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by

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Abstract. Several bounds and approximation methods for computing the distribution function of the maximum absolute component of the multivariate t distribution are compared to obtain the P value in multiple comparison procedure. The results show the superiority of the improved Monte-Carlo method to the other tested methods, and its accuracy is evaluated to determine the required simulation size.

1.Bounds and Monte-Carlo approximations of multivariate t probabilities.

Let $\boldsymbol{X} = (X_1, X_2, \cdots, X_r)'$ be a multivariate normal random variable following $N(\boldsymbol{\mu}, \sigma^2\Sigma)$. The mean $\boldsymbol{\mu}$ is unknown and to be tested, σ^2 is an unknown positive parameter, and Σ is a known nonnegative definite matrix. We assume the availability of V/ν, an estimate of σ^2 such that V/σ^2 is a chisquare variable with d.f. ν.

Let A be a matrix with r rows and m columns such that $A'\mathbf{1} = \mathbf{0}$. Put $\boldsymbol{Y} = (Y_1, Y_2, \cdots, Y_m)' = A'\boldsymbol{X} \sim N(A'\boldsymbol{\mu}, \sigma^2\Lambda)$ where $\Lambda = A'\Sigma A$, which is assumed to be positive definite. Further, put $\Delta = (\mathrm{Diag}\Lambda)^{-1/2}$, where $\mathrm{Diag}\Lambda$ is the diagonal matrix with the diagonal components of Λ. If $A'\boldsymbol{\mu} = \mathbf{0}$, then $\boldsymbol{T} = (T_1, T_2, \cdots, T_m)' = \Delta\boldsymbol{Y}/\sqrt{V/\nu}$ is a multivariate t random variable with the correlation matrix $R = \Delta\Lambda\Delta$.

Put $T_{max} = \max_{1\le j\le m}|T_j|$. For an observed value t_0 of T_{max}, $P_{max}(t_0) = \Pr(T_{max} > t_0 | A'\boldsymbol{\mu} = \mathbf{0})$ is its P value, namely the sample significance level. The purpose of this paper is the computation of $P_{max}(t)$ for some t, and the following three types of bounds and Monte-Carlo approximations are examined.

Let A_j denote the event $|T_j| > t$, $1 \le j \le m$, and K the number of events which occur. Put $p = \Pr(K > 0) = \Pr(\bigcup_{j=1}^{m} A_j)$ and $S_l = \sum_{1\le j_1<\cdots<j_l\le n}\Pr(A_{j_1}\cdots A_{j_l})$. S_l is the binomial moment of K of order l, namely $\mathrm{E}[\binom{K}{l}]$.

(1) Simple Bonferroni bounds:

$$S_1 - S_2 + \cdots - S_{2j} \le p \le S_1 - S_2 + \cdots + S_{2j+1}. \tag{1.1}$$

The bounds of single, two and three terms of (1.1) are used. The values of S_l's are computed by numerical integration, (See Appendix 1).

(2) A bound by Hunter(1976):

$$p \le S_1 - \sum_{e_{kl}\in\mathbb{T}}\Pr(A_kA_l), \tag{1.2}$$

where the sum runs over such (k, l) that e_{kl} is an edge of a tree $\mathbb{T}$ of the graph whose vertices correspond to the events $(A_j)_{j=1}^m$. The tree $\mathbb{T}$ should be the 'maximum spanning' to get smaller bound.

(3) Bounds by Kwerel(1975 a, 1975 b):

$$\frac{2S_1}{\lambda+1}(1-\frac{\alpha}{\lambda}) \leq p \leq S_1 - \frac{2}{m}S_2, \tag{1.3}$$

Where $\alpha = S_2/S_1$, $\lambda = [2\alpha+1]$, and [ ] denotes the Gauss symbol.

$$\begin{aligned} &\frac{\omega(\omega+1)\beta_1 + (2\omega+1)(m\beta_1-\beta_2) - (m\beta_2-\beta_3)}{\omega(\omega+1)m} \leq p \\ &\leq \frac{\mu(\mu+1)\beta_1 - (2\mu+1)(\beta_2-\beta_1) + (\beta_3-\beta_2)}{\mu(\mu+1)}, \end{aligned} \tag{1.4}$$

Where $\beta_1 = S_1$, $\beta_2 = S_1 + 2S_2$, $\beta_3 = S_1 + 6S_2 + S_3$, $\omega = [(m\beta_2-\beta_3)/(m\beta_1-\beta_2)]$, and $\mu = [(\beta_3-\beta_2)/(\beta_2-\beta_1)]$.

(4) Monte-Carlo approximations:

Let $N(\mathbf{0}, R)$ be an m-variate normal distribution with the 'known standard variance 1' and the known correlation matrix R. Let $\boldsymbol{Z}_i = (Z_{i1}, ..., Z_{im})', 1 \leq i \leq N$, be a sequence of random numbers following $N(\mathbf{0}, R)$. $\boldsymbol{Z}_i$ can be obtained by using m-variate normal random numbers $\boldsymbol{W}_i = (W_{i1}, \cdots, W_{im})'$ following $N(\mathbf{0}, I_m)$. Let B be the matrix such that $B'B = R$. For example, $B'B$ is the Cholesky decomposition of R. Then $B'\boldsymbol{W}_i$ follows $N(\mathbf{0}, R)$, (See Appendix 2).

For generating a sequence $T_{max,i}, 1 \leq i \leq N$, of the random numbers of our interest, $\xi_i = \max_{1 \leq j \leq m} |Z_{ij}|$ is Studentized by the following two methods.

1) Naive Monte-Calro.

This is just to divide ξ_i by $\sqrt{V^*/\nu}$, where V^* is an independent chi-square variable with d.f. ν. The complementary distribution function $P_{max}(t)$ of T_{max} is estimated by counting $\#\{i : T_{max,i} > t\}$ for some selected values of t.

2) Improved Monte-Calro.

Let Ψ_N^* be the empirical distribution function of the simulation values $(\xi_i)_{i=1}^N$, and let g and G be the probability density and the distribution functions of $\sqrt{V^*/\nu}$, respectively. Then $P_{max}(t)$ is estimated by

$$1 - \int_0^\infty \Psi_N^*(ts)g(s)ds = 1 - \frac{1}{N}\sum_{i=1}^N \mathrm{G}(\xi_i/t). \tag{1.5}$$

2. Comparison of the accuracy of each method.

To compare the accuracy of each method, Dunnet's multiple comparisons with unequal observations on n populations are considered. That is, multiple comparisons with a control group. Put $\lambda_j = (1 + r_0/r_j)^{-1/2}, j = 1, 2, \cdots, m = n-1$, where r_j is the number of observations on the j-th population (r_0 is that on the control group). Only the ratio r_0/r_j

is significant as λ_j shows. Then, the (k,l)-th element of the correlation matrix R is 1 if $k=l$, and $\lambda_k\lambda_l$ if $k \neq l$.

For this problem, $P_{max}(t)$ is explicitly expressed by a double integral, Hochberg and Tamhane(1987):

$$(2.1) \qquad 1-\int_0^\infty \int_{-\infty}^\infty \prod_{j=1}^m \{\Phi[\frac{\lambda_j z+ts}{(1-\lambda_j^2)^{1/2}}]-\Phi[\frac{\lambda_j z-ts}{(1-\lambda_j^2)^{1/2}}]\}\phi(z)dzg(s)ds,$$

where $\phi(z)$ and $\Phi(z)$ are the probability density and distribution function of the standard normal distribution, respectively.

