsf{O} = (X = \{Y, N\}, 2^X, F)$ in $C(\Omega)$ is satisfied with

(H) $[F(\{Y\})](\omega) = p \quad [F(\{N\})](\omega) = 1 - p \qquad (\forall \omega \in \Omega_{M_1})$

Assume that

- by the parallel measurement $\bigotimes_{i=-n}^{N} \mathsf{M}_{C(\Omega)}$ $(\mathsf{O}_i := (X, \mathcal{P}(X), F_i),\ S_{[\omega_i]})$, we get "green $(= Y_{gr} = Y)$ at time -10", "not green $(= N_{gr} = N)$ at time -9",,,, and thus we get the following table:

Table 9.7 (≈ Table 9.5): Mine $_1$; time; -10, -9, -8,...,-1

time	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	$\cdots$
'Green', or 'not Green'	Y	N	Y	N	Y	Y	N	Y	Y	Y	$\cdots$

The, we see, by the assumption (G′), that

$$p \approx 0.8$$

It wouldn't be surprising if this part $[\cdots]$ in the above table looked like the table below.

Table 9.8 (= Table 9.6): Mine M_1time; 0,1,...,9

time	0	1	2	3	4	5	6	7	8	9
'Green', or 'not Green'	Y	Y	Y	N	Y	Y	Y	Y	N	Y

////

9.7* Grue paradox cannot be represented in quantum language

♠**Note 9.18.** I've read several books that explain Goodman's arguments concerning "grue Paradox" (*cf.* ref. [25]), but I've never been impressed with them. Rather, I think that the grue paradox is a strange paradox that no one understands, yet everyone knows its importance (If this paradox wasn't so important, it wouldn't be so famous.) I think that grue Paradox evokes the inevitability of the quantum mechanical worldview. That is, grue paradox can not be understood without quantum language.

This section improves upon the following literature

- [78]:Ishikawa, S: Philosophy of science for scientists; The probabilistic interpretation of science, Journal of quantum information science, Vol. 9, No.3, 140-154, DOI: 10.4236/jqis.2019.93007 (`https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447`)

9.7.1 Grue paradox (explanation without quantum language)

Let us extend the discussion in Example 9.8 of the previous section a little. Suppose that mine M_2 is located near mine M_1. Assume the following.

(I_1) In mine M_1, we assume that the probability that the excavated emerald is green is $p(0 \ll p \le 1)$.

Further,

(I_2) Suppose that the probability that an emerald excavated in mine M_2 is blue is p.

This is a somewhat unnatural assumption, but not impossible. For example, if we consider that a part of the original mine M_1 became mine M_2 due to tectonic movement, and the green emerald turned blue, the above condition is also possible.

First, let's excavate emeralds at the mine M_1. Let's take $t_0 (= 0)$ as a reference point. For example, "green ($= Y_{green} = Y$) at time -10", "not green ($= N_{green} = N$)at time -9", ... , we obtain the following table.

Table 9.9: Mine M_1; time; -10,-9,...,-1

time	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
'Green', or 'not Green'	Y	N	Y	Y	Y	Y	N	Y	Y	Y

If so, by the assumption (I_1), it is natural to consider

$$p \approx 0.8$$

And further, if so, by the assumption (I_2), we can know that the probability p in (I_2) is also equal to 0.8.

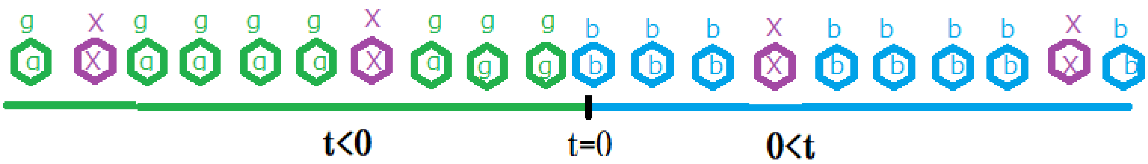
Thus, when excavating emeralds in mine M_2, we get, for example, "blue ($= Y_{blue} = Y$) at time 0, ... , not blue ($= N_{blue} = N$) at time 3,...", and so, we get the following table. This is of course, natual.

Table 9.10 : Mine M_2; time; 0,1,...,9

time	0	1	2	3	4	5	6	7	8	9
'Blue', or 'not Blue'	Y	Y	Y	N	Y	Y	Y	Y	N	Y

So far, the above makes some sense, even if there are some unnaturalness of (I_1) and (I_2). However, the following opinion is natural.

(J) The above argument asserts the following
- Since most of them were green before time $t = 0$, most of them are blue after time $t = 0$



Is this really "inductive reasoning" under "the uniformity principle of nature" ?

Now rewrite "the uniformity principle of nature (E)" in Sec. 9.6:

(E) (in Sec. 9.6) [**Hume's principle of the uniformity of nature**] Events that occur normally in this world do not occur completely randomly, but there is some kind of order. If it is confirmed that the same phenomenon occurs over and over again under similar conditions, the same phenomenon will occur again under similar conditions.

This must have been the starting point (Hume's original intention). The (J) above seems to be far from the original intention.

Here, Goodman asserted (*cf.* ref. [25]) that

(K) the (J) is surely "inductive reasoning"

He argued for (K) with the following very original idea.

(L_1) There is a dialect "grue" in the area of mine M_1 and mine M_2, which means "green" in the area of mine M_1 and "blue" in the area of mine M_2.

Put it this way.

(L_2) (J) above can be rewritten as
- If most of them are grue before time $t = 0$, most of them will be grue after time $t = 0$ as well

This would be in accordance with "Hume's original intention". Nevertheless, is Goodman's (K) correct? (proceed to next section).

9.7.2 Grue paradox (explanation with quantum language)

There are pros and cons to Goodman's arguments [(L_1) and (L_2)]. The reason there are pros and cons is that we do not yet understand "inductive reasoning" very well. In such a case, we can describe it in quantum language. Then we can know whether it is right or wrong.

Suppose that mine M_2 is located near mine M_1. Assume the following as in the previous (I_1) and (I_2).

(I'_1) In mine M_1, we assume that the probability that the excavated emerald is green is $p(0 \ll p \leq 1)$.

Further,

(I'_2) Suppose that the probability that an emerald excavated in mine M_2 is blue is p.

Then,

$$\Omega_1 = \{\omega_1 \mid \omega_1 \text{ is a state of an emerald in mine } M_1\}$$
$$\Omega_2 = \{\omega_2 \mid \omega_2 \text{ is a state of an emerald in mine } M_2\}$$
$$\text{where, } \Omega_1 \bigcap \Omega_2 = \emptyset \text{ (Strictly speaking, no two different emeralds are in the same state)}$$

Here, define the observable $\mathsf{O}_1^{green} = \{\{Y, N\}, 2^{\{Y,N\}}, F_1^{green})$ in $C(\Omega_1)$ as

$$[F_1^{green}(\{Y\})](\omega_1) = p \quad (\forall \omega_1 \in \Omega_1)$$

Thus, we see:

(M_1) The probability that a measured value "Y" is obtained by the measurement $\mathsf{M}_{C(\Omega_1)}(\mathsf{O}_1^{green} = \{\{Y, N\}, 2^{\{Y,N\}}, F_1^{green}), S_{[\omega_1]})$ is

$$[F_1^{green}(\{Y\})](\omega_1) = p \qquad (\forall \omega_1 \in \Omega_1)$$

Similarly, define the observable $\mathsf{O}_2^{blue} = \{\{Y, N\}, 2^{\{Y,N\}}, F_2^{blue})$ in $C(\Omega_2)$ as

$$[F_2^{blue}(\{Y\})](\omega_1) = p \quad (\forall \omega_2 \in \Omega_2)$$

Thus, we see:

(M_2) The probability that a measured value "Y" is obtained by the measurement $\mathsf{M}_{C(\Omega_2)}(\mathsf{O}_2^{blue} = \{\{Y, N\}, 2^{\{Y,N\}}, F_2^{blue}), S_{[\omega_2]})$ is

$$[F_2^{blue}(\{Y\})](\omega_2) = p \qquad (\forall \omega_2 \in \Omega_2)$$

Here, putting $\Omega = \Omega_1 \bigcup \Omega_2$, define the observable $\mathsf{O}^{grue} = \{\{Y, N\}, 2^{\{Y,N\}}, F^{grue})$ in $C(\Omega)$ by

$$[F^{grue}(\{Y\})](\omega) = \begin{cases} [F_1^{green}(\{Y\})](\omega) & (\omega \in \Omega_1) \\ \\ [F_2^{blue}(\{Y\})](\omega) & (\omega \in \Omega_2) \end{cases}$$

and $F^{grue}(\{N\}) = 1 - F^{grue}(\{Y\})$.

Let's take $t_0(=0)$ as a reference point. At time $t=-n,-n+1,...,-1,0,1,...,N$ a measurement $\mathsf{M}_{C(\Omega_1 \bigcup \Omega_2)}(\mathsf{O}^{grue}, S_{[\omega_i]})$ is taken. (Here, $\omega_i \in \Omega_1$ $(i=-n,-n+1,...,-1)$, $\omega_i \in \Omega_2$ $(i=0,1,...,N)$).

That is, a parallel measurement $\bigotimes_{i=-n,-n+1,...,0,...,N} \mathsf{M}_{C(\Omega_1 \bigcup \Omega_2)}(\mathsf{O}^{grue}, S_{[\omega_i]})$ is taken. For example, assuming $n=10, N=9$, we get "grue $(=Y_{grue}=Y)$ at time -10, not grue$(=N_{grue}=N)$ at time -9, ..." Then, we get the following table.

Table 9.11 (≈ Table 9.7): Mine $_1$; time; -10, -9, -8,...,-1, 0,1,...,9

time	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	$\cdots$
'Grue', or 'not Grue'	Y	N	Y	N	Y	Y	N	Y	Y	Y	$\cdots$

If so, from the assumption (I_1'), it is reasonable to infer the following .

$$p \approx 0.8$$

Furthermore, if so, from the assumption (I_2'), it is reasonable to infer that the part $[\cdots]$ in the above table is as follows.

Table 9.12 (≈ Table 9.8): Mine M_2 time; 0,1,...,9

time	0	1	2	3	4	5	6	7	8	9
'Grue', or 'not Grue'	Y	Y	Y	N	Y	Y	Y	Y	N	Y

Thus we can conclude that

(N) Goodman's arguments [(L_1) and (L_2)] is true

The readers here may not need any more "seediness", but I will mention it here in case they are interested. To restate Definition 9.5 [The principle of the uniformity of nature], we see

(O) Consider $[C(\Omega) \subseteq B(L^2(\Omega,\nu))]$. A family of measurements $\{\mathsf{M}_{C(\Omega)}(\mathsf{O}_i := (X, \mathcal{P}(X), F_i), S_{[\omega_i]}) \mid i=-n,-n+1,...,-1,0,1,2,...,N\}$ is said to be satisfied with the condition "the uniformity principle of nature", if it satisfies that
 (♯) there exists a probability space $(X, \mathcal{P}(X), \mu)$ that satisfies

$$[F_i(\Xi)](\omega_i) = \mu(\Xi) \qquad \forall \Xi \in \mathcal{P}(X), \forall i=-n,-n+1,...,-1,0,1,2,...,N$$

This says
- the measurable space $(X, \mathcal{P}(X))$ must be common, but F_i and ω_i are arbitrary but it must satisfy the condition (♯).

In short, if we rely on the quantum language Definition 9.5 (uniformity principle of nature) instead of the vague "Hume's uniformity principle of nature (E)," we know that Goodman's argument is correct.

♠**Note 9.19.** In the above discussion, the reader should have easily understood how Grue's paradox works and how interesting it is. The author believes that it is not easy to understand the workings and interest of Grue's paradox without using the quantum language formulation.

Chapter 10

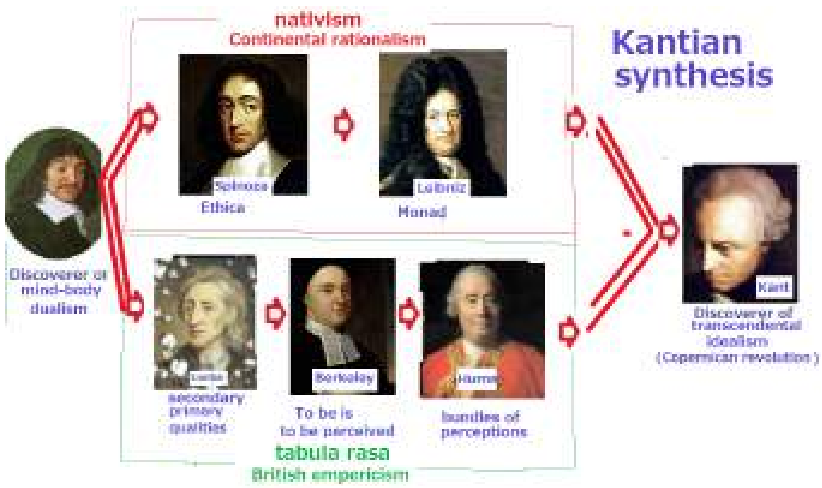
Kant: Copernican revolution

[I]: The following is "Kantian synthesis", which is called the flower of modern philosophy:

($\sharp_1$) mind-body problem — Descartes (the father of modern phil.) — subjectivity problem $\longrightarrow$

- Deduction (due to Descartes): Continental Rationalism (nativism) [Spinoza, Leibniz]
- Induction (due to Bacon): British Empiricism (tabula rasa) [Locke, Berkeley, Hume]

$\xrightarrow{\text{(synthesis)}}$ Kant



I think that Kant dreamed of a good relationship between philosophy and natural science. Specifically, he dreamed that philosophy would be the foundation of Newtonian mechanics. In other words, his dream was to create the philosophical foundation of Newtonian mechanics, and he tried to realize it by "the Copernican revolution (regarding cognition)". He wanted to write about this dream in "Critique of Pure Reason". In other words

- Knowledge gained from various experiences (British empiricism)
- Innate reason integrating miscellaneous knowledge (Continental rationalism)

$\xrightarrow[\text{synthesis}]{\text{Kantian}}$ Critique of Pure Reason

Although his challenge was not successful theoretically, his work was very successful as a kind of enlightening book that promoted a good relationship between philosophy and natural science.

In other words, I would like to think that his work had a great influence on the acceptance of natural science by the general public.

However, the above " Synthesis of Continental Rationalism and British Empiricism" will be realized in the following "QL Synthesis".

From a scientific point of view, I believe that this ($\sharp_1$) should be replaced by the following QL synthesis ($\sharp_2$) (=Quantum Linguistic Synthesis).[*1]

($\sharp_2$) QL-synthesis

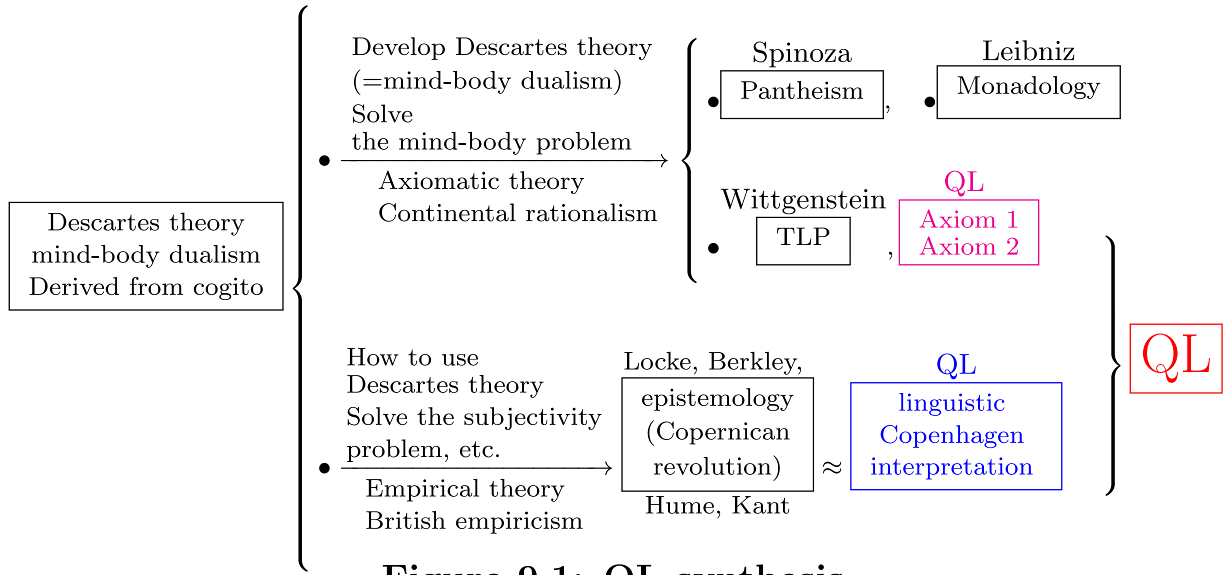
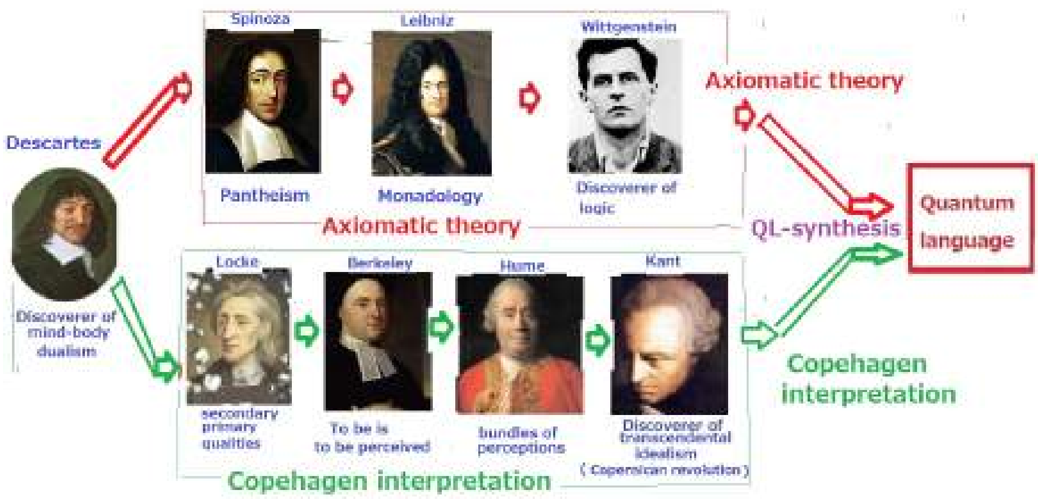


Figure 9.1: QL-synthesis



(However, attempts at Axiomatic theory were too difficult at the time)

[*1] This was, for the first time, proposed in ref. [84]: S. Ishikawa, (2022) *Empiricism, Rationalism, and Kantian synthesis in quantum linguistic point of view* OJPP, Vol. 12, No. 2, 182-198, (https://www.scirp.org/journal/paperinformation.aspx?paperid=117114)

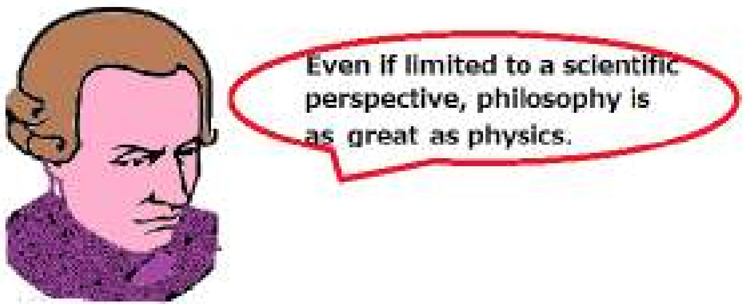
[[II]: But miracles do happen. This was not to say that Kant's "Copernican revolution" was a bad idea.

- **With the "Copernican revolution," Kant proposed scientific idealism (the so-called "transcendental idealism").**

That is, Descartes, Locke, and Hume strictly speaking belong to "pseudo-scientific realism" and not scientific idealism. Kant discovered another conceptual science, which is different from the realistic science (=physics). Quantum language is one of the most successful examples of the Copernican revolution (= scientific idealism). (*cf.* Sec. 11.7 in Chapter 11).

Therefore, if Kant said this, we would agree.

Philosophy (idealism) is as useful and great as physics, even if limited to the scientific point of view.



Summing up,

- Kant's synthesis is enlightenment, but the Copernican revolution is scientific.

10.1 Critique of Pure Reason

10.1.1 Three Critiques

Immanuel Kant (1724 - 1804), a professor of at the University of Königsberg, was one of the most influential philosophers in the history of Western philosophy. His main work is "Critique of Pure Reason (1781)", "Critique of Practical Reason (1788)", "Critique of Judgment(1790)", whose theme is respectively "truth" (i.e., "pseudo- truth" in the sense of this text), "virtue", "beauty".

Kant, who is at the top of the three fields, may look like a superhuman (for example, just as one person simultaneously wins three Nobel Prizes (physics, chemistry, physiology or medicine), however, in the case of philosophy, the three ("truth, goodness, and beauty") are one. As I said before, concerning "good" and "beauty", most philosophers have similar arguments. Therefore, they appeal a clear difference in the part of "true". It is the wisdom of Western philosophy (not found in Eastern philosophy) to keep fresh by model change.

That is, he followed the formulation of an idealistic worldview (*cf.* Sec.1.1) as follows.

$$(\mathrm{A}_1) \quad \underset{\text{preface, introduction, (fictional)premise, expedient}}{\overset{\text{"Critique of Pure Reason (1781)"}}{\boxed{\text{fictional worldview (literary truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\substack{\text{"Critique of Practical Reason (1788)"}\\ \text{"Critique of Judgment(1790)"}}}{\boxed{\text{ethics, morals, aesthetics}}}}$$

Of course, the interest of this text is concentrated to the worldview (i.e., "Critique of Pure Reason"). Have said many times in this text, philosophy of the worldview is a "preface" of (A_1). Thus, it must be built with as "meticulous logic" as a detective novel.

In fact,

(A_2) The Critique of Pure Reason succeeds as literature (or, enlightenment philosophy) in impressing upon its readers that it has succeeded in laying the foundation for Newtonian mechanics.

Considering the state of the world at that time, it is not surprising that it was favorably accepted by the general public. What kind of arguments Kant used to draw this conclusion is a testament to his skill. In fact, as a philosophy of the Enlightenment, it was a great success.

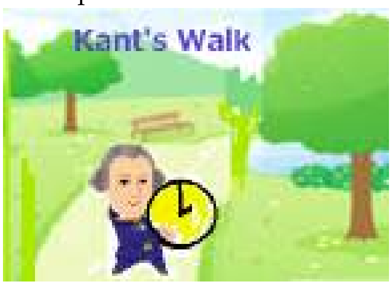
However, even the meaning of such things as "the foundation of Newtonian mechanics" is unclear, and there was no chance whatsoever that the "Critique of Pure Reason" would succeed scientifically.

That is,

(A$_3$) The Critique of Pure Reason (whose purpose is the "Enlightenment of Science") and Quantum Language (whose purpose is the "Language of Science") have little in common.

♠**Note 10.1.** As mentioned frequently up to this point, the idealistic worldview is really "asserted fiction", however, we must pretend not to regard the fictional worldview as "non-logical", or we must accept it as "logical in a wide sense". That is because it must be prohibited that the difference between philosophy and religion becomes fuzzy. Therefore, philosophers must use the terms such as "logic", "reason", etc. a lot in the idealistic worldview (e.g., "Critique of Pure Reason", "Tractatus Logico philosophicus (=TLP)", etc.). I think that the formulation of an idealistic worldview (*cf.* Sec.1.1) is a survival strategy to prevent Western philosophy from being swallowed by Christianity. Philosophers have to say "logical" even if it is a lie. Of course, philosophy enthusiasts know this and enjoy reading it, so it may not be a "lie".

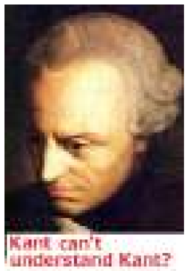
♠**Note 10.2.** It is often held that Kant lived a very strict and disciplined life, leading to an oft-repeated story that neighbors would set their clocks by his daily walks. In fact, he may have been a cheerful and sociable man. However, the image of Kant as strict and honest promotes an understanding of Kant's philosophy. For the philosopher, his image itself is a part of his work. Therefore, if you have seen a painting of Kant chatting with friends at a luncheon, it is better to pretend you did not see it and forget about it. To have a strong impact on his word "Copernican Revolution," Kant must be a modest and non-joking philosopher.




Kant chatting with friends at a luncheon

10.1.2 The purpose of "Critique of Pure Reason" (1781)



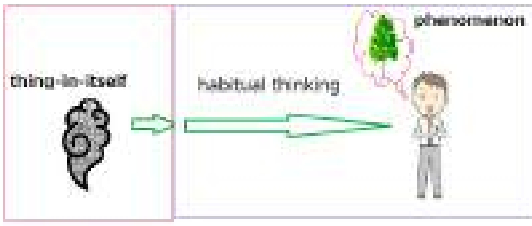


The following is a story that philosophy buffs will love.

(B_1) with the advent of Newton, the natural sciences have grown exponentially. People believe in that systematic knowledge as "objective truth" and come to have great expectations. It was Hume who made a bitter blow to it. He argues that

(♯) scientific wisdom is a product of "habitual thinking", not objective truth.

Kant is shocked by Hume's opinion. One of the central issues of "Critique of Pure Reason" is to rebuild the objectivity and reliability of the natural sciences that were shattered by Hume.



Therefore,

(B_2) Kant's purpose is to answer the following

- Why does Newtonian mechanics hold?
- Why can Newtonian mechanics be understood?

There is no way that philosophy can answer such a question, but Kant has come up with a solution that satisfies philosophy enthusiasts. Of course, it does not have to be a solution that satisfies physicists. Because, remember, modern philosophy is philosophy for the general public.

♠**Note 10.3.** (i): If we replace "habitual thinking" with "Copenhagen Interpretation" in (♯) in (B_1) above, i.e.,

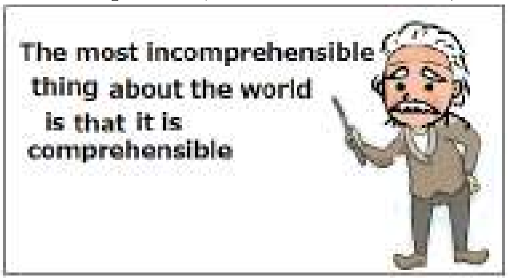
(♯) scientific wisdom is a product of "Copenhagen Interpretation", not objective truth,

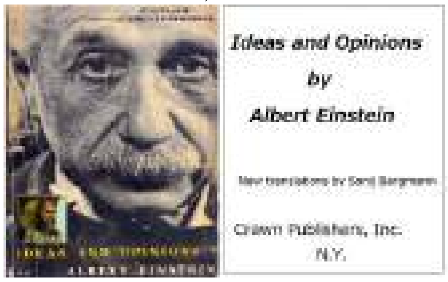
this is true for the explaination of quantum language. Thus, it is tempting to think that Kant envisioned a quantum language, not Newtonian mechanics.

(ii): Next is Einstein's words:

- *The most incomprehensible thing about the world is that it is comprehensible.*
  Or
  One may say "*the eternal mystery of the world is its comprehensibility.*" It is one of the great realizations of Immanuel Kant that the postulation of a real external world would be senseless without this comprehensibility. (on page 292 of ref. [18]:

Ideas and Opinions, Crown Publishers, Inc . . New York 1954)





Note that the above (B_2) is essentially the same as the following:

- Why is the world comprehensible?

With Einstein's endorsement above, it seems certain that Kant is the discoverer of "the most incomprehensible problem in the history of science" even if his theory is scientifically inadequate.
I don't think any researcher would challenge this today, because everyone would think that such a problem is impossible to solve. This is a problem to enjoy not knowing, not a problem to solve.

(iii): To mount against Newtonian mechanics, Kant may have proposed the following question:

- Why does Newtonian mechanics hold? Why can we understand Newtonian mechanics?

Even if the answer is uncertain, it is better to just discuss it. We may have the feeling that "philosophy is better than physics." If so, it is a success for the philosophy of the Enlightenment.

(iv): As seen in Chap. 12 later, I assert that the purpose of Wittgenstein's TPL (i.e., ref. [125]: "Tractatus Logico-Philosophicus") is to explain

- Why does logic work in our usual world?

My solution to this problem is not enlightening.

10.1.3 Thing-in-itself, Copernican revolution; from copy theory to constitution theory, Kantian synthesis

In order to solve the problem (B), Kant thought that

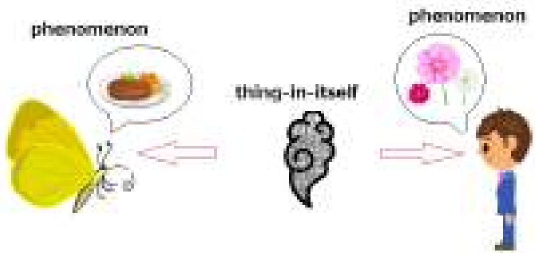
(C) We can understand the "world" only through the human perception. Also, cats can understand the "world" only through the cat perception. Thus,

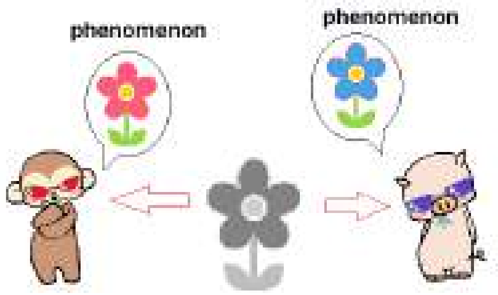
- There is "cat's world" for cats. and further, there is "butterfly's world" for butterflies.

If there are aliens who have better cognitive abilities than we humans do, their world is different from ours. Although the difference of the worlds is made by that of the cognitive ability, it is sure there exists something, which is called "thing-in-itself" by Kant.

That is, Kant thought as follows.

(D) we do not perceive the world as copy, but we perceive the world as it is constituted by cognition ability.





That is, Kant proposed so called Copernican revolution such that

$$\underset{\text{realism}}{\text{"copy theory"}} \xrightarrow[\text{Copernican revolution}]{} \underset{\text{(transcendental) idealism}}{\text{"constitution theory"}}$$

namely,

(E_1) It's not "the world first, cognition later", but "cognition first, the world later".

or

(E_2) Cognition is not about painting a photorealistic picture, but it is similar to painting an abstract picture.

Then our problem is as follows.

Problem 10.1.
[Kant's metaphysical foundation for Newtonian mechanics (natural science)]:
Assume the above Copernican revolution is true. Then, study the following problems:

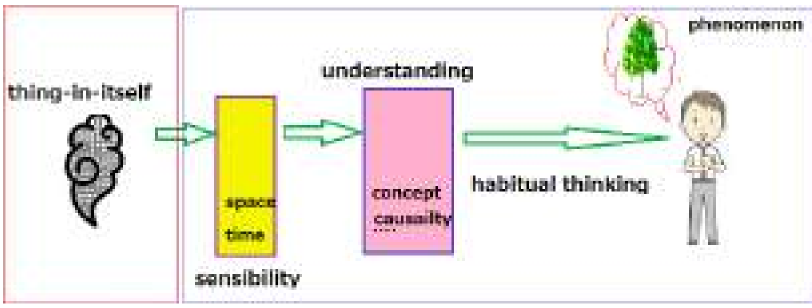
(F_1) Why can we make and understand Newtonian mechanics?
(F_2) Is there a question we can't answer?

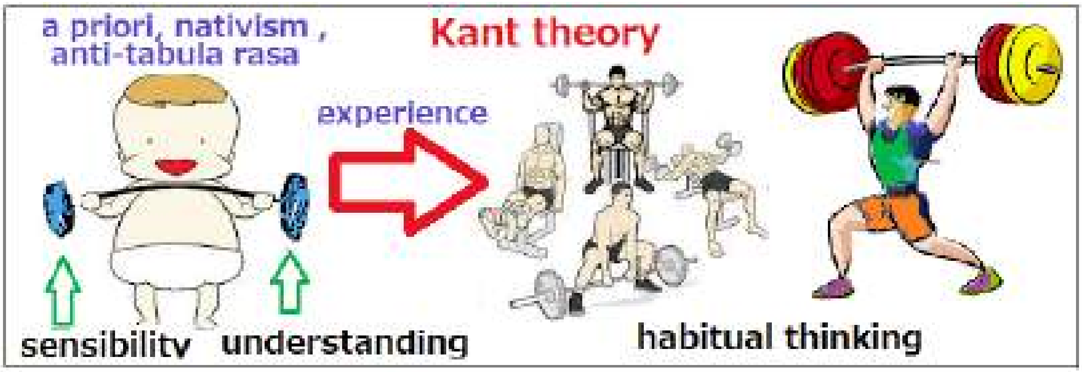
[A rough sketch of Kant's Answer for (F_1)]:
In order to understand Newtonian mechanics, we need the scientific ability, that is,

(G_1) Sensibility: The cognitive ability to organize the various sensations received through the sense organs within the framework of "space-time"
(G_2) Understanding: The ability to judge and understand material obtained through sensibility based on concepts such as quantity and cause-effect relationships.
(G_3) These abilities (sensitivity and understanding) are a priori, i.e., innate, and should be common to all human beings.

And,

(G_4) With experience and training, these abilities will increase.





Hence, our habitual thinking is also common to all human beings at its core, and in this sense, it is universal. **Therefore, we can understand Newton mechanics** (though it depends on our sensibility and understanding).

[A rough sketch of Kant's Answer for (F_2)]:
Kant describes four examples of unanswerable problems.

(H_1) Is the universe finite or infinite?
(H_2) Does the atom exist?
(H_3) Does free will exist?
(H_4) Does God exist?

Kant proves these by deriving antinomies, but this is a play on words rather than a proof. If a proof is to be given, it must be preceded by the declaration of a premise (or worldview), but since no premise (or worldview) has been declared, it is not a proof but a play on words.

□

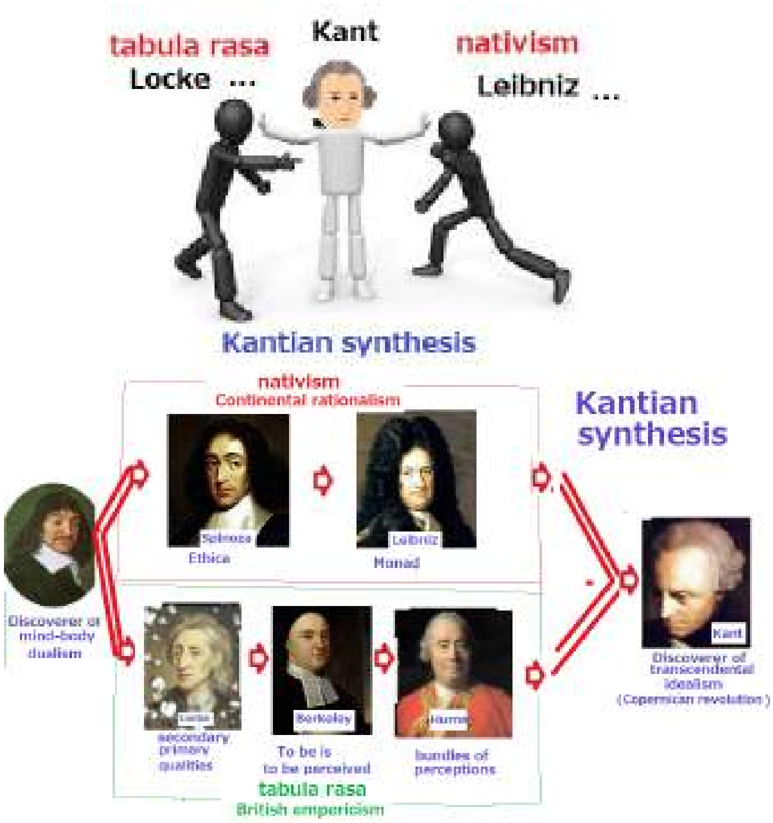
The above proof is not a scientific one. However, it is certain that many philosophical fans agreed with Kant's explanation (G) in Problem 10.1. In particular

$$\begin{cases} (G_3) \cdots \text{Sensibility, Understanding (nativism)} \\ (G_4) \cdots \text{experience and training (experiment)} \end{cases}$$

This is usually called "Kantian synthesis" such as

(I) Descartes [mind-body dualism] ⟶ { [tabula rasa , experience] British Empiricism [Locke, Berkeley, Hume] ; [nativism , a priori] Continental Rationalism [Spinoza, Leibniz] } —Kantian synthesis→ ["Critique of Pure Reason"] Kant philosophy [summing-up, compromise]

In this sense, it is usually said that Kant integrated British empiricism and continental rationalism.



10.1.4 Summary

As we can see from the above discussion,

(J) the purpose of Kant's Critique of Pure Reason (= CPR) was to provide a foundation for Newtonian mechanics from a philosophical perspective.

To begin with, this objective was a scientifically unintelligible objective. However, by adding "common human sensitivity and understanding" to subjectivism such as Hume's bundle of perceptions, it was concluded that humans possess the intelligence to understand Newtonian mechanics. This is an argument "with a conclusion at the beginning," and "the common sensitivity and understanding of mankind" is only an afterthought. Nevertheless, "CPR" was a great success because it was the "philosophy of the Enlightenment. It is because the general public was eager to know "the position of Newtonian mechanics" and "CPR" proposed "the position of Newtonian mechanics" as the general public wished.

After all,

Get along with science!

To begin with, philosophy for the general public would have to be Enlightenment philosophy to be a hit.

No one would dispute that Kant's philosophy was a great success as an Enlightenment philosophy. However, it is not easy to evaluate Kant's philosophy from a scientific viewpoint.

(K) Is the following true?

$$\boxed{\text{Locke}} \xrightarrow[\text{progress (QL)}]{} \boxed{\text{Kant}}^{*2}$$

The Critique of Pure Reason is pseudo-scientific, and it is a bit of a stretch to agree with it.
However, if we think in terms of "language" rather than "recognition," it was as simple as shown in Note 9.3. That is,

> **Repost of [Note 9.3]**
> Thinking in terms of "language" instead of "recognition"
>
> ($\sharp_1$) "Everyday language" is similar to tabla-rasa. When a baby is born, it does not know any ordinary language at all (= tabula rasa). The baby acquires everyday language through repeated trial and error.
>
> ($\sharp_2$) Mathematics is similar to Continental Rationalism. All mathematical theorems can be proved (in principle) if only the axioms of set theory are determined.
>
> ($\sharp_3$) The quantum language is similar to the compromise between ($\sharp_1$) and ($\sharp_2$) above as follows.
>
> $$\boxed{\text{Quantum language}} = \overset{\text{rationalism}}{\boxed{\begin{array}{c}\text{Axiom 1 (measurement)} \\ \text{Axiom 2 (causality)}\end{array}}} + \overset{\text{(empiricism)}}{\boxed{\begin{array}{c}\text{linguistic} \\ \text{Copenhagen interpretation}\end{array}}}$$

Also, as noted in ($\sharp$) of (B_1) (or, (i) in Note 10.3), the following is a central claim of quantum language.

($\sharp$) scientific wisdom is a product of "Copenhagen Interpretation", not objective truth,

Therefore, if we consider that Kant wrote the spirit of quantum language (i.e., the theory of the reach of dualistic idealism) in the "Critique of Pure Reason," Kant's reputation rises to the highest level. If philosophy fans also read "Critique of Pure Reason" as a book of dualistic idealism, we can understand why they were so enthusiastic about it. If we take this into account and judge the overall situation, we can agree with (k) above. In addition, this chapter does not elaborate on Kant's achievement, the Copernican revolution.

After the discussion on the Copernican turn, we have to answer (K), and the final answer is given in Answer 11.22 of Sec. 11.7.

[*2] "$X \xrightarrow[\text{progress(QL)}]{} Y$" means ""Y" is closer to quantum language than "X".

10.2 * Kant's Copernican revolution: What is causality?

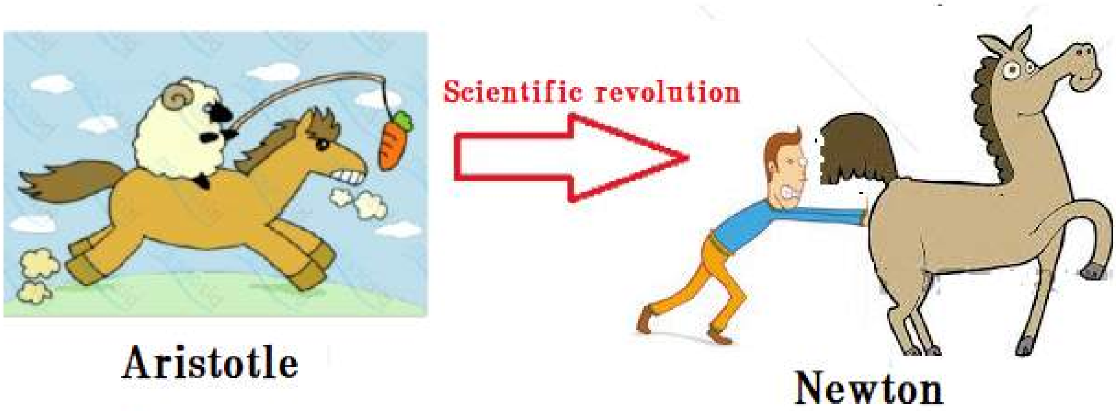
Sec. 11.7 will discuss the details of the Copernican revolution [II] in detail, but in this section, I will give a light introduction to the Copernican revolution in connection with "causality".

As mention in Sec 4.2, Aristotle considered the cause of the movement to be the "purpose" of the movement. Although this was what should be praised, it was not able to be said that "the purpose was to the point." For human beings to discover that the essence of movement and change is "causal relationship", we had to wait for the appearance of Galileo, Bacon, Descartes, Newton, etc.

Revolution to "Causality" from "Purpose"

is the greatest paradigm shift in the history of science. It is not an overstatement even if we call it "**birth of modern science**".

the birth of worldview — Movement (Hērakleitos, Parmenides, Zeno) —"purpose"→ Aristotle :(About 1500 years) — the birth of modern science — Causality (Galileo, Bacon, Descartes, Newton)



♠**Note 10.4.** I cannot emphasize too much the importance of the discovery of the term: "causality". That is,

($\sharp_1$) Science is the discipline about phenomena can be represented by the term "causality".

Thus, I consider that the discovery of "causality" is equal to that of science. However, my opinion is not the above ($\sharp_1$) but

($\sharp_2$) Science is a discipline that describes the world in terms of "measurement" and "causality".

10.2.1 Four answers to "what is causality?"

As mentioned above, about "what is an essence of movement and change?", it was once settled with the word "causality." However, not all were solved now. We do not yet understand "causality" fully. In fact,

Problem 10.2. Problem:

"What is causality?"

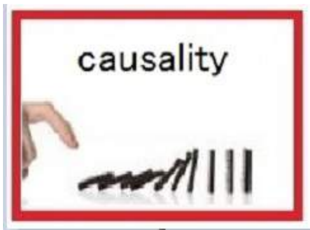
is the most important outstanding problems in modern science.

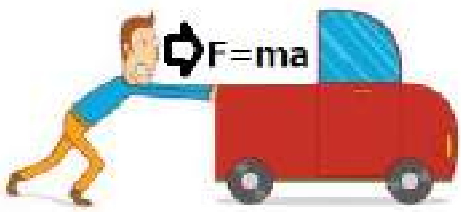
Answer this problem!

There may be some readers who are surprised with saying like this, although it is the outstanding problems in the present. Below, I arrange the history of the answer to this problem.

(A) **[Realistic causality]:** Newton advocated the realistic describing method of Newtonian mechanics as a final settlement of accounts of ideas, such as Galileo, Bacon, and Descartes, and he thought as follows. :

"Causality" actually exists in the world. Newtonian equation described faithfully this "causality". That is, Newtonian equation is the equation of a causal chain.





This realistic causality may be a very natural idea, and you may think that you cannot think in addition to this. In fact, probably, we may say that the current of the realistic causal relationship which continues like

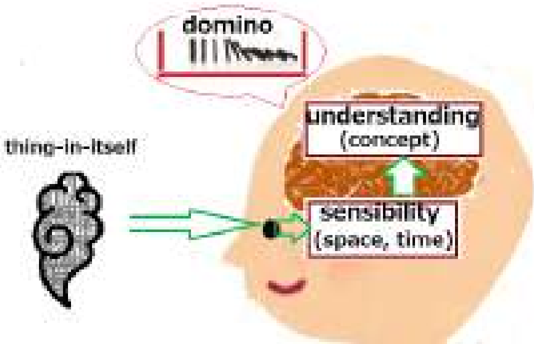
"Newtonian mechanics$\longrightarrow$ Electricity and magnetism$\longrightarrow$ Theory of relativity$\longrightarrow$ $\cdots$"

is the mainstream of science.

However, there are also other ideas, i.e. three "non-realistic causalities" as follows.

(B) **[Cognitive causality]:** David Hume, Immanuel Kant, etc. thought as follows. :

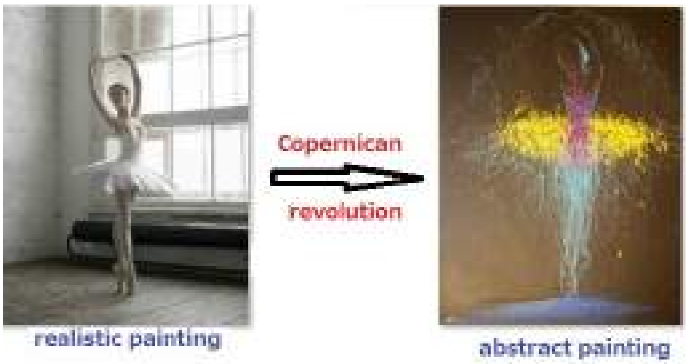
We can not say that "Causality" actually exists in the world, or that it does not exist in the world. And when we think that "something" in the world is "causality", we should just believe that it has "causality".



Most readers may regard this as "a kind of rhetoric", however, some readers may believe it. It may look like that, because you are looking through the prejudice of "causality." This is Kant's famous "Copernican revolution" (i.e., "Kant was awakened from his dogmatic slumber by Hume's idea and came up with the Copernican revolution"), that is,

"cognition constitutes the world."

which is considered that the cognition circuit of causality is installed in the brain, and when it is stimulated by "something" and reacts, "there is causal relationship."



♠**Note 10.5.** About his discovery of "the Copernican revolution", Kant says in his book "Prolegomena" (1783):

(♯) *I freely admit that it was the remembrance of David Hume which, many years ago, first interrupted my dogmatic slumber and gave my investigations in the field of speculative philosophy a completely different direction.*

Readers may ask, "Why did Kant, an honest and humble man, make such an exaggerated statement?" It is a matter of course that Kant had great confidence such that it was the greatest discovery in the history of philosophy. I agree to his opinion. For additional explanation about this, see Problem 10.5 later. Also, see Section 11.5.2. □

(C) **[Mathematical causality(Dynamical system theory)]:**

Since dynamical system theory has developed as the mathematical technique in engineering, they have not investigated "What is causality?" thoroughly. However,

In dynamical system theory, we start from the state equation (i.e., simultaneous ordinary differential equation of the first order) such that

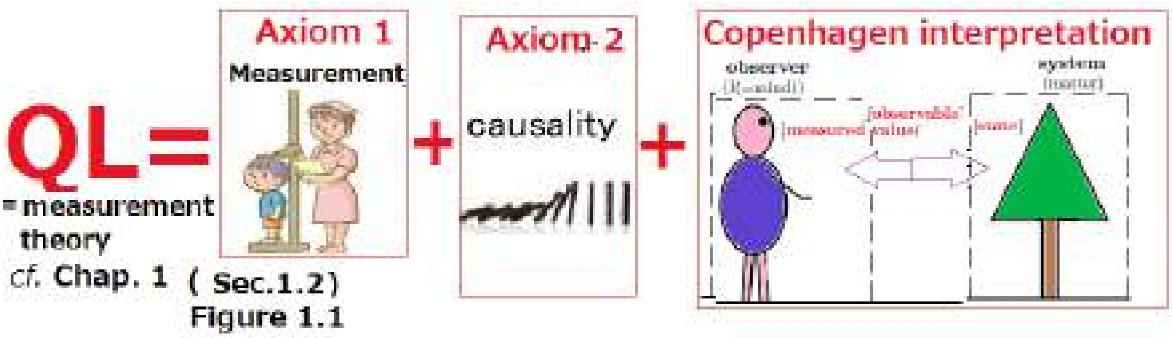
$$\begin{cases} \frac{d\omega_1}{dt}(t) = v_1(\omega_1(t), \omega_2(t), \ldots, \omega_n(t), t) \\ \frac{d\omega_2}{dt}(t) = v_2(\omega_1(t), \omega_2(t), \ldots, \omega_n(t), t) \\ \cdots\cdots \\ \frac{d\omega_n}{dt}(t) = v_n(\omega_1(t), \omega_2(t), \ldots, \omega_n(t), t) \end{cases} \tag{10.1}$$

and, we think that

(♯) the phenomenon described by the state equation has "causality."

This is the spirit of dynamical system theory (= statistics). Although this is proposed under the confusion of mathematics and worldview, it is quite useful. In this sense, I think that (C) should be evaluated more.

(D) **[Linguistic causal relationship (Measurement Theory)]:**



The causal relationship of measurement theory is decided by the Axiom 2 (causality; Sec. 1.1) of Chap. 1. If I say in detail,:

- Although measurement theory (= quantum language) consists of the two Axioms 1 and 2, it is the Axiom 2 that is concerned with causal relationship. When describing something in quantum language and using Axiom 2 (causality; Sec. 1.1) , we think that it has causality.

Summary 10.3. The above is summarized as follows.

(A) World is first

(B) Recognition is first

(C) Mathematics(buried into ordinary language) is first

(D) Language (= quantum language) is first

Now, in measurement theory, we assert the next as said repeatedly:

Quantum language is a basic language which describes various sciences.

Supposing this is recognized, we can assert the next. Namely,

In science, causality is just as mentioned in the above (D).

This is my answer to "What is causality?".

10.3 From Kantian synthesis to quantum linguistic synthesis

In Preface, I said that

> Without seeing the finished product, it is impossible to know exactly what it is.

A typical example of this is Kant's synthesis. Let us discuss this below.

It is usually said that there were two trends in modern philosophy. That is,

- Continental rationalism
  - Descartes, Spinoza, Leibniz, ...
- British empiricism
  - Bacon, Descartes, Locke, Berkley, Hume, ...

And the following is the established theory.

(A_1) "Continental rationalism vs. British empiricism" is due to "nativism vs. tabula rasa". And Kant synthesized the two.

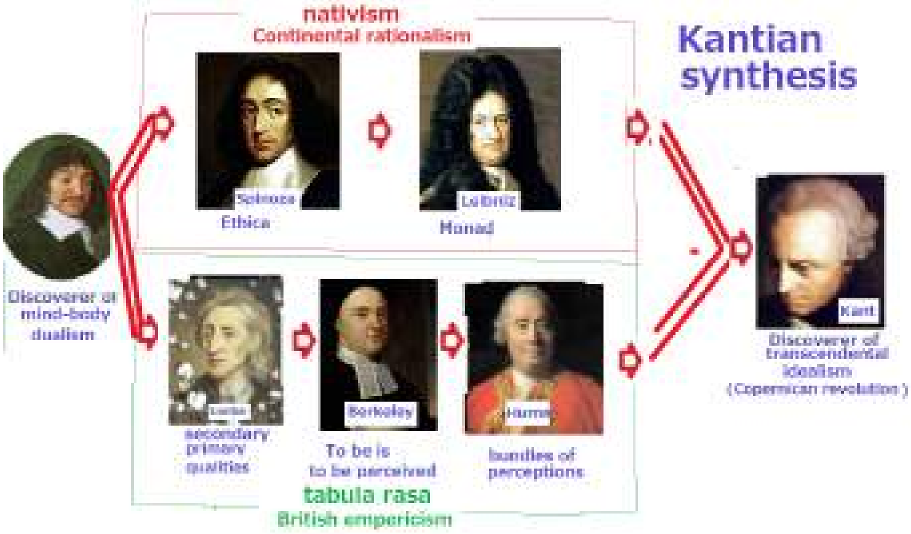
That is, The following is called the flower of modern philosophy:

(A_2) : **[Establish theory]: Kantian synthesis**

mind-body problem
Descartes (the father of modern phil.)
subjectivity problem
$\longrightarrow$
- Deduction (due to Descartes)
  Continental Rationalism (nativism)
  [Spinoza,Leibniz]
- Induction (due to Bacon)
  British Empiricism (tabula rasa, experience)
  [Locke, Berkeley, Hume]

(nativism) +(experience) $\xrightarrow{\text{(Kantian synthesis)}}$ Kant

More precisely,



I think that Kant dreamed of a good relationship between philosophy and natural science. Specifically, he dreamed that philosophy would be the foundation of Newtonian mechanics. In other words, his dream was to create the philosophical foundation of Newtonian mechanics, and he tried to realize it by "the Copernican revolution (regarding cognition)". He probably wanted to write about his dream in the Critique of Pure Reason. That is,

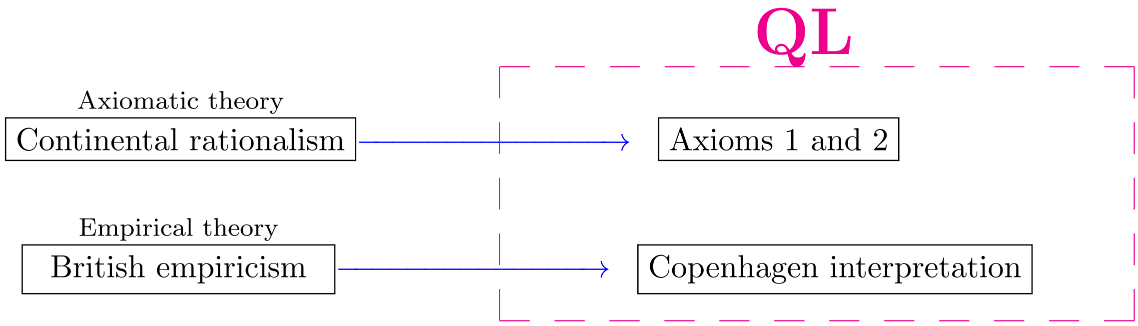
$$\left.\begin{array}{l} \boxed{\begin{array}{c}\text{The innate human reason to integrate}\\ \text{miscellaneous knowledge}\end{array}} \\ \text{(Continental rationalism)} \\ \\ \boxed{\begin{array}{c}\text{Miscellaneous knowledge gained}\\ \text{from various experiences}\end{array}} \\ \text{(British empiricism)} \end{array}\right\} \xrightarrow[\text{Kantian synthesis}]{} \boxed{\text{Critique of Pure Reason}}$$

Although his challenge was not scientifically successful, his work was very successful as a kind of enlightening book that promoted a good relationship between philosophy and natural science. In other words, I would like to think that this work had a great influence on the acceptance of natural science by the general public.

For further information, see my homepage

However, the oppositional structure [nativism vs. tabula rasa] seems to be pseudoscientific. Thus, in this text I think that

(B_1) [*3]



Here, recall that

(B_2) [QL (quantum language)] = [Axiom 1 (measurement) Axiom 2 (causality)] + [linguistic Copenhagen interpretation]

Thus, (B_1) implies that

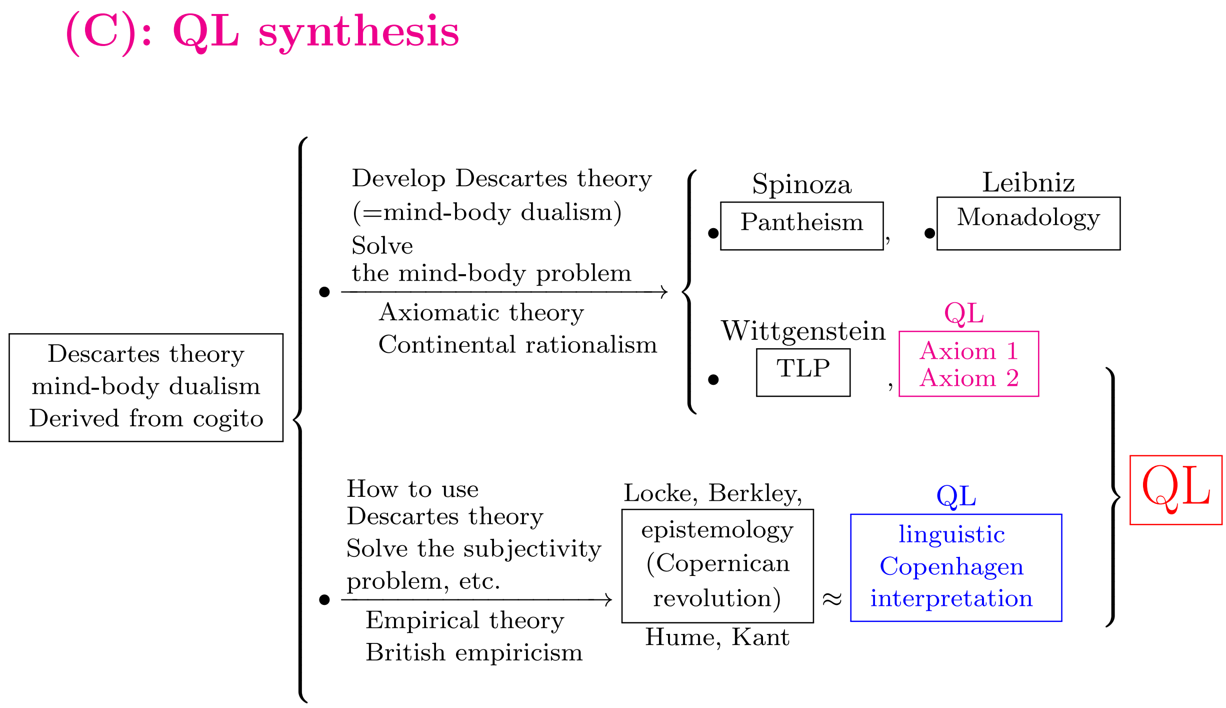
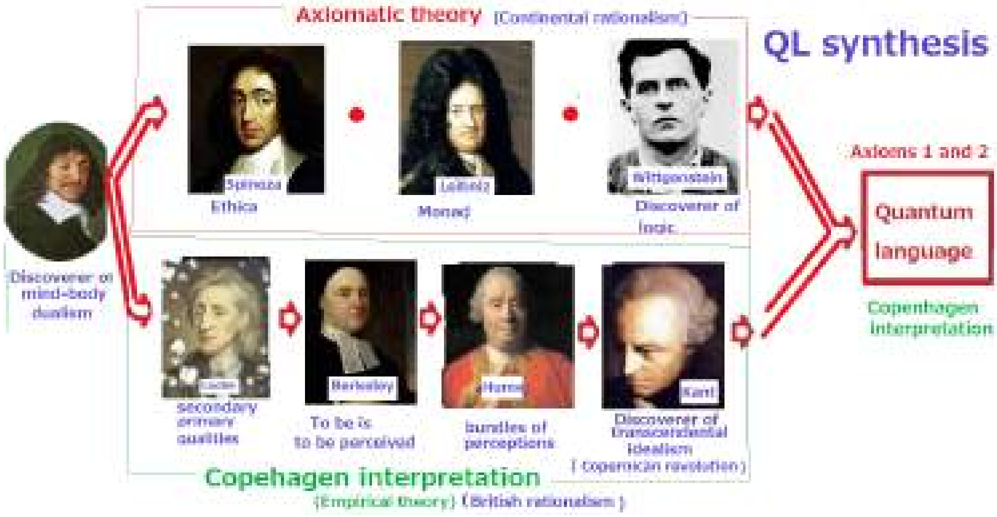


Figure 10.1(=Figure 9.1): QL-synthesis
We have not yet discussed Wittgenstein, but will do so in chapters 11 and 12.

[*3] This was, for the first time, proposed in ref. [84]: S. Ishikawa, (2022) *Empiricism, Rationalism, and Kantian synthesis in quantum linguistic point of view* OJPP, Vol. 12, No. 2, 182-198, (https://www.scirp.org/journal/paperinformation.aspx?paperid=117114)

That is,



Since the difficulty of axiom 1 (measurement) of QL is almost the same as the difficulty of quantum mechanics, they (Spinoza, Leibniz, Wittgenstein) tried something that was too difficult for the level of science at the time, and of course they did not achieve any scientific results. However, many philosophy enthusiasts were moved by the way they sincerely challenged the most important and difficult problem. Their philosophies are scientifically understood to anyone (perhaps even to themselves), yet they are celebrated by many philosophy enthusiasts, a strange thing that continues to this day.

British empiricism philosophers, on the other hand, have had some success. The task of creating a Copenhagen Interpretation (= manual for the use of Axiom 1) without knowing Axiom 1 may seem impossible, but their results were not so far off the mark. I don't know exactly why such a miracle occurred, but I find it very interesting. It is very interesting that it was possible to imagine dualistic idealism without knowing the axioms (i.e., principle).

With the above discussion, I reject the Kantian synthesis and adopt the QL synthesis as follows.

However, it was Kant who first realized that continental rationalism and British empiricism were to be integrated.

QL synthesis explains the following question:

(D) Why does modern philosophy consist of two theories (i.e., continental rationalism and British empiricism)?

That is,

(E) the two theories (i.e., continental rationalism and British empiricism) are actually one theory such as (B).

Remark 10.4. Summing up, in chapters 8,9,10, we see that[*4]

($\sharp_1$) Descartes: mind-body dualism (mind-body problem and subjectivity problem are the most important questions in modern philosophy)
(nevertheless, cogito proposition is a kind of wordplay)

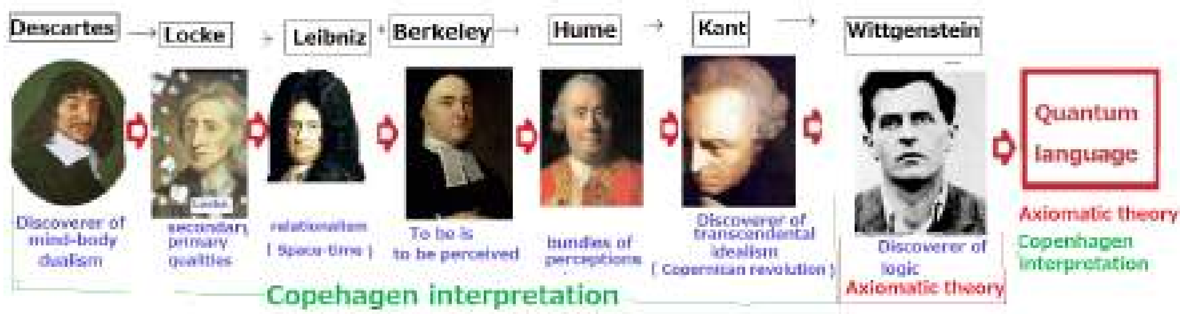
($\sharp_2$) Locke: secondary quality ($\approx$ universal by Anselmus) is the most important in dualistic idealism
(nevertheless, "tabula rasa" is nonsense)

($\sharp_3$) Leibniz: relationalism of space-time is a kind of Copenhagen interpretation
(nevertheless, nativism is nonsense, and monadology is incomprehensible)

($\sharp_4$) Berkeley: "To be is to be perceived" is the most important spirit of QL
(nevertheless, his subjective idealism is rather religion)

($\sharp_5$) Hume: the Copenhagen interpretation is a kind of "habitual thinking"
(nevertheless, "a bundle of perceptions" is a pseudoscientific fiction)

($\sharp_6$) Kant: transcendental idealism is the greatest discovery in modern philosophy (this will be discussed in Se, 11.7)
(nevertheless, "thing-in-itself" is a kind of wordplay, also, Kantian synthesis is his pseudoscientific fiction)



These achievements are the essence of the linguistic Copenhagen interpretation (= how to use Axiom 1 (measurement) and Axiom 2 (causality). What follows is astonishing.

($\flat$) **they arrived at the essence of the Copenhagen interpretation without knowing Axioms 1 and 2.**

On the other hand, it was too difficult to propose an axiom theory to solve the mind-body problem. Even with geniuses such as Spinoza, Leibniz, and Wittgenstein, they could not move a millimeter forward.

However, the following problems still remain.

Problem 10.5. If Kantian synthesis is Kant's pseudoscientific fiction, is the next "progress" believable?

[*4] When I was young, I was taught by my teacher that researcher's achievement should be evaluated by their "max".

$$\boxed{\text{Descartes, Locke, Berkeley, Hume}} \xrightarrow[\text{progress}]{} \boxed{\text{Kant}}$$

Or is Kant really great in the first place?

This will be answered in Answer 11.22

Chapter 11

Linguistic philosophy (Before TLP)

In this chapter, we discuss analytic philosophy and its surroundings (the [gray] and ⑫, ⑬, ⑭) in Figure 0.1 (in Preface) below (= Figure 0.1 in the Introduction).

Figure 0.1 (in Preface): The location of QL in the history of western philosophy

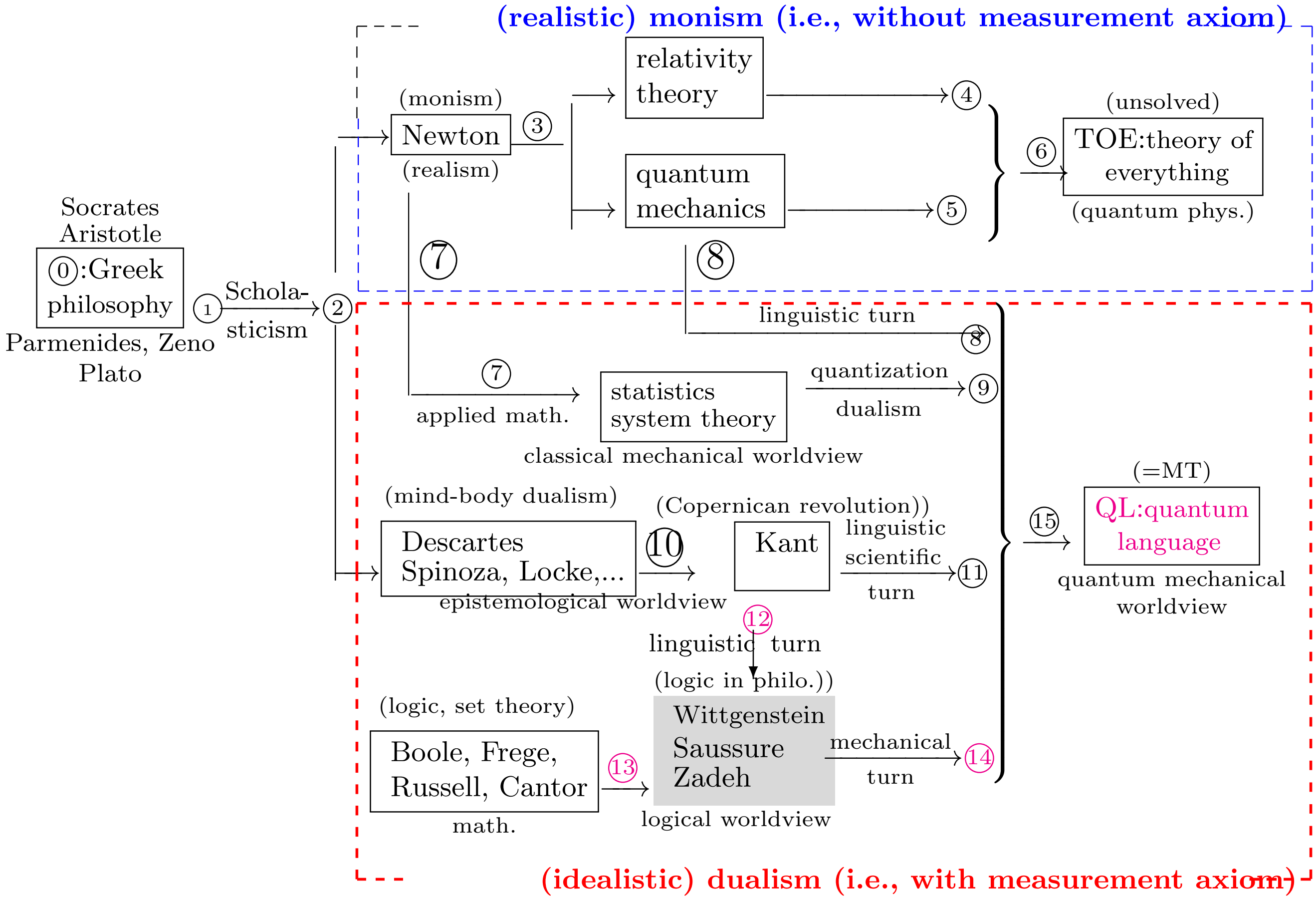


Figure 0.1 (in Preface): The history of the world-descriptions
Philosophy (≈ dualistic idealism) has progressed toward QL
(i.e., ⓪→ ①→ ②→ ⑩→ ⑫→ ⑭→ ⑮)

It is usually said that there is a gap between Kant philosophy and analytic philosophy. In other words, ⑫ above is not theoretically connected. But this is a superficial conclusion. From a quantum linguistic point of view, studying Kant philosophy and analytic philosophy reveals that the two are closely related. This will be studied in this and next chapters (in particular, Sec. 12.1 in next chapter). That is, the above figure asserts **"philosophy is one"**, just as physics is one.

To put it provocatively,

- ⑫ is more essential than ⑬.

11.1 Elementary knowledge of symbolic logic in mathematics

11.1.1 Propositional logic and predicate logic

Let's review the basics of logic. That's enough to read this text.

Postulate 11.1. [Symbolic logic (i.e., Propositional logic and predicate logic: G. Boole, G. Frege)]
For any proposition P, the truth function $\phi(P)$ is determined such that

$$\phi(P) = \begin{cases} 1 \;\; (\text{or, "T", "Yes"}) & (\text{if } P \text{ is true}) \\ 0 \;\; (\text{or, "F", "No"}) & (\text{if } P \text{ is false (i.e., not true)}) \end{cases}$$

Propostional logic

(A$_1$) Assume that P, P_1, P_2 are propositions. Then, $\neg P$, $P_1 \wedge P_2$, $P_1 \vee P_2$, $P_1 \to P_2$ are propositions.
And it holds that

$$\phi(\neg P) = \begin{cases} 1 & (\text{if } \phi(\neg P) = 0) \\ 0 & (\text{if } \phi(\neg P) = 1) \end{cases}$$

$$\phi(P_1 \wedge P_2) = \begin{cases} 1 & (\text{if } \phi(P_1) = \phi(P_2) = 1) \\ 0 & (\text{otherwise}) \end{cases}$$

$$\phi(P_1 \vee P_2) = \phi(\neg(\neg P_1 \wedge \neg P_2)) = \begin{cases} 1 & (\text{if } (\phi(P_1), \phi(P_2)) = (1,1), (1,0), (0,1)) \\ 0 & (\text{otherwise}) \end{cases}$$

$$\phi(P_1 \to P_2) = \phi(\neg P_1 \vee P_2)) = \begin{cases} 1 & (\text{if } (\phi(P_1), \phi(P_2)) = (1,1), (0,1), (0,0)) \\ 0 & (\text{otherwise}) \end{cases}$$

where $\wedge$, $\vee$, $\neg$, $\to$ respectively is called "and", "or", "not", "implies".

Also, assume that P_θ is a proposition ($\theta \in \Theta \equiv \{1, 2, ..., n\}$), then it holds
(i): $P_1 \wedge P_2 \wedge ... \wedge P_n$ (denoted by $\underset{\theta \in \Theta}{\wedge} P_\theta$,) is a proposition
(ii): $P_1 \vee P_2 \vee ... \vee P_n$ (denoted by $\underset{\theta \in \Theta}{\vee} P_\theta$,) is a proposition.
Here, $\phi(P_1 \wedge P_2 \wedge ... \wedge P_n) = \min_{i=1,...,n} \phi(P_i)$, $\phi(P_1 \vee P_2 \vee ... \vee P_n) = \max_{i=1,...,n} \phi(P_i)$.

(A$_2$) The above finite set $\Theta (\equiv \{1, 2, ..., n\})$ can be extended to an infinite set Θ.

Predicate logic

(i): $\underset{\theta \in \Theta}{\wedge} P_\theta$ is denoted by $\forall \theta [[\theta \in \Theta] \to [P_\theta]]$
(ii): $\neg[\forall \theta[\neg[Q_\theta]]]$ is defined by $\exists \theta[Q_\theta]$.
Therefore, we see

$$\underset{\theta \in \Theta}{\vee} P_\theta = \neg[\underset{\theta \in \Theta}{\wedge} \neg P_\theta] = \neg[\forall \theta[[\theta \in \Theta] \to [\neg P_\theta]] = \exists \theta[[\theta \in \Theta] \wedge [P_\theta]]$$

Remark 11.2. The point to note above is the sudden appearance of the word

"Proposition P". The end result is that the "proposition" is not defined until the end. This is supplemented in (i)-(iv) below.
(i):In mathematics, predicate logic is common knowledge. However, in the fields of philosophy and science, predicate logic is not yet understood by anyone. This fact was first pointed out in "Hempel's raven paradox (*cf.* Sec. 12.7). Thus, depending on one's point of view, the logic of philosophy and science is more difficult than the logic of mathematics. Relying on the "ZFC axiom system (1921)" proposed by Zermelo and Frenkel, we already have the answer to "What is a (mathematical) proposition?". On the other hand, in philosophy and science, the answer to "What is a proposition? is unknown in philosophy and science.

(ii):Descartes considered the unintelligible proposition "I think, therefore I am" to be the first proposition of philosophy, so he may not have seriously considered the question "What is a proposition? Since Kant discussed the distinction between "analytic propositions" and "synthesis propositions," he may not have realized the importance of the question, "What is a proposition? The problem of "analytic propositions" and "synthesis propositions" raised by Kant is still an unresolved issue today as the Carnap-Quine controversy (*cf.* Sec. 12.3).

(iii): The history of the question "What is a (scientific) proposition?" in science is not very old. In Newtonian mechanics, the question is "Is it a proposition of Newtonian mechanics, or not?" was often self-evident and did not become a serious problem (personally, I am interested in [Galileo's free fall] in section 7.5). In quantum mechanics, however, things have changed. Quantum mechanics assumes a "observer (i.e., mind-body dualism)," which complicates the story. Therefore, it became necessary to have rules for judging "is it a proposition of quantum mechanics or not?". This is what has been described so far in this text. Or rather, my motivation for entering this field was "Why does quantum mechanics have a (Copenhagen) interpretation?" because I wondered.

(iv): As readers may have already noticed, I think

- the most important question in philosophy is "What is a proposition in this world?"

which might be, for the first time, founed by Wittgenstein. Of course, there is not always one answer, but this is the main theme of this text.

(v): Thus, in this text (chapters 11 and 12) we show the following

($\flat_1$) **What is a proposition (of this world)?**

($\flat_2$) **Why does propositional logic hold in this world?**
(As for predicate logic, it is strictly incomplete.)

Exercise 11.3. The truth table below is well known. Wittgenstein was one of the contributors to the popularization of truth tables.

(i): The proof of syllogism: $[(p \to q) \wedge (q \to r)] \to (p \to r)$

Truth Table

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$(p \to q) \wedge (q \to r) \to (p \to r)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	0	1	0	1	1	0	1
1	0	0	0	1	0	0	1
0	1	1	1	1	1	1	1
0	1	0	1	0	1	0	1
0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1

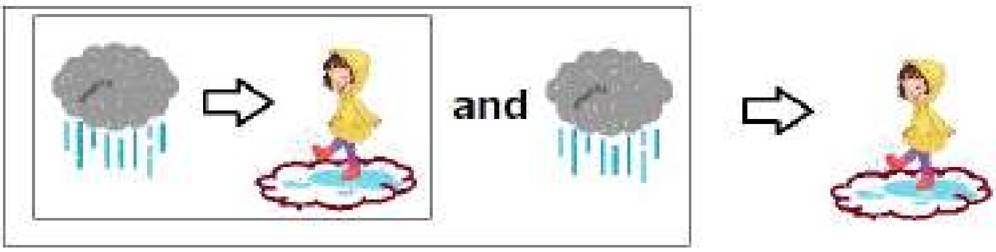
Thus, syllogism: $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is always true.
(ii):Modus ponens $((p \to q) \wedge p) \to q$ is always true, but $((p \to q) \wedge q) \to p$ is not always true.

Proof of (ii)

p	q	$p \to q$	$(p \to q) \wedge p$	$(p \to q) \wedge q$	$((p \to q) \wedge p) \to q$	$((p \to q) \wedge q) \to p$
1	1	1	1	1	1	1
1	0	0	0	0	1	1
0	1	1	0	1	1	0
0	0	1	0	0	1	1

For example, put $p :=$ "it rains", $q :=$ "the ground is wet". Modus ponens says that

$$[[\text{"it rains"} \to \text{"the ground is wet"}] \wedge \text{"it rains"}] \to \text{"the ground is wet"}.$$



(iii):[The elementary mathematical problem concerning predicate logic]
"$\lim_{n\to\infty} a_n = a$ (i.e., "A real-valued sequence $\{a_n\}_{n=1}^{\infty}$ converges to a") is defined by

$$\forall \epsilon (> 0) \exists N (\text{natural number}) \forall n (\text{natural number}) [[n > N] \to |a_n - a| < \epsilon]$$

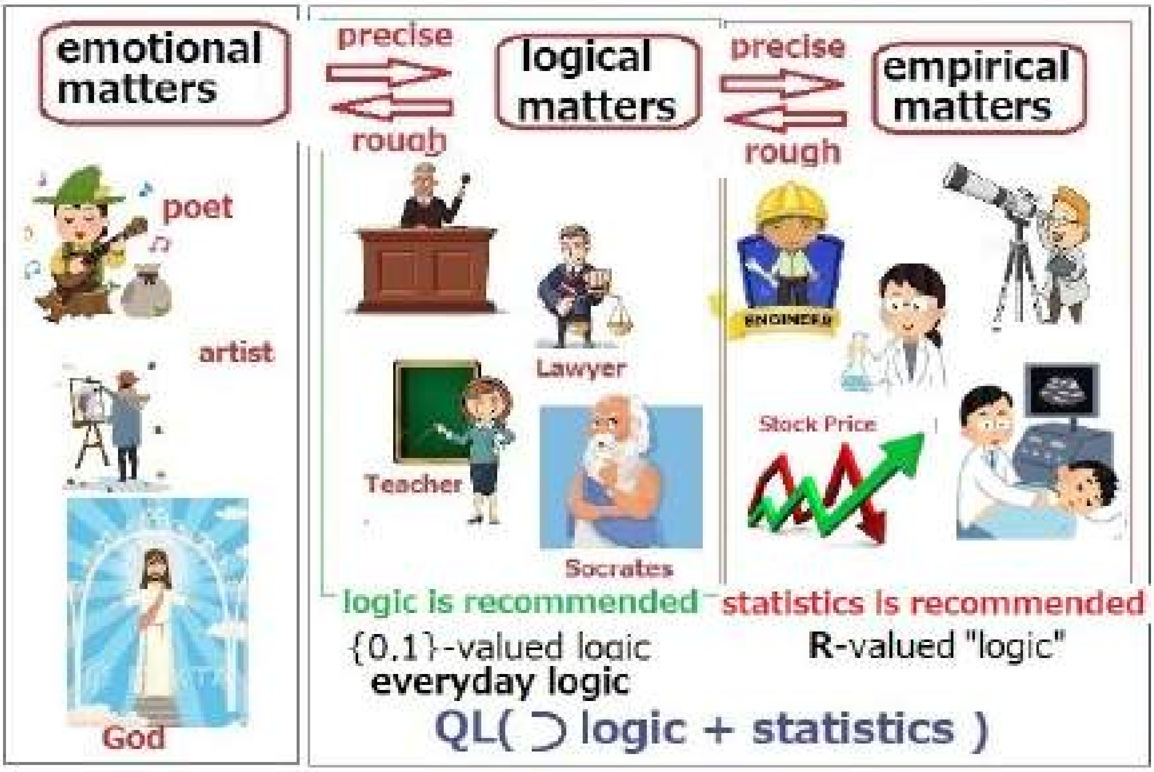
This was due to the great mathematicians Cauchy(1789-1857) and Weierstrass(1815-1897) etc.
(iv):The following is not a syllogism.(*cf.* Exercise 4.5 in Sec. 4.3.2).

(♯) Socrates is a man, and all men are mortal, so Socrates dies.

□

11.1.2 Which do you trust more, logic or statistics?

In a word, logic is used in fields where quantitative discussion is difficult, and statistics is used in fields where detailed data is available as follows.



In motto terms, this figure says the following.

(B) ”Logic for the humanities, statistics for the sciences”

which is common sense that everyone knows.

Thus we have three kinds of ”logics”

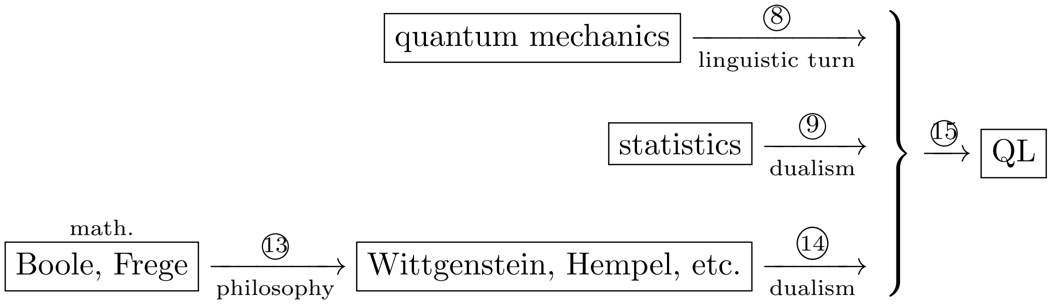
(C_1) logic for the humanities: everyday logic (QL logic (=fuzzy logic))
(C_2) logic for the sciences: statistics (a kind of many-valued logic)
(C_3) logic for mathematicians: mathematical logic

Frege and Russell may have deliberately used the confusion of the two logics of (C_1) and (C_3) to promote analytic philosophy. In this and the next chapters, (C_1) and (C_2) will be explained. For this,

(D) It suffices to derive statistics (C_2) and everyday logic ($\approx$ fuzzy logic)(C_1) from QL.

Recall the following part of Figure 0.1 in Preface, that is,

(E)



The simple derivation (⑨←⑮) of statistics from QL is described in Note 1.3 (or, generally, ref.[74]), so in this text (chapters 11, 12, especially, in Sec, 12.1) we devote ourselves to the derivation (⑭←⑮) of everyday logic (fuzzy logic) from QL.

♠**Note 11.1.** I am in the department of mathematics and know many brilliant mathematicians, many of whom do not know much about mathematical logic. I was surprised that philosophers know difficult theorems in mathematical logic (e.g., Gödel's incompleteness theorem, Löwenheim-Skolem theorem, Banach-Tarski's Theorem, etc.). This fact was strange to me because I did not think that mathematical logic and philosophy are closely related. There can be a philosophy of mathematics, but it should not be the center of philosophy.

11.2 Probabilistic symbolic logic in mathematics

This section consists of excerpts from the following paper:

(A) Ref. [81]: Ishikawa, S. (2021) Fuzzy Logic in the Quantum Mechanical Worldview ; Related to Zadeh, Wittgenstein, Moore, Saussure, Quine, Lewis Carroll, etc. JAMP, Vol. 9, No.3, 140-154, (https://www.scirp.org/journal/paperinformation.aspx?paperid=110830)

Now let us introduce probabilistic symbolic logic, which is a slight generalization of mathematical symbol logic mentioned in the previous section. Probabilistic symbolic logic plays an essential role in QL logic (= fuzzy logic) (in Sec. 12.1). Since I am convinced that this fuzzy logic is what Wittgenstein had wanted to insist, I encourage readers to read this section. This section is written in such a way that it can be read without knowledge of quantum language. It's very easy and I think it's readable by normal high school students. However, it should be noted that this probabilistic symbolic logic, like mathematical symbolic logic, is not sufficient from a philosophical perspective in the sense that the "proposition" is defined only in mathematics (or, equivalently in the sense that it is not discussed in a certain worldview).

11.2.1 Easy example

Let us start from the following easy example.

Example 11.4. For example, consider a proposition P_1 such that

(B) P_1 = "this tomato is red", $\neg P_1$ = "this tomato is not red",

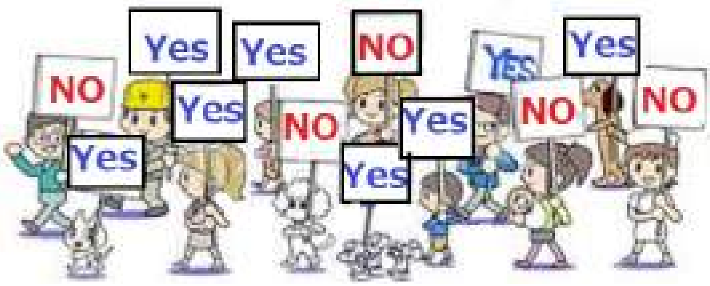
And suppose that there are 100 respondents, and furthermore, the following question is asked to them.

(C) Is this tomato red? (i.e., is the proposition P_1 true or not?)

Assume that the results of the responses are as follows.

(D) $\begin{cases} \text{70 respondents say "Yes, this tomato is red" (i.e., the proposition } P_1 \text{ is true, i.e. "T")} \\ \text{30 respondents say "No, this tomato is not red" (i.e., the proposition } P_1 \text{ is false, i.e. "F")} \end{cases}$

It is possible to consider that T = "Yes" = 1 and F = "No" = 0.



This can be probabilistically interpreted as follows.

(E) When any respondent is *randomly* selected out of 100, the probability that this respondent will answer "yes" to question (C) is $p_1 (= 0.7)$. Or simply, the probability that the proposition P_1 is true is p_1. In symbolic form,

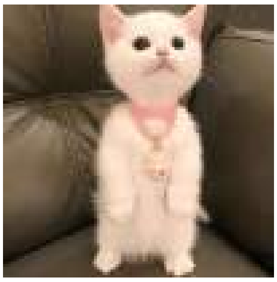
$$\text{Prob}[P_1; \{T\}] = p_1 (= 0.7)$$

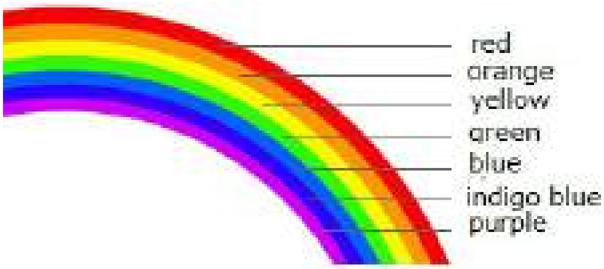
Then we generally denote that

$$\text{Prob}[P_1; \{T\}] = p_1, \quad \text{Prob}[P_1; \{F\}] = 1 - p_1 \qquad (\text{where } 0 \le p_1 \le 1)$$

Also, note that $\text{Prob}[\neg P_1; \{T\}] = \text{Prob}[P_1; \{F\}] = 1 - p_1$.

Remark 11.5. (i):We do not plan to do anything unusual. Propositions" are the same as what we have usually called "propositions. However, it might have been better to use a more ambiguous proposition such as "cats are cute" or "the rainbow has seven colors" as an example.





(ii): Zadeh often emphasized that fuzziness and probability are different concepts. However, I believe that fuzziness without a probabilistic interpretation cannot be a scientific concept. In this sense, (e) above is essential

11.2.2 Quasi-product probability

In addition, assume another proposition P_2 such that $\text{Prob}[P_2; \{T\}] = p_2, \text{Prob}[P_2; \{F\}] = 1 - p_2$. Thus, we have two probability spaces $(\{T, F\}, 2^{\{T,F\}}, \mu_i)$, $(i = 1, 2)$, such that

$$\mu_1(\{T\}) = p_1, \qquad \mu_1(\{F\}) = 1 - p_1, \qquad \mu_2(\{T\}) = p_2, \qquad \mu_2(\{F\}) = 1 - p_2$$

And consider the quasi-product probability space $(\{T, F\}^2, \mathcal{P}(\{T, F\}^2), \mu_1 \times^{qp} \mu_2)$, which satisfies the following marginal conditions:

$$(\mu_1 \times^{qp} \mu_2)(\{T\} \times \{T, F\}) = \mu_1(\{T\}), \qquad (\mu_1 \times^{qp} \mu_2)(\{F\} \times \{T, F\}) = \mu_1(\{F\}),$$

and

$$(\mu_1 \times^{qp} \mu_2)(\{T, F\} \times \{T\}) = \mu_2(\{T\}), \qquad (\mu_1 \times^{qp} \mu_2)(\{T, F\} \times \{F\}) = \mu_2(\{F\}),$$

Putting the above together, we get the following table (i.e., Table 11.1).

Table 11.1: quasi-product probability measure $\mu_1 \times^{qp} \mu_2$

\	$\mu_1(\{T\})$	$\mu_1(\{F\})$
$\mu_2(\{T\})$	$(\mu_1 \times^{qp} \mu_2)(\{(T,T)\})(\equiv \alpha)$	$(\mu_1 \times^{qp} \mu_2)(\{(F,T)\})(\equiv \mu_2(\{F\}) - \alpha)$
$\mu_2(\{F\})$	$(\mu_1 \times^{qp} \mu_2)(\{(T,F)\})(\equiv \mu_1(\{F\}) - \alpha)$	$(\mu_1 \times^{qp} \mu_2)(\{(F,F)\})(\equiv 1 - \mu_1(\{F\}) - \mu_2(\{F\}) + \alpha)$

where

$$\max\{\mu_1(\{T\}) + \mu_2(\{T\}) - 1, 0\} \leq \alpha \leq \min\{\mu_1(\{T\}), \mu_2(\{T\})\}$$

Thus, the quasi-product probability is not unique in general.

Remark 11.6. The followings (i) and (ii) are typical. Assume that $P_1 = P_2$. Thus, $\mu_1 = \mu_2$.
(i):[Product probability space]
Assume that each respondent randomly chooses "T" [resp. "F"] with probability 0.7 [resp. 0.3] in the same way for the two questions P_1 and P_2. Then, the $\mu_1 \times^{qp} \mu_2$ is considered as the ordinary product probability $\mu_1 \times \mu_2$ such that

$$(\mu_1 \overset{qp}{\times} \mu_2)(\{(x_1, x_2)\}) = \mu_1(\{x_1\}) \times \mu_1(\{x_2\}) \qquad (\forall (x_1, x_2) \in \{T, F\}^2)$$

If we write the above in a table, we get the following table (i.e., Table 11.2).

Table 11.2: product probability measure $\mu_1 \times^{qp} \mu_1 (= \mu_1 \times \mu_1)$

\	$\mu_1(\{T\})$	$\mu_1(\{F\})$
$\mu_1(\{T\})$	$\mu_1(\{T\}) \times \mu_1(\{T\})(\equiv \alpha)$	$\mu_1(\{F\}) \times \mu_1(\{T\})(\equiv \mu_1(\{F\}) - \alpha)$
$\mu_1(\{F\})$	$\mu_1(\{T\}) \times \mu_1(\{F\})(\equiv \mu_1(\{F\}) - \alpha)$	$\mu_1(\{F\}) \times \mu_1(\{F\})(\equiv 1 - \mu_1(\{F\}) - \mu_1(\{F\}) + \alpha)$

(ii):[Standard situation when $P_1 = P_2$] It is natural to think that the respondent selected in (C) will give the same answer to the same question (i.e., P_1 and P_1). In this case we see $\mu_1 \times^{qp} \mu_1$ such that $(\mu_1 \times^{qp} \mu_1)(\{(T,T)\}) = p_1$ and $(\mu_1 \times^{qp} \mu_1)(\{(F,F)\}) = 1 - p_1$. Thus,

$$(\overset{qp}{\underset{i=1,2}{\times}} \mu)(\{(x_1, x_2)\}) = \begin{cases} p_1 & (\text{ if } (x_1, x_2) = (T, T) \\ 1 - p_1 & (\text{ if } (x_1, x_2) = (F, F) \\ 0 & (\text{ if } (x_1, x_2) = (T, F), (F, T) \end{cases}$$

If we write the above in a table, we get the following table (i.e., Table 11.3).

Table 11.3: quasi-product probability measure $\mu_1 \times^{qp} \mu_1$

\	$\mu_1(\{T\})$	$\mu_1(\{F\})$
$\mu_1(\{T\})$	$(\mu_1 \times^{qp} \mu_1)(\{(T,T)\}) = p_1$	$(\mu_1 \times^{qp} \mu_1)(\{(F,T)\}) = 0$
$\mu_1(\{F\})$	$(\mu_1 \times^{qp} \mu_1)(\{(T,F)\}) = 0$	$(\mu_1 \times^{qp} \mu_1)(\{(F,F)\}) = 1 - p_1$

In this text, unless otherwise stated, this quasi-product probability measure $\mu_1 \times^{qp} \mu_1$ will be used. However, if so, it is not necessary to ask the same question twice; we only need to ask it once. For further discussion, see Example 11.8 later. Also, this $\times^{qp}_{i=1,2} \mu_i (= \mu)$ is easily extended to the case that $\times^{qp}_{i=1,2,...,n} \mu_i$ (where $\mu_i = \mu$, $i = 1, 2, ..., n$). such that

$$(\overset{qp}{\underset{i=1,2,\dots,n}{\times}} \mu)(\{(x_i)_{i=1}^n\}) = \begin{cases} p_1 & (\text{ if } (x_i)_{i=1}^n = (T)_{i=1}^n \\ 1 - p_1 & (\text{ if } (x_i)_{i=1}^n = (F)_{i=1}^n \\ 0 & (\text{ others }) \end{cases}$$

11.2.3 Logic symbols and logical operations

For the sake of convenience, we will define as follows (also, see (G) for the formal definition):

(F) $P_1 \wedge P_2 =$ "P_1 and P_2", $P_1 \vee P_2 =$ "P_1 or P_2", $P_1 \to P_2 =$ "$\neg P_1$ or P_2", $\neg P =$ not P

Also, $P_1 \wedge P_2$, $\neg P_1 \wedge P_2$, $P_1 \wedge \neg P_2$, $\neg P_1 \wedge \neg P_2$) are located in the following table (i.e., Table 11.4).

Table 11.4: quasi-product probability measure $\mu_1 \overset{qp}{\times} \mu_2$ concerning $P_1 \wedge P_2$, $\neg P_1 \wedge P_2$, $P_1 \wedge \neg P_2$, $\neg P_1 \wedge \neg P_2$

\ \	P_1; Prob$[P_1;\{T\}]$ $p_1 = \mu_1(\{T\})$	$\neg P_1$; Prob$[\neg P_1;\{T\}]$ $p_{\bar{1}} = \mu_1(\{F\})$
P_2; Prob$[P_2;\{T\}]$ $p_2 = \mu_2(\{T\})$	$P_1 \wedge P_2$; Prob$[P_1 \wedge P_2;\{T\}]$ $p_{12}(= (\mu_1 \times^{qp} \mu_2)(\{(T,T)\}) \equiv \alpha)$	$\neg P_1 \wedge P_2$; Prob$[\neg P_1 \wedge P_2;\{T\}]$ $p_{\bar{1}2}(= (\mu_1 \times^{qp} \mu_2)(\{(F,T)\}) \equiv p_2 - \alpha)$
$\neg P_2$; Prob$[\neg P_2;\{T\}]$ $p_{\bar{2}} = \mu_2(\{F\})$	$P_1 \wedge \neg P_2$; Prob$[P_1 \wedge \neg P_2;\{T\}]$ $p_{1\bar{2}}(= (\mu_1 \times^{qp} \mu_2)(\{(T,F)\})$ $\equiv p_1 - \alpha)$	$\neg P_1 \wedge \neg P_2$; Prob$[\neg P_1 \wedge \neg P_2;\{T\}]$ $p_{\bar{1}\bar{2}}(= (\mu_1 \times^{qp} \mu_2)(\{(F,F)\})$ $\equiv 1 - p_1 - p_2 + \alpha)$

where $\max\{p_1 + p_2 - 1, 0\} \le \alpha \le \min\{p_1, p_2\}$.
Now we can explain the following example:

Example 11.7. [Simple probabilistic truth table] The following table (i.e., Table 11.5: Simple probabilistic truth table) is the same as a well-known truth table, except for the "probability column".

Table 11.5: Simple Probabilistic Truth (Elementary propositions P_1, P_2)

P_1	P_2	probability $p = \times^{qp}_{i=1,2}\mu_i$	$\neg P_1$	$P_1 \to P_2$	$P_1 \wedge P_2$	$P_1 \vee P_2$
T	T	$p_{12} = \times^{qp}_{i=1,2}\mu_i(\{(T,T)\})$	F	T	T	T
T	F	$p_{1\bar{2}} = \times^{qp}_{i=1,2}\mu_i(\{(T,F)\})$	F	F	F	T
F	T	$p_{\bar{1}2} = \times^{qp}_{i=1,2}\mu_i(\{(F,T)\})$	T	T	F	T
F	F	$p_{\bar{1}\bar{2}} = \times^{qp}_{i=1,2}\mu_i(\{(F,F)\})$	T	T	F	F

Thus, it will be enough to explain hoe to use the "probability column" as follows. For example, consider the above proposition $\neg P_1$, which can be regarded as the map from $\{T,F\}^2$ to $\{T,F\}$ such that

$$[\neg P_1](x_1,x_2) = \begin{cases} T & \text{if } (x_1,x_2) = (F,T), \text{ or } (F,F) \\ F & \text{if } (x_1,x_2) = (T,T), \text{ or } (T,F) \end{cases}$$

(or $\neg P_1 : \{T,F\} \to \{T,F\}$ such that $[\neg P_1](T) = F$, $[\neg P_1](F) = T$).
Thus, we see that Prob$[\neg P_1;\{T\}]$ (i.e., the probability that $\neg P_1$ is true) is equal to

$$(\mu_1 \overset{qp}{\times} \mu_2)([\neg P_1]^{-1}(\{T\})) = p_{\bar{1}2} + p_{\bar{1}\bar{2}}$$

Next, consider $[P_1 \to P_2]$ $(= [\neg P_1 \vee P_2])$, which is regarded as the map from $\{T,F\}^2$ to $\{T,F\}$ such that

$$[P_1 \to P_2](x_1,x_2) = \begin{cases} T & \text{if } (x_1,x_2) = (T,T) \text{ or } (F,T) \text{ or } (F,F) \\ F & \text{if } (x_1,x_2) = (T,F) \end{cases}$$

Thus, we see that $\mathrm{Prob}[P_1 \to P_2; \{T\}]$ (i.e., the probability that $[P_1 \to P_2]$ is true) is equal to

$$(\mu_1 \overset{qp}{\times} \mu_2)([P_1 \to P_2]^{-1}(\{T\})) = p_{12} + p_{\bar{1}2} + p_{\bar{1}\bar{2}}$$

Similarly we see

$$\mathrm{Prob}[P_1 \wedge P_2; \{T\}] = p_{12}, \qquad \mathrm{Prob}[P_1 \vee P_2; \{T\}] = p_{12} + p_{\bar{1}2} + p_{1\bar{2}}$$

Furthermore, we must note that

(G) $\neg P_1$, $[P_1 \wedge P_2]$, $[P_1 \vee P_2]$ and $[P_1 \to P_2]$ can be respectively regarded as maps from $\{T, F\}^2$ to $\{T, F\}$ as shown in the above Table 11.5. Rather than (F), this map is the formal definition of logic symbols (i.e., $\neg$, $\wedge$, $\vee$, $\to$).