For two- and three-dimensional numerical integrations, including that of S_2 and S_3, 'good lattice point method' is used, Mori(1987).

The parameters in the calculation are selected as follows.

a) Number n of populations compared, that is number $m=n-1$ of paired comparisons.

$$n=3,5 \text{ and } 7; \quad m=2,4 \text{ and } 6.$$

b) Pattern of number of observations. (Only the rations are meaningful.)

$$(1)1:1:\cdots:1, \quad (2)n-1:1:\cdots:1, \quad (3)1:2:\cdots:n, \quad \text{and} \quad (4)n:n-1:\cdots:1.$$

c) Degree of freedom of the error variance.

$$\nu=5,10 \text{ and } 20.$$

d) t value.

$$t=0.125,0.25,0.5,1.0,1.5,2.0,2.5,3.0,4.0 \text{ and } 6.0.$$

e) Simulation size $N=500\times 5$, repeated 5 times for variance estimation.

For generating normal random variables and for the other computations, the New S(1988) was used.

Examples of the absolute errors of P values for each method are shown in Table 2.1. The other results are shown in Appendix 3.

Table 2.1. Absolute errors of the bounds and approximations. (The parameter values are restricted to $n=7$, no. of obs $=1:1:1:1:1:1:1$, $\nu=10$).

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.418	6.991	0.331	0.028	0.331	-7.843	-0.011	-0.001	0.000	0.000	0.000	0.000
0.250	1.000	3.846	4.849	0.547	0.088	0.547	-6.050	-0.044	-0.011	-0.001	0.002	0.000	0.000
0.500	0.989	2.778	2.372	0.689	0.149	0.689	-3.489	-0.109	-0.044	0.001	0.005	0.001	0.003
1.000	0.816	1.229	0.647	0.444	0.112	0.444	-1.127	-0.186	-0.091	0.001	0.010	-0.002	0.014
1.500	0.507	0.480	0.188	0.199	0.053	0.199	-0.363	-0.130	-0.077	0.004	0.024	0.003	0.021
2.000	0.263	0.177	0.055	0.079	0.018	0.079	-0.116	-0.067	-0.031	0.006	0.017	0.005	0.019
2.500	0.124	0.065	0.017	0.030	0.006	0.030	-0.038	-0.032	-0.011	0.006	0.011	0.002	0.012
3.000	0.056	0.024	0.005	0.012	0.002	0.012	-0.013	-0.013	-0.004	0.001	0.010	0.000	0.007
4.000	0.011	0.004	0.000	0.002	0.000	0.002	-0.002	-0.002	-0.001	0.001	0.001	-0.001	0.002
6.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.000	0.000

tval: t value, Exact: true P value, Bonf: Bonferroni bounds(1st, 2nd, 3rd order) Hunt: Hunter's bound, Kw: Kwerel's bounds(2nd, 3rd order; U: upper bound, L: lower bound), N.M.C.: Naive Monte-Carlo approximation, I.M.C.: Improved Monte-Carlo approximation, sd: standard deviation of Simulation.

The above result and others, tabulated in Appendix 3, show that the Monte-Carlo method, especially improved one, is superior to the other methods. Edwards and Berry(1987), stated similar observations in relation with the computation of critical values.

Note that the error variance of Improved Monte-Calro method is uniformly smaller than that of Naive Monte-Carlo method. In fact, if π denotes the expectation value of $G(\xi/t)$

$$\mathrm{Var}[G(\xi/t)] = E[G(\xi/t)^2] - E[G(\xi/t)]^2 \leq \pi - \pi^2 \tag{2.2}$$

The last expression times $1/N$ is the variance of Naive Monte-Carlo estimate of π.

3. Evaluation of Improved Monte-Carlo approximation.

In order to estimate the required simulation size for accurate enough P value, the evaluation of the accuracy of Improved Monte-Carlo method is necessary, and the distribution of ξ and $G(\xi/t)$ should be studied.

An important and difficult point is its dependence on the correlation matrix R of Z_i. If Z_i is independent, then the distribution function of ξ becomes

$$\Psi_m(\xi) = (2\Phi(\xi) - 1)^m, \tag{3.1}$$

where Φ is the standard normal distribution function. If all the correlation coefficients among the components of Z_i are ± 1, then ξ_i is distributed as $|Z_{ij}|$ and

$$\Psi_m(\xi) = 2\Phi(\xi) - 1. \tag{3.2}$$

It is conjectured that for any positively correlated m-variate normal variable, $\Psi_m(\xi)$ is between these two distribution functions.

Another difficulty is the evaluation of the variance of ξ and $G(\xi/t)$. In order to avoid it, its inter-quantile distance is considered. In general, the inter-quartile distance of a distribution is rather stable for some change of parameter values. In the present case also. Some numerical experiences support this conjecture.

First, the distribution of ξ is studied by Monte-Carlo method. The parameter of the simulation is selected as follows.

(1) Correlation matrix R of Z_i.

a) Intra-class correlation model. b) AR(1) model.

$$R = \begin{pmatrix} 1 & \rho & \dots & \rho \\ \rho & 1 & \dots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \dots & 1 \end{pmatrix} \qquad R = \begin{pmatrix} 1 & \rho & \dots & \rho^{m-1} \\ \rho & 1 & \dots & \rho^{m-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{m-1} & \rho^{m-2} & \dots & 1 \end{pmatrix}$$

(2) The number of observations $m = 5$.

(3) Simulation size $N = 500$.

Figs. 3.1 and 3.2 show the box plots of $\xi_i, 1 \leq i \leq N$, and Figs. 3.3 and 3.4 show the inter-quartile distance of ξ_i for each ρ. In these figures, it is shown that the median of ξ becomes smaller as ρ increases, and the distribution of ξ is almost the same when ρ is smaller than 0.4. Further, it is shown that the inter-quartile distance of ξ is stable against the change of correlation ρ.

Next, the distribution of $G(\xi/t)$ is calculated as follows. Let $\xi_{0.25}$ and $\xi_{0.75}$ be the first and the third quantile of ξ. Then, the inter-quartile distance of $G(\xi/t)$ is

$$G(\xi_{0.75}/t) - G(\xi_{0.25}/t). \tag{3.3}$$

By using $\widehat{\xi}_{0.75}$ and $\widehat{\xi}_{0.25}$ estimated from the Monte-Carlo method, the inter-quartile distance of $G(\xi/t)$ is estimated. Figs. 3.5, 3.6 and 3.7 show the inter-quartile distance of $G(\xi/t)$ in I.C. correlation model for $\nu = 5, 10$ and 20, respectively. The distance is unstable for the change of ρ, but when ρ is smaller than 0.4, it can be roughly approximated by that for $\rho = 0$, and the relative error of the approximation for the inter-quartile distance of $G(\xi/t)$ is not greater than 10 %. Figures for AR(1) model are almost the same.

Figs. 3.8, 3.9 and 3.10 show the true P values of inter-quartile distance of $G(\xi/t)$ in the independent case of $m = 1, 2, \cdots, 6$ for $\nu = 5, 10$ and 20, respectively. It can be calculated by (3.3) and used to determine the simulation size.

The following procedure shows a way for determining the simulation size to obtain the P value of which inter-quartile distance is not greater than a predetermined value d^* :

(1) Make a rough estimate of the P value by a small simulation, for example, with simulation size $N^* = 100$.
(2) Evaluate the inter-quartile distance of $G(\xi/t)$, d, for the independent case by using (3.3) and the estimated P value at stage (1).
(3) The required simulation size N is nearly equal to $(d/d^*)^2$.

For example, the required simulation size becomes as shown in Table 3.1 and 3.2 for $d^* = 0.01$, $\nu = 5$ and 20, respectively. It should be noted that d is regarded as a scale parameter in (3) above. The larger ν is, the larger simulation size must be.

Table 3.1 Required simulation size for $d^* = 0.01$ and $\nu = 5$.

P value	$m = 1$	$m = 3$	$m = 5$
0.05	1000	200	100
0.1	3000	600	400
0.25	7000	2000	1500
0.5	9000	4000	3000

Table 3.2 Required simulation size for $d^* = 0.01$ and $\nu = 20$.

P value	$m = 1$	$m = 3$	$m = 5$
0.05	9000	2500	1000
0.1	10000	5000	3000
0.25	10000	7000	5000
0.5	10000	9000	7000

References

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Appendices

Appendix 1.

S_2 and S_3 in case of Dunnet's multiple comparison can be calculated by integrals of bivariate and trivariate t distributions with d.f. ν, Hochberg and Tamhane(1987):

$$S_2 = \sum_{1\le i<j\le m} \int_{-t}^{t}\int_{-t}^{t} \frac{\Gamma((\nu+m)/2)|R_{ij}|^{-1/2}}{\Gamma(\nu/2)(\pi\nu)^{m/2}}[1+(s'R_{ij}^{-1}s)/\nu]^{-(k+\nu)/2}ds_1ds_2,$$

where $s' = (s_1, s_2)$ and $R_{ij} = \begin{pmatrix} 1 & \lambda_i\lambda_j \\ \lambda_j\lambda_i & 1 \end{pmatrix}$, and

$$S_3 = \sum_{1\le i<j<k\le m} \int_{-t}^{t}\int_{-t}^{t}\int_{-t}^{t} \frac{\Gamma((\nu+m)/2)|R_{ijk}|^{-1/2}}{\Gamma(\nu/2)(\pi\nu)^{m/2}}[1+(s'R_{ijk}^{-1}s)/\nu]^{-(k+\nu)/2}ds_1ds_2ds_3,$$

where $s' = (s_1, s_2, s_3)$ and $R_{ijk} = \begin{pmatrix} 1 & \lambda_i\lambda_j & \lambda_i\lambda_k \\ \lambda_j\lambda_i & 1 & \lambda_j\lambda_k \\ \lambda_k\lambda_i & \lambda_k\lambda_j & 1 \end{pmatrix}$.

Appendix 2.

In case of Dunnet's multiple comparison, Z_i can be also obtained by using n-variate normal random numbers $W_i^* = (W_{i0}^*, W_{i1}^*, \cdots, W_{im}^*)$ following $N(\mathbb{0}, I_n)$. Let Z_{ij} be $(\lambda_j W_{i0}^* - \lambda_j^* W_{ij}^*)$, where $\lambda_j^* = (1 + r_j/r_0)^{-1/2}, j = 1, 2, \cdots m$. Then, $Z_i = (Z_{i1}, Z_{i2}, \cdots, Z_{im})$ follows $N(\mathbb{0}, R)$. In this paper, Z_i is obtained by this method.

Appendix 3.

The absolute errors of P values for each method are shown in the following tables. In case of $n = 3$, the errors of 2nd and 3rd order bounds are all zero. So these are omitted in the following tables.

n: population number, no. of. obs.: pattern of number of observations, d.f.: degree of freedom of the error variance ν, tval: t value, Exact: true P value, Bonf: Bonferroni bounds(1st, 2nd, 3rd order) Hunt: Hunter's bound, Kw: Kwerel's bounds(2nd, 3rd order; U: upper bound, L: lower bound), N.M.C.: Naive Monte-Carlo approximation, I.M.C.: Improved Monte-Carlo approximation, sd: standard deviation of Simulation.

n= 3

no. of obs. = 1:1:1

d.f.= 5

tval	Exact	Bonfi	N.M.C.	sd	I.M.C.	sd
0.125	0.989	0.822	-0.001	0.008	-0.002	0.005
0.250	0.956	0.669	-0.007	0.014	-0.005	0.012
0.500	0.841	0.436	-0.001	0.013	0.001	0.013
1.000	0.545	0.182	-0.001	0.023	0.004	0.012
1.500	0.310	0.077	-0.004	0.011	0.001	0.011
2.000	0.169	0.035	0.000	0.019	-0.001	0.011
2.500	0.092	0.017	0.004	0.024	-0.002	0.009
3.000	0.052	0.008	0.000	0.013	-0.002	0.006
4.000	0.018	0.003	0.000	0.007	-0.001	0.002
6.000	0.003	0.000	0.000	0.002	0.000	0.000

df= 10

tval	Exact	Bonfi	N.M.C.	sd	I.M.C.	sd
0.125	0.989	0.817	-0.001	0.008	-0.002	0.007
0.250	0.956	0.660	-0.007	0.014	-0.005	0.012
0.500	0.838	0.418	0.000	0.011	0.001	0.013
1.000	0.525	0.157	0.001	0.018	0.005	0.013
1.500	0.273	0.056	-0.004	0.015	0.000	0.012
2.000	0.127	0.019	0.000	0.020	-0.002	0.014
2.500	0.056	0.007	0.000	0.016	-0.003	0.009
3.000	0.024	0.002	-0.002	0.008	-0.002	0.004
4.000	0.005	0.000	-0.001	0.001	-0.001	0.001
6.000	0.000	0.000	0.000	0.001	0.000	0.000

d.f.= 20

tval	Exact	Bonfi	N.M.C.	sd	I.M.C.	sd
0.125	0.989	0.815	-0.001	0.008	-0.001	0.007
0.250	0.955	0.655	-0.007	0.014	-0.006	0.013
0.500	0.836	0.409	0.000	0.010	0.001	0.013
1.000	0.514	0.145	0.001	0.015	0.005	0.014
1.500	0.252	0.046	-0.005	0.015	0.000	0.013
2.000	0.105	0.013	0.000	0.021	-0.002	0.016
2.500	0.039	0.003	-0.004	0.011	-0.004	0.009
3.000	0.013	0.001	-0.003	0.005	-0.002	0.003
4.000	0.001	0.000	-0.001	0.001	0.000	0.000
6.000	0.000	0.000	0.000	0.000	0.000	0.000

no. of obs. = 2:1:1

d.f.= 5

tval	Exact	Bonfi	N.M.C.	sd	I.M.C.	sd
0.125	0.990	0.821	-0.001	0.004	-0.001	0.004
0.250	0.959	0.666	-0.003	0.009	-0.001	0.008
0.500	0.851	0.426	-0.002	0.013	0.000	0.009
1.000	0.550	0.166	0.008	0.025	0.004	0.013
1.500	0.322	0.066	-0.004	0.018	0.001	0.011
2.000	0.176	0.028	-0.001	0.017	-0.001	0.010
2.500	0.096	0.013	-0.001	0.016	-0.002	0.008
3.000	0.054	0.006	0.000	0.020	-0.002	0.005
4.000	0.019	0.002	-0.003	0.006	-0.001	0.002
6.000	0.003	0.000	-0.001	0.001	0.000	0.000

df= 10

tval	Exact	Bonfi	N.M.C.	sd	I.M.C.	sd
0.125	0.990	0.816	-0.001	0.004	-0.001	0.004
0.250	0.959	0.656	-0.003	0.009	-0.001	0.009
0.500	0.848	0.407	-0.002	0.012	0.000	0.009
1.000	0.542	0.140	0.006	0.026	0.006	0.014
1.500	0.284	0.045	0.000	0.016	0.001	0.011
2.000	0.133	0.014	-0.001	0.017	-0.002	0.012
2.500	0.058	0.004	0.000	0.018	-0.003	0.009
3.000	0.025	0.001	-0.004	0.008	-0.002	0.004
4.000	0.005	0.000	0.000	0.001	-0.001	0.001
6.000	0.000	0.000	0.000	0.001	0.000	0.000