Example 11.8. [Probabilistic truth table ((i): $P_1 \neq P_3$, (ii): $P_1 = P_3$)]
(i) (Simplest case: $P_1 \neq P_3$): The following table (i.e., Table 11.6) is the preparation of the next (ii) (i.e., Tables 11.7, 11.8, 11.9). Consider the truth table of $[P_1 \to P_2] \wedge P_3$ as follows.

Table 11.6: Probabilistic Truth Table (Elementary propositions P_1, P_2, P_3)

P_1	P_2	P_3	probability: $p = \times^{qp}_{i=1,2,3} \mu_i$	$[P_1 \to P_2] \wedge P_3$
T	T	T	$p_{123} = \times^{qp}_{i=1,2,3} \mu_i(\{(T,T,T)\})$	T
T	T	F	$p_{12\bar{3}} = \times^{qp}_{i=1,2,3} \mu_i(\{(T,T,F)\})$	F
T	F	T	$p_{1\bar{2}3} = \times^{qp}_{i=1,2,3} \mu_i(\{(T,F,T)\})$	F
T	F	F	$p_{\bar{1}\bar{2}\bar{3}} = \times^{qp}_{i=1,2,3} \mu_i(\{(T,F,F)\})$	F
F	T	T	$p_{\bar{1}23} = \times^{qp}_{i=1,2,3} \mu_i(\{(F,T,T)\})$	T
F	T	F	$p_{\bar{1}2\bar{3}} = \times^{qp}_{i=1,2,3} \mu_i(\{(F,T,F)\})$	F
F	F	T	$p_{\bar{1}\bar{2}3} = \times^{qp}_{i=1,2,3} \mu_i(\{(F,F,T)\})$	T
F	F	F	$p_{\bar{1}\bar{2}\bar{3}} = \times^{qp}_{i=1,2,3} \mu_i(\{(F,F,F)\})$	F

This says that the proposition $[P_1 \to P_2] \wedge P_3$ is the map from$\{T, F\}^3 \to \{T, F\}$ such that

$$\begin{aligned} & [[P_1 \to P_2] \wedge P_3](x_1, x_2, x_3) \\ = & \begin{cases} T & \text{if } (x_1, x_2, x_3) = (T,T,T), (F,T,T), (F,F,T) \\ F & \text{if } (x_1, x_2, x_3) = (T,T,F), (T,F,T), (T,F,F), (F,T,F), (F,F,F) \end{cases} \end{aligned}$$

Thus,

$$\mathrm{Prob}[[[P_1 \to P_2] \wedge P_3]; \{T\}] = (\underset{i=1,2,3}{\overset{qp}{\times}} \mu_i)([[P_1 \to P_2] \wedge P_3]^{-1}(\{T\})) = p_{123} + p_{\bar{1}\bar{2}3} + p_{1\bar{2}\bar{3}}$$

(ii) (Case; $P_1 = P_3$ in the above (i)): Furthermore, assume that $P_1 = P_3$ in the above. Then, recalling Remark 11.6 (ii), we usually assume that $\times^{qp}_{i=1,2,3} \mu_i(\{(x_1, x_2, x_3)\}) = 0$ (if $x_1 \neq x_3$). Thus, putting $\times^{qp}_{i=1,2} \mu_i(\{(x_1, x_2)\}) = \times^{qp}_{i=1,2,3} \mu_i(\{(x_1, x_2)\} \times \{T.F\})$, we see the following: (Note that Table 11.6= Table 11.7 except the probability column).

Table 11.7: Probabilistic Truth Table
(Overlapping elementary propositions P_1, P_2, $P_3(=P_1)$)

P_1	P_2	$P_3(=P_1)$	probability: $p=\times_{i=1,2}^{qp}\mu_i$	$[P_1 \to P_2] \wedge P_3(=P_1)$
T	T	T	$p_{123}(=p_{12})=\times_{i=1,2}^{qp}\mu_i(\{(T,T)\})$	T
T	T	F	$p_{12\bar{3}}(=0)$	F
T	F	T	$p_{1\bar{2}3}(=p_{1\bar{2}})=\times_{i=1,2}^{qp}\mu_i(\{(T,F)\})$	F
T	F	F	$p_{1\bar{2}\bar{3}}(=0)$	F
F	T	T	$p_{\bar{1}23}(=0)$	T
F	T	F	$p_{\bar{1}2\bar{3}}(=p_{\bar{1}2})=\times_{i=1,2}^{qp}\mu_i(\{(F,T)\})$	F
F	F	T	$p_{\bar{1}\bar{2}3}(=0)$	T
F	F	F	$p_{\bar{1}\bar{2}\bar{3}}(=p_{\bar{1}\bar{2}})=\times_{i=1,2}^{qp}\mu_i(\{(F,F)\})$	F

Since the case of the probability 0 (i.e., $p_{12\bar{3}}=p_{\bar{1}23}=p_{1\bar{2}\bar{3}}=p_{\bar{1}\bar{2}3}=0$) can be omitted, we have the following table:

Table 11.8: Probabilistic Truth Table
(Overlapping elementary propositions P_1, P_2, $P_3(=P_1)$)

P_1	P_2	$P_3(=P_1)$	probability: $p=\times_{i=1,2}^{qp}\mu_i$	$[P_1 \to P_2] \wedge P_3(=P_1)$
T	T	T	$p_{123}(=p_{12})=\times_{i=1,2}^{qp}\mu_i(\{(T,T)\})$	T
T	F	T	$p_{1\bar{2}3}(=p_{1\bar{2}})=\times_{i=1,2}^{qp}\mu_i(\{(T,F)\})$	F
F	T	F	$p_{\bar{1}2\bar{3}}(=p_{\bar{1}2})=\times_{i=1,2}^{qp}\mu_i(\{(F,T)\})$	F
F	F	F	$p_{\bar{1}\bar{2}\bar{3}}(=p_{\bar{1}\bar{2}})=\times_{i=1,2}^{qp}\mu_i(\{(F,F)\})$	F

Therefore, we see that Prob$[[P_1 \to P_2] \wedge P_3(=P_1); \{T\}]$ (i.e., the probability that $[P_1 \to F_2] \wedge P_1$ is true) is equal to

$$(\mu_1 \overset{qp}{\times} \mu_2)([[P_1 \to P_2] \wedge P_3(=P_1)]^{-1}(\{T\})) = p_{12}$$

Note that this is essentially the same as the following table.

Table 11.9: Probabilistic Truth Table
(Non-overlapping elementary propositions P_1, P_2)

P_1	P_2	probability: $p=\times_{i=1,2}^{qp}\mu_i$	$[P_1 \to P_2] \wedge P_1$
T	T	$p_{12}=\times_{i=1,2}^{qp}\mu_i(\{(T,T)\})$	T
T	F	$p_{1\bar{2}}=\times_{i=1,2}^{qp}\mu_i(\{(T,F)\})$	F
F	T	$p_{\bar{1}2}=\times_{i=1,2}^{qp}\mu_i(\{(F,T)\})$	F
F	F	$p_{\bar{1}\bar{2}}=\times_{i=1,2}^{qp}\mu_i(\{(F,F)\})$	F

Hence, we conclude that Tables 11.7, 11.8 and 11.9 are essentially the same. And therefore, we see;

(H) the calculation will be more concise if we start with non-overlapping propositions (such as Table 11.9). (For example, if $P_1, P_2, P_3 = P_4 = P_5 = P_6, P_7$, then it suffices to consider $P_1, P_2, P_3.P_7$)

Example 11.9. [Modus ponens in probabilistic truth table] The following table (i.e., Table 11.10: Probabilistic truth table) is the same as a well-known truth table, except for the "probability column".

Table 11.10: Probabilistic Truth Table (Elementary propositions P_1, P_2)

P_1	P_2	probability: $p = \bigtimes_{i=1,2}^{qp} \mu_i$	$[P_1 \to P_2] \wedge P_1$	$[P_1 \to P_2] \wedge P_1 \to P_2$
T	T	$p_{12} = \bigtimes_{i=1,2}^{qp} \mu_i(\{(T,T)\})$	T	T
T	F	$p_{1\bar{2}} = \bigtimes_{i=1,2}^{qp} \mu_i(\{(T,F)\})$	F	T
F	T	$p_{\bar{1}2} = \bigtimes_{i=1,2}^{qp} \mu_i(\{(F,T)\})$	F	T
F	F	$p_{\bar{1}\bar{2}} = \bigtimes_{i=1,2}^{qp} \mu_i(\{(F,F)\})$	F	T

For example, consider the proposition $[P_1 \to P_2] \wedge P_1$, which is regarded as the map from $\{T,F\}^2$ to $\{T,F\}$ such that

$$[[P_1 \to P_2] \wedge P_1](x_1,x_2) = \begin{cases} T & \text{if } (x_1,x_2) = (T,T) \\ F & \text{if } (x_1,x_2) = (T,F),(F,T),(F,F) \end{cases}$$

Thus, we see that $\text{Prob}[[[P_1 \to P_2] \wedge P_1]; \{T\}]$ (i.e., the probability that $[[P_1 \to P_2] \wedge P_1]$ is true) is equal to

$$(\mu_1 \overset{qp}{\bigtimes} \mu_2)([[P_1 \to P_2] \wedge P_1]^{-1}(\{T\})) = p_{12}$$

By the same way, we can calculate as follows.

$$(\mu_1 \times^{qp} \mu_2)([((P_1 \to P_2) \wedge P_1) \to P_2]^{-1}(\{T\})) = p_{12} + p_{1\bar{2}} + p_{\bar{1}2} + p_{\bar{1}\bar{2}}) = 1$$

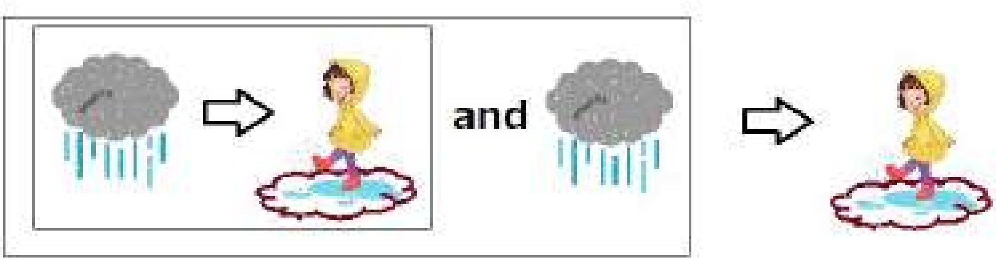
That is,

$$\text{Prob}[((P_1 \to P_2) \wedge P_1) \to P_2]; \{T\}] = p_{12} + p_{1\bar{2}} + p_{\bar{1}2} + p_{\bar{1}\bar{2}} = 1$$

Thus, modus ponens is always true even in probabilistic logic.

For example, put P_1 :="it rains", P_2 :="the ground is wet". Modus ponens says that

[["it rains" $\to$ "the ground is wet"]$\wedge$ "it rains"]$\to$"the ground is wet".



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The statement (H) in Examples 11.8 says that it suffices to consider the case of non-overlapping propositions. Thus we have the following theorem.

Theorem 11.10. [Logical sample space] Let $P_1, P_2, ..., P_i, ..., P_n$ be non-overlapping propositions (i.e., $P_i \neq P_j (\forall i, j$ such that $i \neq j$, *cf.* (H) above), and consider the probability space $(\{T,F\}, 2^{\{T,F\}}, \mu_i)$ such that $\mu_i(\{T\}) = \text{Prob}[P_i; \{T\}]$, $\mu_i(\{F\}) = \text{Prob}[P_i; \{F\}]$. Consider a quasi-product probability space $(\{T,F\}^n, \mathcal{P}(\{T,F\}^n), \times^{qp}_{i=1,2,...,n} \mu_i)$ such that

$$(\underset{i=1,2,\ldots,n}{\overset{qp}{\times}} \mu_i)(\{T,F\}^{k-1} \times \{x\} \times \{T,F\}^{n-k}) = \text{Prob}[P_k; \{x\}] \qquad (x \in \{T,F\}, \quad k = 1,2,...,n)$$

Then, the pair $\big[\{P_1, P_2, ..., P_n\}; (\{T,F\}^n, \mathcal{P}(\{T,F\}^n), \times^{qp}_{i=1,2,...,n} \mu_i)\big]$ is called the *logical sample space*. Let P be a proposition which is constructed by $P_1, ..., P_n$. Note that P is regarded as the map from $\{T,F\}^n \to \{T,F\}$. Then, we see that

$$\text{Prob}[P; \{T\}] = (\underset{i=1,2,\ldots,n}{\overset{qp}{\times}} \mu_i)(P^{-1}(\{T\})), \quad \text{Prob}[P; \{F\}] = (\underset{i=1,2,\ldots,n}{\overset{qp}{\times}} \mu_i)(P^{-1}(\{F\}))$$

Also, it is clear that the above Theorem 11.10 implies the following corollary.

Corollary 11.11. [Tautology in probabilistic logic] Let P be a proposition constructed from non-overlapping propositions $P_1, P_2, ..., P_n$. Then, the followings are equivalent:

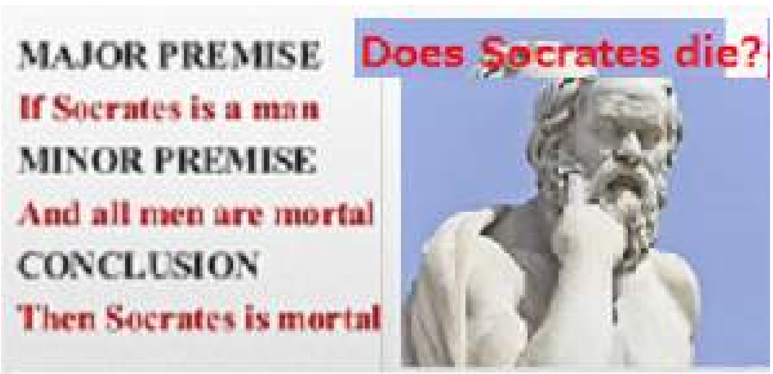
(i) P is a tautology in the sense of usual logic.
(ii) $\text{Prob}[P; \{T\}] = 1$.

That is, tautology always holds even in probabilistic logic. For example,

($\flat$) syllogism (i.e., $[[P_1 \to P_2] \wedge [P_2 \to P_3]] \to [P_1 \to P_3]$ always holds.

Remark 11.12. It is usually said that the following is the typical example of logic.

($\sharp$) Since Socrates is human being, and human being is mortal, it follows that Socrates is mortal.



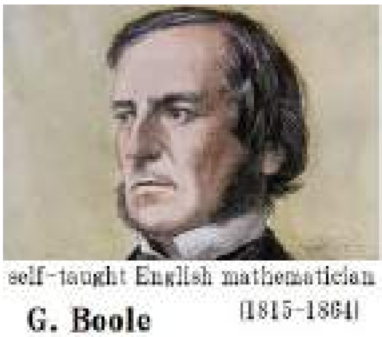
However, so far, we do not know the definition of "proposition" other than "mathematical proposition". For example, as seen in Sec. 8.2, "I think" and "I am" are not propositions in QL. Strictly speaking, there is no guarantee that "Socrates is a man" is a proposition.

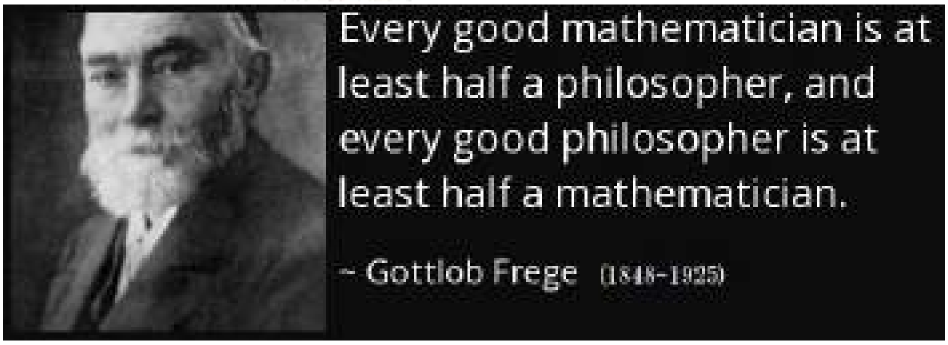
A discussion of mathematics alone, without a worldview, is powerless.

Therefore, the above Corollary 11.11 does not guarantee that (♯) holds. This will be discussed in Sec. 12.1.

11.3 George Boole, Gottlob Frege and Bertrand Russell: Mathematics or philosophy?

George Boole (1815-1864) was a mathematician, philosopher and logician in England. Particularly, it's said that the propositional logic (i.e., Boolean algebra) was proposed by him. Also, Gottlob Frege (1848-1925) was a German philosopher, logician, and mathematician. He is called the father of analytic philosophy , concentrating on the philosophy of language, logic, and mathematics. Particularly, it's said that the predicate logic was proposed by him. Bertrand Russel (1872-1970) was mathematician, philosopher, logician, social critic in England. With A. N. Whitehead he wrote *Principia Mathematica*, an attempt to create a logical basis for mathematics.

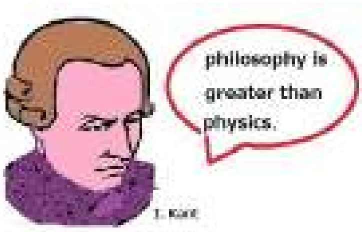




Many times we have discussed the following idealism:

(A)
introduction part:
- Plato: Idea theory
- Descartes: I think, therefore I am
- Kant: thing-in-itself
  Kant's philosophy provided the foundation for Newtonian mechanics

$\xrightarrow[\text{trust me}]{\text{Therefore}}$ common: spirit, ethics, moral



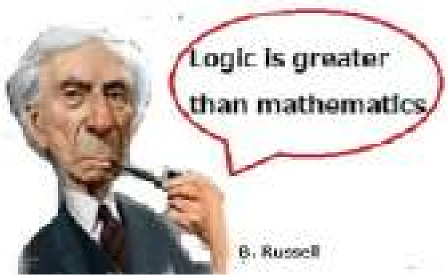
That is, it was the method of philosophy (i.e., Plato's way of telling philosophy) to find something impossible to doubt, and then declare that they had "deduced" various things from it.

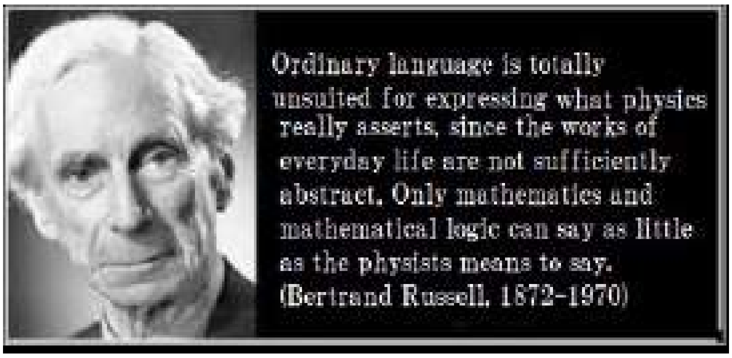
Following the example of Kant, B. Frege and B. Russell must have thought the following (or they must have thought that the general public would support them if they said the following):

(B_1) Mathematical logic provides the foundation for mathematics. Mathematics is just one area of mathematical logic. Mathematical logic is so powerful that there is no doubt about it. Thus, mathematical logic is greatest. Therefore, we have to be "logical".

That is,

(B_2) Frege, Russell — mathematical logic — Logic is so powerful that it is the foundation of mathematics $\xrightarrow[\text{trust logic!}]{\text{Therefore}}$ "Think everything logically!"





B. Russell wrote in "The Basic Writings of Bertrand Russell, 1903-1959", (p.608, Psychology Press) such as

(C) *Ordinary language is totally unsuited for expressing what physics really asserts, since the words of everyday life are not sufficiently abstract. Only mathematics and mathematical logic can say as little as the physicist means to say.*

I think Russell was sociable and interacted with many scientists in different fields, so of course he knew that "Einstein was not familiar with mathematical logic". So why did he make the above statement? My guess is as follows. He knew the method of philosophy (i.e., Plato's way of telling philosophy), that is,

- Philosophy is "asserted fiction" that is, to say "Therefore" in the (B), even if it's a lie.

Moreover, Russell was a man of integrity, so I think he wanted to say somewhere that

(D_1) mathematics (mathematical logic) alone cannot be a worldview,

because he could not lie. At the end of Wittgenstein's Tractatus Logico-philosophicus (TLP: 1921), he wrote, "What we cannot speak about we must pass over in silence.)".

(D_2) Don't talk about anything other than what we can talk about (= scientific propositions)!

Therefore. I think Wittgenstein was saying on Russell's behalf what Russell wanted to argue. If so, I can understand Russell's tolerance for Wittgenstein.

Russell must have been concerned about scientific propositions. Otherwise, Russell's generosity to Wittgenstein cannot be understood.

♠**Note 11.2.** The rule of philosophy (≈ idealism) is as follows:

($\sharp_1$) Only discussion, no experimentation.
($\sharp_2$) Winners are determined by the popularity of the general public (or philosophical enthusiasts).

Therefore, philosophers are not necessarily in pursuit of truth. Sometimes it is necessary to bluff a little to gain a lot of support. Russell did not mean to say that logic is greater than mathematics. However, I think Russell succeeded in making the average philosophy enthusiasts feel that "logic is greater than mathematics." Gaining the support of the general public is not an easy task. Even if you write about meaningless things in an esoteric and lofty way, the general public will not follow you unless there is something in your writing that attracts them. Wittgenstein had a gift for writing philosophy.

11.4 Peirce (1839-1914): Abduction

This section was written with reference to the following.

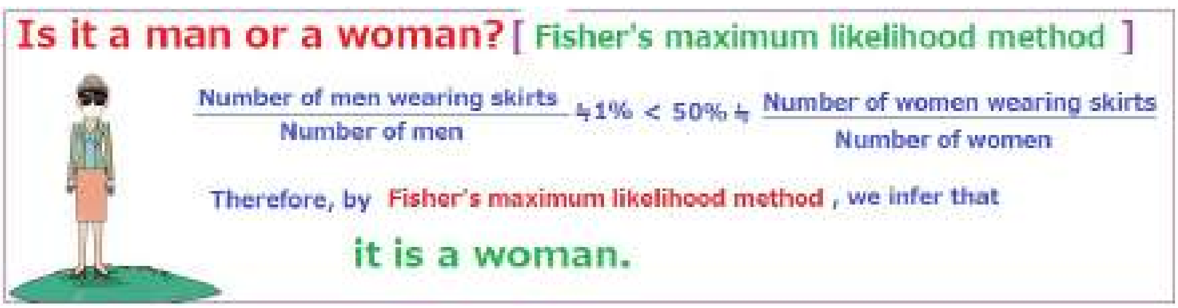
- [78]:Ishikawa, S: Philosophy of science for scientists; The probabilistic interpretation of science Journal of quantum information science, Vol. 9, No.3, 140-154, DOI: 10.4236/jqis.2019.93007 (https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

Charles Sanders Peirce (1839–1914)

Charles Sanders Peirce (1839-1914) was an American philosopher, logician, mathematician, and scientist who is sometimes known as "the father of pragmatism". This paper will not touch on pragmatism, but will speak of his "abduction" (also called abductive reasoning, abductive inference, or retroduction). In ref. [104] (Vol. II p.375), as the typical example of "abduction", Peirce mentioned as follows.

($\sharp_1$) Fossils are found; say, remains like those of fishes, but far in the interior of the country. To explain the phenomenon, we suppose the sea once washed over this land.

This kind of reasoning was called "abduction" by Peirce. As most readers will immediately realize, abduction is essentially the same as the qualitative representation of Fisher's maximum likelihood method. Recall the following figure in Sec. 1.4.4:



($\sharp_2$) A person in the distance is standing. I am not sure of the person's gender, but the person is wearing a skirt. Therefore, I presume the person to be a woman.

Such an estimation (($\sharp_1$), ($\sharp_2$)) is called "abduction" by Peirse. As many readers will immediately notice, abduction is essentially the same as the qualitative expression of

Fisher's maximum likelihood method.

The purpose of this section is to explain the following table:

Logical worldview [no measurement (logical approach))]		Quantum mechanical worldview [measurement (probabilistic approach)]
deduction (cf. Sec. 11.4.1.1 (i))	←→	measurement
abduction (cf. Sec. 11.4.1.1 (ii))	←→	inference (maximum likelihood method)
induction (cf. Sec. 11.4.1.1 (iii))	←→	the law of large numbers

where

$$(\sharp) \begin{cases} \text{the logical approach} & \text{(rough, handy)} \\ \text{the probabilistic approach} & \text{(exact, calculation required)} \end{cases}$$

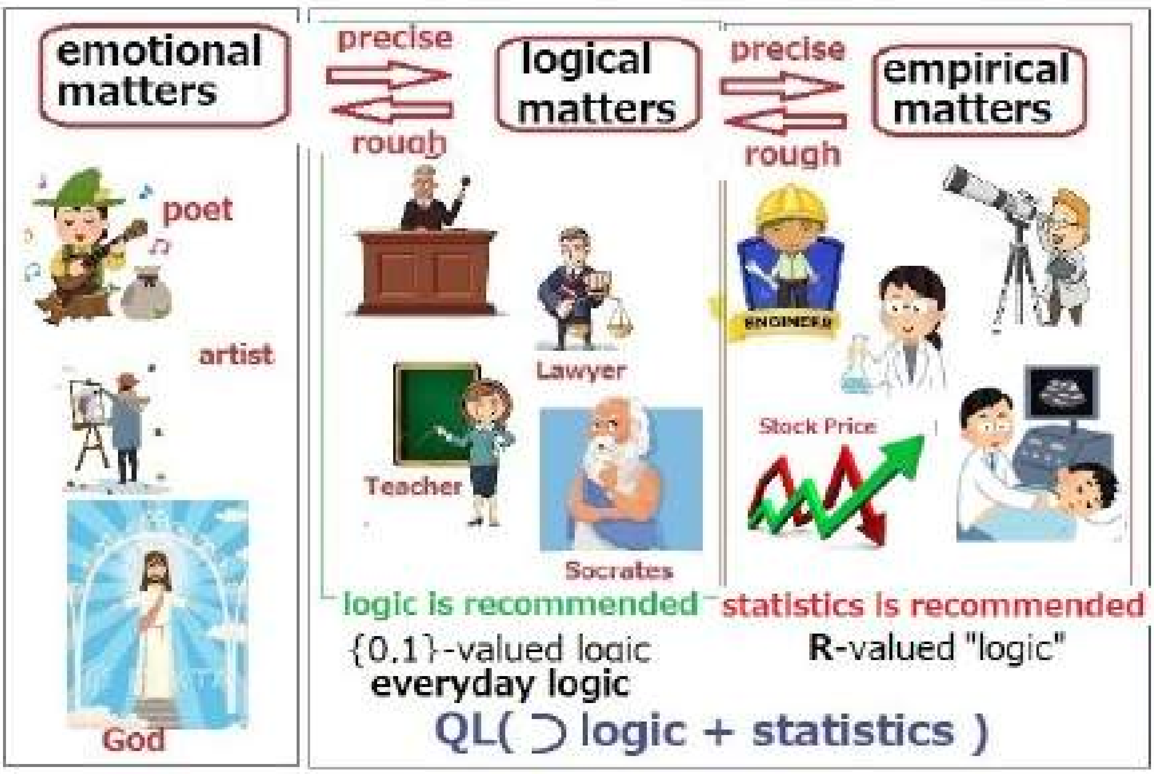
♠**Note 11.3.** In the table above (or Figure 0.1 in Preface), I used the term "logical worldview," but it is quite difficult to explain what this means. In a nutshell, think of it as follows.

- Arguments that use many [and, or, not, ⇒] to advance the argument.

Or, think of it as the same as the "set-theoretic worldview" of Sec. 4.3.4. I know it may seem lax, but I think

- In dualistic idealism, only the quantum mechanical worldview (quantum language) is solid

Recall the figure in Sec. 11.1.2



♠**Note 11.4.** Peirce's work was done before 1900. The pioneering nature of his work can be clearly seen in the following time series

(i) Born's discovery "the probabilistic interpretation of quantum mechanics" in ref. [6] (1926)
which is essentially the same as Axiom 1 (measurement) of quantum language

(ii) Fisher's great book "Statistical Methods for Research Workers" in ref. [22] (1925) (in which Fisher's maximum likelihood method was introduced)

These two are among the major works of the top ten most influential writings of the 20th century. Recall that Axiom 1 (measurement) of quantum language and Fisher's maximum likelihood method are essentially the same (*cf.* Sec. 1.4.4). Therefore, I believe that Peirce's work was ahead of its time.

11.4.1 * What is Peirce's abduction?

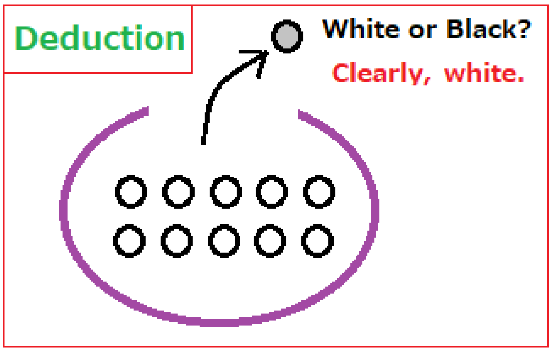
11.4.1.1 Deduction, abduction and induction in "logic"

According to Peirce, three kinds of inferences (i.e., deduction, abduction, induction) are important. Let us explain deduction, abduction and induction as follows.

[(i):Deduction]

A typical example of deduction is as follows: (In the following, (A_1') and ($\widehat{A}_1'$) are often omitted.)

(A_1) All the beans in this bag B_1 are white: [bag B_1 $\longrightarrow$"w"($\approx$ white)]
(A_1') All the beans in that bag B_2 are white or black fifty-fifty (or generally, the ratio of white beans to black beans is $p/(1-p)$ where $0 < p < 1$): [bag B_2 $\longrightarrow$"w"($\approx$ white) or "b"($\approx$ black)]
(A_2) This bean is from this bag B_1: [bag B_1]
(A_3) Therefore, this bean is white: ["w"($\approx$ white)]



It is, of course, obvious and ordinary.

♠**Note 11.5.** It is clear that the following is a tautology:

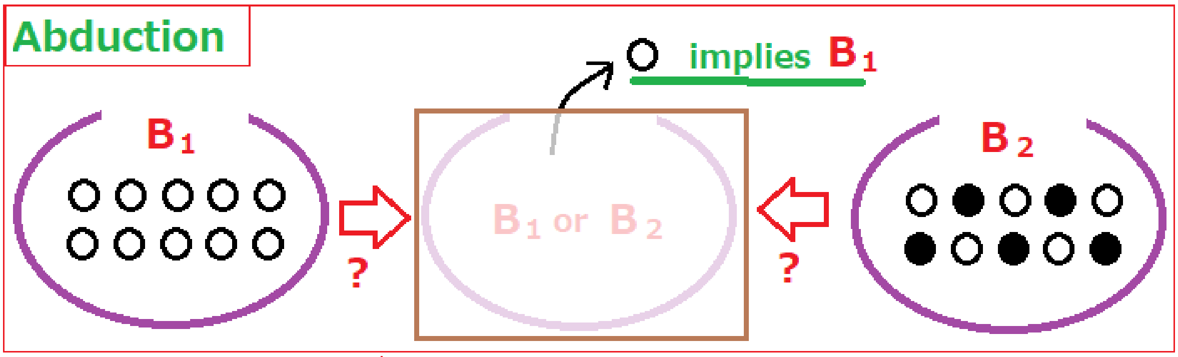
($\sharp_1$) $$[[B_1 \longrightarrow w] \wedge [B_2 \longrightarrow [w \vee b]] \wedge B_1] \longrightarrow [w]$$

Thus, the above conclusion (A_3) can be understood as a consequence of this tautology.

[(ii): Abduction]

C.S, Peirce (*cf.* ref. [104]) proposed abduction as follows.. The example of abduction is as follows:

($\widehat{A}_1$) All the beans in this bag B_1 are white: [bag B_1 $\longrightarrow$"w"($\approx$ white)]
($\widehat{A}_1'$) All the beans in that bag B_2 are white and black fifty-fifty (or generally, the ratio of white beans to black beans is $p/(1-p)$): [bag B_2 $\longrightarrow$"w"($\approx$ white) or "b"($\approx$ black)]
($\widehat{A}_2$) This bean (from B_1 or B_2 (i.e., it is not known whether it is B_1 or B_2)) is white: ["w"($\approx$ white)]
($\widehat{A}_3$) Therefore, this bean is from this bag B_1 : [bag B_1]



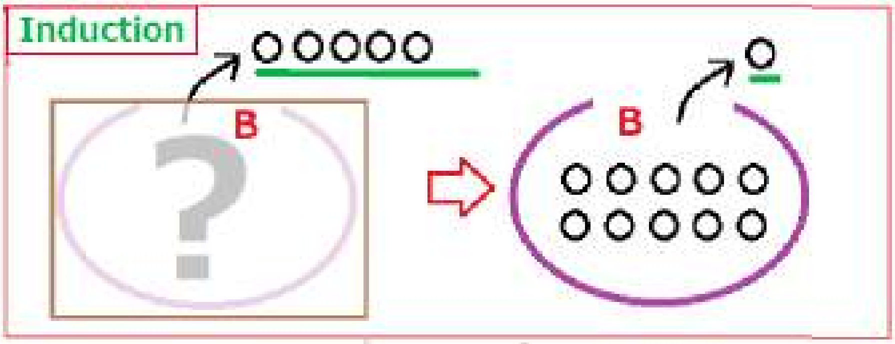
[(iii): Induction]

Further, induction (inductive reasoning) is as follows.

($\widetilde{\mathrm{A}}_1$) $1000p$ white beans and $1000(1-p)$ black beans are mixed well in this bag B_3 (here, $0 < p < 1$). Assume that we do not know the value p $(0 < p < 1)$.

($\widetilde{\mathrm{A}}_2$) When we took 20 beans out of this bag B_3, every bean was white.

($\widetilde{\mathrm{A}}_3$) Therefore, the bean picked out from this bag B_3 next can be presumed to be white.



This will be again discussed in the following section.

11.4.1.2 Deduction, abduction and induction in quantum language (i.e., the quantum mechanical worldview)

In our worldview (i.e., the quantum mechanical worldview ≈ the quantum linguistic worldview), the relation among deduction, abduction and abduction is characterized as follows. First, we will show that the abduction $[(\widehat{\mathrm{A}}_1)$-$(\widehat{\mathrm{A}}_3)]$ can be justified in quantum language. Consider the state space $\Theta = \{\theta_1, \theta_2\}$ with the discrete topology, and the classical basic structure $[C(\Theta) \subseteq B(L^2(\Theta, \nu))]$, where $\nu(\{\theta_1\}) = \nu(\{\theta_2\}) = 1/2$. Assume that

$$\theta_1 \approx \text{the state of the bag B}_1, \quad \theta_2 \approx \text{the state of the bag B}_2,$$

Assume that 1000 white beans belong to bag B_1, and further, $1000p$ white beans and $1000(1-p)$ black beans belong to the bag B_2 (where $0 < p < 1$). Thus we have the observable $\mathsf{O} = (\{w, b\}, 2^{\{w,b\}}, F)$ in $C(\Theta)$ such that

$$\begin{aligned} &[F(\{w\})](\theta_1) = 1 \qquad &&[F(\{b\})](\theta_1) = 0 \\ &[F(\{w\})](\theta_2) = p \qquad &&[F(\{b\})](\theta_2) = 1-p \qquad (0 < p < 1) \end{aligned}$$

where "w" and "b" means "white" and "black" respectively.
Thus, we have the measurement $\mathsf{M}_{C(\Theta)}(\mathsf{O} := (\{w, b\}, 2^{\{w,b\}}, F), S_{[\theta_i]})$, $i = 1, 2$.

[(i): Measurement ≈ Deduction]

Axiom 1 [measurement] (in Section 1.1) says that

- $(\widehat{\mathrm{B}}_1)$ [measurement]: The probability that the measured value w is obtained by $\mathsf{M}_{C(\Theta)}(\mathsf{O} := (\{w, b\}, 2^{\{w,b\}}, F), S_{[\theta_1]})$ is equal to 1

This is the same as the deduction (i.e., (A_1)–(A_3)).

[(ii): Inference ≈ Abduction]

Next, under the circumstance that bags B_1 and B_2 cannot be distinguished, we consider the following inference problem:

- $(\widehat{\mathrm{B}}_2)$ [inference problem]: When the measured value w is obtained by the measurement $\mathsf{M}_{C(\Theta)}(\mathsf{O} := (\{w, b\}, 2^{\{w,b\}}, F), S_{[*]})$, which do you infer, $[*] = \theta_1$ or $[*] = \theta_2$?
  Fisher's maximum likelihood method Theorem 1.15 [Fisher's maximum likelihood method] says that $[*] = \theta_1$, since

  $$\max\{F(\{w\})](\theta_1), F(\{w\})](\theta_2)\} = \max\{1, p\} = 1 = [F(\{w\})](\theta_1)$$

  This implies $(\widehat{\mathrm{A}}_3)$.

Therefore, the above $(\widehat{\mathrm{B}}_2)$ is the quantum linguistic representation of abduction (i.e., $(\widehat{\mathrm{A}}_1)$–$(\widehat{\mathrm{A}}_3)$).

[(iii): Induction ≈ the law of large numbers]

This has already been discussed in Sec, 9.6, "Hume's problem of induction," so we omit it here.

11.4.1.3 Summary; Logic-like understanding (by Peirce) vs. quantum linguistic understanding

Generally speaking,

(C) Quantum linguistic method (in Sec. 11.4.1.2) is natual, and logic-like method (≈ pseudo-logical method in Sec.)(by Peirce) is incomplete.

However, the world is not only about "theory". In many cases, quantitative data can not be obtained, thus, logic-like method (by Peirce) is often used.

- In fact, Sherlock Holmes, the famous detective, must have been a master in the use of Peirce's abduction.
  If Holmes had used the maximum likelihood method, his detective novels would not have become bestsellers.

The slogan is "logic for rough arguments, statistics for precise arguments," to put it roughly,

Logic for the humanities, statistics for the sciences

When quantitative data are available, the "quantum mechanical worldview" is superior to the "logical worldview. However, the "logical worldview" has an advantage that it can be used without calculation. Therefore, in daily life, the "logical worldview" is often superior to the "quantum mechanical worldview".

In summary, it is as follows.

Correspondence 11.13.

$$
\begin{array}{ccc}
\begin{array}{c}(\mathrm{B}_1)\text{: measurement} \\ \text{Axiom 1 [measurement] (in Section 1.2.1)}\end{array} & \xrightarrow[\text{simplified form}]{} & \begin{array}{c}[(\mathrm{A}_1),(\mathrm{A}_1'),(\mathrm{A}_2) \longrightarrow (\mathrm{A}_3)] \\ \text{deduction}\end{array} \\
\Big\downarrow \text{reverse} & & \Big\downarrow \text{reverse} \\
\begin{array}{c}(\widehat{\mathrm{B}}_2)\text{: inference} \\ \text{(Fisher's maximum likelihood method)}\end{array} & \xrightarrow[\text{simplified form}]{} & \begin{array}{c}[(\widehat{\mathrm{A}}_1),(\widehat{\mathrm{A}}_1'),(\widehat{\mathrm{A}}_2) \longrightarrow (\widehat{\mathrm{A}}_3)] \\ \text{abduction}\end{array}
\end{array}
\tag{11.1}
$$

That is,

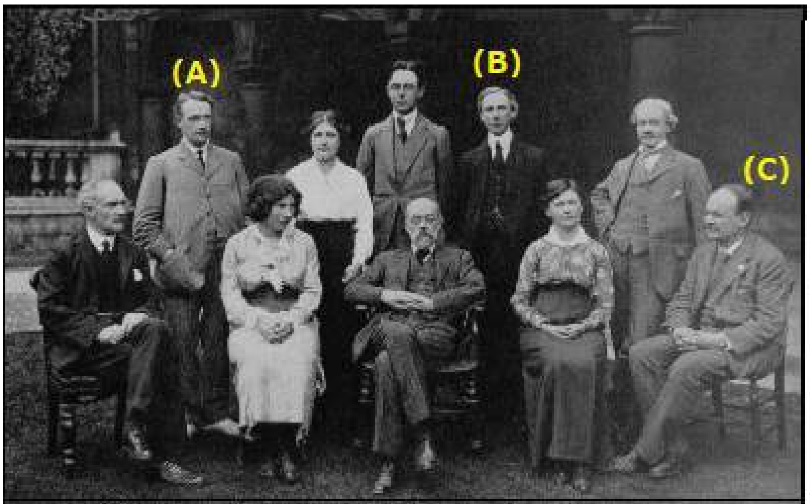
Logical worldview [no measurement (logical approach))]		Quantum mechanical worldview [measurement (probabilistic approach)]
deduction (cf. Sec. 11.4.1.1 (i))	←→	measurement
abduction (cf. Sec. 11.4.1.1 (ii))	←→	inference (maximum likelihood method)
induction (cf. Sec. 11.4.1.1 (iii))	←→	the law of large numbers

11.5 Bertrand Russell: five-minute hypothesis, McTaggart's paradox, Moore's paradox

This section consists of excerpts from the following paper:

- Ref. [81]: Ishikawa, S. (2021) Fuzzy Logic in the Quantum Mechanical Worldview ; Related to Zadeh, Wittgenstein, Moore, Saussure, Quine, Lewis Carroll, etc. JAMP, Vol. 9, No.3, 140-154, (https://www.scirp.org/journal/paperinformation.aspx?paperid=110830)

Bertrand Russell (1872-1970) was a great intellectual and one of the founders of analytic philosophy along with Gottlob Frege (and student Ludwig Wittgenstein). Russell is a multi-talented philosopher who has produced many achievements. However,



Cambridge Moral Science Club, 1915
(A): Moore,(B): Russell,(C):McTaggart

In this section, I mention "Russell's paradox", "Five-minute hypothesis", and "McTaggart's paradox", Moore's paradox. And we see that these are closely related to the linguistic Copenhagen interpretation (or equivalently, to the problem "What is a 'proposition'?"). Thus, it can be said that

- the three of them prepared the ground for the appearance of Wittgenstein at Cambridge University.

11.5.1 Russell's paradox in set theory

As mentioned frequently in this text, I think that mathematical logic is one of fields of mathematics and thus, Russell's study of logic has little to do with traditional Western philosophy (i.e., Descartes-Kant philosophy).

However, some things should be mentioned below. The following is famous as Russell's paradox (or, Russell-Zermelo paradox).

The naive set theory (i.e., Cantor's set theory) involves contradictions. For example, Russell showed Russell's paradox such that

(A) if it is assumed that $\{U \mid U \notin U\}$ is a set, then it leads contradiction (1902).[*1]

To avoid such a paradox, Alfred North Whitehead and Bertrand Russell proposed "type theory" (i.e., a kind of axiomatic set theory), which was published as the Principia Mathematica on the foundations of mathematics in 1910–1913. Also, the axiomatic system of set theory was developed by Zermelo and others. Most modern mathematicians study mathematics developed under a system of axioms called ZFC (1921), which consists of eight axioms by Zermelo and Fraenkel plus an axiom called the Axiom of Choice.

Remark 11.14. The above is closely related to the problem: "What is a 'mathematical proposition'?". That is because this problem is essentially the same as the problem: "What is a 'set'?". That is, from the theoretical point of view, we see

$$\text{What is a 'mathematical proposition'?} \underset{\text{equiv.}}{\Longleftrightarrow} \text{What is a 'set'?}$$

Thus, the discovery of "something that like a set but is not a set" (such as Russell discovered) promoted the birth of axiomatic set theory (e.g., Zermelo-Fraenkel set theory). It is believed that the problem:"What is a 'mathematical proposition'?" was solved by Zermelo and Fraenkel.

Therefore, the next biggest and only remaining philosophical question is

- "What is a 'non-mathematical proposition'?"

This, of course, was the problem Wittgenstein pursued in ref. [125]: "TLP (=Tractatus Logico-Philosophicus)".

////

Summing up, we see that

(B) Russell's paradox (i.e., the discovery of something like a set but not a set) prompted the birth of axiomatic set theory (e.g., Zermelo-Fraenkel set theory), in which "mathematical proposition" can be completely defined.

This (B) should be compared with the following (C).

[*1] If it is assumed that "the set of all sets" is a set, it leads contradiction. This is called Cantor's paradox.

(C) Similarly, the discovery of something like a proposition but not a proposition (e.g., "I think, therefore I am", Moore's paradox, etc.) prompted the development of the linguistic Copenhagen interpretation (which serves to distinguish between "scientific proposition" and "non-scientific proposition").

Recall the final chapter of Wittgenstein's TLP (i.e., ref. [125]: "Tractatus Logico-Philosophicus") in which only " *What we cannot speak about we must pass over in silence*" is written. This implies that Wittgenstein's central theme in his TLP is to draw a line between what we can speak about (= scientific propositions) and what we cannot speak about (= non-scientific propositions), which is the same as the theme of this text, that is,

the linguistic Copenhagen interpretation
=the rule of how to use axioms 1 (measurement) and 2 (causality)
=the rule of how to draw a line between scientific propositions and
= non-scientific propositions

♠**Note 11.6.** Note that

Russell's paradox(1902) $\xrightarrow[\text{about 20 years later}]{}$ Zermelo-Fraenkel set theory
Descartes' "I think therefore I am"(1637) $\xrightarrow[\text{about 350 years later}]{}$ linguistic Copenhagen interpretation

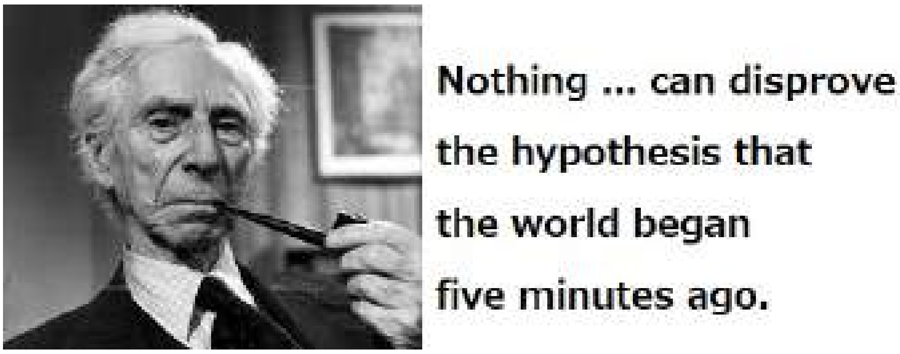
This is due to the fact:

- There was a strong belief in mathematics that paradox should be avoided at all costs. On the other hand, philosophers have enjoyed and exploited paradoxes.

11.5.2 * Five-minute hypothesis

In summary, it is as follows.

($\sharp_1$) Russell discovered the Russell-Zermelo paradox. This discovery led to the biggest question in the history of mathematics: "What is a mathematical proposition?".
($\sharp_2$) He solved this problem with Whitehead.
($\sharp_3$) Next, Russell challenged the question "What is a scientific proposition?" It was only a proposal of a "Five-minute hypothesis," and no noteworthy results were obtained.
($\sharp_3$) Therefore, he entrusted the problem to his disciple, Wittgenstein. However, Wittgenstein could not solve this problem either.
($\sharp_4$) But Wittgenstein's words had power. He was notably gifted as a poet. As a result, he became a symbolic philosopher in the propagation of analytic philosophy.



Bertrand Russell was a very eloquent philosopher who communicated many thought-provoking things to the general public. For a layman like me, he is a very helpful philosopher. If the following quote had not come from Russell, I don't think it would have been as famous as it is.