d.f.= 20

tval	Exact	Bonfi	N.M.C.	sd	I.M.C.	sd
0.125	0.990	0.814	-0.001	0.004	-0.001	0.004
0.250	0.959	0.651	-0.002	0.009	-0.002	0.009
0.500	0.847	0.398	-0.002	0.011	-0.001	0.009
1.000	0.531	0.127	0.003	0.021	0.006	0.016
1.500	0.263	0.035	-0.001	0.018	0.001	0.012
2.000	0.110	0.009	-0.004	0.014	-0.002	0.014
2.500	0.041	0.002	-0.002	0.018	-0.003	0.009
3.000	0.014	0.000	-0.005	0.003	-0.002	0.003
4.000	0.001	0.000	-0.001	0.001	0.000	0.000
6.000	0.000	0.000	0.000	0.000	0.000	0.000

no. of obs. = 1:2:3

d.f.= 5

tval	Exact	Bonfl	N.M.C.	sd	I.M.C.	sd
0.125	0.986	0.825	-0.003	0.004	-0.003	0.003
0.250	0.947	0.678	-0.005	0.010	-0.003	0.011
0.500	0.816	0.461	-0.004	0.022	0.001	0.019
1.000	0.511	0.215	-0.014	0.020	0.000	0.013
1.500	0.287	0.101	-0.001	0.018	-0.001	0.013
2.000	0.156	0.048	0.001	0.025	-0.002	0.012
2.500	0.085	0.024	0.009	0.020	-0.002	0.008
3.000	0.048	0.013	0.005	0.013	-0.002	0.005
4.000	0.017	0.004	0.005	0.008	-0.001	0.002
6.000	0.003	0.001	0.004	0.003	0.000	0.000

df= 10

tval	Exact	Bonfl	N.M.C.	sd	I.M.C.	sd
0.125	0.986	0.820	-0.003	0.004	-0.003	0.004
0.250	0.946	0.669	-0.005	0.010	-0.004	0.011
0.500	0.812	0.444	-0.002	0.021	0.002	0.021
1.000	0.489	0.193	-0.013	0.023	0.000	0.015
1.500	0.251	0.078	0.002	0.015	0.000	0.014
2.000	0.117	0.030	0.006	0.019	-0.002	0.013
2.500	0.051	0.011	0.001	0.011	-0.003	0.008
3.000	0.022	0.004	0.003	0.009	-0.002	0.004
4.000	0.004	0.001	0.002	0.002	-0.001	0.001
6.000	0.000	0.000	0.000	0.001	0.000	0.000

d.f.= 20

tval	Exact	Bonfl	N.M.C.	sd	I.M.C.	sd
0.125	0.986	0.817	-0.003	0.004	-0.003	0.004
0.250	0.946	0.664	-0.005	0.010	-0.004	0.011
0.500	0.810	0.435	0.000	0.021	0.002	0.022
1.000	0.477	0.182	-0.007	0.024	-0.001	0.016
1.500	0.231	0.068	0.005	0.016	0.000	0.016
2.000	0.096	0.022	0.000	0.022	-0.003	0.015
2.500	0.036	0.007	0.002	0.009	-0.003	0.007
3.000	0.012	0.002	-0.001	0.003	-0.002	0.002
4.000	0.001	0.000	0.000	0.001	0.000	0.000
6.000	0.000	0.000	0.000	0.000	0.000	0.000

no. of obs. = 3:2:1

d.f.= 5

tval	Exact	Bonfl	N.M.C.	sd	I.M.C.	sd
0.125	0.990	0.821	0.000	0.004	-0.001	0.004
0.250	0.959	0.666	-0.001	0.007	-0.001	0.007
0.500	0.852	0.425	-0.004	0.014	-0.002	0.011
1.000	0.562	0.165	0.008	0.020	0.004	0.014
1.500	0.323	0.065	-0.004	0.018	0.001	0.011
2.000	0.176	0.027	-0.002	0.014	-0.001	0.010
2.500	0.096	0.013	-0.001	0.017	-0.002	0.008
3.000	0.054	0.006	-0.002	0.018	-0.002	0.005
4.000	0.019	0.002	-0.002	0.006	-0.001	0.002
6.000	0.003	0.000	-0.001	0.001	0.000	0.000

tval	Exact	Bonfl	N.M.C.	sd	I.M.C.	sd
0.125	0.990	0.816	0.000	0.004	-0.001	0.004
0.250	0.959	0.656	-0.001	0.007	-0.001	0.007
0.500	0.849	0.407	-0.003	0.014	-0.002	0.012
1.000	0.543	0.139	0.015	0.021	0.006	0.016
1.500	0.285	0.044	-0.001	0.021	0.001	0.012
2.000	0.133	0.014	-0.003	0.015	-0.002	0.012
2.500	0.059	0.004	-0.003	0.020	-0.003	0.009
3.000	0.025	0.001	-0.002	0.009	-0.002	0.005
4.000	0.005	0.000	-0.001	0.002	-0.001	0.001
6.000	0.000	0.000	0.000	0.001	0.000	0.000

d.f.= 20

tval	Exact	Bonfl	N.M.C.	sd	I.M.C.	sd
0.125	0.990	0.814	0.000	0.004	0.000	0.004
0.250	0.959	0.651	-0.001	0.007	-0.001	0.007
0.500	0.848	0.397	-0.002	0.014	-0.003	0.013
1.000	0.533	0.126	0.014	0.023	0.006	0.016
1.500	0.264	0.034	-0.001	0.017	0.000	0.013
2.000	0.110	0.008	-0.005	0.014	-0.003	0.014
2.500	0.041	0.002	-0.005	0.013	-0.003	0.009
3.000	0.014	0.000	-0.002	0.005	-0.002	0.005
4.000	0.001	0.000	-0.001	0.001	0.000	0.000
6.000	0.000	0.000	0.000	0.000	0.000	0.000

n= 5

no. of obs. = 1:1:1:1:1

d.f.= 5

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	2.622	0.684	0.155	0.008	0.155	-2.311	-0.011	-0.001	0.000	0.001	0.000	0.000
0.250	0.996	2.254	0.471	0.246	0.024	0.246	-1.762	-0.041	-0.008	0.001	0.003	0.000	0.002
0.500	0.958	1.595	0.237	0.287	0.044	0.287	-1.020	-0.118	-0.037	0.004	0.004	0.001	0.003
1.000	0.726	0.727	0.073	0.182	0.037	0.182	-0.363	-0.120	-0.036	-0.020	0.015	0.003	0.004
1.500	0.452	0.324	0.026	0.091	0.020	0.091	-0.141	-0.090	-0.015	-0.010	0.011	0.006	0.005
2.000	0.259	0.149	0.011	0.045	0.010	0.045	-0.060	-0.056	-0.007	0.003	0.008	0.003	0.006
2.500	0.146	0.072	0.005	0.022	0.005	0.022	-0.027	-0.027	-0.003	0.010	0.008	0.001	0.004
3.000	0.083	0.037	0.002	0.012	0.002	0.012	-0.014	-0.014	-0.002	0.007	0.003	0.000	0.003
4.000	0.030	0.012	0.000	0.004	0.000	0.004	-0.004	-0.004	-0.001	0.019	0.004	0.000	0.001
6.000	0.005	0.002	0.000	0.001	0.000	0.001	-0.001	-0.001	0.000	0.012	0.001	0.000	0.000