- *There is no logical impossibility in the hypothesis that the world sprang into being five minutes ago, exactly as it then was, with a population that "remembered" a wholly unreal past. There is no logically necessary connection between events at different times; therefore nothing that is happening now or will happen in the future can disprove the hypothesis that the world began five minutes ago.*
Ref. [114] "The Analysis of Mind", p.223, Bertrand Russell

This section was written with reference to the following.

- [76]:Ishikawa, S: Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language, Open Journal of philosophy, Vol. 8, No.5, 466-480, 2018, DOI: 10.4236/ojpp.2018.85032 (https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862)

  [Revised version] (https://philpapers.org/rec/ISHLCB)
  (http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf)
- [85] S. Ishikawa, K. Kikuchi, (2021) *Quantum Fuzzy Logic and Time, Journal of Applied mathematics and physics,* Vo.9 No.11 2021, 2609-2622
  (https://www.scirp.org/journal/paperinformation.aspx?paperid=112972)

The five-minute hypothesis is a skeptical hypothesis put forth by the philosopher Bertrand Russell. However, as seen later, I do not think that this hypothesis is not related to skepticism though my understanding to skepticism may be insufficient.

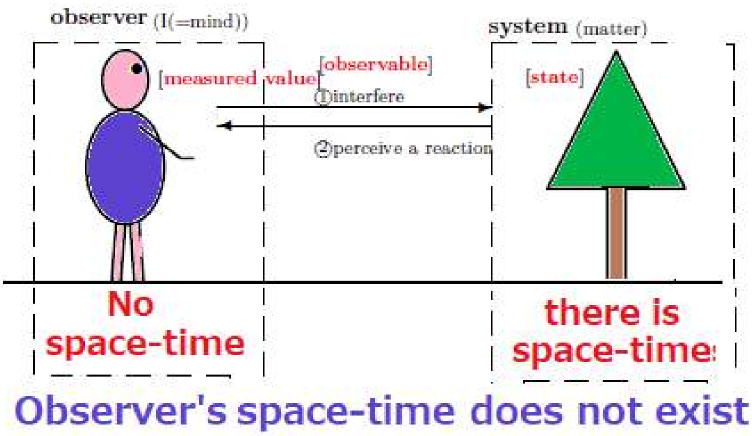
The five-minute hypothesis, proposed by B. Russell (*cf.* ref. [114]), is as follows.

(A_1) the universe was created five minutes ago. Or equivalently, the universe was created ten years ago.

Now we show that this (A_1) is not the statement in quantum language as follows (i.e., The first answer (i) and the second answer (ii))

Answer:

Recall the linguistic Copenhagen interpretation:



Thus, observer's time (e.g., "tense", "now",...) can not used in quantum language. Note that this hypothesis (A_1) is related to "tense". Thus, the linguistic Copenhagen interpretation (E_2) in Sec. 1.2.1 says that this (A_1) is not a statement in quantum language. Thus, the (A_1) is not scientific, that is, there is no experiment to verify the statement (A_1).

Therefore, we can conclude that

(A_2) **"the universe was created five minutes ago" is not a proposition in QL.**

Some may want to relate this hypothesis to skepticism (*cf.* ref. [114]), However we do not think that this direction is productive.

Remark 11.15. (i): Also, the above (A_2) should be compared to the following (A_3)

(A_3) The universe was created in A.D. 2010. (Or equivalently, now is A.D. 2020, and the universe was created ten years ago.)

This (A_3) can be denied by experiment, that is, it is different from the fact. Thus, this is a proposition in quantum language.
(ii): The following is a trick question;

In the diagram below, which is less scientific, Russell or Wittgenstein?

In the above, we say that

($\sharp_1$) Russell's statement is scientific, but it is wrong.
($\sharp_2$) Wittgenstein is making a logical error. That is, he misunderstands Russell's scientific statements as unscientific.

11.5.3 * McTaggart's paradox

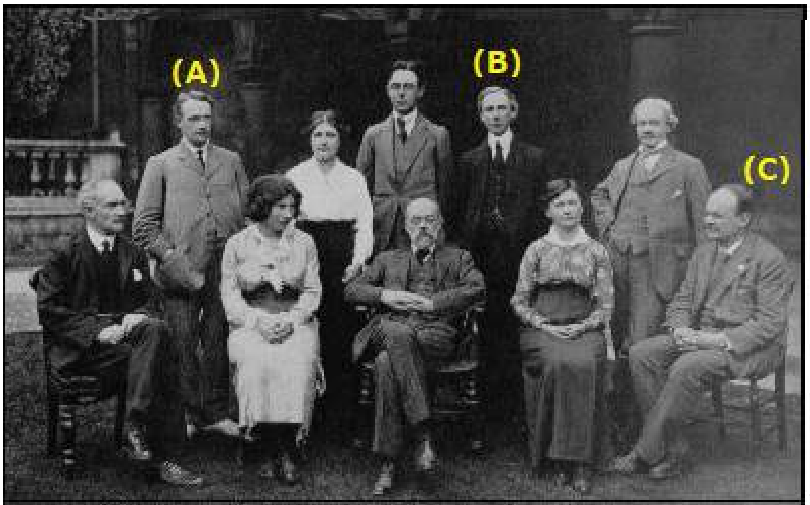
This section was written with reference to the following.

(A) [76]:Ishikawa, S: Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language Open Journal of philosophy, Vol. 8, No.5, 466-480, 2018, DOI: 10.4236/ojpp.2018.85032 (https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862)

[Revised version] (https://philpapers.org/rec/ISHLCB) (http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf)

- [85] S. Ishikawa, K. Kikuchi, (2021) *Quantum Fuzzy Logic and Time, Journal of Applied mathematics and physics,* Vo.9 No.11 2021, 2609-2622 (https://www.scirp.org/journal/paperinformation.aspx?paperid=112972)

J.M.E. McTaggart (1866–1921) was an English philosopher.



Cambridge Moral Science Club, 1915
(A): Moore, (B): Russell, (C): McTaggart

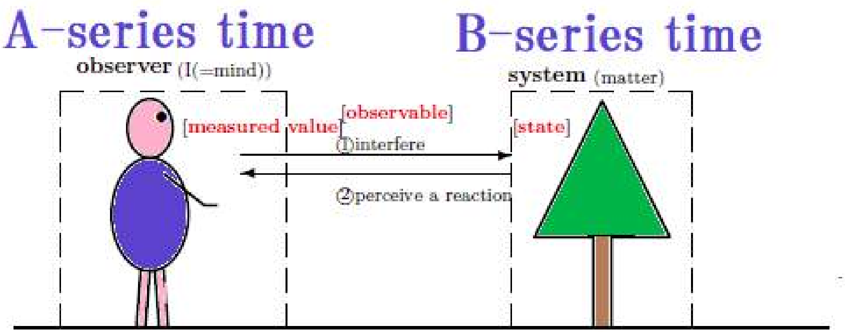
He was a member of the Cambridge Apostles, along with B. Russell (1877-1970). In ref. [98], McTaggart asserted "the Unreality of Time" as follows.

The sketch of McTaggart's proof

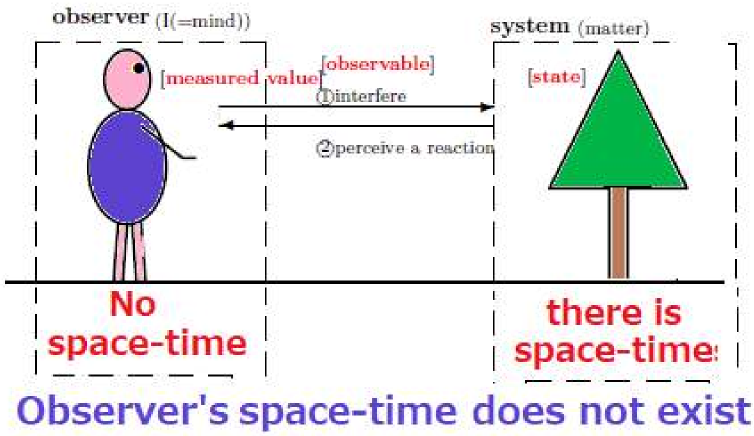
(B_1) Assume that there are two kinds of times. i.e. "observer's time (A-series)" and "objective time (B-series)". (Note that this assumption is against the linguistic Copenhagen interpretation (E_2) in Sec. 1.2.1.)
(B_2) ······
(B_3) After all, the contradiction is obtained

Therefore, by the reduction to the absurd (i.e., the proof by contradiction), we get;

(B_4) A-series does not exist (in science).



Recall the linguistic Copenhagen interpretation:



This implies that the above McTaggart's proof is non-sense.

For completeness, we add the following. About this proof, there are various opinions also among philosophers. Although I cannot understand the above part (B_2) (since the properties of A-series are not clear), I agree to him if his assertion is (B_4) (*cf.* ref. [49]). That is, I agree that McTaggart noticed first that observer's time is not scientific. Recall the linguistic Copenhagen interpretation (E_2) in Sec. 1.2.1:

- While "matter" is in the space-time, the observer is not.

Thus, I agree to the opinion that McTaggart is one of discoverers of the linguistic Copenhagen interpretation. I think, from the quantum linguistic point of view, that he should be estimated more highly.

Therefore, we can say that

(C) **McTaggart was also related to the linguistic Copenhagen interpretation ($\approx$ the problem such that "What is a 'proposition'?")**

11.5.4 Moore's paradox: "It is raining, but I do not believe it is raining"

11.5.4.1 "It is raining, but I do not believe it is raining" and Wittgenstein

G.E. Moore (1873-1958) was an English philosopher. He was, with Bertrand Russell, Ludwig Wittgenstein, and Gottlob Frege, one of the founders of analytic philosophy.

As his quote, the following is famous:

(A) "It is raining, but I do not believe it is raining"

That is, Moore raised the following question (i.e., Moore's paradox).

(B) Is the (A) a proposition? That is, can we define the truth value of (A)?

This problem is famous because Wittgenstein was very interested in it. I don't remember where it was written, but I heard that Wittgenstein said that the discovery of (A) was the greatest of all Moore's achievements. The reason why Wittgenstein was interested in (A) is clear as follows.

(C_1) His purpose at TLP was to answer "What is a proposition?" And at the end of his TLP, he wrote, "For something that is not a proposition, you must be silent." However, TLP was not theoretically successful, and "what is a proposition?" was the philosophical theme of his life.

And Wittgenstein thought:

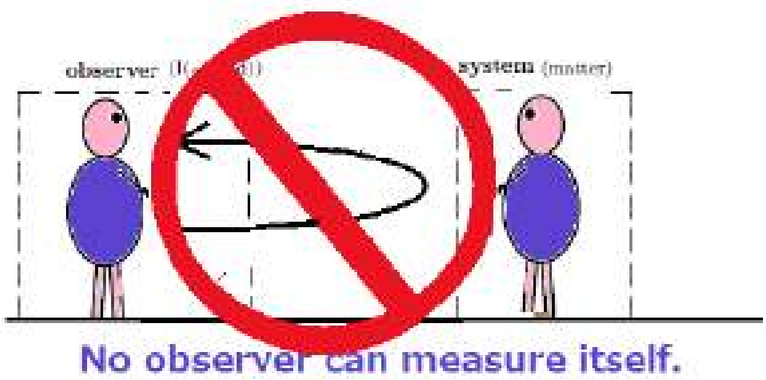
(C_2) The first thing to do in order to answer "what is a proposition?" is to study a pseudo-proposition such as "It is raining, but I do not believe it is raining". It would be great if we could make a theory of pseudo-propositions (and pseudo-truth values).

Now that I think about it, there are various detailed themes in this direction (ordinary language school), which are practically interesting (relationship between everyday language and AI, etc.), but there was not much possibility that a big theory would be born. Of course, the future is still undecided, and it's too early to say for sure.

11.5.4.2 "It is raining, but I do not believe it is raining" in the linguistic Copenhagen interpretation

Remember the linguistic Copenhagen interpretation (*cf.* Sec. 1.1) as follows.

(D_1) Audience should not go on stage
(D_2) The measurement is not dependent on the observer
(D_3) the observer cannot measure the observer himself
(D_4) Don't use terms "I", here", "now".



"It is raining, but I do not believe it is raining" can be interpreted in many ways, but here I will explain the following scientific interpretation.
First, let us assume that

(E_1) "I" in "It is raining, but I do not believe it is raining" is "Mr. X".
(E_2) "(D_1):Audience should not go on stage" implies that the observable is not Mr X.

That is, it implies that

(F) The sentence "It is raining, but I do not believe it is raining" is regarded as the measuered value of [Weather conditions, Mr. X's state of mind]

If so, this is scientific. But, it is somewhat unnatural.
Therefore, it is natural to assume that

(G) "It is raining, but I do not believe it is raining" is not considered as a scientific proposition, but as a literary art that enjoys playing with words.

11.5.4.3 Wittgenstein was interested in Moore's paradox

However, the purpose here is not to examine Moore's paradox in detail. What I have focused on is the following

(H) *when Wittgenstein first heard this paradox one evening (which Moore had earlier stated in a lecture), he rushed round to Moore's lodgings, got him out of bed and insisted that Moore repeat the entire lecture to him.*

If this anecdote is true, it is because it clearly shows that the central theme of Wittgenstein's research is "What is a scientific proposition?"

(I) "It is raining, but I do not believe it is raining" is *"what we cannot speak about"*.

Furthermore, he must have thought Descartes' cogito proposition "I think, therefore I am" (which is called the first principle of philosophy) was not also a proposition (*cf.* the linguistic Copenhagen interpretation (E_1) in Sec. 1.3.3).

At the time, Russell, Moore, and McTaggart were all professors in the Department of Philosophy at Cambridge University. Moore's paradox, Russell's "five-minute hypothesis," and McTaggart's "non-existence of time" were all based on questions such as "What is a scientific proposition? (Equivalently, "What is the Copenhagen Interpretation?) . This was the environment in which analytic philosophy (symbolically, "Wittgenstein") was born.

In other words, they showed an unusual interest in "nonsense phrases". Since the Copenhagen Interpretation is the principle to eliminate these "nonsense phrases" from science, we can say that their interest was in the Copenhagen Interpretation. However, I would like to think that Wittgenstein's real purpose was "axiomatic theory". Axioms 1 (measurement) and 2 (causality) in quantum language are difficult to discover without knowing quantum mechanics, so it does not matter whether Wittgenstein was a scientific success or failure. Therefore, I would like to argue the following diagram

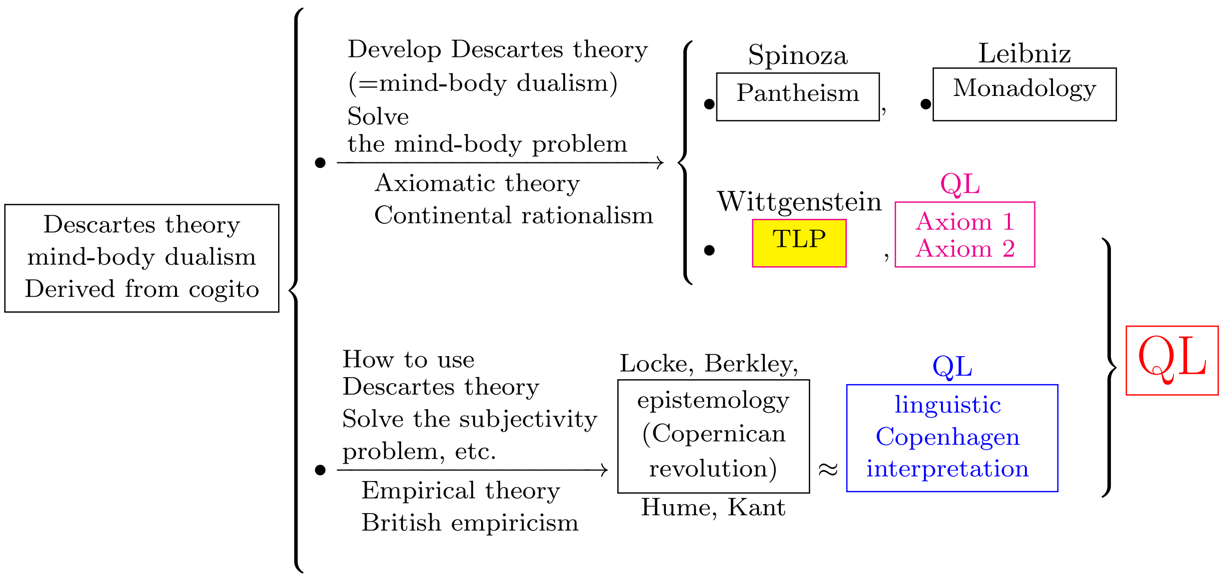


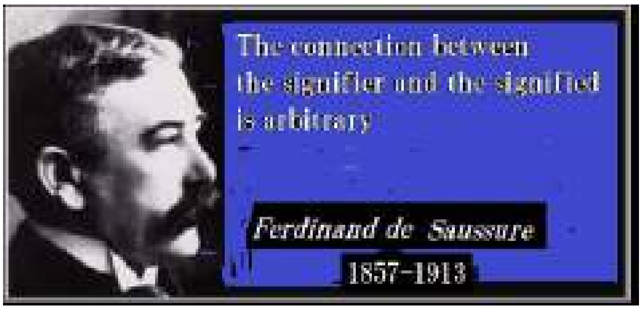
Figure 9.1: QL-synthesis

That is, the above figure explains the following

(J) The reason why the studies on the Copenhagen interpretation in analytic philosophy (Russell with the "world five minutes before" hypothesis, McTaggart's paradox, and Moore's paradox) were not connected with TLP (see the discussion) is that TLP belonged to the axiomatic approach.

Up to this point, the "pre-Wittgenstein" period has ended, and after Wittgenstein, philosophy of science is still considered to play the leading role.

11.6 Saussure: Copernican revolution in language



Ferdinand de Saussure (1857-1913) was a Swiss linguist. He is widely considered one of the founders of 20th-century linguistics.

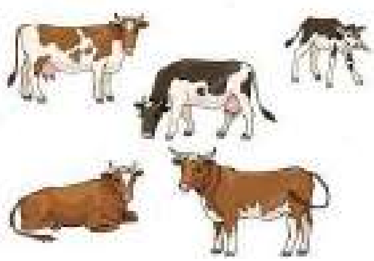
11.6.1 Saussure's linguistics: What comes first, things or words?

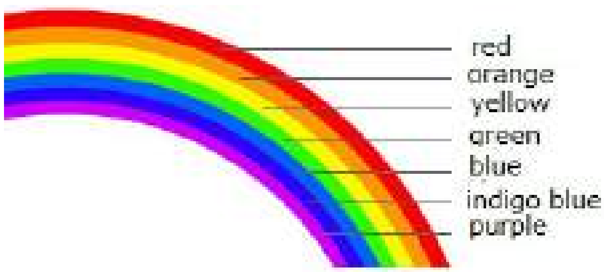
Let's think a little more about the implications of Saussureian linguistics. We tend to think that there are things at the beginning, and that we give each thing a name, just like we put a label on it. However, that is not the case. Rather, Saussureian linguistics says that we understand the order of things by the act of giving them names. I think that his theory is the linguistic version of the Copernican revolution of Kant. That is,

$$
\underset{}{\overset{\text{copy theory (realism)}}{\boxed{\text{things} \longrightarrow \text{words}}}}
\xrightarrow[\text{Copernican revolution in language}]{\text{Saussure's revolution}}
\underset{\text{Saussure's linguistics}}{\overset{\text{constructionism (transcendental idealism)}}{\boxed{\text{words} \longrightarrow \text{things}}}}
\tag{11.2}
$$

For example,

(A_1) I, who live in Japan, an island nation, know the names of many fish, but rarely distinguish between "cow, bull, ox, calf," etc.





(A_2) Also, in Japan, the rainbow has seven colors (i.e., red, orange, yellow, green, blue, indigo, violet(=purple)), and I was surprised when I first heard that there is a country where the rainbow does not have seven colors.

(A_3) Quantum language changes the way we see the world. In other words, quantum language is like a kind of worldview. That is, we see the world through the colored glasses of quantum language.

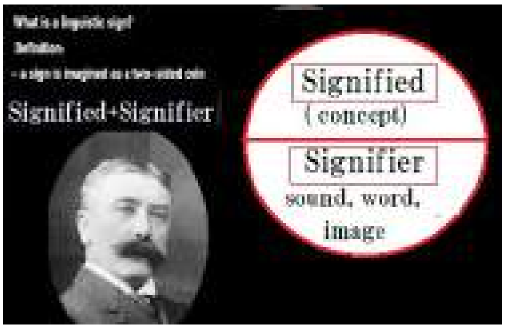
In the above, I'd like to assume that (A_1)$\sim$(A_3) are about the same.

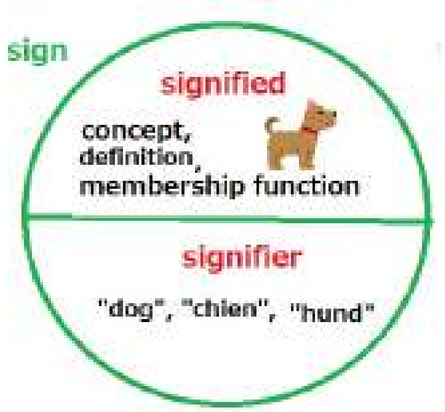
11.6.2 * The quantitative expressions of "signified"

Now let us explain the terms "signifier" and "signified", which were introduced by Saussure.

Definition 11.16. ["signifier" and "signified" in ordinary language]
For example, we explain "signifier" and "signified" concerning "dog"

(B_1) The "dog" in front of you is itself a physical being, isn't it? The image of the dog you have in mind, the barking, or the image of the dog in your head, is "signified".
(B_2) When this becomes a word (letter/sound) such as "dog", it is called "signifier".





Although Saussure's proposal (i.e., the above definition) is very significant, his ideas are not quantitative, so they are not very useful from a scientific standpoint. As emphasized frequently throughout the paper, I consider

(C) Without a quantitative worldview, we can't say anything solid from a scientific standpoint.

In fact, Saussure's idea does not play an important role in science.

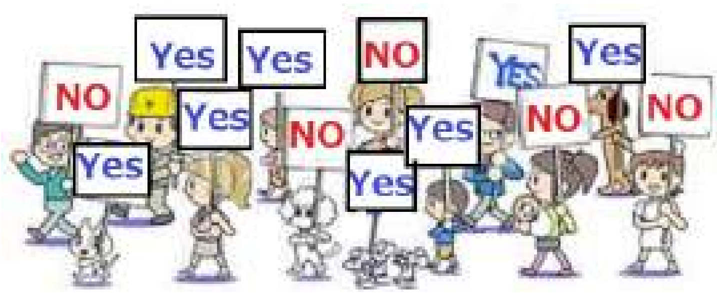
Thus, I will propose Definition 11.17, in which Saussure's idea is realized in quantum language as follows.

Definition 11.17. [Membership function (= Fuzzy set, *cf.* [41, 42, 43, 44, 129])] Let Ω be a state space. A continuous function $m : \Omega \to [0, 1]$ (i.e., the closed interval in the real $\mathbb{R}$) is called a membership function. Assume that the state (i.e., quantitative property) of any animal can be expressed by a point in the state space Ω. Define the membership functions $m_D : \Omega \to [0, 1]$ of dogs as follows. Suppose that there are 100 zoologists and the following question is made them.

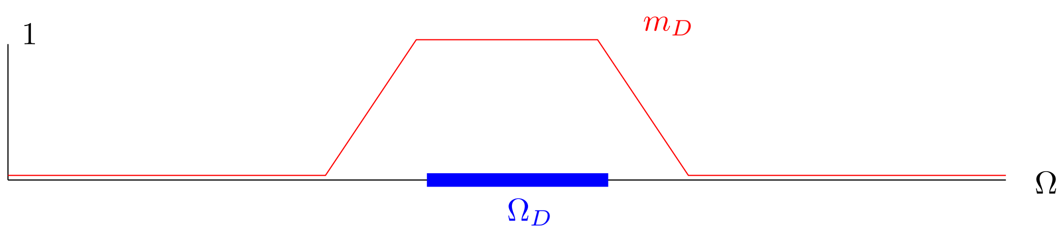
(D) Is this animal with the sate ω_1 $(\in \Omega)$ a dog or not?

The answer is as follows.

(E) $\begin{cases} \text{70 zoologists say that this bird is a dog.} \\ \text{30 zoologists say that this animal is not a dog.} \end{cases}$



Then the value of $m_D(\omega_1)$ is defined by 0.7. For many animals with the state ω_i $(i = 2, 3, ...N)$, repeating the experiment in the same way, the value of $m_D(\omega_i)$ $(i = 2, 3, ...N)$ is determined. And the membership function $m_D : \Omega \to [0, 1]$ of dogs is defined by the interpolation method (which may be rather subjective). Put $\Omega_D = \{\omega \in \Omega \mid m_D(\omega) = 1\}$, which is called the dog state class.



[membership function $m_D : \Omega \to [0, 1]$, and Ω_D]

Thus we have the dog-like observable $\mathsf{O}_{\rm D} = (\{T, F\}, 2^{\{T,F\}}, G_{\rm D})$ in $C(\Omega)$ with the membership function $G_{\rm D}(\{T\})$ $(= m_{\rm D})$ (*cf.* Definition 12.3).

With the above preparation, we get the following definitions.

Definition 11.18. [”signifier” and ”signified” in quantum language]

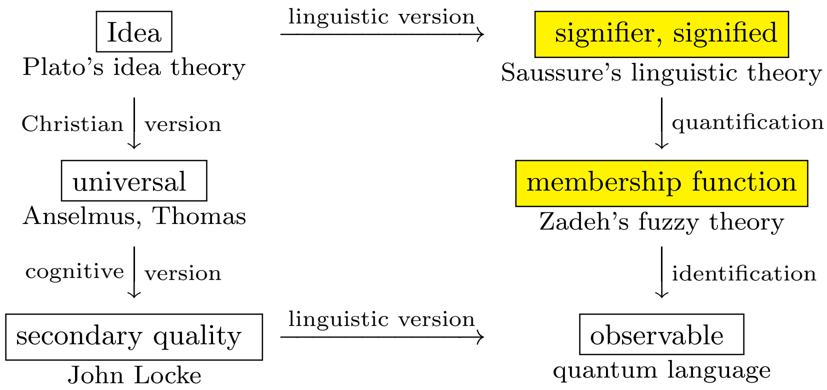
($\rm F_1$) ”signifier”——”dog”
($\rm F_2$) ”signified”—— membership function $m_D : \Omega \to [0, 1]$ (or roughly, Ω_D)

The above will play an essential role in Hempel's raven problem in Sec. 12.7.

♠**Note 11.7.** (i); The above definition is essential to the solution of Hempel raven problem. See Section 12.7
(ii): From the quantum linguistic point of view, Saussure's theory is located in the follow-

ing.

•



Some readers may disagree with me about Plato's Idea, as I've made a rather dogmatic assumption about it. However, I like the diagram above because it makes me feel like I understand the history of Western philosophy as the history of dualistic idealism.

11.7 Several Copernican revolutions [II]

Continued from [Copernican revolution [I] (*cf.] Sec. 10.2)].*

11.7.1 Copernican revolution as a method of making idealism from realism

The figure below is the same as Figure 0.1 in Preface.

The "Copernican revolution" was discussed in section 10.2, but let us consider the following problem again.

Problem 11.19. The ⑩ below is Kant's famous Copernican revolution. However, there are "two other Copernican revolutions than ⑩" in this figure. Find them. The answer is given in Answer 11.20.

Figure 0.1: The location of QL in the history of western philosophy

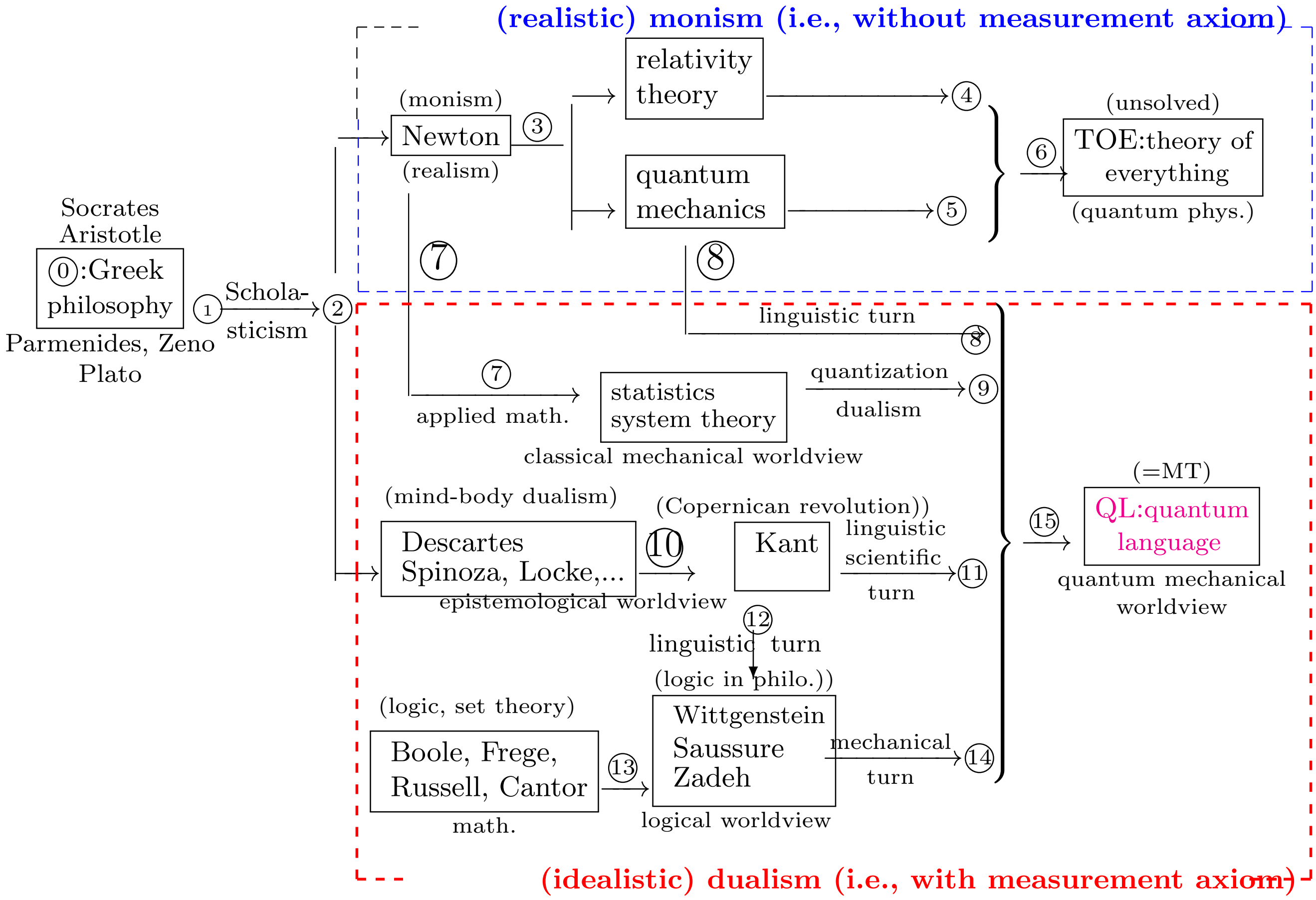
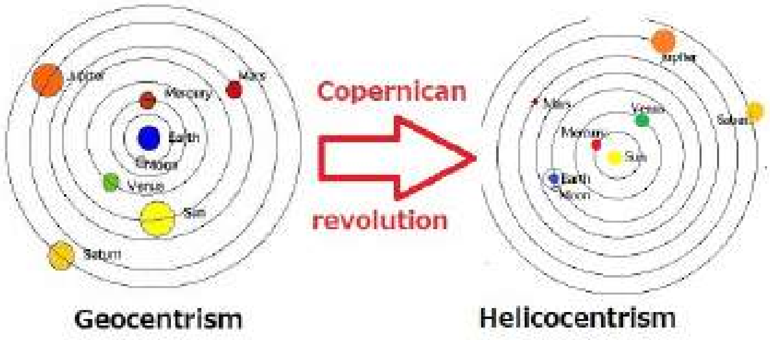


Figure 0.1 (in Preface): The history of the world-descriptions
Philosophy (≈ dualistic idealism) has progressed toward QL
(i.e., ⓪→ ①→ ②→ ⑩→ ⑫→ ⑭→ ⑮)

Answer: The answer is written in Answer 11.20. Since there are three Copernican revolutions in the above figure, you can understand how great a discovery Kant's Copernican revolution is.

♠**Note 11.8.** The original Copernican revolution is of course as follows (*cf.* Chap. 7):



Thus, the original Copernican revolution means

Developing science
realism
Ptolemaic system
$\xrightarrow[\text{scientific progress}]{\text{the original Copernican revolution}}$
More advanced science
realism
Copernican system

////

However, Kant's original Copernican revolution implies the following

copy theory [The world first, cognition later]
Hume's pseudoscience, pseudo realism

$\xrightarrow[\text{scientific progress}]{\text{Kantian Copernican revolution}}$

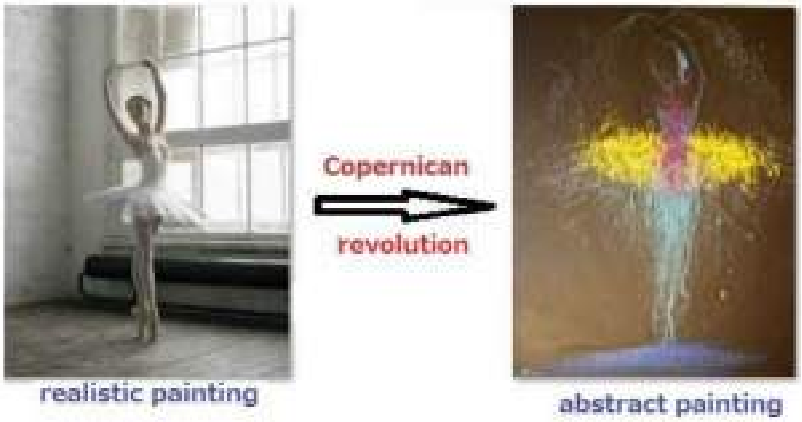
constructionism [Cognition first, the world later]
(transcendental) idealism

In this text, the term is used to describe the shift from "realism (the world comes first and then we make a theory of it)" to "idealism (the theory comes first and then we understand the world).

copy theory [World First, Theory Second]
realism
realistic painting

$\xrightarrow[\text{Copernican revolution}]{\text{as a way of making idealism from realism.}}$

constructionism [Theory first, world second]
transcendental idealism
abstract painting



In short,

(A) **the Copernican revolution is a method of turning realism into idealism.**

♠**Note 11.9.** In this text, the idealism generated by the "Copernican turn" is called "transcendental idealism. I believe that scientific idealism is always transcendental idealism.

(♯) The discovery of scientific idealism that is not transcendental idealism would be the greatest discovery in the history of philosophy.

Previously, I thought that probability theory belongs to scientific idealism and not transcendental idealism. However, since probability theory is a part of QL. Thus, probability theory may be transcendental idealism. I think it is worth challenging (♯), because there might be a position that mathematics or evolutionary theory can be regarded as scientific idealism that is not transcendentalist idealism.

Answer 11.20. Answer of Problem 11.19:
As discussed above, in this text, we understand the "Copernican revolution" as follows.

copy theory [World First, Theory Second]
[realism]
realistic painting

as a way of making idealism from realism. —Copernican revolution→ constructionism [Theory first, world second]
[transcendental idealism]
abstract painting

Thus, the answer is the following three "⑦, ⑧, ⑩".

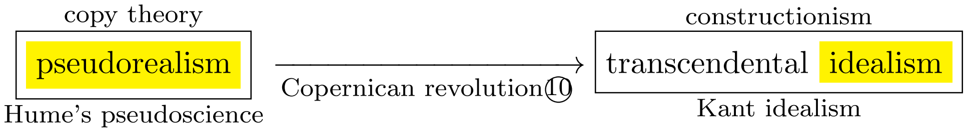
⑦:

copy theory [realism] Newtonian mechanics —Copernican revolution⑦→ constructionism [transcendental idealism] statistics

⑧:

copy theory [realism] quantum language —Copernican revolution⑧→ constructionism [transcendental idealism] quantum language

⑩:



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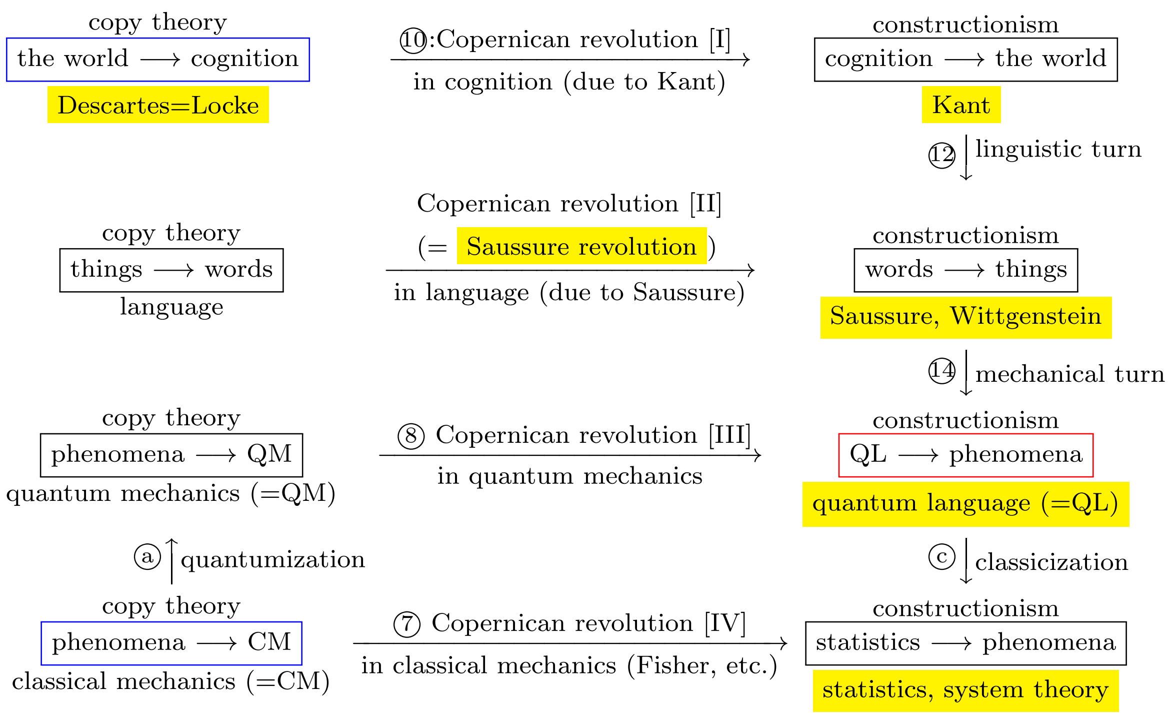
Assertion 11.21. Four Copernican revolutions: the above ⑦, ⑧, ⑩ and Saussure's revolution
Summing up as follows;



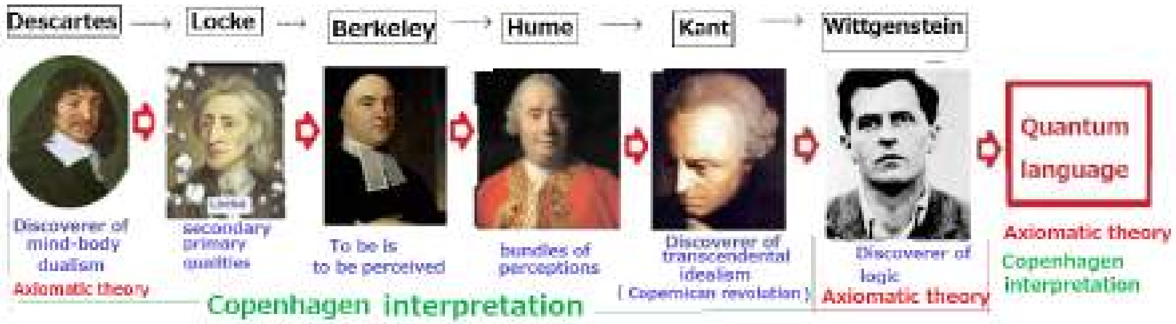
Thus, we say:

(B) There are various types of idealism, such as spiritual, religious, and anti-scientific, and it is true that all of them are "useful in some sense." However, there are ideologies such as [II]∼[IV] above (especially statistics) that can become the basic theories of the ongoing scientific revolution. Moreover, it should be noted that their prototype is Kant's Copernican revolution [I] (*cf.* Sec. 10.2).

11.7.2 From Descartes to Kant: It is a progress?

Now that you understand the meaning of the Copernican turn, I will give the answer to Problem 10.5 "Is Descartes to Kant a progress?"

Answer 11.22. [The answer of Problem 10.5: In the figure below, is Descartes to Kant a progress?]

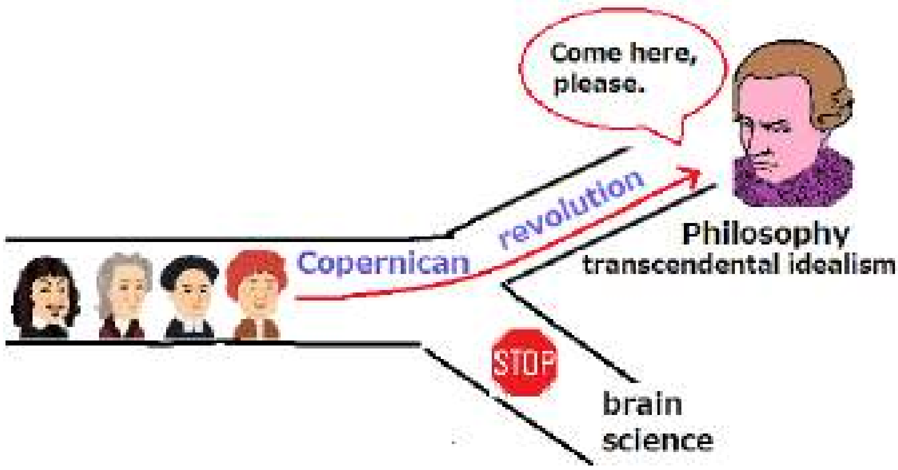


The immature state of a discipline at the time of its birth is sometimes regarded as philosophy. For example, in ancient Greece, mathematics, astronomy, and atomic theory were philosophies. It is also reasonable to consider the epistemology of Locke and Hume as the birth of brain science. Kant, however, rejected this trend as follows.

($\sharp_1$) Greatness of Kant is to have prevented that epistemology faces the direction of the brain science by the showy name called "Copernican revolution".

That is, Kant understand that, even if the epistemology is clarified by the brain science, this is non-sense from the philosophical point of view. We think that

($\sharp_2$) **using the term "Copernican revolution", Kant prevented that epistemology enters into the zone of brain science.**



Naming of "Copernican revolution" does not mean that self-congratulation of Kant. I'd like to believe that

- strong intention of Kant which says "Epistemology is not experimental science, but philosophy" (or, "philosophy should not aim at experimental sciences"), is included in the term: "Copernican revolution".

Therefore, I think that

- **"The Copernican revolution (due to Kant)" is the greatest discovery in the history of philosophy.**
  That is,

the Copernican revolution = the discovery of "scientific idealism"

So, in conclusion, I think the following is obvious.

$$\boxed{\text{Descartes, Locke, Berkeley, Hume}} \xrightarrow[\text{progress(QL)}]{} \boxed{\text{Kant}}^{*2}$$

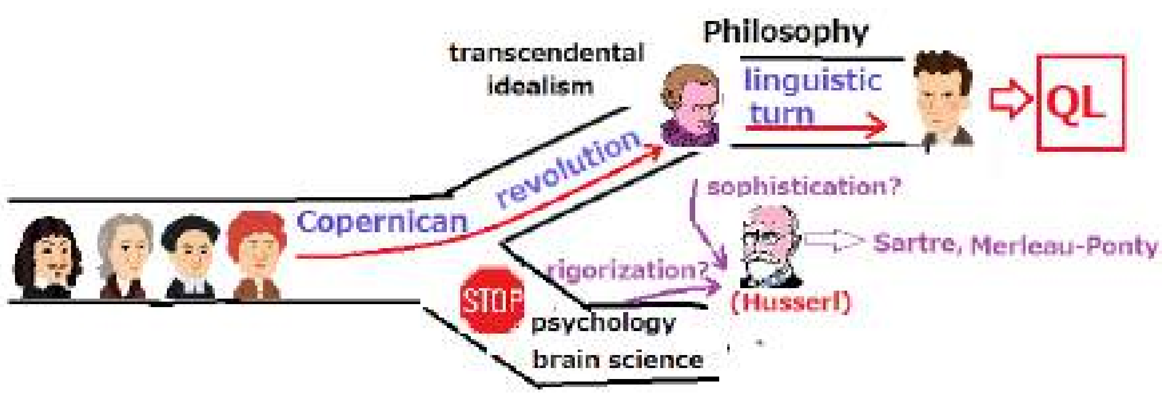
since the term "Copernican revolution" prevented epistemology from entering the realm of brain science.

♠**Note 11.10.** Kant's pseudo-scientific work ("thing-in-itself", reason, understanding, antinomy, etc.) is considered more Enlightenment than scientific. Here, we did not count anything other than the "Copernican turn" as either positive or negative. However, the analytical and comprehensive propositions are highly evaluated (*cf.* Sec.12.3).

Remark 11.23. Someone may want to consider the following progress:

$$\boxed{\text{Descartes=Kant}} \xrightarrow[\text{progress}]{} \boxed{\text{Husserl}}$$

This is because it is widely accepted that Husserl overcame the heights of Kant. In terms of literary enjoyment, I think the above is correct. If so, then Husserl's position is as follows



*2 The symbol "$X \xrightarrow[\text{progress (QL)}]{} Y$" is defined by ["Y" is closer to quantum language than "X"]

Thus I conclude that

$$\boxed{\text{Descartes=Kant}} \xrightarrow[\text{regress (QL)}]{} \boxed{\text{Husserl}}^{*2}$$

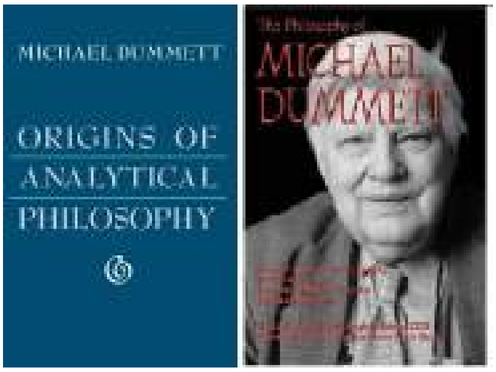
(For $\boxed{\text{Kant}} \xrightarrow[\text{progress (QL)}]{} \boxed{\text{Wittgenstein}}$, see the following Chapter.)
I think Husserl developed the pseudo-scientific aspects of Kant. I think he enjoyed the term "rigor" from a literary point of view. Husserl's phenomenology is one of the great achievements of Western philosophy, but it has little to do with science. If you see it as a kind of literature, it is fine if there are philosophy lovers who prefer this style of philosophy.

Chapter 12

Linguistic philosophy (After TLP)

I agree with the following Dummett's opinion: (*cf.* ref. [15]: Origins of analytical philosophy):

$(\natural_1)$ *If we identify the linguistic turn as the starting-point of analytical philosophy proper, there can be no doubt that, to however great an extent Frege, Moore and Russell prepared the ground, the crucial step was taken by Wittgenstein in the Tractatus Logico-philosophicus (= TLP) of 1922* (127 page)



Since I can not understand the non-mathematical work of Frege and Russell, I have not judged them as comprehensively as Dummett, but I consider the following.

$(\natural_2)$ Frege, Russell: [mathematical logic] — Logic is so powerful that it is the foundation of mathematics $\xrightarrow{\text{philosophy}}$ Wittgenstein: [TLP] — What we cannot speak about we must pass over in silence

Many philosophers may not agree with the above opinion $(\natural_1)$ $(=(\natural_2))$. Because Wittgenstein failed to fulfill the purpose of TLP, i.e. to answer the following questions:

$(\natural_3)$ What is the definition of "what we can speak about" (="proposition")?