d.f.= 10

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	2.612	0.675	0.160	0.009	0.160	-2.292	-0.011	-0.001	0.000	0.001	0.000	0.000
0.250	0.997	2.234	0.455	0.254	0.025	0.254	-1.725	-0.041	-0.008	0.001	0.003	0.000	0.002
0.500	0.961	1.550	0.215	0.297	0.047	0.297	-0.957	-0.123	-0.038	0.005	0.001	0.002	0.003
1.000	0.719	0.645	0.057	0.174	0.037	0.174	-0.297	-0.124	-0.031	-0.014	0.014	0.003	0.006
1.500	0.415	0.243	0.016	0.075	0.016	0.075	-0.094	-0.089	-0.011	-0.004	0.007	0.008	0.006
2.000	0.206	0.088	0.005	0.029	0.005	0.029	-0.030	-0.030	-0.004	0.007	0.011	0.003	0.007
2.500	0.094	0.031	0.001	0.011	0.001	0.011	-0.010	-0.010	-0.001	0.002	0.009	0.000	0.005
3.000	0.042	0.011	0.000	0.004	0.000	0.004	-0.003	-0.003	-0.001	0.015	0.005	-0.001	0.003
4.000	0.008	0.002	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.008	0.002	0.000	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.000	0.000

d.f.= 20

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	2.607	0.670	0.162	0.009	0.162	-2.282	-0.011	-0.001	0.000	0.000	0.000	0.000
0.250	0.997	2.223	0.447	0.259	0.025	0.259	-1.706	-0.042	-0.008	0.000	0.003	0.000	0.002
0.500	0.963	1.527	0.205	0.301	0.049	0.301	-0.925	-0.120	-0.039	0.005	0.001	0.003	0.002
1.000	0.715	0.602	0.050	0.168	0.037	0.168	-0.266	-0.126	-0.029	-0.009	0.015	0.002	0.007
1.500	0.393	0.204	0.012	0.065	0.012	0.065	-0.074	-0.074	-0.009	0.005	0.006	0.010	0.007
2.000	0.176	0.061	0.003	0.021	0.003	0.021	-0.019	-0.019	-0.003	0.008	0.010	0.002	0.008
2.500	0.068	0.017	0.000	0.006	0.000	0.006	-0.004	-0.004	-0.001	0.006	0.007	-0.002	0.005
3.000	0.024	0.004	0.000	0.002	0.000	0.002	-0.001	-0.001	0.000	0.004	0.004	-0.001	0.002
4.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.003	0.000	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

no. of obs. = 4:1:1:1:1

d.f.= 5

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	2.622	0.677	0.159	0.008	0.159	-2.303	-0.010	-0.001	0.000	0.000	0.000	0.000
0.250	0.998	2.252	0.452	0.259	0.023	0.259	-1.734	-0.037	-0.007	0.001	0.002	0.000	0.001
0.500	0.971	1.582	0.206	0.318	0.048	0.318	-0.946	-0.112	-0.034	0.000	0.007	0.000	0.004
1.000	0.773	0.680	0.043	0.204	0.043	0.204	-0.272	-0.122	-0.036	-0.007	0.007	-0.001	0.006
1.500	0.499	0.277	0.012	0.095	0.012	0.095	-0.086	-0.086	-0.013	-0.007	0.009	0.001	0.005
2.000	0.290	0.117	0.004	0.043	0.004	0.043	-0.031	-0.031	-0.005	0.005	0.010	0.001	0.003
2.500	0.164	0.054	0.001	0.020	0.001	0.020	-0.013	-0.013	-0.002	0.007	0.008	0.000	0.003
3.000	0.094	0.026	0.000	0.010	0.000	0.010	-0.006	-0.006	-0.001	-0.003	0.009	-0.001	0.003
4.000	0.034	0.008	0.000	0.003	0.000	0.003	-0.002	-0.002	0.000	-0.005	0.009	-0.001	0.002
6.000	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.006	0.000	0.000

d.f.= 10

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	2.612	0.668	0.164	0.008	0.164	-2.284	-0.010	-0.001	0.000	0.000	0.000	0.000
0.250	0.998	2.233	0.436	0.268	0.024	0.268	-1.697	-0.038	-0.007	0.001	0.001	0.000	0.001
0.500	0.974	1.538	0.176	0.330	0.051	0.330	-0.878	-0.105	-0.035	0.000	0.006	0.000	0.005
1.000	0.772	0.592	0.027	0.196	0.027	0.196	-0.201	-0.127	-0.030	-0.004	0.007	-0.002	0.007
1.500	0.467	0.191	0.004	0.074	0.004	0.074	-0.044	-0.044	-0.008	0.001	0.014	0.002	0.006
2.000	0.235	0.058	0.001	0.024	0.001	0.024	-0.010	-0.010	-0.002	0.003	0.012	0.001	0.003
2.500	0.108	0.018	0.000	0.008	0.000	0.008	-0.003	-0.003	-0.001	0.000	0.009	-0.001	0.004
3.000	0.048	0.006	0.000	0.002	0.000	0.002	-0.001	-0.001	0.000	-0.006	0.011	-0.001	0.004
4.000	0.009	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.007	-0.001	0.001
6.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	0.000	0.000	0.000

d.f.= 20

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	2.607	0.663	0.166	0.008	0.166	-2.275	-0.010	-0.001	0.000	0.000	0.000	0.000
0.250	0.998	2.222	0.427	0.273	0.025	0.273	-1.677	-0.038	-0.007	0.001	0.001	0.000	0.001
0.500	0.975	1.515	0.164	0.336	0.053	0.336	-0.843	-0.101	-0.035	0.001	0.007	0.000	0.006
1.000	0.771	0.546	0.020	0.189	0.020	0.189	-0.167	-0.131	-0.027	-0.002	0.007	-0.002	0.008
1.500	0.447	0.150	0.002	0.061	0.002	0.061	-0.027	-0.027	-0.005	0.006	0.014	0.003	0.007
2.000	0.202	0.035	0.000	0.015	0.000	0.015	-0.004	-0.004	-0.001	0.004	0.007	0.002	0.003
2.500	0.078	0.007	0.000	0.003	0.000	0.003	-0.001	-0.001	0.000	-0.005	0.013	-0.002	0.006
3.000	0.027	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.002	0.009	-0.002	0.005
4.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	-0.001	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

no. of obs. = 1:2:3:4:5

d.f.= 5

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	0.999	2.622	0.702	0.144	0.010	0.142	-2.334	-0.015	-0.002	0.000	0.001	0.000	0.001
0.250	0.991	2.259	0.521	0.210	0.024	0.204	-1.838	-0.049	-0.015	-0.005	0.005	-0.002	0.003
0.500	0.920	1.633	0.323	0.218	0.034	0.202	-1.198	-0.115	-0.041	-0.022	0.014	-0.005	0.006
1.000	0.631	0.821	0.141	0.137	0.025	0.119	-0.547	-0.119	-0.031	-0.033	0.021	0.004	0.016
1.500	0.372	0.403	0.062	0.078	0.014	0.064	-0.252	-0.074	-0.016	-0.011	0.027	0.007	0.015
2.000	0.208	0.199	0.028	0.040	0.008	0.034	-0.119	-0.043	-0.008	0.020	0.018	0.004	0.010
2.500	0.116	0.102	0.013	0.022	0.004	0.018	-0.059	-0.024	-0.005	0.026	0.008	0.001	0.005
3.000	0.066	0.054	0.004	0.012	0.001	0.010	-0.031	-0.014	-0.004	0.032	0.007	0.000	0.003
4.000	0.023	0.018	0.003	0.004	0.001	0.003	-0.010	-0.005	0.000	0.033	0.008	0.000	0.001
6.000	0.004	0.003	0.001	0.001	0.001	0.001	-0.002	-0.001	0.000	0.024	0.005	0.000	0.000

d.f.= 10

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	0.999	2.613	0.693	0.149	0.010	0.147	-2.315	-0.015	-0.002	0.000	0.001	0.000	0.001
0.250	0.992	2.238	0.507	0.218	0.025	0.211	-1.803	-0.051	-0.013	-0.003	0.003	-0.003	0.003
0.500	0.922	1.589	0.307	0.221	0.036	0.204	-1.147	-0.123	-0.043	-0.018	0.015	-0.006	0.007
1.000	0.614	0.750	0.124	0.131	0.024	0.112	-0.488	-0.117	-0.029	-0.013	0.020	0.005	0.018
1.500	0.333	0.325	0.047	0.065	0.013	0.055	-0.194	-0.067	-0.014	0.006	0.025	0.009	0.017
2.000	0.162	0.132	0.017	0.029	0.006	0.024	-0.073	-0.034	-0.006	0.020	0.013	0.003	0.009
2.500	0.074	0.052	0.006	0.012	0.003	0.010	-0.027	-0.016	-0.002	0.020	0.008	0.000	0.004
3.000	0.033	0.021	0.001	0.005	0.001	0.004	-0.010	-0.007	-0.002	0.021	0.006	-0.001	0.002
4.000	0.007	0.003	0.000	0.001	0.000	0.001	-0.002	-0.002	0.000	0.017	0.005	-0.001	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.003	0.000	0.000

d.f.= 20

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	0.999	2.608	0.688	0.151	0.010	0.149	-2.305	-0.015	-0.002	0.000	0.001	0.000	0.001
0.250	0.993	2.228	0.499	0.221	0.026	0.214	-1.785	-0.051	-0.013	-0.002	0.004	-0.003	0.003
0.500	0.924	1.566	0.299	0.223	0.037	0.205	-1.121	-0.127	-0.045	-0.012	0.010	-0.006	0.008
1.000	0.604	0.713	0.116	0.127	0.024	0.108	-0.459	-0.117	-0.028	-0.002	0.020	0.006	0.020
1.500	0.311	0.286	0.039	0.060	0.012	0.050	-0.166	-0.064	-0.012	0.012	0.019	0.011	0.017
2.000	0.137	0.101	0.012	0.024	0.005	0.019	-0.053	-0.030	-0.005	0.013	0.012	0.002	0.009
2.500	0.053	0.032	0.003	0.008	0.002	0.007	-0.015	-0.012	-0.002	0.010	0.008	-0.001	0.004
3.000	0.019	0.009	0.000	0.003	0.000	0.002	-0.004	-0.004	-0.001	0.015	0.005	-0.001	0.002
4.000	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.003	0.000	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000

no. of obs. = 5:4:3:2:1

d.f.= 5

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	2.622	0.679	0.158	0.008	0.158	-2.305	-0.010	-0.001	0.000	0.001	0.000	0.000
0.250	0.997	2.253	0.457	0.253	0.024	0.254	-1.741	-0.038	-0.007	0.001	0.001	0.000	0.001
0.500	0.968	1.585	0.209	0.311	0.047	0.306	-0.954	-0.116	-0.035	0.001	0.005	0.000	0.003
1.000	0.760	0.693	0.050	0.199	0.042	0.191	-0.295	-0.121	-0.036	-0.011	0.011	0.000	0.005
1.500	0.485	0.290	0.015	0.095	0.015	0.089	-0.100	-0.098	-0.014	0.002	0.008	0.004	0.004
2.000	0.281	0.127	0.005	0.044	0.005	0.041	-0.038	-0.038	-0.006	-0.001	0.010	0.003	0.001
2.500	0.159	0.059	0.002	0.021	0.002	0.019	-0.017	-0.017	-0.003	0.008	0.005	0.001	0.001
3.000	0.091	0.029	0.001	0.011	0.001	0.010	-0.008	-0.008	-0.001	0.002	0.010	0.000	0.002
4.000	0.032	0.009	0.001	0.003	0.001	0.003	-0.002	-0.002	0.000	0.007	0.006	0.000	0.001
6.000	0.006	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.006	0.004	0.000	0.000

d.f.= 10

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	2.612	0.670	0.163	0.008	0.162	-2.286	-0.010	-0.001	0.000	0.001	0.000	0.001
0.250	0.998	2.233	0.440	0.265	0.024	0.263	-1.704	-0.039	-0.007	0.001	0.001	0.001	0.001
0.500	0.971	1.541	0.185	0.322	0.050	0.316	-0.898	-0.109	-0.036	0.003	0.002	0.001	0.003
1.000	0.757	0.607	0.034	0.191	0.034	0.182	-0.225	-0.125	-0.031	-0.008	0.010	-0.001	0.006
1.500	0.452	0.206	0.007	0.075	0.007	0.069	-0.056	-0.056	-0.009	0.009	0.011	0.006	0.005
2.000	0.227	0.067	0.001	0.026	0.001	0.023	-0.015	-0.015	-0.003	0.004	0.008	0.003	0.001
2.500	0.104	0.022	0.000	0.009	0.000	0.008	-0.004	-0.004	-0.001	-0.001	0.008	0.000	0.003
3.000	0.046	0.007	0.000	0.003	0.000	0.002	-0.001	-0.001	0.000	0.000	0.007	-0.001	0.003
4.000	0.009	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.006	-0.001	0.001
6.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.000	0.000

d.f.= 20

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	2.607	0.665	0.165	0.008	0.165	-2.277	-0.010	-0.001	0.000	0.001	0.000	0.001
0.250	0.998	2.223	0.432	0.269	0.025	0.268	-1.684	-0.039	-0.007	0.001	0.001	0.001	0.001
0.500	0.972	1.518	0.173	0.327	0.052	0.322	-0.864	-0.106	-0.036	0.003	0.002	0.001	0.003
1.000	0.756	0.561	0.027	0.185	0.027	0.175	-0.192	-0.129	-0.028	-0.007	0.009	-0.002	0.008
1.500	0.431	0.166	0.004	0.063	0.004	0.058	-0.039	-0.039	-0.007	0.010	0.012	0.008	0.007
2.000	0.195	0.043	0.000	0.017	0.000	0.015	-0.008	-0.008	-0.002	0.007	0.009	0.003	0.002
2.500	0.075	0.010	0.000	0.004	0.000	0.003	-0.002	-0.002	-0.001	0.000	0.011	-0.002	0.005
3.000	0.026	0.002	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.001	0.006	-0.001	0.003
4.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.000	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