$(\natural_4)$ Why does logic work in our daily lives?[*1]

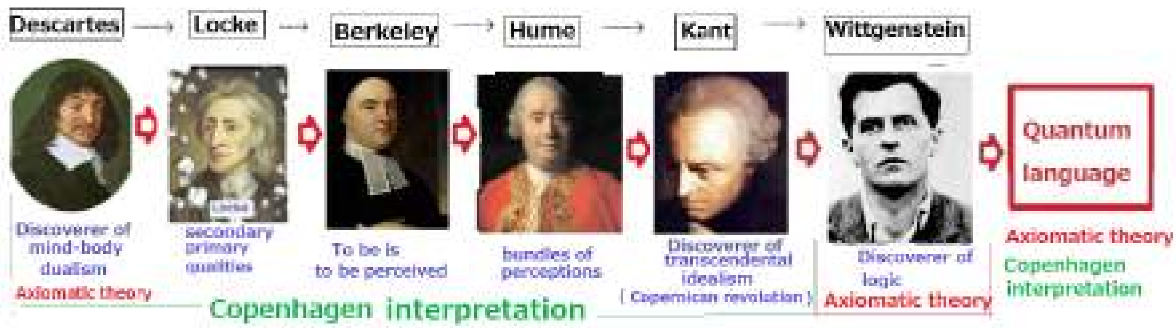
[*1] About the question "Why does statistics work in our daily lives?", we already answered in refs. [42], [44], [50],...,etc. These are summarized in [74]

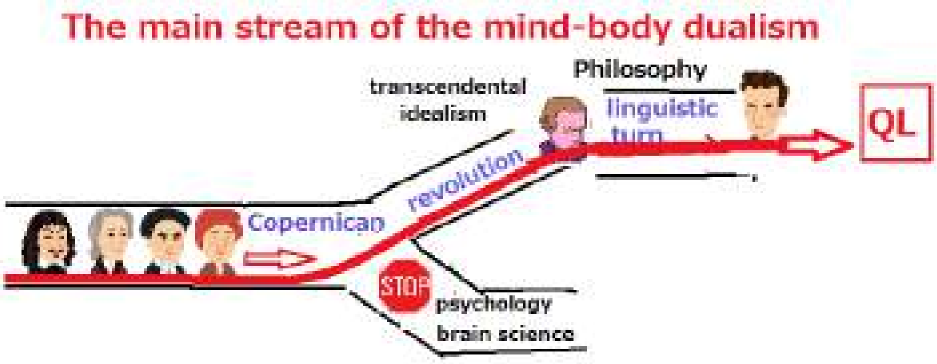
Although he failed in this task, his aspirations were enthusiastically praised by the general public, and he laid the foundation for today's analytic philosophy.
I agree with opinion ($\sharp_1$) because I think that the philosopher's job is not only to solve problems, but also to "present problems and create new trends in philosophy[*2].

In Sec. 12.2, I answer the above questions ($\sharp_3$) and ($\sharp_4$) in QL. And thus, the reader can be sure that Wittgenstein's direction was the right one.
Just as "physics is one," I like to think, "philosophy is one". For this, I assert that Wittgenstein must be located in the mainstream of philosophy (i.e., the mind-body dualism) as follows.

the mainstream of philosophy (i.e., the mind-body dualism)

Descartes → Locke → Berkeley → Hume → Kant → Wittgenstein → QL





[*2] Wittgenstein may have only agitated and did nothing substantial. However, we consider "agitating and involving the general public" to be of special importance as an achievement of philosophy. In this text, most of "what Wittgenstein did in science" is "what the author thinks 'what Wittgenstein wanted to do in science'".

12.1 QL-logic (i.e., Fuzzy logic in QL); My scientific understanding of TLP

This section consists of excerpts from the following paper:

- Ref. [81]: Ishikawa, S., (2021) Fuzzy Logic in the Quantum Mechanical Worldview ; Related to Zadeh, Wittgenstein, Moore, Saussure, Quine, Lewis Carroll, etc. JAMP, Vol. 9, No.3, 140-154, (https://www.scirp.org/journal/paperinformation.aspx?paperid=110830)

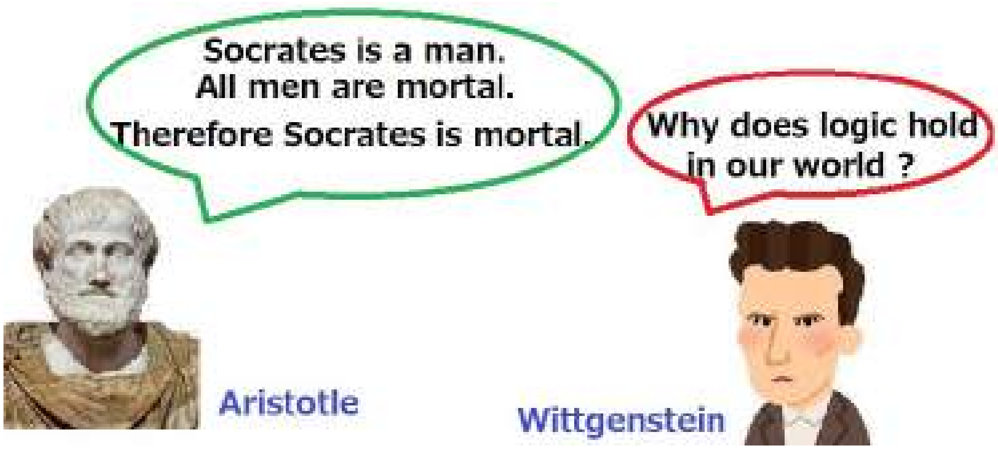
12.1.1 Wittgenstein: Tractatus Logico Philosophicus (=TLP)

In ref. [125] (i.e., Wittgenstein's "Tractatus Logico Philosophicus" (abbreviated as TLP)), which is one of the most famous philosophy books of the 20th century, Wittgenstein studied "logic" in philosophy (and not in mathematics). However, in hindsight, he may have written literary work on the subject of "logic" (*cf.* refs. [13], [93]). It is a fact, however, that TLP was accepted by much of the general public. I think the general public felt that Boole and Frege's "symbolic logic" was just mathematics, and that this was not sufficient as philosophy. In other words, they expected TLP to answer the question, such that

($\sharp_1$) **Why does logic work in our world?**

and

($\sharp_2$) **What is the definition of non-mathematical proposition?**

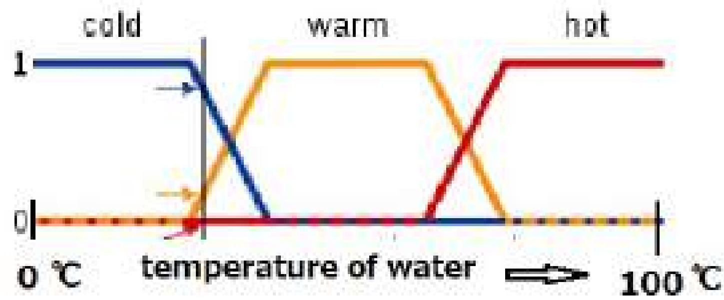


Wittgenstein's motivation for "TLP" should have been to answer these questions (($\sharp_1$) and ($\sharp_2$)), but he was too much a poet and a confident man. Moreover, "TLP" was published in 1921, and Wittgenstein did not know the discovery of quantum mechanics (around 1925). Therefore, Wittgenstein's chances of scientific success were very small.

12.1.2 Zadeh and Kalman; The problem of universals in 20th century

In 1965, L.A. Zadeh proposed an engineering concept called "fuzzy sets"(*cf.* ref. [129]), which was enthusiastically supported by some engineers. However, R.E. Kalman did not recognize "fuzzy sets" as a scientific concept, and argued as follows (1972):

> *"Let me say quite categorically that there is no such thing as a fuzzy concept... We do talk about fuzzy things but they are not scientific concepts. Some people in the past have discovered certain interesting things, formulated their findings in a non-fuzzy way, and therefore we have progressed in science"* (*cf.* ref. [130]).



I consider this the "problem of universals of the 20th century" .

Even now, more than 50 years later, I don't think Kalman's claim can be denied. In fact, the concept of "fuzzy sets" has not yet acquired the status of more than a convenient engineering method. Kalman might have thought that the basic philosophy of engineering is a mechanical worldview, and thus, "scientific concepts"="concepts in the (classical) mechanical worldview". Note that dynamical system theory (which is essentially the same as statistics) is considered as the mathematical representation of the (classical) worldview. That is,

$$(A_1) \quad \underset{(= \text{statistics })}{\overset{(\text{classical mechanical worldview})}{\boxed{\boxed{\text{Dynamical system theory}}}}} = \underset{(\text{ causality})}{\boxed{\text{state equation}}} + \underset{(\text{probability})}{\overset{(\text{measured value, state})}{\boxed{\text{measurement equation}}}}$$

(See ⑦ in Figure 0.1 (in Preface))

On the other hand, quantum language is characterized as follows:

$$
(\mathrm{A}_2) \quad \underset{\text{(= measurement theory)}}{\overset{\text{(quantum mechanical worldview)}}{\boxed{\boxed{\text{Quantum language}}}}} = \underset{\text{(causality)}}{\boxed{\text{Axiom 2}}} + \underset{\text{(measurement)}}{\overset{\text{(measured value, observable, state)}}{\boxed{\text{Axiom 1}}}} + \underset{\text{(how to use Axioms 1 and 2)}}{\boxed{\text{linguistic (Copenhagen) interpretation}}}
$$

(See ⑧ in Figure 0.1 (in Preface))

which is a mathematical representation of quantum mechanical worldview (*cf.* refs. [46], [74], [69]).

In fact, "fuzzy sets" correspond to "observables" in QL. If so, it is understandable that Kalman did not recognize "fuzzy sets". Zadeh's ref.[129] is one of the most cited papers of the 20th century. This means that many engineers felt that "mathematical sets" were not sufficient and expected his "fuzzy sets".

I think Wittgenstein and Zadeh were in a very similar position in the sense that they were enthusiastically supported by many ordinary people (or engineers in the field). I would rather believe in the sense of the general public than in the sense of a few outstanding experts. But what Wittgenstein and Zadeh lacked, I think, was a world view. Therefore, their arguments are vague and difficult to understand, but if we understand them under the quantum mechanical worldview, they seem to be arguing almost the same thing. In this section, I will explain them. In other words, we explain the validity of Wittgenstein's TLP and Zadeh's fuzzy set theory at the same time.

12.1.3 QL proposition (= fuzzy proposition in QL = TF-measurement) in classical system

Now let us explain "Fuzzy logic in QL" (i.e., my understanding of TLP). Let us start from easy example as follows.

Although our theory is valid for quantum systems as well as classical systems, in this Sec. 12.1 we explain our idea in classical systems (i.e., the commutative case that $\mathcal{A} = C(\Omega)$). Again let us start from the following example (= Example 11.4).

Example 12.1. [= Example 11.4]: For example, consider a proposition P_1 such that

(B) $P_1 =$ "this tomato is red", $\neg P_1 =$ "this tomato is not red",

And suppose that there are 100 respondents, and furthermore, the following question is asked to them.

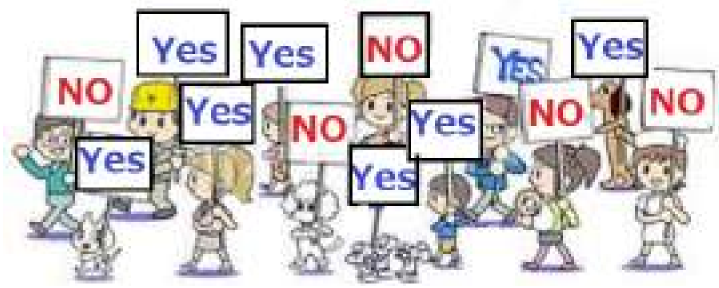
(C) Is this tomato red? (i.e., is the proposition P_1 true or not?)

Assume that the results of the responses are as follows.

(D)

$$\begin{cases} \text{70 respondents say "Yes, this tomato is red" (i.e., the proposition } P_1 \text{ is true, i.e."T")} \\ \text{30 respondents say "No, this tomato is not red" (i.e., the proposition } P_1 \text{ is false, i.e."F")} \end{cases}$$

For simplicity, we consider $T =$ "Yes" $= 1$ and $F =$ "No" $= 0$.



This can be probabilistically interpreted as follows.

(E) When any respondent is *randomly* selected out of 100, the probability that this respondent will answer "yes" to question (C) is $p_1 (= 0.7)$. Or simply, the probability that the proposition P_1 is true is p_1. In symbolic form,

$$\mathrm{Prob}[P_1; \{T\}] = p_1 (= 0.7)$$

Then we generally denote that

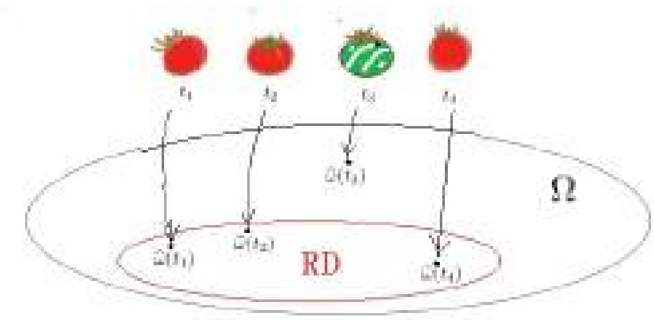
(F) $$\mathrm{Prob}[P_1; \{T\}] = p_1, \quad \mathrm{Prob}[P_1; \{F\}] = 1 - p_1 \qquad (\text{where } 0 \le p_1 \le 1)$$

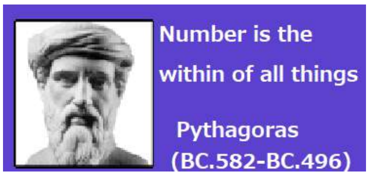
Also, note that $\mathrm{Prob}[\neg P_1; \{T\}] = \mathrm{Prob}[P_1; \{F\}] = 1 - p_1$.

////

The above will be formulated in terms of QL as follows. Let Ω a state space, which is, for simplicity, assumed to be a compact set $\Omega \subseteq \mathbb{R}^N$ (= N-dimensional real space), where N is sufficiently large natural number. Consider many tomatoes, that is, roughly speaking, consider T as the set of all tomatoes. Assume that any tomato $t(\in T)$ is represented by a state ω, which is an element of the state space Ω. Thus, we have the map $\widehat{\omega} : T \to \Omega$. That is, the quantitative property of a tomato t is represented by $\widehat{\omega}(t)$. For example, it suffices to consider Ω such that

$$
\text{(G)} \quad \omega = \Big(\omega^{(1)}(=\text{weight}), \omega^{(2)}(=\text{diameter}), \omega^{(3)}(=\text{color value}), \\
\omega^{(4)}(=\ \text{calorie}), \omega^{(5)}(=\ \text{sugar content}), ..., \omega^{(N)}(=\ ...)\Big) \in \Omega \subseteq \mathbb{R}^N
$$





Consider a binary observable (or, $\{T, F\}$-valued observable) $\mathsf{O}_1 = (\{T, F\}, 2^{\{T,F\}}, G_1)$ in $C(\Omega)$. The measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}_1, S_{[\delta_\omega]})$ is called a (TF)-*measurement*, which is also called a *fuzzy proposition*. Axiom 1 says that

(H) the probability that measured value T is obtained by the (TF)-measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}_1, S_{[\delta_\omega]})$ is given by $\delta_\omega(G_1)(= {}_{C(\Omega)^*}\langle \delta_\omega, G_1\rangle_{C(\Omega)} = G_1(\omega))$

This is the quantum linguistic representation of the above (E). That is, we identify a proposition P_1 with a (TF)-measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}_1, S_{[\delta_\omega]})$.

Remark 12.2. (i): Someone might say that the term "the set of all tomatoes" is as ambiguous as "the set of all dinosaurs". However, for the sake of convenience, here we use the term "the set of all tomatoes". This problem is the same as that of the Hempel' raven paradox (*cf.* Sec. 12.7, i.e. "the set of all ravens" leads to contradiction).
(ii): If we want to consider another proposition $P_2(=\mathsf{M}_{C(\Omega)}(\mathsf{O}_2, S_{[\delta_{\omega'}]})$), we must define $\mathsf{M}_{C(\Omega)}(\mathsf{O}_1, S_{[\delta_\omega]}) \wedge \mathsf{M}_{C(\Omega)}(\mathsf{O}_2, S_{[\delta_{\omega'}]}) \quad (\omega \neq \omega')$. This will be explained in Sec. 12.1.7
(iii): If we want to consider both tomato's world Ω_1 and apple's world Ω_2, it suffices to start from the tensor space $C(\Omega_1) \otimes C(\Omega_2)$ $(=C(\Omega_1 \times \Omega_2))$. This will be also explained in Sec. 12.1.7.

12.1.4 Fuzzy logic in QL

Let's start with the following definition.

Definition 12.3. [(TF)-measurement (=Fuzzy proposition), Fuzzy set (= Membership function)]

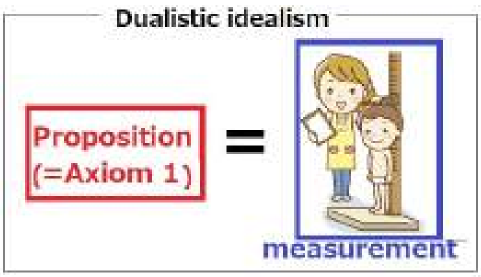
Let $\mathsf{O} = (\{T, F\}, 2^{\{T,F\}}, G)$ be a binary observable (or, (TF)-observable, $\{T, F\}$-valued observable) in a C^*-algebra $\mathcal{A}$. A measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]})$ is called a *(TF)-measurement*, which is also called a *fuzzy proposition*.
Since Axiom 1 says that the probability that a measured value T is obtained by (TF)-measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]})$ is given by $\rho(G(\{T\}))$, we say that

(I_1) a (TF)-measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]})$ is true with probability $\rho(G(\{T\}))$
Or,
(I_2) $\mathrm{Prob}[\mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]}); \{T\}] = \rho(G(\{T\})) \; (= {}_{\mathcal{A}^*}\langle \rho, G(\{T\}) \rangle_{\mathcal{A}})$

Also, $G(\{T\}) (\in \mathcal{A})$ is called the membership function of O (*cf.* Definition 11.17).

♠**Note 12.1.** (i): We sometimes use a binary observable $\mathsf{O} = (\{y, n\}, 2^{\{y,n\}}, G)$ be a binary observable (or, (yes-no)-observable, $\{y, n\}$-valued observable) instead of a binary observable $\mathsf{O} = (\{T, F\}, 2^{\{T,F\}}, G)$ (or, (TF)-observable, $\{T, F\}$-valued observable).
(ii): Let $\mathsf{O} = (\{T, F\}, 2^{\{T,F\}}, G)$ be an (TF)-observable in C^*-algebra $\mathcal{A}$. The observable $\mathsf{O} = (\{T, F\}, 2^{\{T,F\}}, G)$(or, in classical case, a membership function (= fuzzy set) $G(\{T\})$ itself is not related to "probability", as Zadeh frequently emphasized. However, when we take a measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]}); \{T\}$, a probability occurs. (See (E) above.)
(iii): More generally, I think that

(♯) "Proposition in QL"= "measurement" (i.e., Axiom 1)"



though it is essentially the same as a proposition in statistics. As is repeatedly emphasized in this text,

- Use statistics for rigorous considerations and logic for coarse considerations!

Our interest is to show that

$$\boxed{\text{Kant philosophy}} \xrightarrow[\text{progress(QL)}]{} \boxed{\text{Analytic philosophy}},$$

and thus,

- the main concept "proposition" in analytic philosophy must be formulated in the framework of dualistic idealism. This is realized in the above definition.

That is, Wittgenstein took over the baton of ”dualistic idealism (i.e., the mainstream of philosophy)” from Kant (*cf.* ref. [81], or Sec. 12.1). In other words, I would like to think that ”philosophy is one” just as ”physics is one”.

12.1.5 Calculations in QL-proposition (= TF-measurement)

In general, we must consider many propositions $\{P_i = \mathsf{M}_{\mathcal{A}_i}(\mathsf{O}_i, S_{[\rho_i]}) : \ i=1,2,...,n\}$. In this section, we devote ourselves to the following simple case:

- (J_1) $\mathcal{A}$ is fixed, i.e. $\mathcal{A}_1 = \mathcal{A}_2 = ... = \mathcal{A}_n$
- (J_2) $\mathsf{O}_1, \mathsf{O}_2, ..., \mathsf{O}_n$ commute,
- (J_3) a state ρ is only one, i.e. $\rho_1 = \rho_2 = ... = \rho_n$

For the general case, we discuss in next section (i.e., Sec.12.1.7). That is, in this section, we devote ourselves to $\{P_i = \mathsf{M}_{\mathcal{A}}(\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i), \ S_{[\rho]}) : i=1,2,...,n\}$. However, it should be noted that the above simple case (N) is essential, that is, the general case is an easy consequence of the simple case as seen in the next section.

Definition 12.4. [Fuzzy logic symbols (¬, ∧, ∨, →))] Let $\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i)$ be binary observables (or, $\{T,F\}$-valued observable) in a C^*-algebra $\mathcal{A}$. $(i=1,2)$. Assume that $\mathsf{O}_i (i=1,2)$ commute.

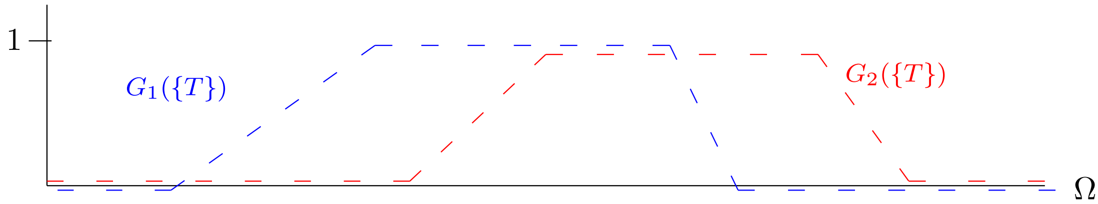


Figure 12.1a: $[G_1(\{T\})$ and $G_2(\{T\})$ in the classical case such that $\mathcal{A} = C(\Omega)$ $]^{*3}$

Fix the quasi-product observable $\mathsf{O}_1 \times^{qp} \mathsf{O}_2 = (\{T,F\}^2, \mathcal{P}(\{T,F\}^2), G_1 \times^{qp} G_2)$. Consider (TF)-measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i), \ S_{[\rho]})$ (which is abbreviated as P_i) in a C^*-algebra $\mathcal{A}$. Put $\mu_i(\Xi) = \rho(G_i(\Xi))$ $(\Xi \in \{T,F\}, i=0,1,2)$, and $(\times^{qp}_{i=1,2}\mu_i)(\Xi_1 \times \Xi_2) = (\rho(G_1 \times^{qp} G_2)(\Xi_1 \times \Xi_2))$ $(\Xi_1, \Xi_2 \subseteq \{T,F\})$.

Definition: [(i): NOT:¬], [(ii): AND:∧], [(iii): OR:∨], [(iv): implication:→] below!

(i): [NOT: ¬]

Put $i=1,2$. Define $\neg \mathsf{M}_{\mathcal{A}}(\mathsf{O}_i, S_{[\rho]})$ such that

$$\neg \mathsf{M}_{\mathcal{A}}(\mathsf{O}_i, S_{[\rho]}) = \mathsf{M}_{\mathcal{A}}(\pi^{\neg}\mathsf{O}_i, S_{[\rho]}) = \mathsf{M}_{\mathcal{A}}(\pi^{\neg}\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, \pi^{\neg}G_i), S_{[\rho]})$$

where the map $\pi^{\neg} : \{T,F\} \to \{T,F\}$ is defined by $\pi^{\neg}(T) = F, \pi^{\neg}(F) = T$. And, $\pi^{\neg}$ may also be used as follows: $\pi^{\neg}G_i(\{T\}) = G_i(\{F\})$, $\pi^{\neg}G_i(\{F\}) = G_i(\{T\})$. Clearly it holds that $\text{Prob}[\neg \mathsf{M}_{\mathcal{A}}(\mathsf{O}_i, \ S_{[\rho]}); \{T\}] = \rho(G_i(\{\{F\})). = \mu_i(\{F\})$.

---

*3 For simplicity, we assume that $\{\omega \in \Omega \mid [G_1(\{T\})](\omega)\} \bigcap \{\omega \in \Omega \mid [G_2(\{T\})](\omega)\} = \emptyset$.

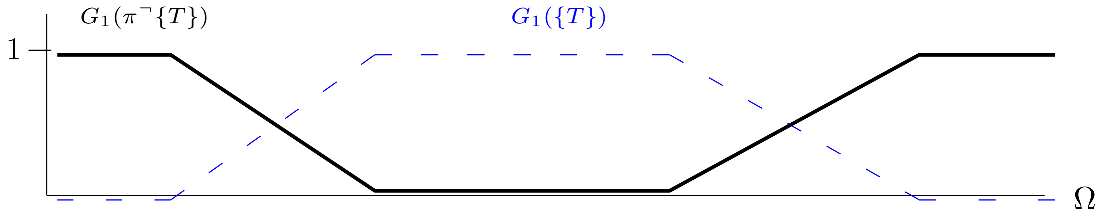


Figure 12.1b: $[G_1(\pi^{\neg}\{T\})$ in the classical case such that $\mathcal{A} = C(\Omega)$]

(ii): [AND: $\wedge$]

Define $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \wedge \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]})$ such that

$$\begin{aligned}\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \wedge \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]}) =& \mathsf{M}_{\mathcal{A}}(\pi^{\wedge}(\mathsf{O}_1 \times^{qp} \mathsf{O}_2), S_{[\rho]}) \\ =& \mathsf{M}_{\mathcal{A}}((\{T,F\}, 2^{\{T,F\}}, \pi^{\wedge}(G_1 \times^{qp} G_2)), S_{[\rho]})\end{aligned}$$

where $\pi^{\wedge} : \{T,F\}^2 \to \{T,F\}$ is defined by $\pi^{\wedge}(T,T) = T, \pi^{\wedge}(T,F) = \pi^{\wedge}(F,T) = \pi^{\wedge}(T,F) = F$.
It holds that $\text{Prob}[\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \wedge \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]}); \{T\}] = \rho(G_1 \times^{qp} G_2)((\pi^{\wedge})^{-1}(\{T\})) = (\mu_1 \times^{qp} \mu_2)(\{(T,T)\})$.

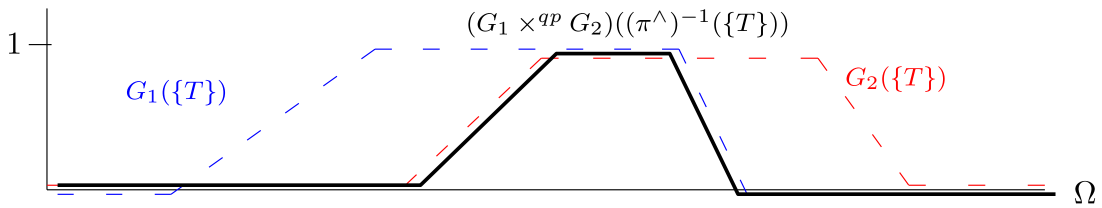


Figure 12.1c: $[(G_1 \times^{qp} G_2)((\pi^{\wedge})^{-1}(\{T\}))$ in the classical case such that $\mathcal{A} = C(\Omega)$]

(iii): [OR: $\vee$]

Define $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \vee \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]})$ such that

$$\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \vee \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]}) = \mathsf{M}_{\mathcal{A}}(\pi^{\vee}(\mathsf{O}_1 \times^{qp} \mathsf{O}_2), S_{[\rho]})$$

where $\pi^{\vee} : \{T,F\}^2 \to \{T,F\}$ is defined by $\pi^{\vee}(T,T) = \pi^{\vee}(T,F) = \pi^{\vee}(F,T) = T, \pi^{\vee}(F,F) = F$.
It holds that $\text{Prob}[\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \vee \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]}); \{T\}] = \rho(G_1 \times^{qp} G_2)((\pi^{\vee})^{-1}(\{T\})) = (\mu_1 \times^{qp} \mu_2)(\{(T,T), (T,F), (F,T)\})$.

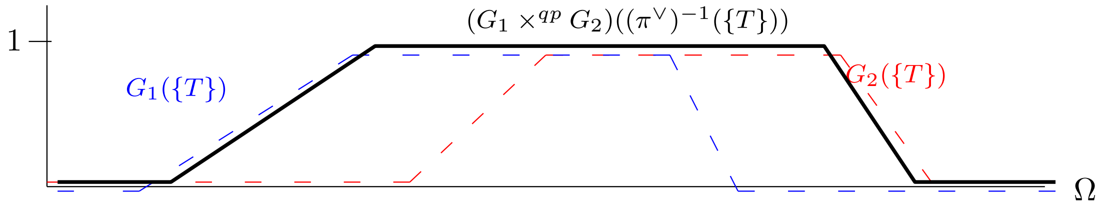


Figure 12.1d: $[(G_1 \times^{qp} G_2)((\pi^{\vee})^{-1}(\{T\}))$ in the classical case such that $\mathcal{A} = C(\Omega)$]

(iv): [implication: $\rightarrow$]

Define $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \rightarrow \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]})$ such that

$$\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \rightarrow \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]}) = \mathsf{M}_{\mathcal{A}}(\pi^{\rightarrow}(\mathsf{O}_1 \times^{qp} \mathsf{O}_2), S_{[\rho]})$$

where $\pi^{\rightarrow} : \{T, F\}^2 \rightarrow \{T, F\}$ is defined by $\pi^{\rightarrow}(T, T) = \pi^{\rightarrow}(F, T) = \pi^{\rightarrow}(F, F) = 1, \pi^{\rightarrow}(T, F) = F$.
It holds that $\mathrm{Prob}[\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \rightarrow \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]}); \{T\}] = \rho(G_1 \times^{qp} G_2)((\pi^{\rightarrow})^{-1}(\{T\}))$ $= (\mu_1 \times^{qp} \mu_2)(\{(T, T), (F, F), (F, T)\})$.

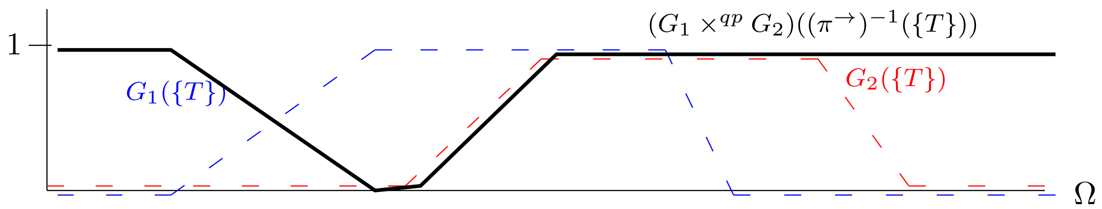


Figure 12.1e: $[(G_1 \times^{qp} G_2)((\pi^{\rightarrow})^{-1}(\{T\}))$ in the classical case such that $\mathcal{A} = C(\Omega)$]

12.1.6 Fundamental theorem in QL logic

With the above preparation (+ Theorem 11.10 [Logical sample space]), we obtain the following theorem.

Theorem 12.5. [Fundamental theorem in Fuzzy logic] Let $\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i)$ be binary observables (i.e., $\{T,F\}$-valued observable) in a C^*-algebra $\mathcal{A}$. $(i = 1, 2, ..., n)$. Assume that $\mathsf{O}_i \neq \mathsf{O}_j$ ($\forall i, j$ such that $i \neq j$) (i.e., non-overlapping condition in Theorem 11.10) and $\mathsf{O}_i (i = 1, 2, ..., n)$ commute. Fix the quasi-product observable $\times^{qp}_{i=1,...,n} \mathsf{O}_i = (\{T,F\}^n, \mathcal{P}(\{T,F\}^n), \times^{qp}_{i=1,...,n} G_i)$. Consider (TF)-measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i), S_{[\rho]})$ (which is abbreviated as P_i) in a C^*-algebra $\mathcal{A}$). And consider the quasi-product measurement $\times^{qp}_{i=1,...,n} \mathsf{M}_{\mathcal{A}}(\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i), S_{[\rho]}) = \mathsf{M}_{\mathcal{A}}(\times^{qp}_{i=1,...,n} \mathsf{O}_i = (\{T,F\}^n, \mathcal{P}(\{T,F\}^n), \times^{qp}_{i=1,...,n} G_i), S_{[\rho]})$. Put

$$\bullet \mu_i(\{x_i\}) = \rho(G_i(\{x_i\})) \qquad (x_i \in \{T,F\}, i = 1, 2, .., n)$$

$$\bullet \underset{i=1,...,n}{\overset{qp}{\times}} \mu_i(\{(x_1, x_2, ..., x_n)\}) = \rho\Big((\times^{qp}_{i=1,...,n} G_i)(\{(x_1, x_2, ..., x_n)\}\Big)$$
$$= (\times^{qp}_{i=1,...,n} \mu_i)(\{(x_1, x_2, ..., x_n)\})$$
$$(\forall (x_1, x_2, ..., x_n) \in \{T,F\}^n)$$

Here, the pair $\big[\{P_1, P_2, ..., P_n\}; (\{T,F\}^n, \mathcal{P}(\{T,F\}^n), \times^{qp}_{i=1,2,...,n} \mu_i)\big]$ is a logical sample space.
Then, by Theorem 11.10 [Logical sample space], we see the following:

- Let P be a proposition which is constructed by $P_1, ..., P_n$. Note that P is regarded as the map from $\{T,F\}^n \to \{T,F\}$. Then, we see that

$$\text{Prob}[P; \{T\}] = (\underset{i=1,2,...,n}{\overset{qp}{\times}} \mu_i)(P^{-1}(\{T\}))$$

Remark 12.6. Since the linguistic Copenhagen interpretation says that "only one measurement is permitted", we only take the measurement: $\times^{qp}_{i=1,...,n} \mathsf{M}_{\mathcal{A}}(\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i), S_{[\rho]}) = \mathsf{M}_{\mathcal{A}}(\times^{qp}_{i=1,...,n} \mathsf{O}_i = (\{T,F\}^n, \mathcal{P}(\{T,F\}^n), \times^{qp}_{i=1,...,n} G_i), S_{[\rho]})$. Therefore, the measurements (i.e., $\mathsf{M}_{\mathcal{A}}(\pi^{\wedge}(\mathsf{O}_1 \times^{qp} \mathsf{O}_2), S_{[\rho]})$ $(= \mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]}) \wedge \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]}))$ etc. in Definition 12.4) are not actually done. To be precise, these measurements are included in the quasi-product measurement $\times^{qp}_{i=1,...,n} \mathsf{M}_{\mathcal{A}}(\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i), S_{[\rho]})$.

Example 12.7. [The QL version of Table 11.10]. Replacing P_1 and P_2 with $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]})$

and $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]})$, we get the following Table 12.1, i.e. the QL version of Table 11.10.

Table 12.1: Probabilistic Truth Table (Elementary propositions $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]})(= P_1)$, $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]})(= P_2)$)

$\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]})$ $(= P_1)$	$\mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]})$ $(= P_2)$	probability: $p = \times^{qp}_{i=1,2} \mu_i$	$[P_1 \to P_2] \wedge P_1$	$[P_1 \to P_2] \wedge P_1 \to P_2$
T	T	$p_{12} = \times^{qp}_{i=1,2} \mu_i(\{(T,T)\})$	T	T
T	F	$p_{1\bar{2}} = \times^{qp}_{i=1,2} \mu_i(\{(T,F)\})$	F	T
F	T	$p_{\bar{1}2} = \times^{qp}_{i=1,2} \mu_i(\{(F,T)\})$	F	T
F	F	$p_{\bar{1}\bar{2}} = \times^{qp}_{i=1,2} \mu_i(\{(F,F)\})$	F	T

Thus, we see that

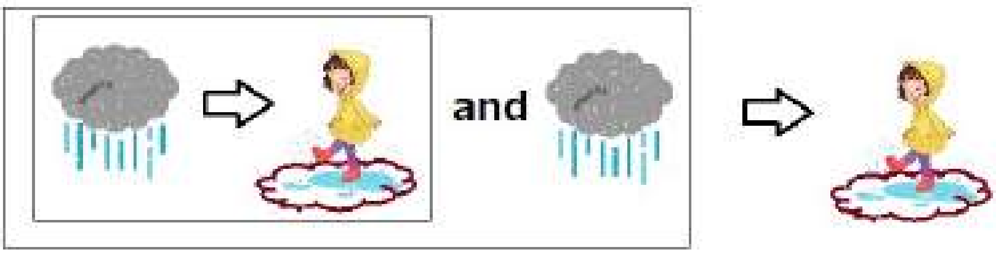
$$\mathrm{Prob}[[P_1 \to P_2] \wedge P_1; \{T\}] = p_{12}$$

Similarly, we see the modus pones:

$$\mathrm{Prob}[[[P_1 \to P_2] \wedge P_1] \to P_2; \{T\}] = 1$$

For example, put P_1 :="it rains", P_2 :="the ground is wet". Modus ponens says that

[["it rains" $\to$ "the ground is wet"]$\wedge$ "it rains"]$\to$"the ground is wet".



The following is the quantum linguistic version of Corollary 11.11.

Corollary 12.8. [Tautology in fuzzy logic] Let $P(= \mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]}))$ be a proposition constructed from elementary propositions $P_1(= \mathsf{M}_{\mathcal{A}}(\mathsf{O}_1.S_{[\rho]})), P_2(= \mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]})), ..., P_n(= \mathsf{M}_{\mathcal{A}}(\mathsf{O}_n, S_{[\rho]}))$. Then, the followings are equivalent:

(i) P is a tautology in the sense of crisp logic.
(ii) $\mathrm{Prob}[P; \{T\}] = 1$.

That is, tautology always holds even in practical fuzzy logic. For example,

(K) syllogism (i.e., $[[P_1 \to P_2] \wedge [P_2 \to P_3]] \to [P_1 \to P_3]$ always holds.

Remark 12.9. We have two results such that

(i) in ref. [79], I showed that syllogism does not always hold in quantum system (*cf.* Sec.4.3.3).
(ii) in Corollary 12.8, I showed that syllogism always holds in classical and quantum systems.

Thus, readers may think that (i) and (ii) are contradictory. However these are not contradictory since Corollary 12.8 requires that $\mathsf{O}_1, \mathsf{O}_2, \mathsf{O}_3$ commute. On the other hand, in ref. [79], the commutativity of O_1 and O_3 is not required. The most important one, of course, is Corollary 12.8.

Remark 12.10. [Cogito proposition is not a proposition in QL; *cf.* Sec. 8.2, also, see refs. [46], [70]]: Examine the cogito proposition "I think, therefore I am", in which it is natural to consider that

"observer" = "I" = "measurement object (=system)"

This is against the linguistic Copenhagen interpretation (E_1) *"No observer can measure itself"* in Sec. 1.2.1. Therefore, the cogito proposition is not a proposition in QL. The fact that the first proposition of philosophy is not a proposition is interesting.

Thus we see the following non-sense table:

Table 12.1′: Probabilistic Truth Table
(non-propositions "I think"$(= P_1')$, "I am"$(= P_2')$)

"I think" $(= P_1')$	"I am" $(= P_2')$	probability: $p = \times_{i=1,2}^{qp} \mu_i$	$[P_1' \to P_2'] \wedge P_1'$	$[P_1' \to P_2'] \wedge P_1' \to P_2'$
?	?	$p_{12} =?$	?	?
?	?	$p_{1\bar{2}} =?$	?	?
?	?	$p_{\bar{1}2} =?$	?	?
?	?	$p_{\bar{1}\bar{2}} =?$	?	?

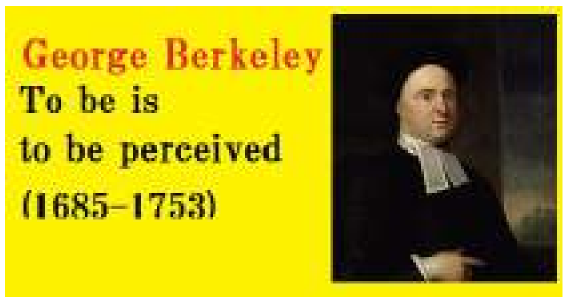
Remark 12.11. The linguistic Copenhagen interpretation (E_0) in Sec.1.2.1 says that

($\sharp_1$) Don't talk about what you can't measure!

Thus, as shown in ref. [65], we see:

Also, the ($\sharp_1$) is equivalent to Berkely's saying

($\sharp_2$) To be is to be perceived:



This implies the following figure:

12.1.7 General case

In previous section, we devote ourselves to $\{\mathsf{M}_{\mathcal{A}}(\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i),\ S_{[\rho]}) : i = 1,2,...,n\}$ under the conditions (N$_1$)$\sim$ (N$_3$), which are not so wide as mentioned in Remark 12.2 (ii) and (iii), In this section, we consider the general case: $\{P_i = \mathsf{M}_{\mathcal{A}_i}(\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i),\ S_{[\rho_i]}) : i = 1,2,...,n\}$. Put

$$\widehat{P}_i = \mathsf{M}_{\bigotimes_{j=1}^n \mathcal{A}_j}(\widehat{\mathsf{O}}_i = (\{T,F\}, 2^{\{T,F\}}, \widehat{G}_i), S_{[\otimes_{j=1}^n \rho_j]})$$

where

$$[\widehat{G}_i](\Xi_i) = (\bigotimes_{j=1}^{i-1} I) \otimes G_i(\Xi_i) \otimes (\bigotimes_{j=i+1}^{n} I) \quad (\forall \Xi_i \in \mathcal{P}(\{T,F\}), i = 1,2,...,n)$$

Here, note that $\{\widehat{P}_i = \mathsf{M}_{\bigotimes_{j=1}^n \mathcal{A}_j}(\widehat{\mathsf{O}}_i = (\{T,F\}, 2^{\{T,F\}}, \widehat{G}_i), S_{[\otimes_{j=1}^n \rho_j]}) : i = 1,2,...,n\}$ satisfies that

(L$_1$) $\widehat{\mathcal{A}} = \bigotimes_{j=1}^n \mathcal{A}_i$ is fixed,
(L$_2$) $\widehat{\mathsf{O}}_1, \widehat{\mathsf{O}}_2, ..., \widehat{\mathsf{O}}_n$ commute,
(L$_3$) a state $\otimes_{j=1}^n \rho_i$ is only one.

Therefore, the general case: $\{P_i = \mathsf{M}_{\mathcal{A}_i}(\mathsf{O}_i = (\{T,F\}, 2^{\{T,F\}}, G_i),\ S_{[\rho_i]}) : i = 1,2,...,n\}$ can be understood.

Remark 12.12. As a special case of measurement, (QL) logic and statistics are derived as follows.

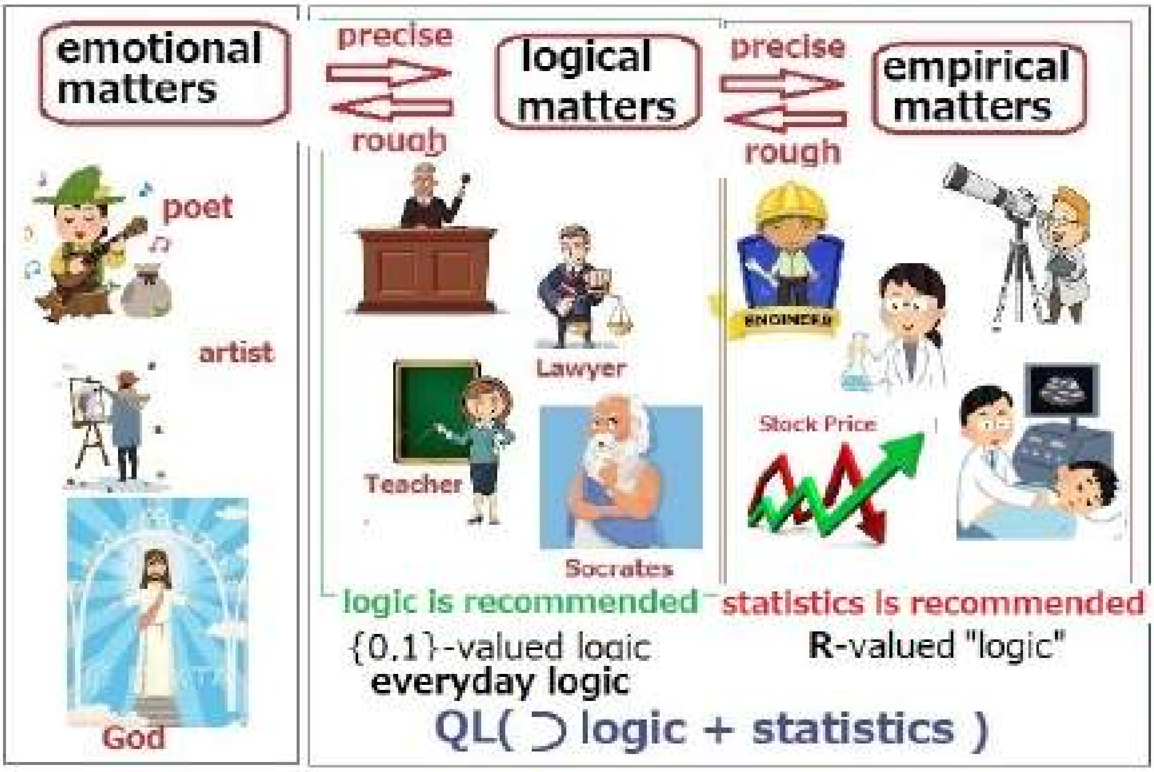
$$\underset{\text{measurement}}{\boxed{\mathsf{M}_{\mathcal{A}}(\mathsf{O} = (X, \mathcal{P}(X), G), S_{[\rho]})}}$$

$$\xrightarrow[\text{derivation}]{} \begin{cases} \overset{\text{binary valued logic}}{\boxed{\mathsf{M}_{\mathcal{A}}(\mathsf{O} = (\{T,F\}, \mathcal{P}(\{T,F\}), G), S_{[\rho]})}} & \cdots \text{QL logic } cf. \text{ this section} \\ \\ \overset{\text{distribution with a parameter } \omega}{\boxed{(\mathbb{R}, \mathcal{B}(\mathbb{R}), [G(\cdot)](\omega))_{\omega \in \mathbb{R}}}} & \cdots \text{statistics } cf. \text{ ref.[74]} \end{cases}$$

Thus, (QL) logic can be seen as a two-valued logic and statistics as a multi-valued logic. Thus, the following motto has a point.

- The broad arguments of the humanities are discussed in "logic,"
- the quantitative arguments of the sciences are discussed in "statistics,"

The actual segregation is shown in the following figure



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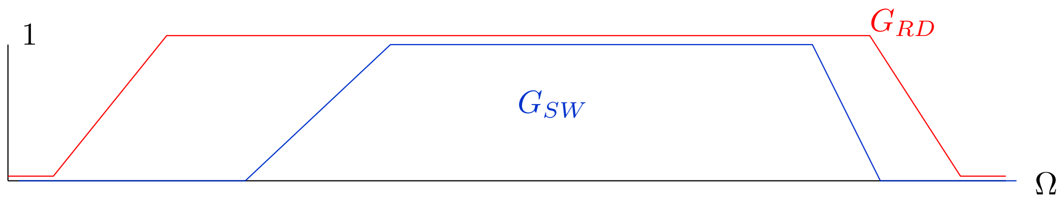
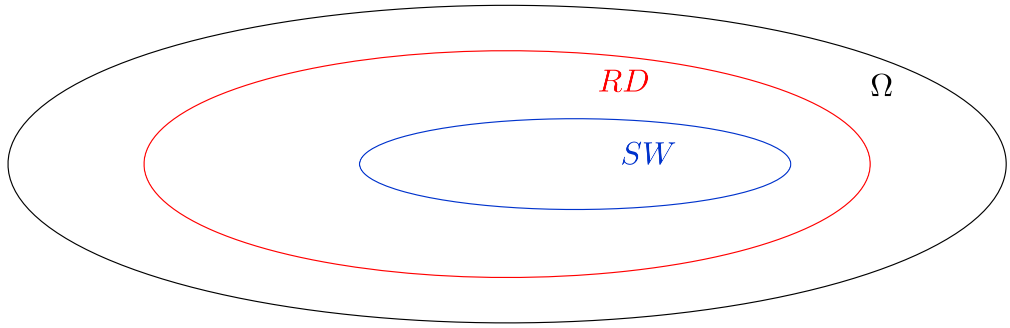
Problem 12.13. [Any sweet tomato is red]

Consider (TF)-valued observables $\mathsf{O}_{SW} = (\{T,F\}, 2^{\{T,F\}}, G_{SW})$ and $\mathsf{O}_{RD} = (\{T,F\}, 2^{\{T,F\}}, G_{RD})$ in $C(\Omega)$, where O_{SW} and O_{RD} is respectively called the sweet observable and the red observable. It is natural to consider that "Any sweet tomato is red" is defined by

(M) $\qquad SW \subseteq RD$

where $SW = \{\omega \in \Omega | G_{SW}(\omega) = 1\}$ and $RD = \{\omega \in \Omega | G_{RD}(\omega) = 1\}$.





In order to examine (M) (i.e., to answer the problem: "Is the (M) a proposition?"), it suffices to check the following:

$$\text{Prob}[\mathsf{M}_{C(\Omega)}(\mathsf{O}_{RD}, S_{[\delta_\omega]}); \{T\}] = 1 \qquad (\forall \omega \in SW)$$

Thus we have the following problem:

(N) **Problem: Show that [SW $\subseteq$ RD] is a QL proposition!**

Now, here is the next division of cases.