n= 7

no. of obs. = 1:1:1:1:1:1:1

d.f.= 5

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.432	7.075	0.322	0.028	0.322	-7.900	-0.011	-0.002	0.000	0.000	0.000	0.000
0.250	1.000	3.876	5.004	0.529	0.085	0.529	-6.164	-0.044	-0.012	0.000	0.001	0.000	0.001
0.500	0.986	2.844	2.591	0.665	0.153	0.665	-3.693	-0.108	-0.044	0.002	0.004	0.001	0.003
1.000	0.814	1.366	0.822	0.457	0.113	0.457	-1.360	-0.178	-0.087	-0.008	0.015	0.000	0.013
1.500	0.527	0.626	0.301	0.238	0.063	0.238	-0.537	-0.149	-0.074	-0.003	0.022	0.002	0.019
2.000	0.319	0.293	0.122	0.119	0.033	0.119	-0.229	-0.085	-0.051	0.001	0.033	0.003	0.017
2.500	0.183	0.144	0.054	0.060	0.016	0.060	-0.106	-0.049	-0.026	-0.007	0.021	0.002	0.012
3.000	0.106	0.074	0.026	0.032	0.008	0.032	-0.053	-0.028	-0.013	-0.015	0.015	0.001	0.008
4.000	0.038	0.024	0.006	0.010	0.002	0.010	-0.016	-0.010	-0.005	-0.006	0.007	0.000	0.003
6.000	0.007	0.004	0.001	0.002	0.001	0.002	-0.003	-0.002	-0.001	-0.001	0.004	0.000	0.001

d.f.= 10

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.418	6.991	0.331	0.028	0.331	-7.843	-0.011	-0.001	0.000	0.000	0.000	0.000
0.250	1.000	3.846	4.849	0.547	0.088	0.547	-6.050	-0.044	-0.011	-0.001	0.002	0.000	0.000
0.500	0.989	2.778	2.372	0.689	0.149	0.689	-3.489	-0.109	-0.044	0.001	0.005	0.001	0.003
1.000	0.816	1.229	0.647	0.444	0.112	0.444	-1.127	-0.186	-0.091	0.001	0.010	-0.002	0.014
1.500	0.507	0.480	0.188	0.199	0.053	0.199	-0.363	-0.130	-0.077	0.004	0.024	0.003	0.021
2.000	0.263	0.177	0.055	0.079	0.018	0.079	-0.116	-0.067	-0.031	0.006	0.017	0.005	0.019
2.500	0.124	0.065	0.017	0.030	0.006	0.030	-0.038	-0.032	-0.011	0.006	0.011	0.002	0.012
3.000	0.056	0.024	0.005	0.012	0.002	0.012	-0.013	-0.013	-0.004	0.001	0.010	0.000	0.007
4.000	0.011	0.004	0.000	0.002	0.000	0.002	-0.002	-0.002	-0.001	0.001	0.001	-0.001	0.002
6.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.000	0.000

d.f.= 20

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.411	6.948	0.336	0.028	0.336	-7.813	-0.011	-0.001	0.000	0.000	0.000	0.000
0.250	1.000	3.831	4.770	0.557	0.090	0.557	-5.992	-0.044	-0.011	0.000	0.001	0.000	0.000
0.500	0.990	2.745	2.262	0.702	0.148	0.702	-3.384	-0.109	-0.044	-0.001	0.006	0.001	0.003
1.000	0.819	1.157	0.568	0.433	0.112	0.433	-1.014	-0.193	-0.093	0.003	0.006	-0.003	0.016
1.500	0.488	0.407	0.142	0.176	0.043	0.176	-0.287	-0.123	-0.072	-0.009	0.035	0.004	0.022
2.000	0.229	0.126	0.032	0.060	0.012	0.060	-0.074	-0.059	-0.021	0.009	0.033	0.006	0.021
2.500	0.092	0.035	0.007	0.018	0.003	0.018	-0.017	-0.017	-0.005	0.004	0.015	0.002	0.012
3.000	0.033	0.009	0.001	0.005	0.001	0.005	-0.004	-0.004	-0.001	0.002	0.011	0.000	0.005
4.000	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	-0.001	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

no. of obs. = 6:1:1:1:1:1

d.f.= 5

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.432	7.020	0.329	0.025	0.329	-7.879	-0.010	-0.001	0.000	0.000	0.000	0.000
0.250	1.000	3.875	4.830	0.555	0.082	0.555	-6.086	-0.039	-0.009	0.000	0.000	0.000	0.000
0.500	0.993	2.836	2.217	0.736	0.190	0.736	-3.466	-0.092	-0.036	0.000	0.004	0.000	0.002
1.000	0.871	1.309	0.486	0.525	0.114	0.525	-1.042	-0.173	-0.087	-0.004	0.010	-0.004	0.009
1.500	0.610	0.553	0.129	0.258	0.046	0.258	-0.332	-0.130	-0.087	-0.013	0.022	0.000	0.014
2.000	0.373	0.238	0.042	0.119	0.021	0.119	-0.120	-0.085	-0.039	-0.004	0.019	0.003	0.013
2.500	0.218	0.109	0.016	0.056	0.010	0.056	-0.050	-0.050	-0.017	-0.010	0.017	0.002	0.009
3.000	0.127	0.054	0.007	0.028	0.005	0.028	-0.023	-0.023	-0.008	-0.016	0.009	0.000	0.006
4.000	0.046	0.016	0.005	0.008	0.003	0.008	-0.007	-0.007	-0.001	-0.007	0.009	-0.001	0.003
6.000	0.009	0.002	0.000	0.001	0.000	0.001	-0.001	-0.001	-0.001	-0.001	0.007	0.000	0.001

d.f.= 10

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.418	6.936	0.338	0.026	0.338	-7.822	-0.010	-0.001	0.000	0.000	0.000	0.000
0.250	1.000	3.846	4.671	0.574	0.084	0.574	-5.971	-0.039	-0.008	0.000	0.000	0.000	0.000
0.500	0.995	2.772	1.973	0.765	0.124	0.765	-3.250	-0.090	-0.035	0.001	0.003	0.000	0.002
1.000	0.879	1.166	0.304	0.516	0.085	0.516	-0.784	-0.166	-0.088	-0.001	0.010	-0.005	0.010
1.500	0.591	0.396	0.047	0.207	0.033	0.207	-0.169	-0.122	-0.061	0.003	0.015	0.000	0.017
2.000	0.318	0.122	0.008	0.069	0.008	0.069	-0.038	-0.038	-0.015	0.006	0.018	0.005	0.014
2.500	0.151	0.037	0.001	0.022	0.001	0.022	-0.010	-0.010	-0.004	0.008	0.019	0.002	0.009
3.000	0.068	0.012	0.000	0.007	0.000	0.007	-0.003	-0.003	-0.001	-0.002	0.005	-0.001	0.005
4.000	0.014	0.001	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.002	0.004	-0.001	0.001
6.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000

d.f.= 20

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.411	6.892	0.343	0.026	0.343	-7.792	-0.010	-0.001	0.000	0.000	0.000	0.000
0.250	1.000	3.831	4.589	0.583	0.085	0.583	-5.912	-0.039	-0.008	0.000	0.000	0.000	0.000
0.500	0.996	2.739	1.848	0.780	0.121	0.780	-3.136	-0.089	-0.034	0.000	0.003	0.000	0.002
1.000	0.885	1.090	0.224	0.508	0.075	0.508	-0.658	-0.151	-0.088	-0.008	0.012	-0.006	0.010
1.500	0.580	0.316	0.023	0.175	0.023	0.175	-0.105	-0.105	-0.041	-0.009	0.026	0.001	0.019
2.000	0.282	0.073	0.002	0.044	0.002	0.044	-0.015	-0.015	-0.006	0.017	0.019	0.008	0.015
2.500	0.113	0.015	0.000	0.009	0.000	0.009	-0.002	-0.002	-0.001	0.003	0.019	0.002	0.009
3.000	0.040	0.003	0.000	0.001	0.000	0.001	-0.001	-0.001	0.000	0.002	0.007	-0.002	0.004
4.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	0.002	-0.001	0.000
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