- $\begin{cases} \text{(I): } SW \text{ is finite} \\ \text{(II): } SW \text{ is infinite} \end{cases}$

[Case (I)] SW is finite

Assume that $SW = \{\omega_1, \omega_2, ..., \omega_n\}$. Recall the linguistic Copenhagen interpretation (L_1) such that only one measurement is permitted. Thus, consider the tensor space $\bigotimes_{i=1}^n C(\Omega)$ $= C(\Omega^n)$, and the tensor product measurement

$$\mathsf{M}_{C(\Omega^n)}(\bigotimes_{i=1}^n \mathsf{O}_{RD} = (\{T, F\}^n, \mathcal{P}\{T, F\}^n, \bigotimes_{i=1}^n G_{RD}), S_{[\delta_{(\omega_1, \omega_2, \dots, \omega_n)}]})$$

where

$$(\bigotimes_{i=1}^n G_{RD})(\{(x_1, x_2, ..., x_n)\}) = \bigotimes_{i=1}^n G_{RD}(\{x_i\}) \qquad (\forall (x_1, x_2, ..., x_n) \in \{T, F\}^n)$$

Assume that the measured value $x = (x_1, x_2, ..., x_n)$ belongs to $\{T\}^n$. Then, we can conclude that (M) is true. Also, as an analogy of (ii) in Definition 12.4, we may consider as follows. Let $\pi^{\wedge} : \{T, F\}^n \to \{T, F\}$ be a map such that

$$\pi^{\wedge}(x_1, x_2, ..., x_n) = \begin{cases} T & \text{if } x_1 = x_2 = ... = x_n = T \\ F & \text{(otherwise)} \end{cases}$$

Then, (M) is equivalent to

$$\text{Prob}[\mathsf{M}_{C(\Omega^n)}(\pi^{\wedge}(\otimes_{i=1}^n \mathsf{O}_{RD}), S_{[\delta_{(\omega_1,\omega_2,\dots,\omega_n)}]}); \{T\}] = 1$$

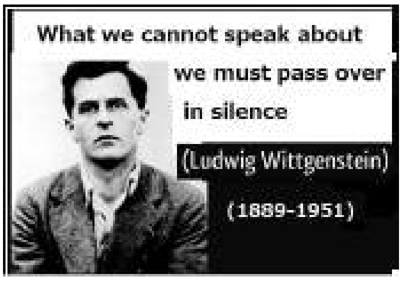
Thus, (M) is a QL proposition.

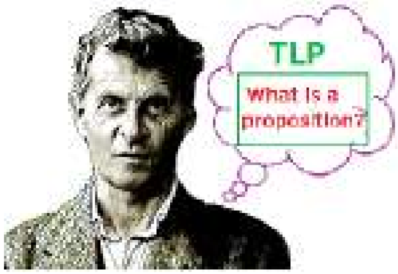
[Case (II)] SW is infinite

This tis the same as the Problem of Hempel's rven problem, which will be answered in Sec. 12.7. □

12.2 Wittgenstein; the biggest star of analytic philosophy.

12.2.1 TLP attempted to answer the question, "What is a proposition?"





Ludwig Wittgenstein (1889 - 1951), who was the student of B. Russel, was the most famous philosopher (in analytic philosophy) in the 20th century.
Wittgenstein wrote two books, i.e., TLP and PI,

(A) [Tractatus Logico-Philosophicus (=TLP)] the former Wittgenstein [1921] $\xrightarrow[\text{gardener}]{\text{teacher}}$ [Philosophisch Investigations (=PI)] the later Wittgenstein [1953]

The former is concerned with world description, while the latter is literary. Thus, I think that the above is similar to the following:

(B) ["Critique of Pure Reason (=CPR)(1781)"] fictional worldview (literary truth) [preface, introduction, (fictional)premise, expedient] $\rightarrow$ ["Critique of Practical Reason (1788)" "Critique of Judgment(1790)"] ethics, morals, aesthetics [main subject]

Thus we devote ourselves to TLP in this text.

It has already been more than 100 years since analytic philosophy was born. Moreover, today, many philosophers specialize in analytic philosophy. However, strangely enough, I don't think that the evaluation of TLP has been settled yet. I think that TLP is theoretically insufficient. In fact, in the last 100 years, no philosopher has been able to read TLP in a theoretical way[*4].

In the preface of his book "Tractatus Logico-Philosophicus", L. Wittgenstein said that

> (C) *This book will perhaps only be understood by those who have themselves already thought the thoughts which are expressed in it — or similar thoughts.*

[*4] This is no mystery in philosophy. No philosopher has ever been able to read Descartes' "Discourse on Method" or Kant's "Critique of Pure Reason" scientifically. We do not even know if they are scientifically decipherable in the first place. That is why I am writing this text!

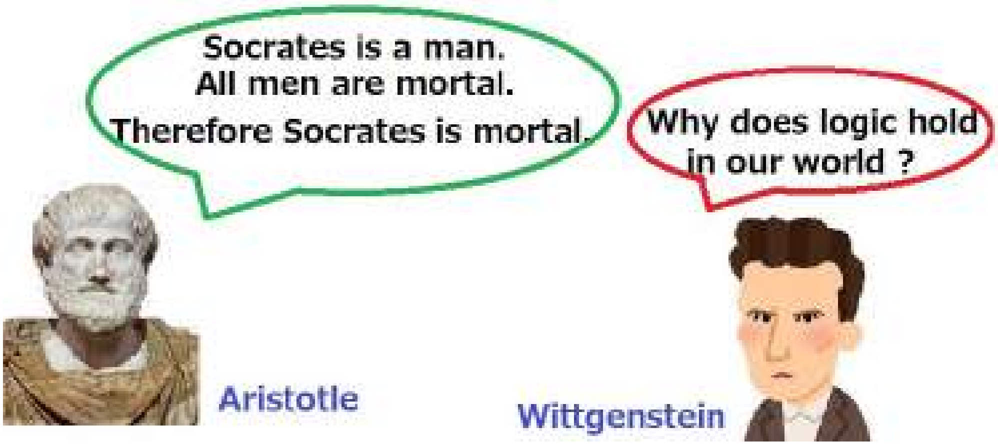
Here, what is "the thoughts which are expressed in it "? I think that his interest is not "mathematical logic", but "non-mathematical logic (i.e., practical logic)".

Thus I believe:

(D) What Wittgenstein wanted to insist on in TLP is to answer the following questions:

(D_1) What is "proposition"? (i.e., what is the definition of "(practical) proposition"?)

(D_2) Why does logic work in our usual world?



If so, we can consider (D) (= (D_1) + (D_2)) directly, without reading the TLP. The reason is that we have many powerful weapons (statistics, quantum mechanics, etc.) that Wittgenstein did not know about. It should be noted that

- **The 20th century was the century of statistics and quantum mechanics.**

though the 21th century may be the century of (quantum) computer.

♠**Note 12.2.** I believe that (D) is due to Wittgenstein. However, even if it were not, it is certain that (D_1) and (D_2) are the most important in philosophy.

If (C) above is true, I would be in a very favorable situation. Because I already know the answer to this problem.

(E) Some twenty years ago, in the papers on the subject "Why does logic work in this world?" (*cf.* refs. [41, 42, 43, 44]). To be precise, I proved that
- both logic and statistics work in this world because quantum language works in this world.

---

*4 It was a reflective paper in the sense that it did not get much response. I should have written it in terms that people who were passionate about "fuzzy" at the time could understand. I also did not realize that this issue is closely related to philosophy.

At the time, I was not aware that "Why logic does work in our world?" was a question of philosophy. After reading (C) in TLP, I immediately rewrote the following papers.

(F_1) Ref. [79]: Ishikawa, S., (2020) Wittgenstein's picture theory in the quantum mechanical worldview, Journal of quantum information science, Vol. 10, No.4, 104-125, DOI:10.4236/jqis.2020.104007 (https://www.scirp.org/journal/paperabs.aspx?paperid=106233)

(F_2) Ref. [81]: Ishikawa, S., (2021) Fuzzy Logic in the Quantum Mechanical Worldview ; Related to Zadeh, Wittgenstein, Moore, Saussure, Quine, Lewis Carroll, etc. JAMP, Vol. 9, No.3, 140-154, (https://www.scirp.org/journal/paperinformation.aspx?paperid=110830) This is essentially the same as Sec. 12.1 in this text.

(F_3) Ref. [85]S. Ishikawa, K. Kikuchi, (2021) *Quantum Fuzzy Logic and Time*, JAMP, Vo.9 No.11 2021, 2609-2622 (https://www.scirp.org/journal/paperinformation.aspx?paperid=112972)

Thus, I can assert the following:

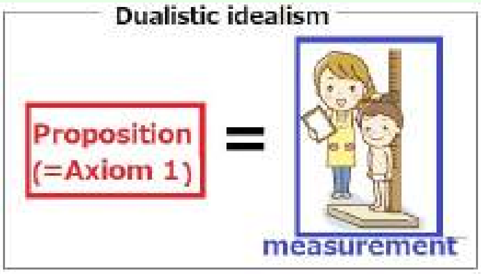
Answers to the above (G)(=(D))

(G_1) (=(D_1)) What is "proposition"? (i.e., what is the definition of "proposition"?)
(G_2) (=(D_2)) Why does logic work in our usual world?

Answer 12.14. The above problems (G) are already solved in the previous section. That is;

($\sharp_1$) Problem (G_1) is already answered in Definition 12.3.

That is, "QL proposition" = "measurement (=Axiom 1)".



(*cf.* Definition 12.3, Note 12.1).
Also,

($\sharp_2$) Problem (G_2) is solved in Theorem 12.5.

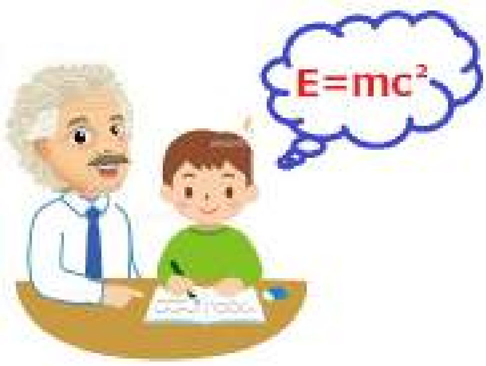
Summing up, we say

- "To speak what we can speak about" = "To speak QL"

□

♠**Note 12.3.** There may be a certain number of people in the world who have a special personality that feels status in reading difficult books that are impossible to understand. I would like to believe that Wittgenstein did not write TLP for those people. I like the following Einstein quote;

- *"If you can't explain it to a six year old, you don't understand it yourself"*



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Therefore, I thought Wittgenstein himself did not understand TLP at all. Reading his confident explanation, I thought he was convinced that he understood TLP. For example, Wittgenstein confidently concluded

(H_1) *"What we cannot speak about we must pass over in silence"*

in TLP. If so,

- The rules for drawing the line between "what can be said" and "what cannot be said" (i.e., the definition of "proposition") should be the main theme of TLP.

However, the definition is nowhere to be found, and without the definition, anyone can say (H_1), etc., which is the worst. However, I was relieved when I read the following (§38 in [126])

(H_2) *"The basic of Russell's logic, as also of mine in the TLP, is that what a proposition is illustrated by a few commonplace examples, and then pre-supposed as understood in full generality"*

When I read this, I knew I could trust Wittgenstein. So, the following §6.54 in TLP means that Wittgenstein knew that TLP was like a ladder that could be thrown down.

(H_3) §6.54: *"He must so to speak throw away the ladder, after he has climbed up on it"*

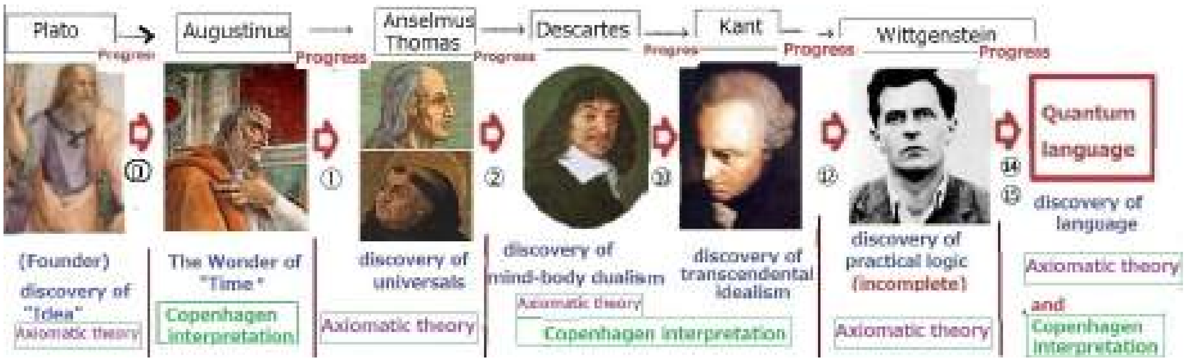
Also, despite the hot love call from logical positivism, Wittgenstein set himself apart from logical positivism because he had an intuition that TPL was incomplete and powerless to science.
It does not help that his TLP is incomplete. No matter how much of a genius Wittgenstein was, it would have been impossible for him to discover "Fuzzy logic in QL". The environment in 1920 was as follows.

(i) Born's discovery "the probabilistic interpretation of quantum mechanics" in ref. [6] (1926)
(ii) Tractatus Logico-Philosophicus by L. Wittgenstein in ref. [125] (1921)
(iii) Fisher's great book "Statistical Methods for Research Workers" in ref. [22] (1925)

The integration of these three is QL, and I don't think any genius at the time could have reached QL. I think that von Neumann's [121] and Davies' [14] are indispensable for QL. If von Neumann had taken an interest in TLP, though, the history of analytic philosophy might have been different.
However, the most important task of the philosopher is

to "create a larger stream of philosophy".

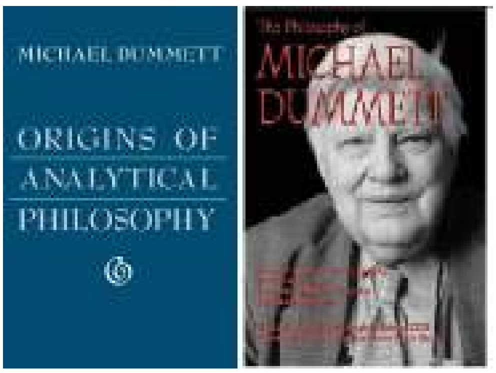
In this sense, it is natural that he is in the position shown in the diagram below.



Even Descartes only raised the issue (i.e., Cartesian problem). Wittgenstein also raised important issues ((G_1) and (G_2)), so it is only natural that he should be depicted in this figure. Only quantum language has the power to solve the various unsolved problems of dualistic idealism.

♠**Note 12.4.** (i):Even if TLP is a poem whose logic is broken, if it raises issues (D_1) and (D_2), TLP is still a top-notch philosophy book. In philosophy, raising a question may be more important than answering. In fact, TLP has changed the mainstream of philosophy. I believe that the correctness of the direction pointed by TLP has been proven by this text.
(ii): As mentioned in the previous section (Sec. 12.1), I believe that QL is the central theory of analytic philosophy, which is also obvious if you look at List (D_1) [Wittgenstein's dream came true] in Preface. Of course, Kripke's possible worlds semantics is one of greatest theories in analytic philosophy. However, I have the same opinion (for Kripke's possible worlds semantics) as Dummett as follows

(♯) *I have been struck by the enormous influence of fashion in philosophy: possible-worlds semantics is an excellent example. Such a fashion seizes almost everyone at a particular moment and they all go haring off after it. I don't think that the vague for possible-worlds semantics was just a mistake. It is occurred because Kripke succeeded in using that apparatus to make some strong points that struck everybody forcibly; most then got themselves into a state of mind in which they could hardly think except in those terms.* (188-189 pages; M. Dummett. Origins of Analytical philosophy)

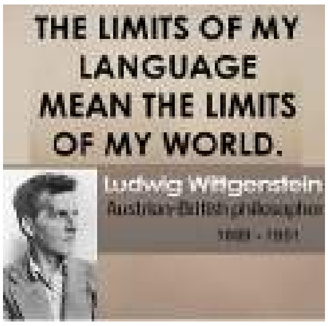


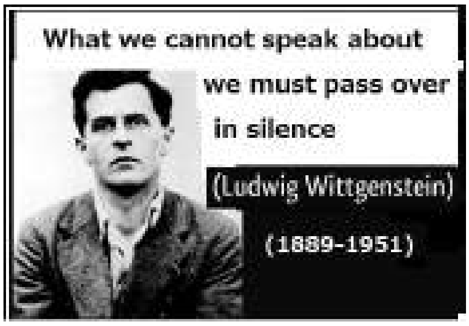
I think Dummett's assessment is harsh because he expected analytic philosophy to be more than statistics and quantum mechanics.

12.2.2 The power of Wittgenstein's word: Linguistic turn

However, in this text, I want to assert that Wittgenstein is one of the greatest philosophers (Plato, Descartes, Kant, etc.). That is because he said the following sayings (I_1) - (I_3):

(I_1) **"The limits of my language mean the limits of my world."**
(however, note that Wittgenstein did not propose "my language").





(I_2) **"What we cannot speak about we must pass over in silence"**
(however, note that Wittgenstein did not define "what we cannot speak about (= proposition)").

(I_3) **"Language-game"**
(however, his "language game" was about everyday language (as mentioned in (I_1), he did not propose his language. The rule of the language game of quantum language is the Copenhagen Interpretation.)).

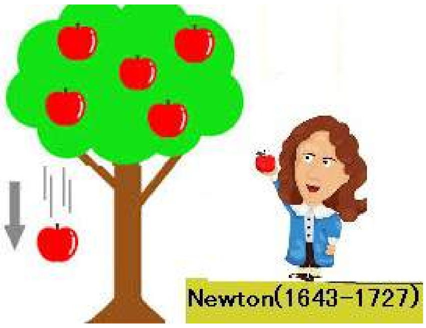
The above is just the spirit of quantum language. Instead of my poor explanation of the spirit of quantum language, I prefer to saying

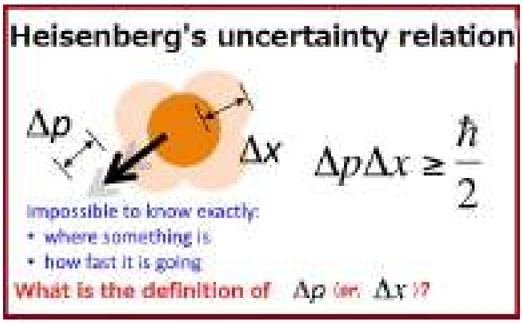
(J) "The spirit of quantum language is represented by the above (I_1) - (I_3)"

Seeing the above (I_1)$\sim$ (I_3), I can understand "Why did Russell support Wittgenstein as his guardian?". Russell must have thought "Without Wittgenstein, we (i.e., Frege and Russell) cannot spread analytic philosophy alone." That is, I think Russell expected Wittgenstein to be an enlightener of the philosophy of language.

♠**Note 12.5.** For each great discovery, an anecdote (or, a catchphrase, stage effect) is left as follows.

($\sharp_1$) Archimedes······golden crown, heureka! (*cf.* Sec.5.4)
($\sharp_2$) Galileo······Leaning Tower of Pisa, "And Yet It Moves" (*cf.* Sec.7.3.4)
($\sharp_3$) Newton······ :Newton's apple, "Geocentrism vs. Heliocentrism"(*cf.* Note 7.8)
($\sharp_4$) Descartes······ :fly on the ceiling (*cf.* Note 8.2), : I think, therefore I am, (*cf.* Sec.8.2)
($\sharp_5$) Kant······clock (*cf.* Note 10.2), dogmatic slumber (*cf.* Note 10.5)
($\sharp_6$) Wittgenstein······Son of a steel magnate, Hitler's schoolmate, Primary school teacher, Gardener, Architect, Guardian: Russell (*cf.* Sec.12.1.1)
($\sharp_7$) Einstein······Elevator
($\sharp_8$) quantum mechanics······Heisenberg's uncertainty principle (*cf.* ref. [38]).





I have the following opinion about ($\sharp_8$):

($\flat$) There is a big gap between Heisenberg's uncertainty relation (refs. [28, 38]) and Kennard=Robertson uncertainty relation (refs [90, 111]). Therefore, just as the "Newton's apple" was used as a symbol for Newton mechanics, "Heisenberg's uncertainty relation" was used as a symbol for quantum mechanics (*cf.* Note 4.1 of ref. [74]).




12.2.3 Philosophical Investigations (1953) and Wittgenstein's paradox

12.2.3.1 the later Wittgenstein

Wittgenstein wrote two books, i.e.

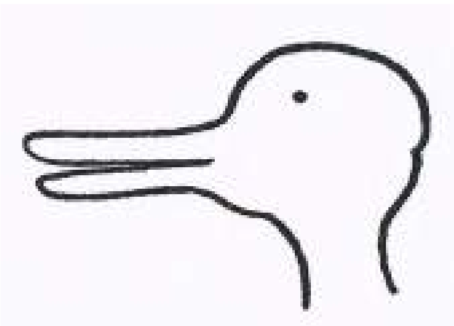
(K) Tractatus Logico-Philosophicus (=TLP) [the former Wittgenstein] 1921 —teacher / gardener→ Philosophisch Investigations (=PI) [the later Wittgenstein] 1953

Despite concluding in TLP that "What we cannot speak about we must pass over in silence," Wittgenstein spoke of "what cannot be spoken about" in PI. This is not to say that he had a "change of heart," since science is not the only target of philosophy.

The outline of the PI (the part 1) is as follows.

(i) Sec.1-88; language game
(ii) Sec.89-133; logic and philosophy
(iii) Sec.134-242; Wittgenstein's paradox
(iv) Sec.242-315; private language
(v) Sec.315-693; psychology

Although PI (the part 2) is famous for rabbit-duck illusion, we are not concerned with psychology and brain science. Psychology (or, cognitive science) is an important discipline, but it is a discipline in which experimentation and observation are essential, and it is not really Wittgenstein's cup of tea. I have an opinion that the "philosophy of mathematics" should be left to mathematicians. That is, I believe that philosophers should concentrate on "world description (= world view)" in the first sense.



All of them (i.e., parts 1 and 2) may be of interest to Wittgenstein fans. From a QL point of view, I think the above (i), (iii), and (iv) are important. (i) has already been mentioned in Section 11.8.2.

12.2.3.2 (iii): Wittgenstein's paradox;

I tried several times to understand this paradox (= Wittgenstein's paradox), but I could not understand it interestingly. About Wittgenstein's paradox, S. Kripke says in ref. [94] S. Kripke says the following about Wittgenstein's paradox

(L) (*Wittgenstein has invented a new form of skepticism. Personally I am inclined to regard it as the most radical and original skeptical problem that philosophy has seen to date, one that only a highly unusual cast of mind could have produced.*)

If Wittgenstein's paradox is something that Kripke admires in this way, I would certainly like to understand it, and I tried several more times to understand it, but could not. However, the following (M), which may or may not be related to Wittgenstein's paradox, but which has been mentioned many times in this text, is interesting.

(L): Wittgenstein's paradox (I like to think so though I could not understand his intentions)

I guess to think that Wittgenstein wanted to say the following in his paradox:

(L_1): [Worldviewism] we must start from a worldview
(L_2): however, no worldview can be accurately described.

□

For skepticism, the most important question is: "What should we be skeptical about?" I think. The object of skepticism is neither "chess game", "quus calculation" nor "private language". That is, we must skeptical about QL (particularly, the linguistic Copenhagen interpretation), i.e.

(= dualistic idealism) **Quantum language** (= measurement theory) = measurement (Axiom 1) + causality (Axiom 2) + linguistic (Copenhagen) interpretation (how to use Axioms 1 and 2)

since Axioms 1 and 2 are kinds of spells (i.e., incantation, magic words, metaphysical statements) as mentioned in Sec. 1.2.1. Also, recall the following sprit of "Copenhagen interpretation" due to Mermin (*cf.* Note 1.6):

($\sharp_1$) If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be **"Shut up and calculate!"**

It is true that many researchers have won Nobel prizes in this spirit. However, I think it is time to become skeptical about QL. In other words, I think it is time to seriously pursue the next as an unsolved problem.

($\sharp_2$) What is "Copenhagen interpretation"?

Thus I believe that the following problems about language game are important:

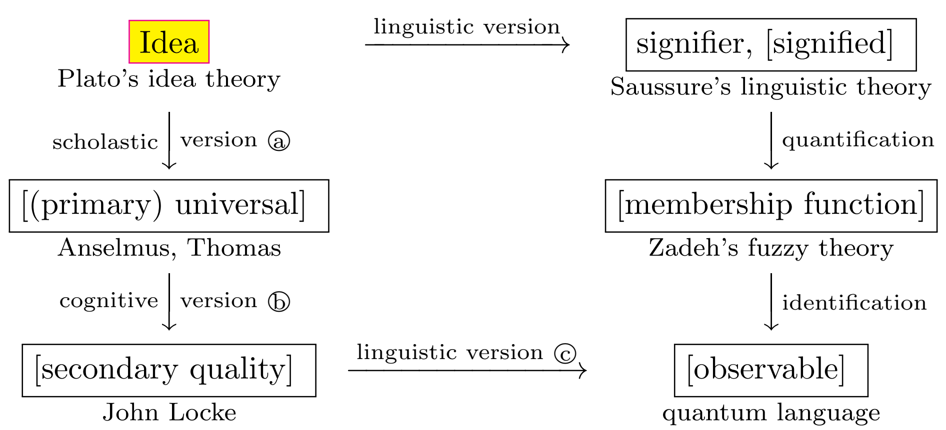
[Question 1 about language game]:

($\flat_1$) Why is "interpretation" needed in QL (or, quantum language)?
Why does Newtonian mechanics need no "interpretation"?
Or, really, Newtonian mechanics needs no "interpretation"?

(monistic realism)
[Newtonian mechanics] = [Newton's Three Laws of Motion] + [No interpretation]

[My answer]: I think that Newtonian mechanics also has an "interpretation". However, I think that Newtonian mechanics does not bother to emphasize "interpretation" because Newtonian mechanics does not have the "observable". One of the arguments of this text was that the difficulty in understanding the concept of "observable (≈Idea)" has perpetuated the history of Western philosophy for 2,500 years, as shown below.

(History of progress from Ideas to observables)



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[Question 2 about language game]:

($\flat_2$) If the Copenhagen Interpretation is a "manual for the successful use of the axioms," isn't it impossible to create a complete manual, no matter how detailed the rules?

[My answer]: I believe that there is no perfect interpretation. No matter how detailed the "rules" are, you cannot make a perfect manual. In the end, we have to use common sense to fill in the gaps in the manual. Then, a new problem arises: What is 'common sense'? This is a reminder of "Hume's habitual thinking.

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[Question 2 about language game]:

($\flat_3$) How are the rules of QL learned?

[My answer]: Note that QL is a kind of idealism (or, a kind of metaphysics). I myself made quantum language as an analogy after learning quantum mechanics. Therefore, I initially thought that it would be difficult to learn quantum language until after I had mastered quantum mechanics. However, in my graduate laboratory, students with no knowledge of quantum mechanics have easily learned quantum languages. This fact surprised me, and I still wonder about it. Just as a baby gradually learns an everyday language, a quantum language can be gradually learned by solving exercises. It is possible to gradually expand the range of objects that can be expressed in a quantum language. I can't help but marvel at the human capacity for language.

////

♠**Note 12.6.** Mathematics requires only axioms and does not require interpretation. Therefore, AI (= artificial intelligence) that understands mathematics better than humans will be a reality in 20 to 30 years. However, I think the emergence of AI that understands QL will be delayed a bit longer. Of course, with a quantum computer, the difference between math and QL may be trivial.

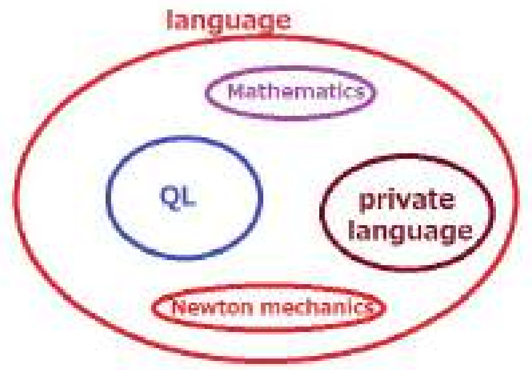
12.2.3.3 (iv): Private language

As mentioned in Sec. 4.3.1, if the ecology of various animals is observed, it will be clear that the base of language was due to intimidation, solidarity, reproduction. Language was one of the strongest arms for the survival and breeding. Such a time have continued for millions of years. Of course, the biggest events in the "history of language" happened one after the other. For example,

(N) "rhythm and song", "logical structure", "quantity concept", "grammar", "tense", etc.

Among these, "logical structure" and "quantity concepts" are relevant to the language of stating science. However, this is a characteristic of only a small portion of language as a whole.

Now, a private language is a language that expresses inner experiences such as senses, emotions, wills, and thoughts. . I feel like it has more affinity with AI than philosophy, but I don't know.


language
Mathematics
QL
private
language
Newton mechanics

12.3 Quine's analytic-synthetic distinction in QL

From a scientific point of view, the late Wittgenstein seems to have been a disappointment. However, It is not that analytic philosophy since TPL has been sterile. From this section onward, we describe the fruitful results within the analytic philosophy.

This section consists of excerpts from the following paper:

- Ref. [81]: Ishikawa, S., (2021) Fuzzy Logic in the Quantum Mechanical Worldview ; Related to Zadeh, Wittgenstein, Moore, Saussure, Quine, Lewis Carroll, etc. JAMP, Vol. 9, No.3, 140-154, (https://www.scirp.org/journal/paperinformation.aspx?paperid=110830)

Rudolf Carnap
1891-1971

W.V.O. Quine
1908-2000

12.3.1 Analytic and Synthetic Propositions in QL

Rudolf Carnap (1891-1971) was a philosopher active in Europe before 1935 and in the United States after that. He was a central member of the Vienna School of logical positivism. The claim of logical positivism is difficult to understand, but in the author's style, it is as follows.

(A) Taking Wittgenstein's "Treatise on the Philosophy of Logic" as a starting point, the aim was to develop it. Thus, they aimed to create **"my language"** in "The limits of my language mean the limits of my world".

If so, their goal is the same as the author's (i.e., quantum language), but logical positivism was exceptionally strong in its commitment to mathematics and logic. The difference may just be that I know that the last 100 years have been the era of quantum mechanics (and not mathematical logic).

Quine (1908-2000) was an American-born philosopher who taught at Harvard University. He was originally a follower of R. Carnap's logical positivism. Later, however, he became the sharpest critic of Carnap's philosophy (and logical positivism). Thus, he formed his own philosophy.

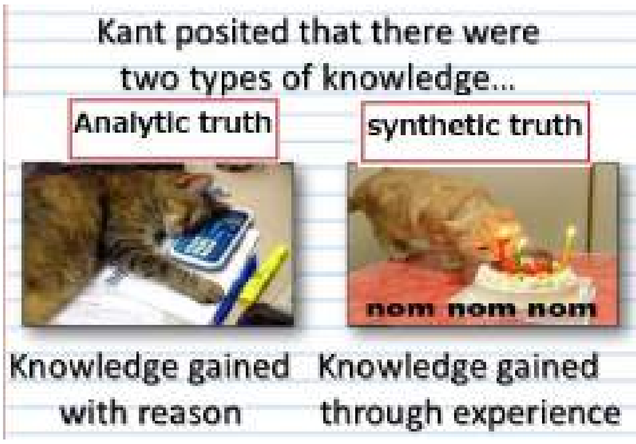
The distinction between "analytic propositions" and "synthetic propositions" begins with Kant. Here,

(B) The classification of "a thing is true or not true" is of interest to any scientist. Moreover, the meaning of " true" differs between analytic propositions and synthetic

propositions. This classification is considered one of the core elements of logical positivism.
"All bachelors are not married." "Horses are animals" "1+3=5"

(C) A " synthetic proposition" is a proposition whose truth or falsity depends on what the facts about the world are. "The cat is on the mat." "Horses are fast", "Jack is a bachelor"

The classification of "a thing is true or not true" is of interest to any scientist. Moreover, the meaning of " true" differs between analytic propositions and synthetic propositions. This classification is considered one of the core elements of logical positivism.



Now, in his 1951 article, "Two Dogmas of Empiricism" ref. [109], Quine rejects the distinction between analytic and synthetic propositions and he asserted

(D) Every proposition is synthetic.

When I learned of Quine's claim (D) some years ago, I remember thinking, "That's a very bold claim," and was surprised. However, I did not quite understand the basis on which Quine made this claim (D).

12.3.1.1 My answer

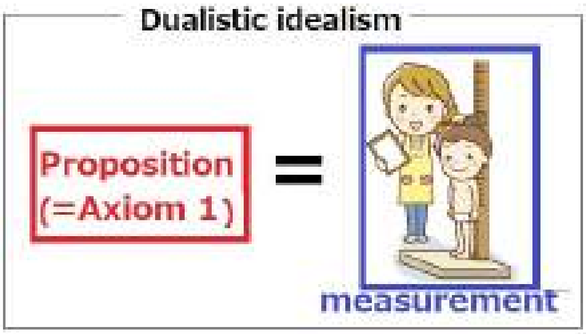
Many philosophers have entered into this issue, but they are all too obscure for the author to be able to give an adequate explanation. As I have said many times, I believe that:

(E_1) If we don't discuss under a worldview, we are just chatting.
(E_1) If there is no definition, there is no discussion.

That is, recalling the following:

(F) "QL-proposition"="(TF)-measurement (= experimental verification)"

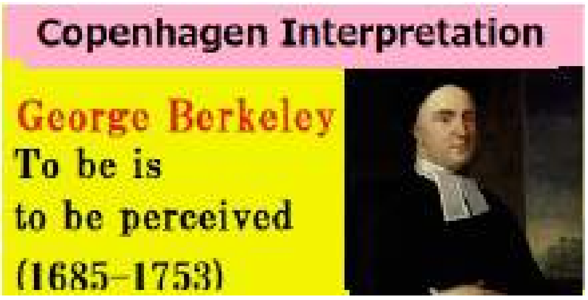
(Or, more generally, "proposition in QL" = "Axiom 1")
(*cf.* Definition 12.3, Note 12.1).
we can naturally conclude that all propositions in QL are synthetic .
Of course, only this definition (F) is important. "Synthesis or analysis?" is not so important since someone may consider (F) analytic.



♠**Note 12.7.** What I mean by (D) is that the worldview and the "definition of propositions" under it are of paramount importance. Without defining propositions, it is meaningless to discuss "whether or not analytic propositions can be distinguished from synthetic propositions".

Of course, different worldviews (=theoretical systems) have different ways of thinking, and my proposal is not the only one. For example, in the theoretical system of mathematics, it is obvious that all propositions are analytic. I hope that many readers will offer various suggestions. I think that philosophers of analytic philosophy should discuss problems after presenting their worldview. Logic without a worldview is, I think, merely a branch of mathematical logic.

The "Carnap-Quine Controversy" is a dispute over the distinction between analytic and synthetic propositions. Quine argued that all propositions are synthetic.



Since in quantum language "every proposition is a total proposition," QL agrees with Quine. The spirit of the Copenhagen Interpretation is " without measurement we cannot know if it is true or false".

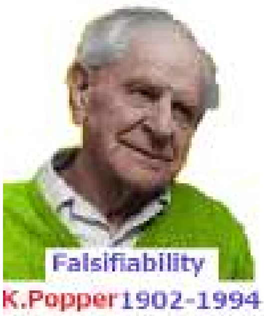
12.4 Popper's falsifiability in the Copenhagen interpretation

From the quantum linguistic point of view, as asserted in Assertion 1.5, we think that

to do science =to describe by QL
=to study in the quantum mechanical worldview

However, when it comes to "what is science?", I can't help but mention Popper. Thus let me mention a little about Karl Popper (1902-1994).

12.4.1 Popper's falsifiability



This may be common to many philosophers of science, if not Popper, but their work is "enlightening" and it is difficult for us in science to understand its true meaning. Popper's books on probability theory and quantum mechanics are also a little far from the interests of science, so I do not fully understand them.

Popper is famous for "falsifiability" as follows. Falsifiability is the following view of science (*cf.* ref. [105]).

(A):Popper's falsifiability

(A) In order to guarantee the objectivity of a scientific theory, there must be a possibility that the hypothesis will be disproved by experiment or observation. In other words, truth must always be subjected to experiments that negate it. And if the denying experiment is confirmed, the truth must be denied.

Many scientists would take this claim for granted. For example,

(B_1) it is a common belief that mammoths are extinct, but if someone discovers a mammoth near the North Pole, this belief will be debunked.

This is a matter of course. Moreover, this is not particularly interesting.

How about the following? (

(B_2) "I think, therefore I am" is against the falsifiability because there is no experiment to deny it. Therefore, it is not a truth.

There may be some philosophy buffs who find this interesting. Popper's falsifiability may not be a statement for scientists, but for philosophy enthusiasts.

12.4.2 Falsifiability for worldviews

It should be obvious that

(C_1) Newtonian mechanics has falsifiability. The power of this is remarkable such as

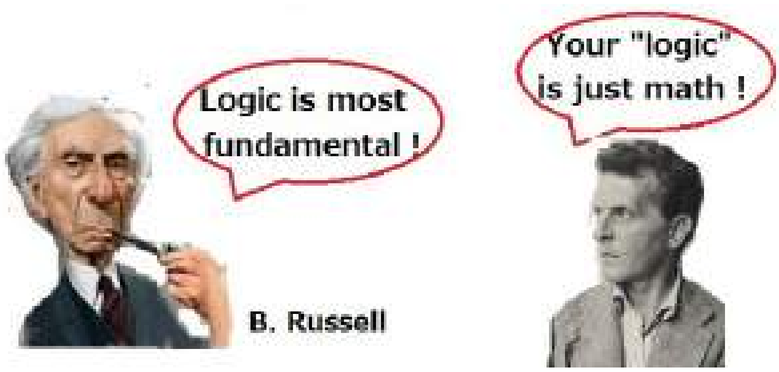
$$\boxed{\text{Newtonian mechanics}} \xrightarrow{\text{Therefore}} \boxed{\text{heliocentrism}}$$

Now, as I have pointed out many times, many good philosophers have followed Plato (or, Euclid) in adopting the following worldview principle.
(C_2)

introduction part (unquestionable truth)

- Descartes: I think, therefore I am
- Kant: thing-in-itself
- Analytic philosophy: mathematical logic

$\xrightarrow{\text{Therefore}}$ common: ethics, moral



In the above, all of the statements on the left (introduction part) are contrary to falsificationism. In other words, philosophers tried to start from "unquestionable truths" for fear of being pointed out in error. In fact, no scientifically productive philosophy was ever born.

If so we want to say

(D) The biggest risk is not taking any risk. Take a risk as Newton!

But, it is not so.

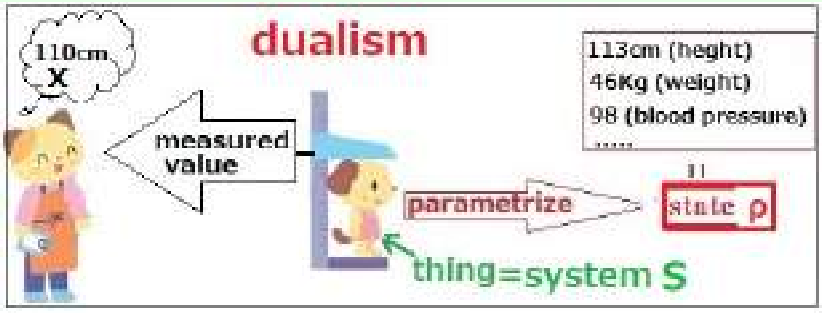
Now let us consider quantum language. For example, let us restate Axiom 1 (in Sec. 1.4.2).

Axiom 1 [State, Measurement] in the C^*-algebraic formulation

•[General (i.e., quantum and classical) case]:

(i) [State]: With any *system* S, a C^*-algebra $\mathcal{A}(\subseteq B(H))$ can be associated. A *state* of the system S is represented by an element $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$.

(ii) [Measurement]: The probability that a measured value x $(\in X)$ obtained by the measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O} = (X, \mathcal{P}(X), G),\ S_{[\rho]})$ is given by $\rho(G(\{x\}))(\equiv {}_{\mathcal{A}^*}\langle \rho, G(\{x\})\rangle_{\mathcal{A}})$.



Again and again, as I have said, Axiom 1 is a nonsensical incantation. This is not the level of satisfying or not satisfying falsifiability. Since it is unintelligible, we can only say that it does not satisfy it.

Therefore,

(E_1) **Newtonian mechanics satisfies falsifiability. However Quantum language does not satisfy falsifiability.**

In other words, quantum language has more power than statistics, yet does not satisfy the same falsifiability as statistics.

Summing up the above (i.e., (C_1), (C_2), and (E_1), we have

(E_2) **Falsifiability is the rule that draws the line between realism and idealism**

This is not a rule for drawing the line between science and pseudo-science.)

This may be a noble claim, but it is not a particularly gratifying claim for scientists.

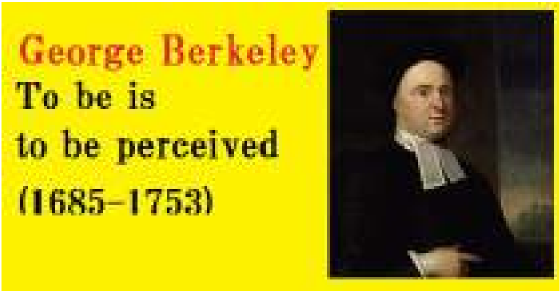
12.4.3 Falsifiability in the Copenhagen interpretation

In conclusion, we would like to argue the following

(F) Falsifiablity (A) should be regarded as one of the Copenhagen interpretations! (cf. (E_0) in Sec. 1.1.2.2)

Because,

Falsifiablity (A) $\approx$ Reinforcement of "To be is to be perceived"



Let us explain it as follows.

(G) The conclusion of Wittgenstein's TLP was "what we cannot speak about we must pass over in silence". In order to do so, we need a rule to distinguish between "what we cannot speak about (=pseudo-science)" and "what we can speak about (=science). Otherwise, the argument is meaningless. It is with this in mind that Popper formulated falsifiability.

Recall that QL has the following form.

(=quantum language) [Axiom 1] [Axiom 2]

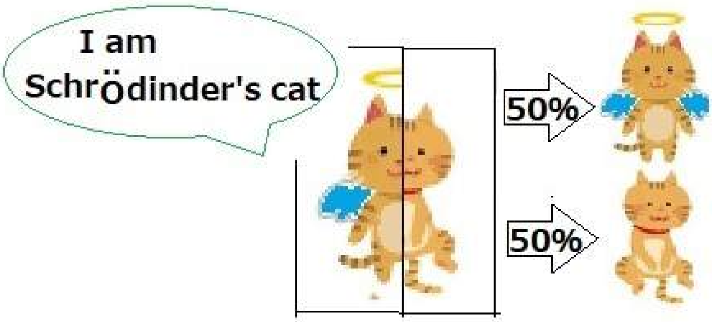
measurement theory = measurement + causal relation

+ linguistic Copenhagen interpretation (*)

[the manual to use Axioms 1 and 2]

Above, Axiom 1 and Axiom 2 alone lead to a messed up conclusion. For example,

(H_1) [Schrödinger's cat]: There is a half-dead cat that we don't know whether it is alive or dead, and the moment we see it, it is determined whether the cat is alive or dead[*5].



(H_2) An incomprehensible non-proposition such as "I think, therefore I am" is regarded as "first proposition of philosophy."

[*5] This was solved in ref. [65]

It is the "Copenhagen interpretation" that contains such pseudoscientific propositions and keeps them out of the open. Therefore, we have two definitions of "Copenhagen interpretation" such as

(I_1) Copenhagen Interpretation is a manual on how to use Axioms 1 and 2
(I_2) Copenhagen interpretation draws a line between scientific and pseudoscientific propositions.

That is, we may consider as follows.

(J) (=(F)) Falsifiablity (A) should be regarded as one of the Copenhagen interpretations! (cf. (E_0) in Sec. 1.1.2.2)

In this light, no scientist would feel uncomfortable with "falsifiability."

♠**Note 12.8.** This is a story about a philosophical study group led by Wittgenstein. Popper was also a participant in that seminar. For Wittgenstein, there were no philosophical problems, only trivial puzzles. All the while, Wittgenstein, seated by the fireplace, had, says Popper, "been nervously playing with the poker". After an exchange of views on ethics, Wittgenstein asked Popper to give an example of an ethical rule. Popper replied, "Not to threaten visiting lecturers with pokers." Wittgenstein then threw away the poker and stormed off.

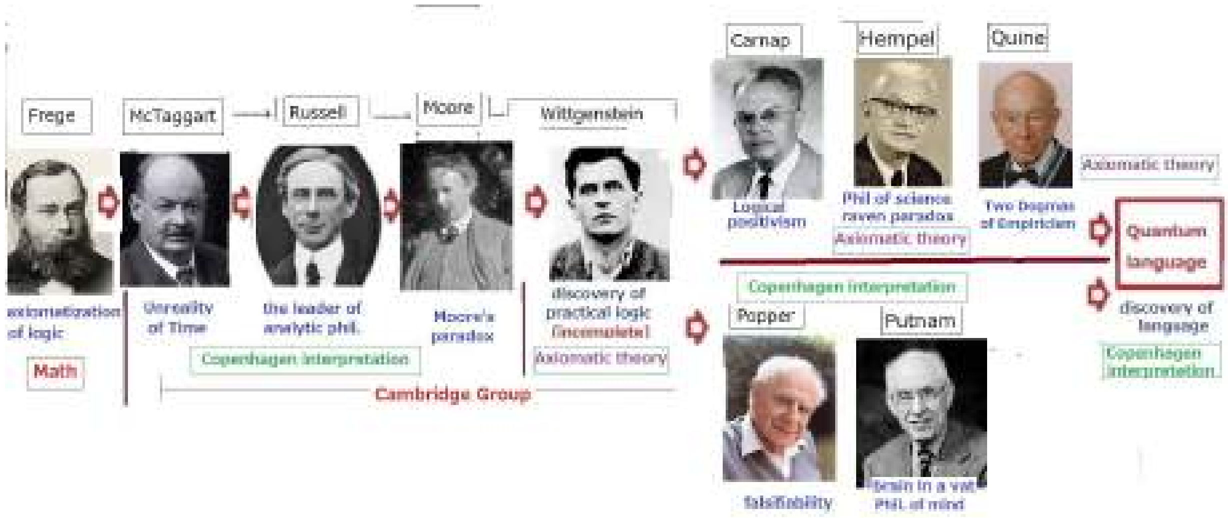
Wittgenstein, Popper and the poker (1946)

This episode implies an adversarial relationship between Wittgenstein and Popper. But what Popper did was to do Wittgenstein's homework (What is "what we cannot speak about"?) as follows.

- using falsifiability, he drew a line between scientific propositions (= what we can speak about) and pseudoscientific propositions (= what we cannot speak about).

However, from the standpoint of quantum language, falsifiability is only one part of the Copenhagen interpretation. Furthermore, a scientific proposition is a QL-proposition (Sec. 12.1).

If we consider falsifiability as one of the rules of Copenhagen interpretation, Popper's place is determined as follows.



12.5 Lewis Carroll's logical paradox

This section is quoted from reference [81], i.e.

- Ishikawa, S., (2021) Fuzzy Logic in the Quantum Mechanical Worldview ; Related to Zadeh, Wittgenstein, Moore, Saussure, Quine, Lewis Carroll, etc. Journal of applied mathematics and physics, Vol. 9, No.7, 1583-1610, (`https://www.scirp.org/journal/paperinformation.aspx?paperid=110830`)

Lewis Carroll (1832–1898) was an English writer of children's fiction, notably "Alice's Adventures in Wonderland" and its sequel "Through the Looking-Glass." He was also a mathematician, photographer, inventor, and Anglican deacon.

In ref. [10] "What the Tortoise said to Achilles" (1895), Lewis Carroll raised the following question.

- Is logic logical?

According to [10], let us explain it as follows.
Achilles says: "Can you understand the following modus ponens?"

> Premise ⓐ: $(P_1 \to P_2)$
> Premise ⓑ: P_1
> Conclusion: ⓩ P_2

the Tortoise says: "I can understand ⓐ and ⓑ. However, why do ⓐ and ⓑ imply ⓩ?"
Achilles says: "I see, then, the following OK?"

> Premise ⓐ: $(P_1 \to P_2)$
> Premise ⓑ: P_1
> Premise ⓒ: ⓐ and ⓑ are "true", therefore ⓩ is "true"
> Conclusion: ⓩ P_2

the Tortoise says: "I can understand Premises ⓐ,ⓑ and ⓒ. However, why do ⓐ, ⓑ and ⓒ conclude ⓩ?"
Achilles says: "I see, then, the following OK?"

> Premise ⓐ: $(P_1 \to P_2)$
> Premise ⓑ: P_1
> Premise ⓒ: ⓐ and ⓑ are "true", therefore ⓩ is "true"

Premise ⓓ: ⓐ, ⓑ and ⓒ are "true", therefore ⓩ is "true"
Conclusion: ⓩ P_2

the Tortoise says: "I can understand Premises ⓐ,ⓑ, ⓒ and ⓓ. However, why do ⓐ, ⓑ, ⓒ and ⓓ conclude ⓩ?"

: (Infinite regress)

This is Lewis Carroll's Paradox

Several philosophers have tried to resolve Carroll's paradox. For example, Bertrand Russell discussed the paradox briefly in ref. [115]. His opinion is as follows.

(A_1) The above ⓐ, ⓑ and ⓩ are propositions, on the other hand, ⓒ and ⓓ are inference rules. And, therefore, both the Tortoise and Achilles are confusing this.

I agree to his opinion. I think that their confusion is due to the lack of a definition of proposition.

♠**Note 12.9.** In this paradox, the following is of essential importance

(♯) Is Lewis Carroll's paradox a mathematical problem or not?

If it is a mathematical problem, it is not a paradox. Because "proposition" and "inference rule" can never be confused since mathematics is formulated under axiomatic set theory. (Lewis Carroll's paradox was proposed in [10] (1895). The axiomatic set theory (e.g., Zermelo-Fraenkel set theory) was proposed was proposed in the early 20th century. Therefore, we can say that, in 1895, Lewis Carroll's paradox was also a paradox within mathematics.)

On the other hand, if Lewis Carroll's paradox is not a mathematical problem, it is not easy. That is because we have no definition of "proposition" in general. However, as indicated before, we already know the definition of "proposition" within QL. Thus, in QL we can argue as follows.

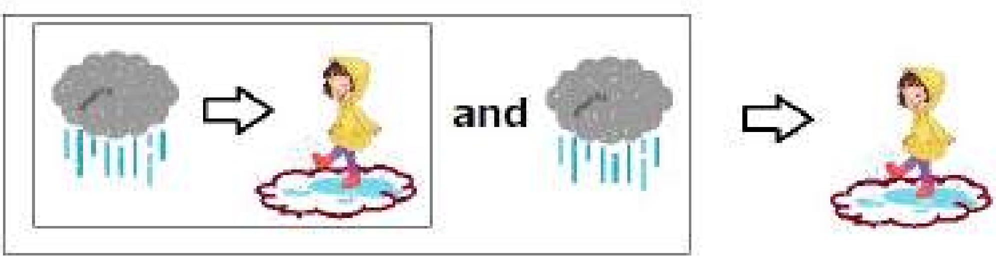
Let us prove that Lewis Carroll's paradox is not a paradox in QL as follows. Note that QL says that

(A_2) "proposition P"="(TF)-measurement $\mathsf{M}_{\mathcal{A}}(\mathsf{O}, S_{[\rho]})$". And the essence of the modus ponens is all described in the following Table 12.2 (= Table 12.1). Therefore, logic is a simple calculation of elementary arithmetic.

Table 12.2: Probabilistic Truth Table (Elementary propositions $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]})(= P_1)$, $\mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]})(= P_2)$)

$\mathsf{M}_{\mathcal{A}}(\mathsf{O}_1, S_{[\rho]})$ $(= P_1)$	$\mathsf{M}_{\mathcal{A}}(\mathsf{O}_2, S_{[\rho]})$ $(= P_2)$	probability: $p = \times^{qp}_{i=1,2} \mu_i$	$[P_1 \to P_2] \wedge P_1$	$[P_1 \to P_2] \wedge P_1 \to P_2$
T	T	$p_{12} = \times^{qp}_{i=1,2} \mu_i(\{(T,T)\})$	T	T
T	F	$p_{1\bar{2}} = \times^{qp}_{i=1,2} \mu_i(\{(T,F)\})$	F	T
F	T	$p_{\bar{1}2} = \times^{qp}_{i=1,2} \mu_i(\{(F,T)\})$	F	T
F	F	$p_{\bar{1}\bar{2}} = \times^{qp}_{i=1,2} \mu_i(\{(F,F)\})$	F	T

For example, put P_1 :="it rains", P_2 :="the ground is wet". Modus ponens says that [["it rains" $\rightarrow$ "the ground is wet"]$\wedge$ "it rains"]$\rightarrow$"the ground is wet".



Maybe the Tortoise thought of logic as a noble discipline and tried to understand it rigorously. However, all we have to do is only what is written in the table above. Note that the above ⓒ and ⓓ are not written in this table.

As mentioned in Note 12.9, in the case of mathematics, the definition of "proposition" is so clear that I think it is rare for mathematicians to take Carroll's logical paradox seriously. When I asked the mathematicians around me, they all just said, "What are you saying stupid things?" or "Isn't that the same as saying, I don't understand '1+1=2'?".

Since the definition of a proposition is not clear in the case of philosophy, we tend to confuse propositions with inference rules. We know the definition of a proposition in QL so clearly that we could avoid any confusion.

I don't know Carroll's own intentions, but I think the reason this paradox has been of interest to people for over 100 years is that it is closely related to the question "what is a scientific proposition?". Carroll's logical paradox is not the childish problem that most mathematicians think it is.

12.6 Flagpole problem: What is a scientific explanation?

This section was written with reference to the following.

- [78]:Ishikawa, S: Philosophy of science for scientists; The probabilistic interpretation of science Journal of quantum information science, Vol. 9, No.3, 140-154, DOI: 10.4236/jqis.2019.93007 (https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

Remark 12.15. Recall that QL logic (= fuzzy logic) (in Sec. 12.1) is practical logic within the quantum mechanical world view. As I have said many times in this text, philosophers like "logic" (or, the word "logic") too much. They always want to "be logical". I feel as follows.

- **Philosophers are more fond of "logic" (or, the word "logic") than mathematicians**

That is, I assert that

- the scientific explanation should not be described in the logical worldview (=the logical spirit=the spirit of "Think logically!"), but in the quantum mechanical worldview.

The flagpole problem below is caused by philosophers' excessive love of logic.

12.6.1 * The quantum linguistic solution of Flagpole problem

Carl Gustav Hempel (1905-1997) was a German philosopher. He was a major figure in logical empiricism, a 20th-century movement in the philosophy of science. His studies of induction, explanation, and rationality in science exerted a profound influence upon a young generation of philosophers of science. He is also known for the raven paradox (also known as "Hempel's paradox").

We think that most modern scientists know Newtonian mechanics, but not mathematical logic. Thus most scientists today would feel that

(A) the basis of science is not "logic" but a "mechanical worldview".

However, for early 20th century philosophers, however, (A) was not common knowledge. It was Hempel who first noticed the incompatibility of logic (or set theory) and science.

12.6.1.1 Flagpole problem in Hempel's model (= the deductive-nomological model (DN model))

D-M explanation (deductive-nomological explanation) is one of the "explanatory methods of science" and has the following forms

D-M explanation (deductive-nomological explanation)

Definition 12.16. [D-M explanation (deductive-nomological explanation)] D-M explanation is the scientific explanation such as

$$\begin{array}{ll} [\text{fol}]_1 & \\ [\text{fol}]_2 & \\ \cdots & \text{where}[\text{fol}]_k\text{means "fact" or "law"} \\ [\text{fol}]_{n-1} & \\ [\text{fol}]_n & \\ \hline C \quad (\text{Conclusion}) & \end{array}$$

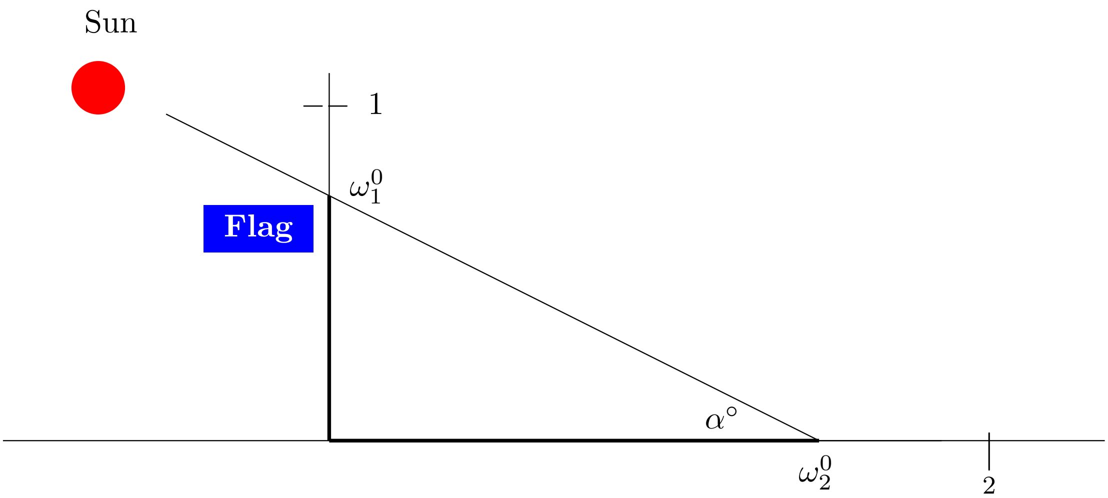


Figure 12.2: [Flagpole problem]

Let us explain the flagpole problem as follows. Suppose that the sun is at an elevation angle $\alpha°$ in the sky. Assume that $\tan\alpha° = 1/2$. There is a flagpole which is ω_1^0 meters tall ($0 \le \omega_1^0 \le 1$. The flagpole casts a shadow ω_2^0 meters long. Suppose that we want to explain the length of the flagpole's shadow. On Hempel's model, we see the following explanation.

(B_1) 1. [fact]: The sun is at an elevation angle $\alpha°$ in the sky.
2. [law]: Light propagates linearly.
3. [fact]: The flagpole is ω_1^0 meters high.

Then,
4. [conclusion]: The length of the shadow is $\omega_2^0 = \omega_1^0/\tan\alpha° = 2\omega_1^0$meters

This is a good explanation of "Why is that shadow $2\omega_1^0$ meters long?"
Similarly, we may consider as follows.

(B_2) 1. [fact]: The sun is at an elevation angle $\alpha°$ in the sky.

2. [law]: Light propagates linearly.
3. [fact]: The length of the shadow is ω_2^0 meters

Then,
4. [conclusion]: The flagpole is $\omega_1^0 (= (\tan \alpha^\circ)\omega_2^0 = \omega_2^0/2)$ meters tall.

This is somewhat disconcerting (since the cause and effect are reversed) as an explanation for the question "Why is the flagpole height $\omega_2^0/2$ meters?"

- **Hempel did not miss this minute discomfort.**

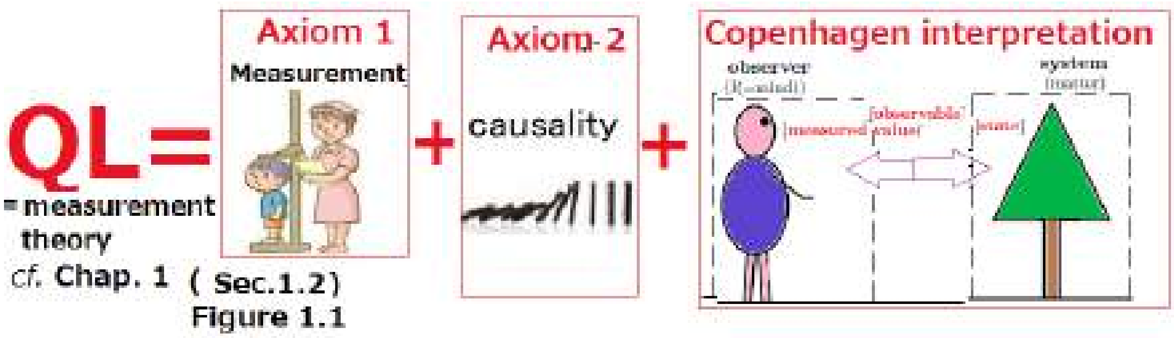
Thus we have the flagpole problem as follows:

Problem 12.17. [Hempel's flagpole problem]

(B$_3$) [**Flagpole problem**]
Why do we feel that the solution (B$_2$) is unnatural?
Is there a deficiency in the D-M description?

Therefore, in what follows, I will present the quantum linguistic explanation.
My opinion is as follows.

(C) the above explanations (B$_1$) and (B$_2$) rely on DN model (i.e., deductive-nomological model), which is due to a kind of the logical worldview (=the logical spirit=the spirit of "Think logically!") (i.e., the spirit that science should be written logically). And thus, the most important concepts "measurement" and "causality" are not used in (B$_1$) and (B$_2$).



Therefore, in what follows, I will present the quantum linguistic explanation.

12.6.1.2 Flagpole problem by the quantum linguistic explanation

Consider two state spaces $\Omega_1 = [0, 1]$ and $\Omega_2 = [0, 2]$. Here, assume that a state $\omega_1 (\in \Omega_1)$ means the height of the flagpole. And an state $\omega_2 (\in \Omega_2)$ means the lenght of the shadow of the flagpole. The relation between ω_1 and ω_2 is represented by the following map:

$$\Omega_1(= [0, 1]) \xrightarrow[\phi_{1,2}]{} \Omega_2(= [0, 2])$$

For simplicity, the speed of light is assumed to be ∞. It would also be easier to use precision measurements, but we will not use them. Since the sun is located at an elevation angle α° in the sky, the causal map $\phi_{1,2} : \Omega_1 \to \Omega_2$ is determined as follows

$$\omega_2 = \phi_{1,2}(\omega_1) = \omega_1 / \tan(\alpha^\circ) = 2\omega_1 \quad (\forall \omega_1 \in \Omega_1 (= [0,1]))$$

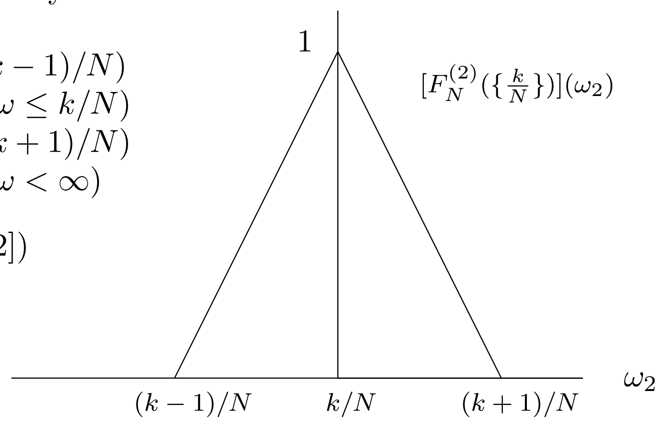
Thus, the causal operator $\Phi_{1,2} : C(\Omega_2) \to C(\Omega_1)$ is defined by

$$(\Phi_{1,2} f_2)(\omega_1) = f_2(\phi(\omega_1)) \quad (\forall f_2 \in C(\Omega_2), \omega_1 \in \Omega_1 (= [0,1]))$$

Let N be sufficiently large number. Put $X_N = \{0 (= \frac{0}{N}), \frac{1}{N}, \frac{2}{N}, ..., \frac{2N}{N}\}$. The observable $\mathsf{O}_N^{(2)} = (X_N, \mathcal{P}(X_N), F_N^{(2)})$ in $C([0,2])$ is defined by

$$[F_N^{(2)}(\{\tfrac{k}{N}\})](\omega_2) = \begin{cases} 0 & (-\infty \le \omega \le (k-1)/N) \\ N\omega + 1 & ((k-1)/N \le \omega \le k/N) \\ -N\omega + 1 & (k/N \le \omega \le (k+1)/N) \\ 0 & ((k+1)/N \le \omega < \infty) \end{cases}$$

$$(k = 0, 1, 2, ..., 2N), \omega_2 \in \Omega_2 (= [0,2])$$



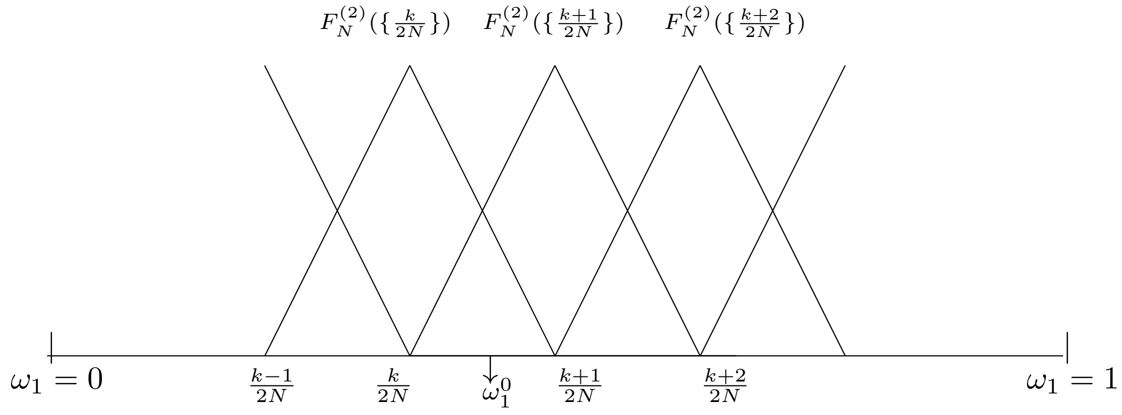
Here, we consider it in terms of the Heisenberg picture. That is,

$$\Phi_{1,2} \mathsf{O}_N^{(2)} = (X_N, \mathcal{P}(X_N), \Phi_{1,2} F_N^{(2)}) = (X_N, \mathcal{P}(X_N), F_N^{(1)})$$

where

$$[F_N^{(1)}(\{\frac{k}{N}\})](\omega_1) = [F_N^{(2)}(\{\frac{k}{N}\})](\phi_{1,2}(\omega_1)) = [F_N^{(2)}(\{\frac{k}{N}\})](2\omega_1) = [F_N^{(2)}(\{\frac{k}{2N}\})](\omega_1)$$
$$(\forall k = 0, 1, ..., 2N, \forall \omega_1 \in \Omega_1 (= [0,1]))$$

Thus, we get the measurement $\mathsf{M}_{C(\Omega_1)}(\Phi_{1,2} \mathsf{O}_N^{(1)} = (X_N, \mathcal{P}(X_N), F_N^{(1)}), S_{[\omega_1^0]})$ with the state $\omega_1^0 (\in \Omega_1)$.



Now, from Axiom 1 we can say the following.

(D$_1$) [Measurement]; We take a measurement $\mathsf{M}_{C(\Omega_1)}(\Phi_{1,2}\mathsf{O}_N^{(1)} = (X_N, \mathcal{P}(X_N), F_N^{(1)}), S_{[\omega_1^0]})$. If it holds that $\frac{k}{2N} \le \omega_1^0 \le \frac{k+1}{2N}$, then we see that

$$\begin{cases} \text{the probability that a measured value } \frac{k}{2N}(\in X) \text{ is obtained is given by} \\ \qquad [F_N^{(2)}(\{\frac{k}{2N}\})](\omega_1^0) \\ \text{the probability that a measured value } \frac{k+1}{2N}(\in X) \text{ is obtained is given by} \\ \qquad [F_N^{(2)}(\{\frac{k+1}{2N}\})](\omega_1^0) \ (=1 - [F_N^{(2)}(\{\frac{k}{2N}\})](\omega_1^0)) \end{cases}$$

This is the quantum linguistic representation of (B$_1$).

Also,

(D$_2$) [Inference]; (*cf.* Sec. 1.4.4): Assume that the measued value $\frac{k}{2N}$ is obtained by a measurement $\mathsf{M}_{C(\Omega_1)}(\Phi_{1,2}\mathsf{O}_N^{(1)} = (X_N, \mathcal{P}(X_N), F_N^{(1)}), S_{[*]})$. Then, Fisher's maximal likelihood method says that

$$\max_{\omega_1 \in \Omega_1} [F_N^{(2)}(\{\frac{k}{2N}\})](\omega_1) = 1 = [F_N^{(2)}(\{\frac{k}{2N}\})](\frac{k}{2N})$$

and thus we can infer that unknown state $[*]$ is equal to $\frac{k}{2N}$.

This is the quantum linguistic representation of (B$_2$).

Therefore, we asset that

(E) "Scientific explanation" is an explanation in quantum language

Of course, since the claim of this text is that "quantum language is the language of science," it would be troubling if it were not (E).

12.7 * Hempel's raven paradox: A milestone in the philosophy of science

As we have repeatedly stated, Wittgenstein's interest was to clarify the problem: "what is a scientific proposition?". For example, the next are doubtful.

- "I think, therefore I am", "thing-in-itself", "$1 + 1 = 2$", etc.

But the most accomplished masterpiece of them all is Hempel's raven paradox, i.e.

"All Ravens Are Black".

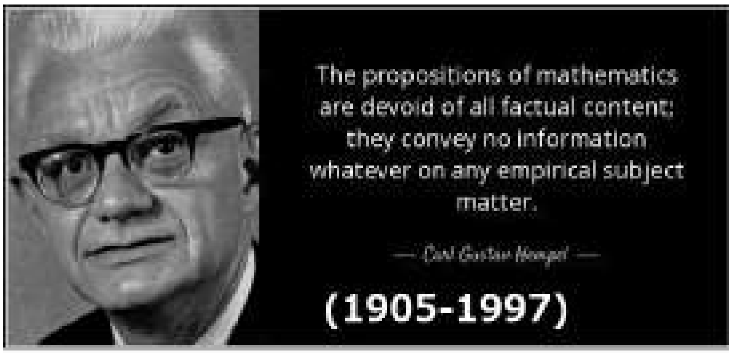
That is, "All Ravens Are Black" is not a scientific proposition.
This section was written with reference to the following.

- [78]:Ishikawa, S: Philosophy of science for scientists; The probabilistic interpretation of science Journal of quantum information science, Vol. 9, No.3, 140-154, DOI: 10.4236/jqis.2019.93007 (`https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447`)

"Hempel's raven" is a paradox that arouses skepticism toward the "set-theoretic world-view. I would like to pay tribute to Hempel's research attitude for pointing out something ("flagpole problem" or "Hempel's raven") that is easy to miss if one does not know the "quantum mechanical worldview." Philosophy of science is a broad field, but I think the direction that Hempel pointed out was the direction that philosophy of science should go.

He said that

- *The propositions of mathematics are devoid of all factual content; they convey no information whatever on any empirical subject matter.*



I think this quote from Hempel suggests that he was exploring something like quantum language.

12.7.1 Is "the set of all tyrannosaurus" meaningful? : the set theoretical worldview

Let us explain "the set theoretical worldview", (which is a kind of spirit of "Think logically!", and thus should not be regarded as a worldiew). Logic and set theory are similar (i.e., "logic + set" = "set theory" ="mathematics"), and thus, it usually believed that set theory as well as logic are considered reliable.
As mentioned Sec. 11.1, the following is the greatest history of mathematics (i.e., the beginning of modern mathematics):

naive set theory		axiomatic set theory		
Cantor (1891)	$\longrightarrow$	ZFC-Axiom (1921)	$\longrightarrow$	modern math.

However, the difference between naive set theory and axiomatic set theory is negligible for most mathematicians (except mathematicians specializing in foundations of mathematics). Therefore, for simplicity, let's assume that "set theory" = "naive set theory" in this section (*cf.* Note 1.1). Therefore, we assume that "set" is defined by "a collection of things". This is the same as the use of "set" in everyday language.

General people (including philosophers) may think

(A_1) Set theory is very reliable because it is used to lay the foundation for mathematics. Therefore, if we use sets to describe the concept of this world, we will not fall into a mistake.

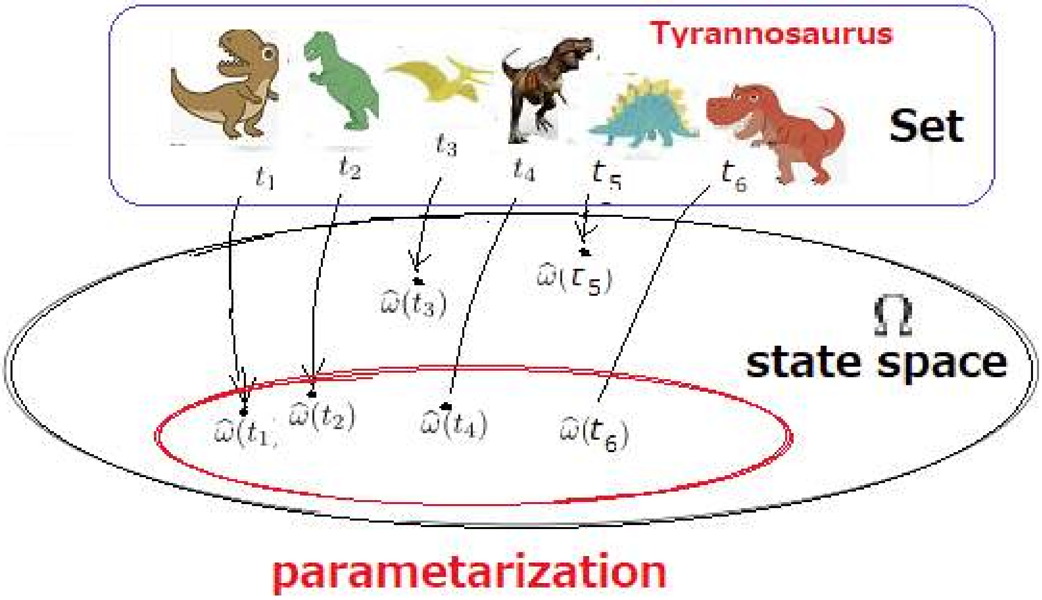
In this text, this is called the set theoretical worldview. However, I am skeptical of the set theoretical worldview ($\sharp_1$). That is, I think that

(A_2) It is true that set theory is a very reliable and solid discipline. However, we must be cautious in using sets to describe the world.

For example,

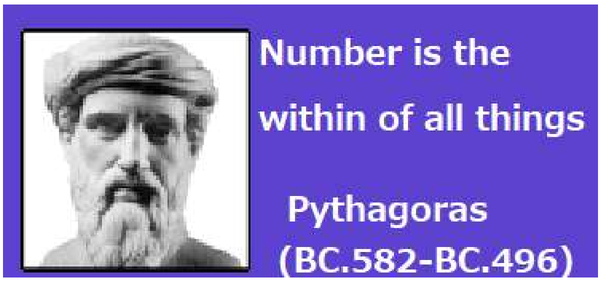
(B) Is "the set of all tyrannosaurus" meaningful? Or, is "the set of all raven" meaningful? If it is not meaningless, how do we represent "All ravens are black"?

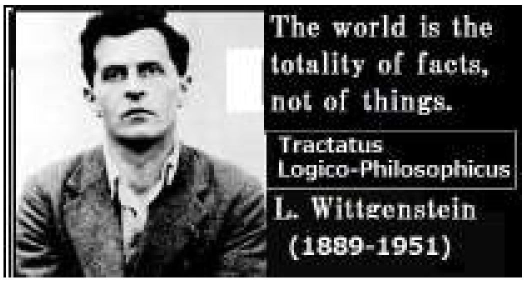
This (B) is Hempel's raven problem.



Remark 12.18. TLP begins with the following statement:

(♯) *The world is the totality of facts (i.e., the totality of states (= state space)), not of things (i.e., the set of things).*





Note that Hempel doubted the "set of things" (i.e., "the set of ravens"). I think that Hempel was one of the philosophers who understood TLP best.

12.7.2 Hempel's raven problem in the set theoretical worldview; What is the problem?

Is "the set of all tyrannosaurs" meaningful?

First, let us review the traditional arguments concerning Hempel's raven problem (*cf.* refs. [31, 32]). Thus, we start from the followings:

(C) Let U be the set of all birds. Let $B(\subseteq U)$ be a set of all black birds. Let $R(\subseteq U)$ be a set of all ravens.

Although these should be doubtful (since these are as ambiguous as "the set of all tyrannosaurs"), we advance towards the next argument. The statement: "Every raven is black" is logically denoted by

(D_1) "Every raven is black" : $(\forall x)[x \in R \longrightarrow x \in B]$ $\quad i.e. R \subseteq B \subseteq U$,

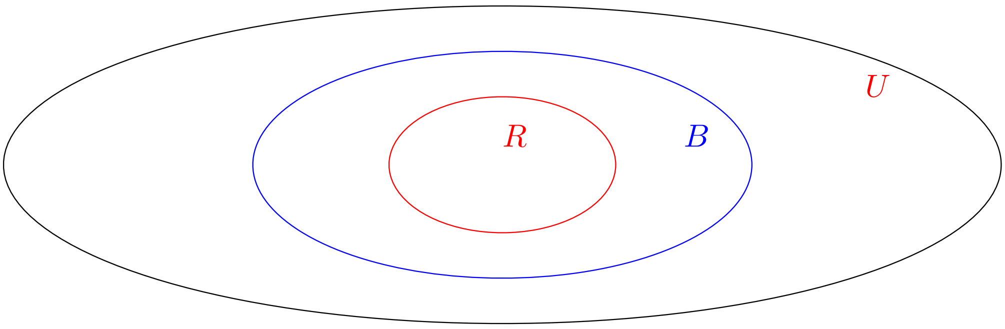


Figure 12.3:The set-theoretical worldview (Sec. 4.3.4)
[U: the set of all birds, B: the set of all black birds, R: the set of ravens]

Also, this is logically equivalent to the following contraposition:

(D_2) "Every non-black bird is a nonraven" : $(\forall x)[x \in U \setminus B \longrightarrow x \in U \setminus R]$
i.e. $U \setminus B \subseteq U \setminus R$

However, if these are equivalent, then we have the following problems (i.e., raven problem):

(E_1) Why is the actual verification of (A_2) much more difficult than the actual verification of (A_1)?
(E_2) Why can the truth of "(A_1): any raven is black" be known by (A_2), i.e. without seeing a raven also at once?
(E_3) Is it possible to experimentally verify "Every raven is black"?

These are so called Hempel's raven paradox. However, there is a reason to consider that "the set of all ravens" is as ambiguous as "the set of all tyrannosaurs". If so, that is, if the above (C) is ambiguous, all other (D_1)-(E_3) are also ambiguous, and thus non-scientific.

Now we think that the most essential problem concerning Hempel's raven problem is as follows:

(F) What is the scientific meaning of "Every raven is black"?

In order to study this problem, we must prepare the quantum linguistic formulation of ornithology, under which the meaning of "Every raven is black" will be clarified in this section.

Remark 12.19. (i): I am impressed by Hempel's powers of observation that he did not overlook "discomfort" that would normally be overlooked. Before I wrote paper [78], I thought the above paradox [(E_1)$\sim$(E_3)] was "mere sophistry."
(ii): Just to be sure, in this text we assume that the followings are the same:

$$\text{"any raven is black"} = \text{"every raven is black"} = \text{"all ravens are black"}.$$

This is the same as the usage in mathematics (i.e., "any" = "every" = "all" = "$\forall$").

12.7.3 Hempel's raven problem in the quantum mechanical worldview

In this section we slightly improve our result in

- Ref. [78]: Ishikawa, S., (2019) Philosophy of science for scientists; The probabilistic interpretation of science, Journal of quantum information science, Vol. 9, No.3, 140-154,
(https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

We think that Hempel's raven problem raises the problem of "What is the scientific meaning of 'Every raven is black'?". In order to answer this problem, we must prepare the quantum linguistic formulation of ornithology

The arguments below are essentially the same as Saussure's "signifier" and "signified" in Sec. 11.6.

12.7.3.1 Quantum linguistic solution of Hempel's raven paradox

Now, we will discuss the continuation of Problem 12.13[All sweet apples are red]. However, since it will be virtually the same as "Hempel's raven problem," we will rewrite "All sweet apples are red" to "All ravens are black" for the discussion.

Problem 12.20. (=Problem 12.13) [All ravens are black]

Consider two (TF)-observables $\mathsf{O}_R = (\{T,F\}, 2^{\{T,F\}}, G_R)$ and $\mathsf{O}_B = (\{T,F\}, 2^{\{T,F\}}, G_B)$ in $C(\Omega)$, where O_R and O_B is respectively called "raven observable" and "black bird observable".

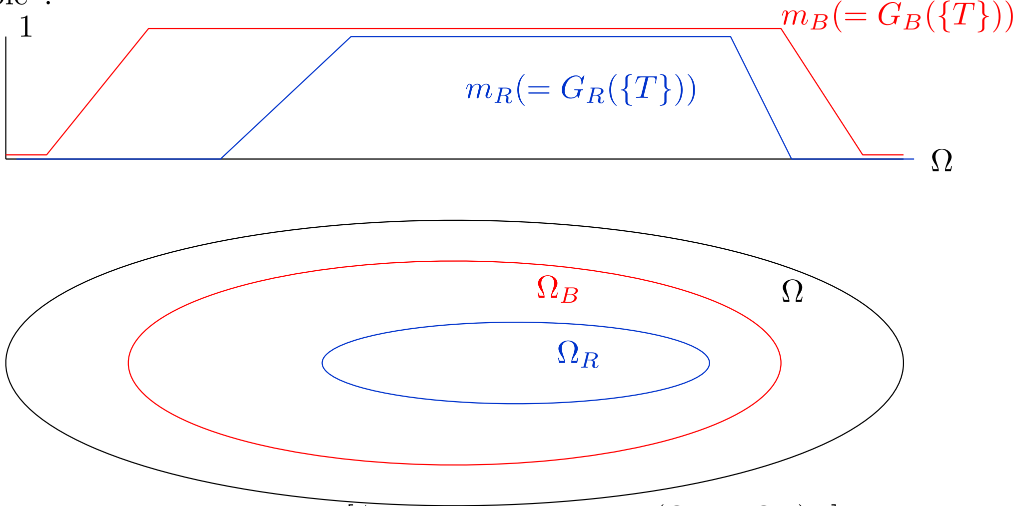


Figure 12.4: [All ravens are black: $(\Omega_R \subseteq \Omega_B)$]

Now, it is natural to define the proposition "all ravens are black" as follows

(G) $\Omega_R \subseteq \Omega_B$

where $m_R = G_R(\{T\})$, $m_B = G_B(\{T\})$, $\Omega_R = \{\omega \in \Omega | m_R(\omega) = 1\}$, $\Omega_B = \{\omega \in \Omega | m_B(\omega) = 1\}$.

Now we have the folowing problem:

(H) **Problem[(Q): Is $[\Omega_R \subseteq \Omega_B]$ a QL-propositin?**.

I will answer this below.

Answer 12.21. Answer of Problem 12.20: Note that $\Omega_R (= \{\omega \in \Omega \mid [G_R(\{T\})](\omega) = 1\})$ is compact. Further, it is natural to assume that Ω_R is separable (i.e., there exists a countable dense set $\Omega'_R (= \{\omega_1, \omega_2, ...\} \subseteq \Omega_R)$. The next is obvious.

$$[\Omega_R \subseteq \Omega_B] \Longleftrightarrow [\Omega'_R \subseteq \Omega_B]$$

Using the notation in "Sec. 1.4.6: Infinite tensor product observable", define the following:

$$\begin{aligned}&\mathcal{P}_0(\bigtimes_{n=1}^{\infty}\{T,F\})\\ =&\Big\{ \bigtimes_{n=1}^{\infty} \Xi_n \;\Big|\; \Xi_n \subseteq \{T,F\}\ (n=1,2,...),\quad \{n(=1,2,...) \mid \Xi_n \neq \{T,F\}\} \text{ is finite } \Big\}\end{aligned}$$

Also, let n_0 be any natural number. And put

$$\begin{aligned}&\mathcal{P}_0^{T,n_0}(\bigtimes_{n=1}^{\infty}\{T,F\})\\ =&\Big\{ \bigtimes_{n=1}^{\infty} \Xi_n \;\Big|\; \Xi_k = \{T\}\ \ (k=1,2,...,n_0), \Xi_k = \{T,F\}\ \ (k = n_0+1, n_0+2, ...)\Big\}\end{aligned}$$

Consider the infinite tensor measurement: $\bigotimes_{n=1}^{\infty} \mathsf{M}_{C(\Omega)}(\mathsf{O}_B, S_{[\delta_{\omega_n}]})$. That is,

$$\begin{aligned}&\bigotimes_{n=1}^{\infty} \mathsf{M}_{C(\Omega)}(\mathsf{O}_B, S_{[\delta_{\omega_n}]}) = \mathsf{M}_{C(\times_{n=1}^{\infty}\Omega)}\Big(\bigotimes_{n=1}^{\infty} \mathsf{O}_B, S_{[\otimes_{n=1}^{\infty}\delta_{\omega_n}]}\Big)\\ =&\mathsf{M}_{C(\times_{n=1}^{\infty}\Omega)}\Big(\bigotimes_{n=1}^{\infty} \mathsf{O}_B\big(=(\bigtimes_{n=1}^{\infty}\{T,F\}, \mathcal{P}_0(\bigtimes_{n=1}^{\infty}\{T,F\}), \bigotimes_{n=1}^{\infty} G_B)\big), S_{[\otimes_{n=1}^{\infty}\delta_{\omega_n}]}\Big)\end{aligned}$$

Assume that, by the measurement, we get the measured value $x = (x_n)_{n=1}^{\infty} (\in \bigtimes_{n=1}^{\infty}\{T,F\})$. Note that

- the probability that $x = (x_n)_{n=1}^{\infty}$ belongs to $\mathcal{P}_0^{T,n_0}(\bigtimes_{n=1}^{\infty}\{T,F\})$ is quaal to 1.

That is, we see that

$$\omega_k \in \Omega_B (= \{\omega \in \Omega : m_B(\omega) = 1\}) \quad (k = 1,2,...,n_0)$$

Here, since n_0 is arbitrary, we get:

$$\omega_k \in \Omega_B \quad (k=1,2,...) \quad \text{thus,} \Omega'_R \subseteq \Omega_B$$

Therefore, we the that the followings are equivalent:

(I$_1$) the measured value $x = (x_n)_{n=1}^{\infty}$ by the infinite tensor product measurement $\bigotimes_{n=1}^{\infty} \mathsf{M}_{C(\Omega)}(\mathsf{O}_B, S_{[\delta_{\omega_n}]})$ satisfiew that $x_n = T(\forall n = 1, 2, ...)$.

(I$_2$) $\Omega_R \subseteq \Omega_B$

This completes the proof. □

♠**Note 12.10.** (i):The condition of separability may not be necessary, but I did so because it is natural to assume separability.
(ii): In fact, it is almost self-evident if we consider the following. Let me reiterate the figure.

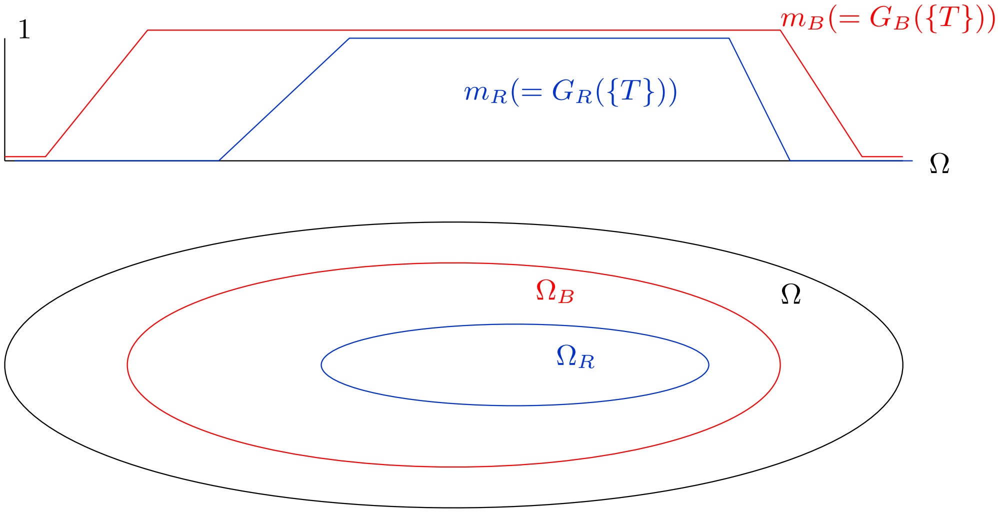


Figure 12.5, which is the same as Figure 12.4: [All ravens are black: $(\Omega_R \subseteq \Omega_B)$]

This figure was created by measurement in the first place. Then it is natural that "$\Omega_R \subseteq \Omega_B$" is also obtained by measurement. The above proof is nothing more than a mathematical expression. Conversely, if you want to regard "$\Omega_R \subseteq \Omega_B$" as a proposition, you must set up an axiom (Axiom' 1) so that you can show it with a mathematical formula.

12.7.3.2 Popper's falsificationism in measurement theory

♠**Note 12.11.** As mentioned in Sec. 12.4.1, Karl Popper (1902-1994) was one of the 20th century's most influential philosophers of science. Popper claims that, in order for something to be considered science, it must be falsifiable. If it is false, it can be shown through observation and experiment to be false. However, I think that the meaning of "observation and experiment" is ambiguous. That is, it must be "observation and experiment that is described by quantum language". This will be done below.

In the previous section, we discussed "$\Omega_{SB} \subseteq \Omega_B$" (i.e., Every small black bird is black). Since this is a priori statement, we can accept this statement without verification by experiment.

In this section we will discuss the statement "$\Omega_R \subseteq \Omega_B$" (i.e., Every raven is black), which is not a priori proposition but a posteriori proposition.

Hence, our problem is as follows:

(J) How can we be sure of $\Omega_R \subseteq \Omega_B$ (i.e., "Every raven is black")?
i.e. What should we do to be sure of "$\Omega_R \subseteq \Omega_B$"?.

In order to do it, we obey Popper's falsificationism (*cf.* ref. [105]) such that

(K) "$\Omega_R \subseteq \Omega_B$" should be accepted, if many experiments which deny "$\Omega_R \subseteq \Omega_B$" are conducted and "$\Omega_R \subseteq \Omega_B$" still cannot be denied.

For instance, we mention the following two tests ([Test I] and [Test II])
[**Test I**]**:** In order to deny "$\Omega_R \subseteq \Omega_B$",

(L) we try to find a bird with the state ω_0 such that $\omega_0 \in \Omega_R \setminus \{\omega \mid m_B(\omega) = 0\}$ (See Figure 11.5 below)

This test is quite natural, and thus, we should try this first.

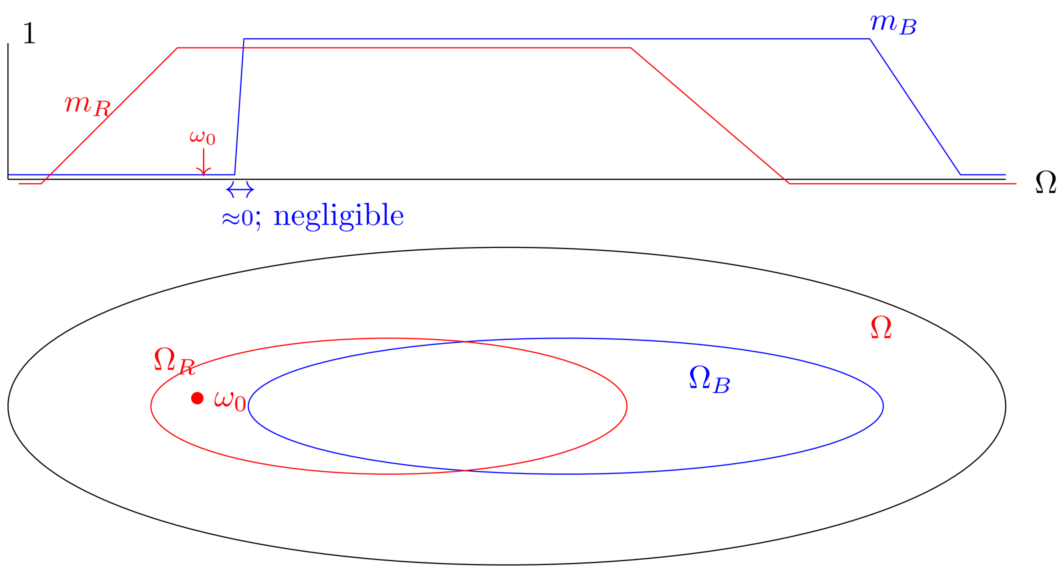


Figure 12.6: [$\omega_0 \in \Omega_R \setminus \{\omega \mid m_B(\omega) = 0\}$, $\rho_0(\{\omega \in \Omega \mid 0 < m_B(\omega) < 1\} \bigcap \Omega_R\}) \approx 0$, i.e. negligible.]

[Test II: statistical hypothesis testing]: In order to deny "$\Omega_R \subseteq \Omega_B$",

(M_0) we try to confirm the hypothesis that there are non-black ravens by 3 percentages in 100 ravens. That is, we take the parallel mixed measurement (*cf.* [74, 78]) $\bigotimes_{i=1}^{100} \mathsf{M}_{C(\Omega)}(\mathsf{O}_B := (\{y,n\}, 2^{\{y,n\}}, F_B),\ S_{[*]}(\rho_0))$, where a mixed state ρ_0 ($\in \mathfrak{S}^m(C(\Omega)^*)$) satisfies $\rho_0(\Omega_R) = 1$ and $\rho_0(\Omega_R \setminus \Omega_B) = 0.03$. Here, we, for simplicity, assume that $\rho_0(\{\omega \in \Omega \mid 0 < m_B(\omega) < 1\} \bigcap \Omega_R\}) \approx 0$, i.e. negligible. (See Figure 11.5 above.)

And assume that

(M_1) as the result of the (M_0), we get that one hundred ravens were black continuously

which is written in terms of quantum language as follows:

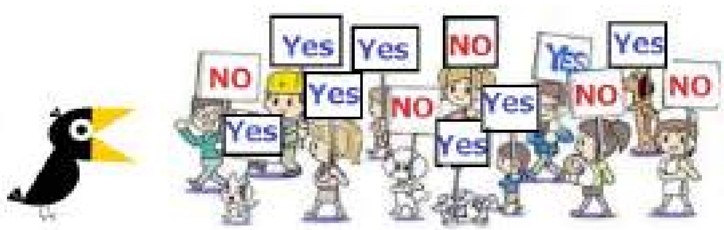
(M_2) By the parallel mixed measurement (*cf.* [74, 78]) $\bigotimes_{i=1}^{100} \mathsf{M}_{C(\Omega)}(\mathsf{O}_B := (\{y,n\}, 2^{\{y,n\}}, F_B),$ $S_{[*]}(\rho_0))$, a measured value $(\underbrace{y, y, y, ..., y}_{100})$ is obtained.

Then, we calculate:

(M_3) the probability that a measured value $(\underbrace{y, y, y, ..., y}_{100})$ is obtained by the parallel mixed measurement $\bigotimes_{i=1}^{100} \mathsf{M}_{C(\Omega)}$ $(\mathsf{O}_B := (\{y,n\}, 2^{\{y,n\}}, F_B), S_{[*]}(\rho_0))$ is given by $(97/100)^{100} (< 0.048)$. That is, the probability that (M_2) is realized (i.e., we meet one hundred black ravens continuously) is less than 0.048 ($> (97/100)^{100}$)).

Thus, if we believe (M_0), a very rare thing (i.e., (M_3)) happened since probability 0.048 is quite rare. Therefore, we should doubt the hypothesis (M_0). That is, we couldn't deny "$\Omega_R \subseteq \Omega_B$ (i.e., any raven is black)". When we can't do such test many times and still deny "$\Omega_R \subseteq \Omega_B$ (i.e., any raven is black)", according to Popper's falsificationism, we will believe this.

Remark 12.22. [Any small black bird is black]
Consider the membership function $m_B : \Omega \to [0,1]$ of "black bird" and the membership function $m_{SB} : \Omega \to [0,1]$ of "small black bird" as follows:



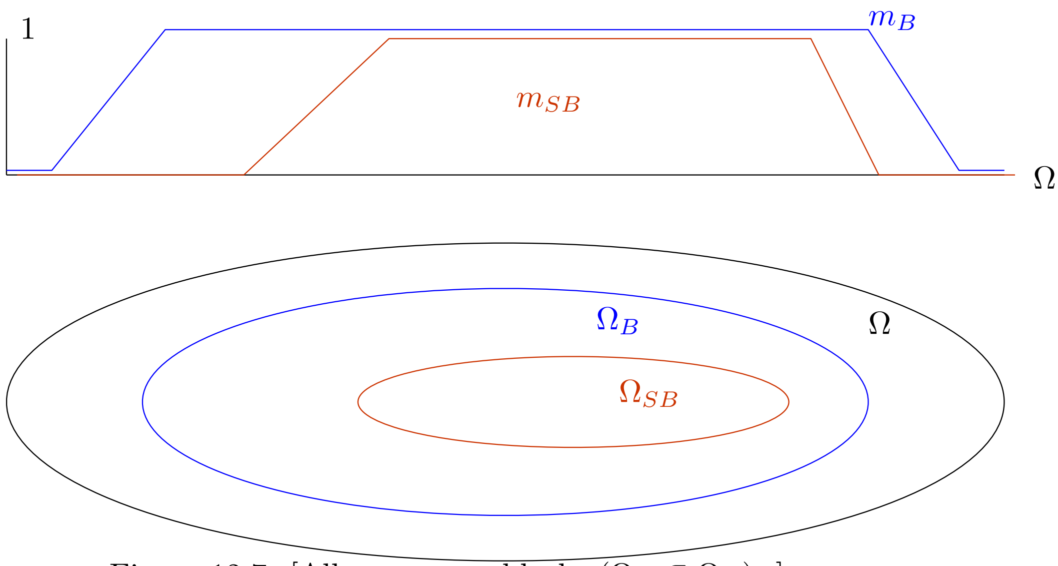
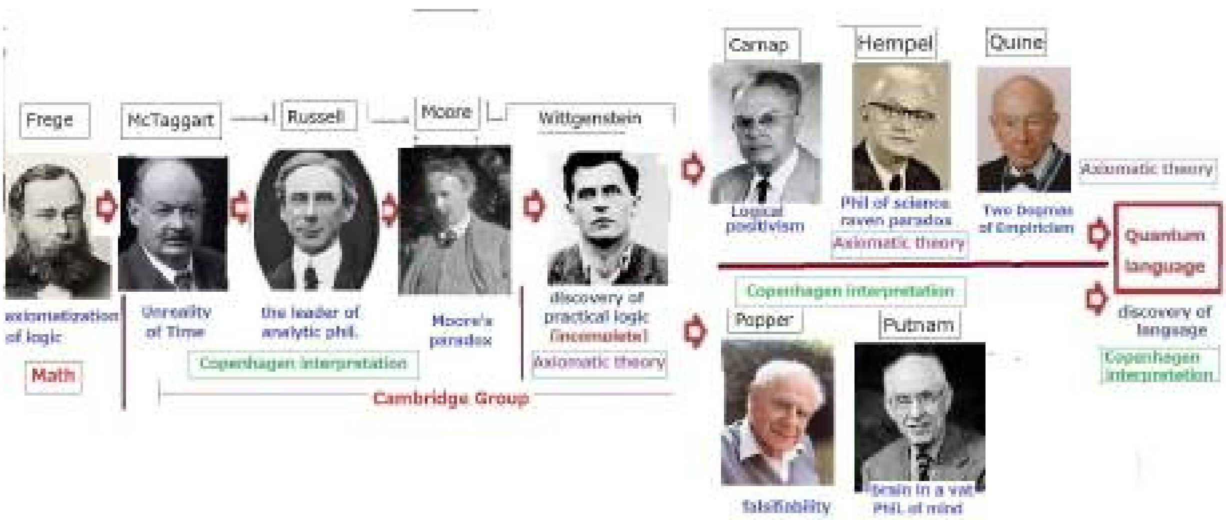


Figure 12.7: [All ravens are black: $(\Omega_R \subseteq \Omega_B)$]

Put $\Omega_B = \{\omega \in \Omega | m_B(\omega) = 1\}$, $\Omega_{SB} = \{\omega \in \Omega | m_{SB}(\omega) = 1\}$. Then, it is natural to consider "$\Omega_{SB} \subseteq \Omega_B$" is self-evident without any need for measurement.

However, as mentioned in Sec.12.3, a proposition in quantum language is a "measurement". Again, see Sec. 4.1.3 ⓕ: Carnap-Quine controversy.

Remark 12.23. I don't think anyone can read (Wittgenstein's) TLP scientifically. But Wittgenstein showed us the direction in which philosophy should go. This is, of course, the most important thing a philosopher can do. Logical positivists such as Carnap, Quine, Hempel, and others, through the problems of analytic and synthetic propositions, the flagpole problem, and Hempel's crow paradox, called for the necessity of a "language of science". And they must have been convinced that the "language of science" was something different from mathematical logic and set theory. However, there was too much distance between their area of expertise and quantum mechanics. I had always wondered how the "philosophy of mind" came about. However, if the "philosophy of mind" can be used as a tool to examine the Copenhagen Interpretation (*cf.* Sec. 9.4.2: Brain in vat), then I can understand its importance.



For further information, see my homepage

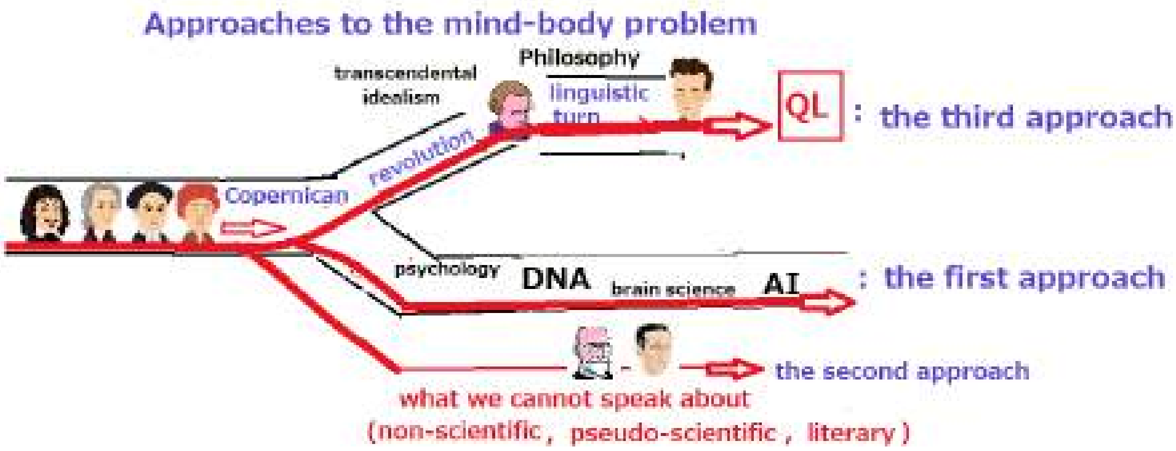
12.8 Three approaches to the mind-body problem

The mind-body problem (i.e., to clarify the relation between "mind" and "body") is the most famous problem in Descartes philosophy. There are three standing positions on whether we see this as a problem of science or a problem of philosophy.

(A) [Science; Sec. 12.8.2: the first approach]: If we are in the position of existential monism (i.e., the scientific position), i.e. if we expect a scientific solution, then the mind-body problem is a problem of brain science, AI, and cognitive science.

(B) [Wordplay in a literary sense; Sec. 12.8.3: the second approach][Illusory problem]: The position that one should enjoy wordplay in a literary sense has historically been most favored by philosophy fans.

(C) [Philosophy; Sec. 12.8.4: the second approach]: if we consider the mind-body problem as a philosophical problem (i.e., dualistic idealism), it is a question of proposing a worldview with "mind" and "body" as the key words.



Of course, our interest is focused on (C). This section was written with reference to the following.

- [70] Ishikawa,S., A Final solution to mind-body problem by quantum language, *Journal of quantum information science*, Vol. 7, No.2, 48-56, 2017, DOI: 10.4236/jqis.2017.72005 (`http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=76391`)

If quantum language is the only scientifically successful theory in dualistic idealism, it is natural to study the mind-body problem in quantum language. This will be discussed in Section 12.8.4 [The third approach].

12.8.1 The mind-body problem

Now let us introduce the mind-body problem, which is said to be the greatest unsolved problem in dualistic idealism.
In spite that the cogito proposition "I think, therefore I am" is non-sense (*cf.* Sec. 8.2), Descartes used it in order to propose Descartes philosophy (i.e., mind-body dualism). That is, his argument is as follows.

(D) If the existence of "I" is deduced from the cogito proposition, the existence of "matter" (which is perceived by "I") is accepted. And further, the medium of "I" and "matter" is automatically accepted as "body (= sensory organ)".

Therefore, the key-words of Descartes philosophy (= mind-body dualism) is

(E) "I"(="mind"), "body"(="sensory organ"), "matter"

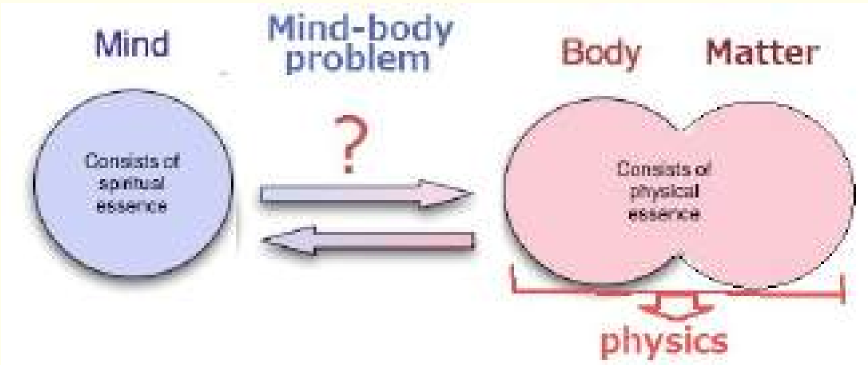
Here, we have the following problem:

(F): Descartes problem (i.e., the mind-body problem) in dualistic idealism

It is natural to consider that "body" and "matter" are ruled in physics.
Then, we have the following problem:

How are "mind" and "body" connected?

(or more generally, how are "mind", "body" and "matter" connected?)

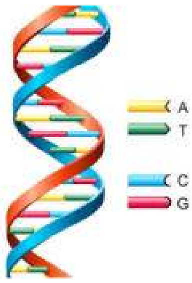


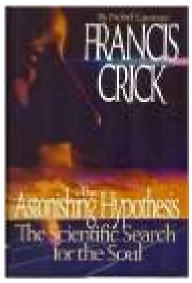
This is generally considered to be the most important problem in Descartes philosophy.

12.8.2 The first approach; Cognitive scientific approach

As mentioned in Note 9.12, Dr. Crick (the most noted for being a co-discoverer of the structure of the DNA molecule in 1953 with James Watson) said in his book "The astonishing hypothesis" [12]) as follows.

(G_1) *You, your joys and your sorrows, your memories and your ambitions, your sense of personal identity and free will, are in fact no more than the behavior of a vast assembly of nerve cells and their associated molecules.*





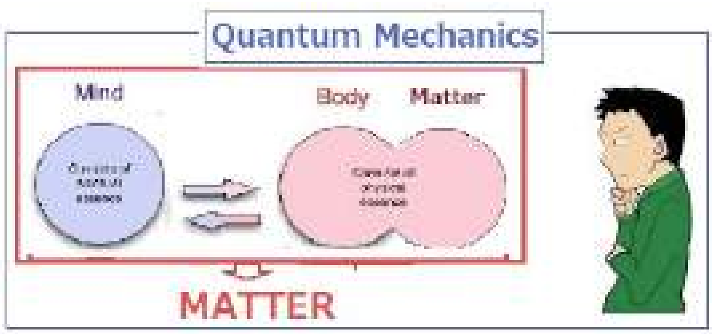
From the scientific point of view, I agree to his opinion (G_1). (i.e., the denial of the substance dualism). Therefore, I believe that the following will be realized.

(G_2) Descartes (Kant,Husserl) $\xrightarrow{\text{progress}}$ Philosophy of mind (based on cognitive science) $\xrightarrow{\text{progress}}$ Brain science $\xrightarrow{\text{progress}}$ Solution (the mind-body problem) .

This means the following.

(G_3) The mind-body problem is solved scientifically (as a matter of quantum mechanics ($\approx$ quantum language))

That is,



But this is what almost all modern scientists believe.

That is because quantum language says:

(H) *Describe any monistic phenomenon (such as* (G_1) *) by dualistic language (=quantum language) !*

12.8.3 The second approach; Illusory problem?

It should be noted that

(I_1) the term 'mind' and 'body' in the mind-body problem (F) is ambiguous in Descartes=Kant epistemology.

Thus, there may be a reason to consider that the mind-body problem is just "what we cannot speak about". Therefore, according to Wittgenstein's famous saying "What we cannot speak about we must pass over in silence" (in [125]), some may conclude that we must speak nothing about the problem (F). That is, the mind-body problem is an illusory problem. As mentioned before, I think that the Wittgenstein's next assertion is spot on.

(I_2) philosophical problems arise from insufficient attention to the variety of natural language use.

12.8.4 * The third approach; Quantum linguistic solution to the mind-body problem

The solution to Descartes' psychosomatic problem with quantum language has already been discussed in Sec.8.3.2, but let us discuss it again here. The mind-body problem was

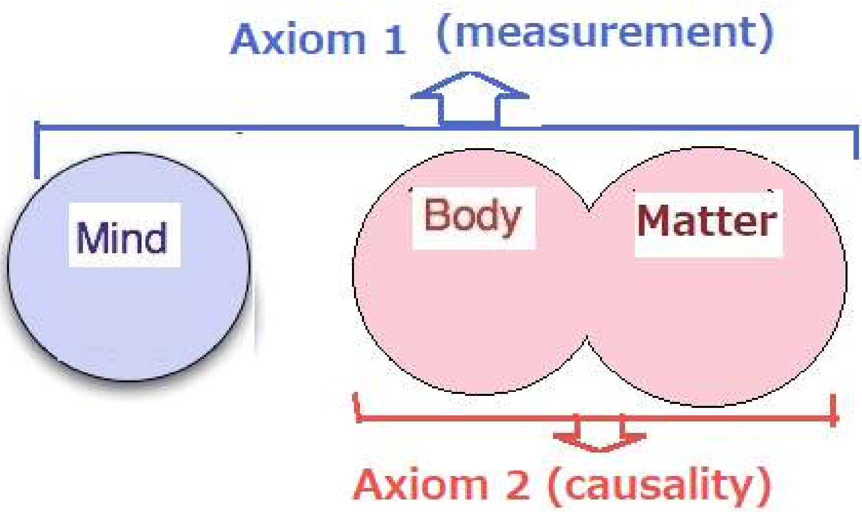
I have already described the solution to Descartes' mind-body problem using quantum language in section ? The mind-body problem was

(J) Describe (in quantum language) the relationship between "mind," "body (sense organs)," and "objects"!

I have already mentioned this many times. I have mentioned this many times, but let us note the following.

$$
\begin{array}{ccccc}
\text{mind} & = & \text{body(sensory organ)} & = & \text{matter} \\
\downarrow{\scriptstyle \text{quantum language}} & & \downarrow{\scriptstyle \text{quantum language}} & & \downarrow{\scriptstyle \text{quantum language}} \\
\text{measured value} & = & \text{observable} & = & \text{state}
\end{array}
$$

Illustrated,



Axiom 1 [measurement]

"I"(="mind", "brain"), "body"(="sensory organ"), "matter"

spiritual dynamics — physical dynamics (Axiom 2 [causality])

Thus, problem (J) can be interpreted as the following problem (K).

(K) Describe in quantum language the relation among "measured value", "observable" and "state"!

Of course, this answer is just Axiom 1 (measurement in Sec. 1.2.1) as follows.

Axiom 1 = The solution to the mind-body problem

- With any system S, a C^*-algebraic basic structure $[\mathcal{A} \subseteq B(H)]$ can be associated in which measurement theory of that system can be formulated. When the observer takes a measurement of an observable (or, by a measuring instrument) $\mathsf{O}{=}(X, \mathcal{P}(X), F)$ for a system $S_{[\rho]}$ i.e. a system S with a state ρ), the probability that a measured value x $(\in X)$ obtained by the measurement is given by $\rho(F(\{x\}))(\equiv {}_{\mathcal{A}^*}(\rho, F(\{x\}))_{\mathcal{A}})$.

Obviously, this Axiom explains the following three relationships

$$
\begin{array}{ccccc}
\text{mind} & = & \text{Body (sensory organs)} & = & \text{matter} \\
\downarrow \text{quantum language} & & \downarrow \text{quantum language} & & \downarrow \text{quantum language} \\
\text{measured value} x (\in X) & = & \text{observable } \mathsf{O} = (X, \mathcal{P}(X), F) & = & \text{state } \rho
\end{array}
$$

This, plus Axiom 2 (causality), is the "solution to the mind-body problem". That is,

($\sharp$) (= (E_5) of Answer 8.7 in Sec. 8.3.2)

(modern) Descartes problem (Establish mind-body dualism!) (Establish QL!)	mind-body problem (Continental rationalism)	$\cdots$	Find Axioms including the key-words (mind, body, matter)
	subjectivity problem (British Empiricism)	$\cdots$	Propose the interpretation concerning this Axioms

Chapter 13

Postscript: Philosophy (of worldviews) has progressed towards quantum language : Philosophy is one

13.1 What is the core of philosophy of science?

Comparing Western and Eastern philosophy, I have the impression that there is something consistently scientific in Western philosophy. The purpose of this text was to clarify something scientific. That is, I concentrated on the question, "What is the core of philosophy of science? I proposed the following Figure 0.1 (in Preface):

Assertion 13.1. (= Figure 0.1 in Preface): The location of QL in the history of western philosophy

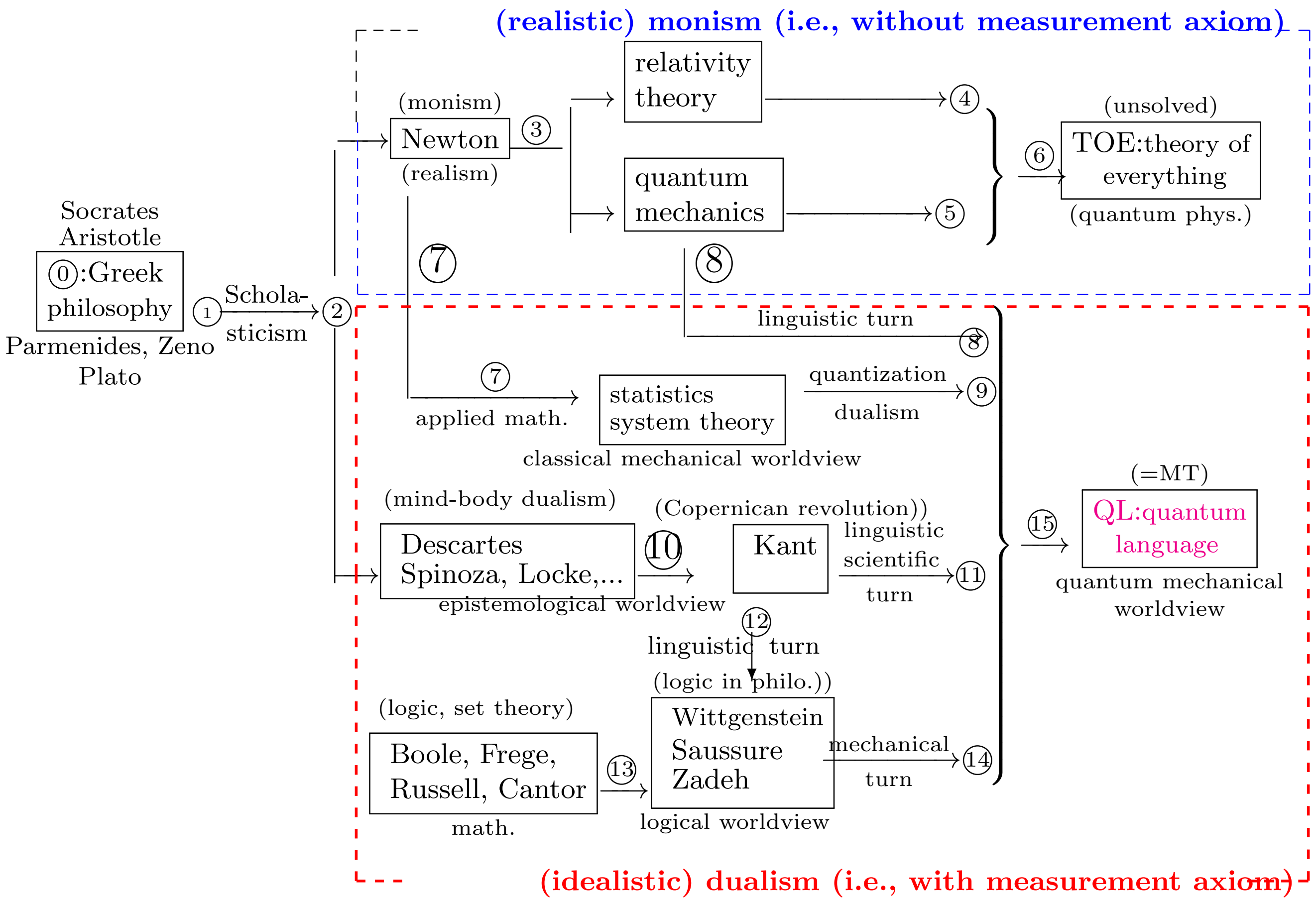
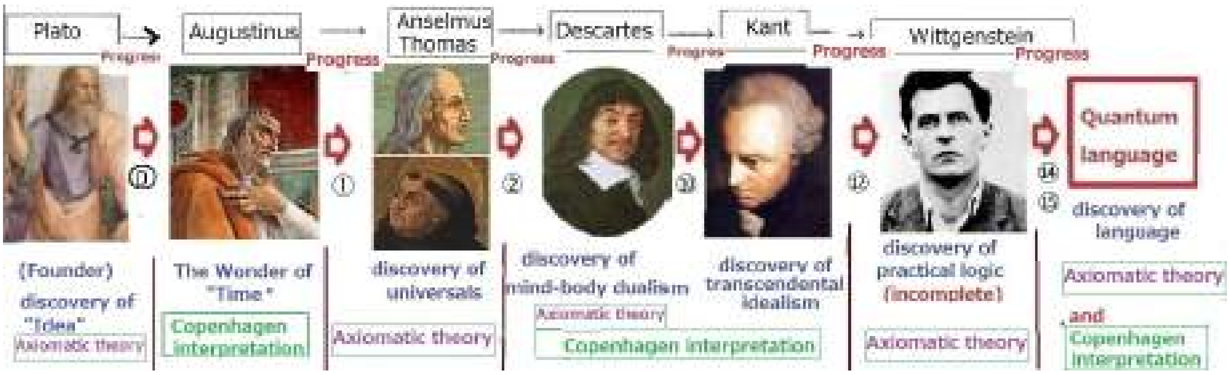


Figure 0.1 (in Preface): The history of the world-descriptions
Philosophy (≈ dualistic idealism) has progressed toward QL
(i.e., ⓪→ ①→ ②→ ⑩→ ⑫→ ⑭→ ⑮)

Writing a history of Western philosophy, a discipline that has produced the greatest number of geniuses, was a rather courageous undertaking for me, a non-specialist in philosophy. Many readers are probably more familiar with the history of Western philosophy than I am. My only advantage is that I am somewhat familiar with quantum language, which is probably the scientific finality of dualistic idealism. If so, I thought that even as a layman I might be able to have some say in the history of Western philosophy.



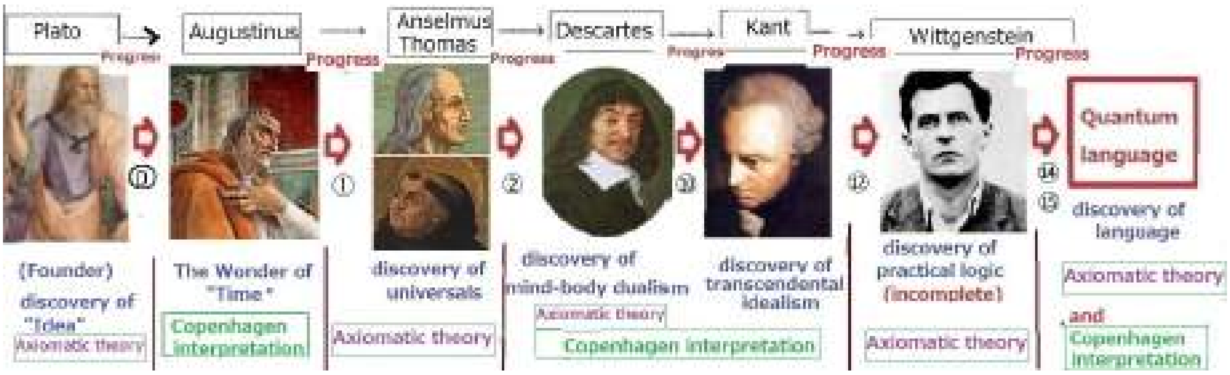
"Does the series **(i.e., ⓪→ ①→ ②→ ⑩→ ⑫→ ⑭→ ⑮)** in the above figure mean progress?" is the biggest question in the History of Western philosophy. Quantum language gives an affirmative answer to this question, if the following definition is acceped:

(B) "$X \xrightarrow[\text{progress(QL)}]{} Y$" is defined that "$Y$ is more like QL than X"

Therefore, in this text, I examined this (B) to confirm the progress of the above series*1.

*1 "History is an unending dialogue between the present and the past." (*cf.* ref. [10]

13.2 Reviews from Plato to Wittgenstein



A summary of their scientific accomplishments follows.

13.2.1 (C_1): Plato

Plato left us with the word "Idea". This was, in fact, the "magic word".

(History of progress from Idea to observables)

Idea — Plato's idea theory —(linguistic version)→ [signifier, [signified]] — Saussure's linguistic theory

Idea ↓ scholastic version ⓐ → [(primary) universal] — Anselmus, Thomas

[(primary) universal] ↓ cognitive version ⓑ → [secondary quality] — John Locke

[signifier, [signified]] ↓ quantification → [membership function] — Zadeh's fuzzy theory

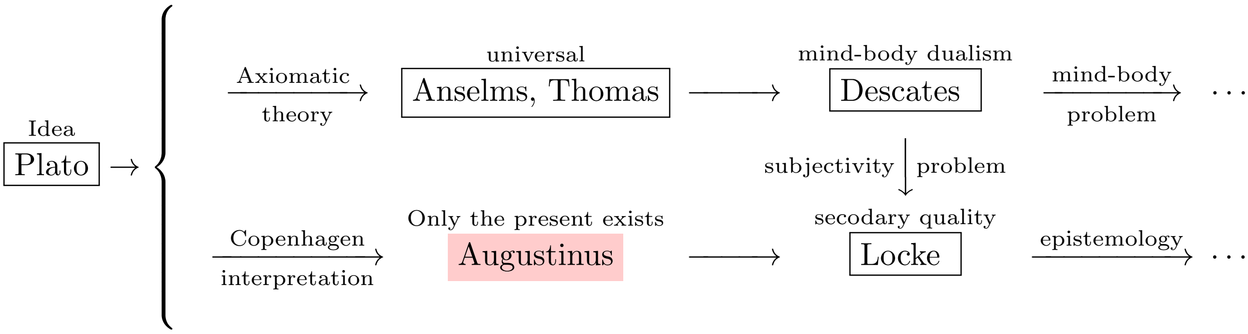
[membership function] ↓ identification → [observable] — quantum language

[secondary quality] —(linguistic version ⓒ)→ [observable]

By scholastic philosophers ⓐ Anselmus (and Thomas), Idea was understood as a scientific concept of "universal". And by Locke ⓑ, Idea became a concept that everyone can understand as "the secondary quality", and in QL ⓒ, Idea became a computable concept as observables.

13.2.2 (C_2): Augustines

St. Augustine offered the hand of God (the hand of salvation) to the dying Platonic philosophy. I think that Parmenides is the founder of "Copenhagen interpretation (*cf.* Sec. 2.3). However, it was Augustine who introduced the Copenhagen Interpretation to Plato's philosophy, arguing that "Only the present exists" as follows:



13.2.3 (C_3): Anselmus and Thomas

Anselmus and Thomas pursued the scientific meaning of "Idea".

(C_3') **Theory of Heaven(by Thomas)**

(C_3^1) :universals exist'after individuals' in the human intellect .
[universalia post rem] as actual world
QL: (i.e., measured value is sensed by the brain (i.e., mind)).

(C_3^2) :universals exist 'before individuals' in the divine intellect, .
[universalia ante res] as Plato's Idea
QL: (i.e., observable (or, measuring instrument) could be in the heavens, like the North Star (*cf.* Note 1.7))

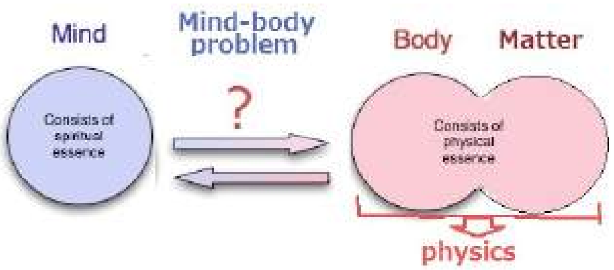
(C_3^3) :universals exist 'in individuals' in the world, .
[universalia in rebus] as Aristotle's eidos
QL: (i.e., state exists in matter)

13.2.4 (C_4): Descartes

Descartes proposed Descartes problem of mind-body dualism (i.e., the mind-body problem and the subjectivity problem)

That is,

($\flat_1$) The mind-body problem: How are mind and body related?
($\flat_2$) The subjectivity problem: Is the world as I see the same as the world as you see it?



This can be stated in relation to quantum language as follows.

(C'_4) **Theory of Earth (by Descartes)**

(modern) Descartes problem
(Establish mind-body dualism!)
(Establish QL!)

- mind-body problem (Continental rationalism) · · · Find Axioms including the key-words (mind, body, matter)
- subjectivity problem (British Empiricism) · · · Propose the interpretation concerning this Axioms

13.2.5 (C_5): Kant

In a Copernican revolution, Kant discovered how to create scientific idealism (transcendental idealism). It is quite powerful, as follows. Especially ⑧ is the most important

copy theory [the world ⟶ cognition] Descartes=Locke —— ⑩:Copernican revolution [I] in cognition (due to Kant) ⟶ constructionism [cognition ⟶ the world] Kant

⑫ ↓ linguistic turn

copy theory [things ⟶ words] language —— Copernican revolution [II] (= Saussure revolution) in language (due to Saussure) ⟶ constructionism [words ⟶ things] Saussure, Wittgenstein

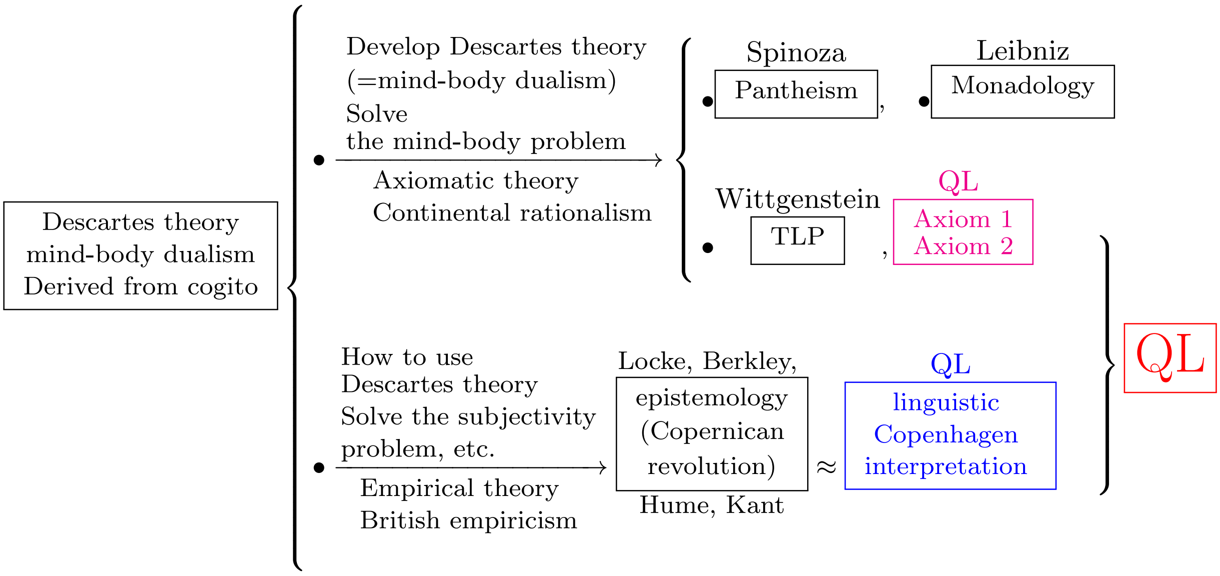
⑭ ↓ mechanical turn

copy theory [phenomena ⟶ QM] quantum mechanics (=QM) —— ⑧ Copernican revolution [III] in quantum mechanics ⟶ constructionism [QL ⟶ phenomena] quantum language (=QL)

ⓐ ↑ quantumization (left) ; ⓒ ↓ classicization (right)

copy theory [phenomena ⟶ CM] classical mechanics (=CM) —— ⑦ Copernican revolution [IV] in classical mechanics (Fisher, etc.) ⟶ constructionism [statistics ⟶ phenomena] statistics, system theory

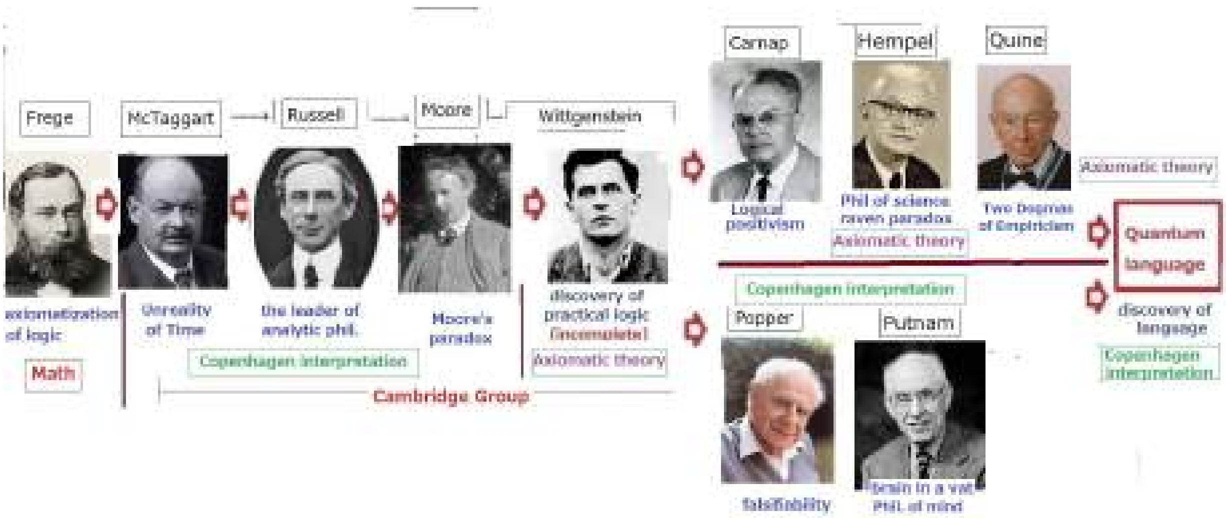
However, Kant's synthesis can be rewritten as QL-synthesis as follows.



QL-synthesis

13.2.6 (C_6): Wittgenstein

I could not read Wittgenstein's work TLP (="Tractatus Logico-Philosophicus") as a scientific academic book. However, as can be seen from the figure below, the influence of TLP was enormous.



Looking at this figure, I think he insisted, "Make up the language of science!" in TLP. Though he said "What we cannot speak about we must pass over in silence" in TPL, he could not define "scientific proposition". However, as shown in the above figure, Wittgenstein changed the trend of philosophy. I think this is what should be most appreciated in philosophy. I believe Wittgenstein's dream was made possible by quantum language (*cf.* Sec. 12.1).

13.3 Almost all open questions of dualistic idealism have been solved; Wittgenstein's dream came true

I thought I would continue the discussion in the previous section a bit further, but the reader may find the following

(B′) "$X \xrightarrow[\text{progress(QL)}]{} Y$" is defined that "$Y$ is more like QL than X"

Aren't you just making up your own arbitrary yardstick like the one above and dogmatically claiming that "quantum language" is the goal, etc.? Enough with the nonsense, if you claim that quantum language is the final form of dualistic idealism, then solve the multitude of unsolved problems! "Proof is better than argument!"
I think you are right. Thus I showed the solutions of unsolved problems ((D$_1$) and (D$_2$)) as follows:

$$\begin{cases} (\text{D}_1)\text{: solutions of unsolved problems in this text} \\ (\text{D}_2)\text{: solutions of unsolved problems in ref. [74]} \end{cases}$$

(D$_1$) **List of unsolved problems in Western philosophy for which solutions are given in this text**

See (D$_1$) in Preface

In other words, this text's argument was the following.

Plato → Christianity (Anselmus, Thomas Aquinas; Augustinus) → Descartes=Kant (Spinoza, Leibnitz; Locke, Berkeley, Hume) → Analytic phil. (Wittgenstein; Phil. of science)

Era of search for dualistic idealism (various unresolved issues arose)

↓

QL

Goal (Solutions in this text)

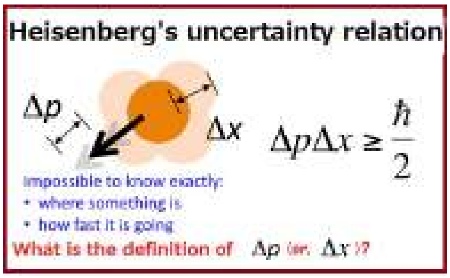
In this text, this was realized. Thus I believe that

QL is the scientific final form of dualistic idealism.

(D$_2$) **The list of my answers for scientific unsolved problems in (D$_2$)**

ref. [74]; Linguistic Copenhagen interpretation of quantum mechanics; Quantum Language [Ver 5], Research Report, Dept. Math. Keio University, KSTS/RR-19/003 (2019); 473 p (http://www.math.keio.ac.jp/academic/research_pdf/report/2019/19003.pdf)

- Kolmogorov's extension theorem in quantum language (Sec.4.1 in ref. [74]) (Sec.4.1 in ref. [74])
- The law of large numbers in quantum language (Sec.4.2 in ref. [74])
- The discovery of Heisenberg's uncertainty relation (*cf.* ref. [38], or, Sec. 4.3 in ref. [74])

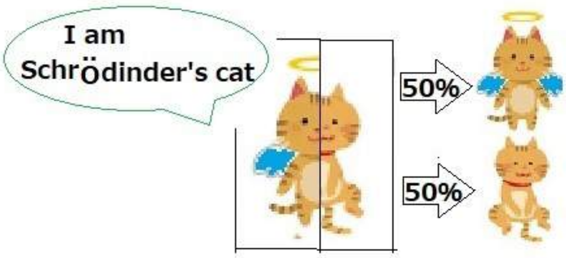


Has Heisenberg's uncertainty relation ever been used effectively in physics? (ref. [38], or, Sec. 4.3 in ref. [74])

- Bell's inequality holds in both classical and quantum systems (Sec. 4.5.2 in ref. [74])
- Measurement theoretical formulation of measurement, inference, control (Sec. 5.2 in ref. [74])
- Monty-Hall problem in quantum language (non-Bayesian approach) (ref. [51], Sec.5.5 in ref. [74])

- Two envelope problem in quantum language (non-Bayesian approach) (ref. [61], Sec.5.6 in ref. [74])
- Confidence interval and statistical hypothesis test (Chapter 6 in ref. [74])
- Analysis of variance (Chapter 7 in ref. [74])
- Syllogism holds in classical systems, but not in quantum systems (Sec.8.6 and Sec.8.7 in ref. [74])
- Mixed measurement theory (Bayesian measurement theory) (Chap. 9 in ref. [74])

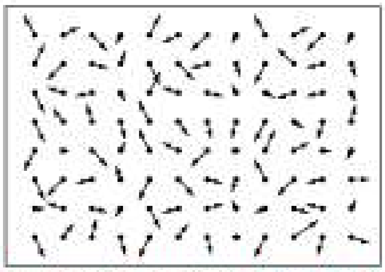
- The measurement theoretical characterization of the wave-function collapse (= projection pustulate) (*cf.* ref. [65], or Sec.11.2 in ref. [74])



(von Neumann-Lüders Projection Postulate can be justified in QL: ref. [65])

- The measurement theoretical characterizations of de Broglie's paradox, quantum

Zeno effect, Schrödinger cat, Wigner's friend, Wheeler's delayed choice experiment, Hardy Paradox, quantum eraser (Sec.11.3∼Sec.11.8 in ref. [74])
- The measurement theoretical characterizations of double-slit experiment, Wilson cloud chamber (Sec.12.2, Sec.12.3 in ref. [74])
- The measurement theoretical characterizations of regression analysis (Sec.13.2 in ref. [74])
- The measurement theoretical characterizations of Brownian motion, Zeno's paradox (Sec.14.2, Sec.14.4 in ref. [74], also, see [47])
- The measurement theoretical characterizations of least-squares method (Chap.15 in ref. [74])
- The measurement theoretical characterizations of Kalman filter (Chap.16 in ref. [74])
- The measurement theoretical characterizations of equilibrium statistical mechanics (ref. [52] or, Chap.17 in ref. [74])



Equilibrium statistical mechanics

- The measurement theoretical characterizations of psychological tests (Chap.18 in ref. [74])
- The measurement theoretical characterizations of belief (Chap.19 in ref. [74])
- The mathematical foundation of science (Hempel's raven paradox) (Chap.20 in ref. [74])

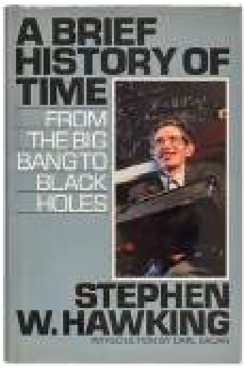
We believe that the primary goal of the philosophy of science is to propose a "language of science." Looking at the above list (D_1) and (D_2), many readers may then agree followings:

(E) As a candidate for the language of science, statistics is currently the most promising candidate, but there is a good chance that quantum language will be more promising in the near future

13.4 Philosophy of science is the scientific part of Western philosophy

13.4.1 Dr. Hawking

Dr. Hawking said, in his best seller "A Brief History of Time", as follows.



- *However, in the nineteenth and twentieth centuries, science became too technical and mathematical for the philosophers, or anyone else except a few specialists. Philosophers reduced the scope of their inquiries so much that Wittgenstein the most famous philosopher this century, said "The sole remaining task for philosophy is the analysis of language." What a comedown from the great tradition of philosophy from Aristotle to Kant!*

I agree to his opinion. Three notable jobs of the 1920s are the following.

(i) Tractatus Logico-Philosophicus by L. Wittgenstein in ref. [125] (1921)
(ii) Fisher's historical masterpiece "Statistical Methods for Research Workers" in ref. [22] (1925)
(iii) Born's discovery "the probabilistic interpretation of quantum mechanics" (*cf.* ref. [6] (1926)

Since then, quantum mechanics and statistics have made great developments. Compared to this, the development of analytic philosophy has been slow. No wonder it is about time that a theory integrating the three has emerged.

13.4.2 The purpose of philosophy of science is to create a language of science

My specialty is nonlinear analysis and mathematical physics (e.g. refs. [36, 37, 38, 39]). Heisenberg's uncertainty principle (*cf* ref. [28]; 1927), derived from his γ ray microscope thought experiment, is expressed as follows:

$$\delta_q \cdot \delta_p \approx \hbar (:= \text{Plank constant}) \tag{HU}$$

The author proposed the mathematical definitions Δ_q and Δ_p, which are considered as the mathematical represenions of δ_q and δ_p. And I proved the following (mathematically, not by thought experiment) (ref. [38]).

$$\Delta_q \cdot \Delta_p \geq \hbar/2 \tag{MU}$$

which is the mathematical representation of (HU).

I immediately shifted my research to the application side to propose a revolutionary measuring instrument based on this principle, but without remarkable results. I still find it curious that the most famous principle in quantum mechanics has no applications, despite the fact that the formula for the calculation of errors has been clearly defined[*2].

It was at this time that I became interested in the Copenhagen interpretation, thinking that it was due to the author's lack of understanding of the Copenhagen interpretation.

This is how I got hooked on the Copenhagen Interpretation, a collection of aphorisms on how to use quantum mechanics.

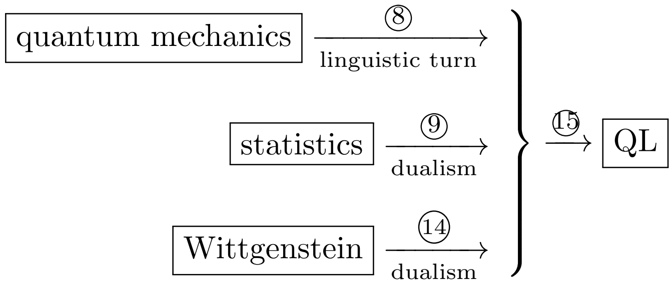
And I proposed:

$$(\mathrm{F})^{*3}: \text{[quantum language]} = \begin{cases} (\mathrm{F}_1)\text{: [mathematical generalization of quantum mechanics]} \\ (\mathrm{F}_2)\text{:[(true) Copenhagen interpretation]} \end{cases}$$

That is, the true Copenhagen interpretation does not belong to quantum mechanics but quantum language.

I put the (F) against the "fuzzy sets" that were popular at the time and claimed the scientific validity of "fuzzy logic" (refs. [41, 42, 43, 44]; 1997-2000, and ref. [46]; 2006) Adding ref.[81]; 2021, the whole picture of quantum language looked like the follows:

(G) (=(E) in Sec. 11.1.2)



Since quantum mechanics, statistics, and logic were the fundamental theories that influenced ordinary scientists in the 20th century, it may be a promising field if all three can be combined in a single theory called quantum language.

Around this time, I learned that there was a discipline called "philosophy of science. At first, the sense of philosophy of science was very humanistic, and there were many aspects of it that I did not understand.

However, the author realized that philosophy of science could be read interestingly from the viewpoint of "quantum language. He thought that the purpose of philosophy of science was "to create a language to talk about science. If so, I thought that this purpose had already been realized in "quantum language.

[*2] If I am allowed to joke, I would like to add "Heisenberg's uncertainty principle is of no practical use" to the Copenhagen interpretation.

[*3] Although not mentioned in this document, (F_1) and (F_2) may be independent.

One book on the philosophy of science said, "The scientific part of Western philosophy is the philosophy of science," so I read some philosophy books, but I did not feel that I understood. However, I did understand that the history of Western philosophy is a treasure trove of unsolved problems concerning dualistic idealism. I thought that the science part of the history of Western philosophy is the history of "creating a language to talk about science. I concentrated on this when I realized that most of the unsolved problems of philosophy could be easily solved by quantum language. The results of this work are described in this text.

The author has taken a very shortcut when it comes to "creating a language that speaks science". He has merely taken advantage of the labor of geniuses (Bohr, Heisenberg, Schrödinger, Born, von Neumann, etc. in quantum mechanics; Fischer, Kolmogorov, Kalman, etc. in statistics; Frege, Cantor, Zadeh, etc. with regard to logic and sets).

13.4.3 QL wants rivals!

Philosophy is a kind of literature that can be enjoyed by the general public. Moreover, even if we limit ourselves to the scientific aspects of philosophy, the general public can enjoy it. This is one of the attractions of philosophy. I have often emphasized in this text that this appeal is due to the following distinction (*cf.* (E) in Sec. 8.1.2).

(H)

(H_1): Philosophy for Professionals · Axiomatic theory
The problem of universals, Continental Rationalism, Statistics, Philosophy of Science (Hempel, etc.), QL (Axiomatic theory)

(A great scientific success has been statistics, but it is not generally regarded as a philosophy since it does not belong to dualism)

(H_2): Philosophy for the Masses · Empirical theory
Descartes-Kant Epistemology, Philosophy of Science (Popper, etc.) Philosophy of Mind, QL (Copenhagen Interpretation)

(Gaining the support of the masses tends to be considered a success)

The caveat above is that the difference between professionals and amateurs is not the level of research. One can think of it as "professional=mathematics required (especially Axioms 1 and 2)" and "amateur=explanation of rules that cannot be expressed in mathematical formulas (Copenhagen Interpretation)". For example, although not mentioned much in this text, various examples discussed in the philosophy of mind (e.g., the qualia problem and the brain in a vat (Sec. 9.4)) are very useful for forming Copenhagen interpretations.

This text was written for the general public, focusing on the Copenhagen interpretation (Ref. [74] is for professional use). Since this text is of type (H_2), there should be a deep discussion of the Copenhagen interpretation, even if the mathematics is treated lightly.

Therefore, I have stuck to a simple C^*-algebraic formulation of the quantum language.

Moreover, in the measurable space $(X, \mathcal{P}(X))$, X was assumed to be a finite set, which made it easier to understand, but also prevented deeper discussion. The W^*-algebraic formulation of the quantum language was also appealing, but to make it clear that this text is intended for the general public, we stuck to the C^*-algebraic formulation.

A more fundamental problem is that " 'Why is quantum language possible?' is not known". In physics, we can proceed by comparing the phenomena/experimental results to confirm their truth or falsity.

However, Axioms 1 and 2 are mere spells, and in the (linguistic) Copenhagen interpretation, there are no clear criteria for "adopting or rejecting a statement.

In extreme terms, it is just a decision based on my preference.

Thus, I feel we need a theory on the criteria for the Copenhagen interpretation, but that may not be possible.

"Is the Copenhagen interpretation uniquely determined in the first place?" and "It may be impossible to create a perfect Copenhagen interpretation (Wittgenstein's paradox; Sec. 12.2.3)," and there are many other things I cannot answer when theorists prod me.

In the end, all I can say now is the following

(I) We can solve a large number of unsolved problems listed in (D_1) in Preface and (D_2) in Sec. 13.3, so let's not get too theoretical and keep solving problems.

Quantum language is a mysterious discipline. Quantum language is a proposed discipline as one attempt to integrate 2,500 years of Western philosophical history, quantum mechanics, and statistics. Therefore, the world the author sees now is a very small part of quantum language. And I believe that QL is the final form of dualistic idealism. Even so, the most desirable thing for quantum language is for rivals to emerge. I believe that comparing with strong rivals will clarify the various issues of quantum language.

I hope that the wise readers will explore the many aspects of quantum language.

December 2022
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*4 For the further information concerning quantum language (e.g. an improved version of this text may be published), see home page: https://ishikawa.math.keio.ac.jp/indexe.html

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