no. of obs. = 1:2:3:4:5:6:7

d.f.= 5

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.432	7.261	0.297	0.035	0.291	-7.975	-0.016	-0.003	0.000	0.000	0.000	0.000
0.250	0.998	3.878	5.544	0.444	0.090	0.425	-6.424	-0.059	-0.018	0.000	0.001	0.000	0.000
0.500	0.950	2.880	3.582	0.475	0.135	0.432	-4.333	-0.140	-0.060	-0.001	0.008	0.001	0.005
1.000	0.679	1.501	1.636	0.312	0.101	0.257	-2.066	-0.164	-0.073	-0.005	0.028	0.000	0.013
1.500	0.409	0.754	0.747	0.177	0.059	0.148	-0.977	-0.116	-0.051	0.000	0.030	0.002	0.021
2.000	0.232	0.378	0.349	0.096	0.031	0.080	-0.470	-0.068	-0.032	-0.007	0.017	0.003	0.017
2.500	0.131	0.196	0.166	0.052	0.015	0.043	-0.235	-0.039	-0.020	-0.006	0.011	0.003	0.011
3.000	0.075	0.106	0.082	0.029	0.009	0.024	-0.124	-0.023	-0.011	-0.012	0.014	0.002	0.007
4.000	0.027	0.035	0.028	0.010	0.003	0.008	-0.040	-0.008	-0.004	-0.006	0.004	0.001	0.003
6.000	0.005	0.006	0.003	0.002	0.000	0.002	-0.006	-0.002	-0.001	-0.001	0.003	0.000	0.001

d.f.= 10

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.418	7.181	0.306	0.036	0.300	-7.918	-0.016	-0.003	0.000	0.000	0.000	0.000
0.250	0.998	3.848	5.412	0.460	0.095	0.440	-6.317	-0.060	-0.017	-0.001	0.002	-0.001	0.000
0.500	0.955	2.813	3.426	0.485	0.142	0.439	-4.171	-0.146	-0.064	0.002	0.010	0.001	0.007
1.000	0.664	1.381	1.459	0.300	0.099	0.253	-1.863	-0.171	-0.075	-0.001	0.019	0.000	0.015
1.500	0.370	0.617	0.572	0.156	0.050	0.129	-0.768	-0.107	-0.049	0.007	0.029	0.002	0.024
2.000	0.183	0.257	0.212	0.072	0.022	0.059	-0.298	-0.055	-0.027	0.004	0.017	0.004	0.018
2.500	0.085	0.104	0.076	0.031	0.009	0.025	-0.113	-0.027	-0.014	0.008	0.007	0.003	0.010
3.000	0.038	0.042	0.017	0.013	0.000	0.011	-0.044	-0.012	-0.008	-0.001	0.010	0.001	0.005
4.000	0.008	0.007	0.004	0.002	0.000	0.002	-0.007	-0.003	-0.002	0.001	0.004	0.000	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000

d.f.= 20

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.411	7.140	0.311	0.037	0.305	-7.889	-0.016	-0.003	0.000	0.001	0.000	0.000
0.250	0.998	3.832	5.345	0.468	0.097	0.448	-6.262	-0.061	-0.017	-0.001	0.001	-0.001	0.001
0.500	0.957	2.778	3.351	0.489	0.146	0.441	-4.090	-0.150	-0.066	0.003	0.011	0.002	0.008
1.000	0.656	1.320	1.372	0.292	0.097	0.246	-1.763	-0.174	-0.077	0.003	0.018	0.000	0.017
1.500	0.347	0.548	0.487	0.144	0.045	0.119	-0.665	-0.101	-0.048	0.004	0.036	0.001	0.027
2.000	0.156	0.199	0.151	0.060	0.017	0.048	-0.220	-0.048	-0.024	0.005	0.025	0.006	0.018
2.500	0.062	0.065	0.042	0.021	0.006	0.017	-0.066	-0.020	-0.010	0.009	0.013	0.003	0.009
3.000	0.022	0.020	0.006	0.007	0.000	0.006	-0.019	-0.007	-0.005	0.004	0.008	0.001	0.004
4.000	0.003	0.002	0.001	0.001	0.000	0.001	-0.001	-0.001	0.000	0.001	0.004	0.000	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

no. of obs. = 7:6:5:4:3:2:1

d.f.= 5

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.432	7.036	0.326	0.026	0.325	-7.885	-0.010	-0.001	0.000	0.000	0.000	0.000
0.250	1.000	3.875	4.882	0.547	0.083	0.543	-6.109	-0.040	-0.010	0.000	0.000	0.000	0.000
0.500	0.991	2.839	2.329	0.714	0.136	0.702	-3.535	-0.097	-0.039	0.002	0.004	0.001	0.002
1.000	0.852	1.328	0.583	0.505	0.116	0.486	-1.142	-0.173	-0.088	-0.002	0.019	0.001	0.014
1.500	0.585	0.579	0.176	0.254	0.053	0.240	-0.396	-0.134	-0.082	-0.003	0.029	0.004	0.018
2.000	0.354	0.258	0.063	0.121	0.023	0.112	-0.154	-0.083	-0.045	0.003	0.024	0.005	0.015
2.500	0.205	0.122	0.026	0.059	0.011	0.054	-0.067	-0.050	-0.020	-0.001	0.016	0.003	0.011
3.000	0.119	0.061	0.012	0.030	0.005	0.028	-0.032	-0.030	-0.010	-0.017	0.012	0.001	0.007
4.000	0.048	0.019	0.005	0.009	0.003	0.009	-0.009	-0.009	-0.002	-0.007	0.007	0.000	0.003
6.000	0.008	0.003	0.000	0.002	0.000	0.001	-0.001	-0.001	0.000	0.001	0.005	0.000	0.001

d.f.= 10

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.418	6.952	0.336	0.025	0.335	-7.828	-0.010	-0.001	0.000	0.000	0.000	0.000
0.250	1.000	3.846	4.724	0.566	0.085	0.562	-5.995	-0.041	-0.009	0.000	0.001	0.000	0.000
0.500	0.993	2.774	2.091	0.742	0.131	0.730	-3.322	-0.096	-0.038	0.003	0.003	0.001	0.002
1.000	0.859	1.187	0.402	0.494	0.102	0.474	-0.892	-0.182	-0.090	0.004	0.010	0.000	0.016
1.500	0.561	0.426	0.084	0.207	0.036	0.193	-0.230	-0.122	-0.073	0.006	0.023	0.005	0.021
2.000	0.298	0.142	0.018	0.075	0.013	0.068	-0.061	-0.061	-0.021	0.004	0.018	0.006	0.016
2.500	0.141	0.047	0.005	0.026	0.004	0.023	-0.018	-0.018	-0.006	0.007	0.009	0.003	0.010
3.000	0.064	0.016	0.001	0.009	0.001	0.008	-0.005	-0.006	-0.002	0.003	0.010	0.000	0.006
4.000	0.013	0.002	0.000	0.001	0.000	0.001	-0.001	-0.001	0.000	0.000	0.004	-0.001	0.002
6.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000

d.f.= 20

tval	Exact	Bonf1	Bonf3	Kw2U	Kw3U	Hunt	Bonf2	Kw2L	Kw3L	N.M.C.	sd	I.M.C.	sd
0.125	1.000	4.411	6.909	0.341	0.027	0.340	-7.799	-0.010	-0.001	0.000	0.000	0.000	0.000
0.250	1.000	3.831	4.643	0.575	0.087	0.571	-5.935	-0.041	-0.009	0.000	0.000	0.000	0.000
0.500	0.994	2.741	1.974	0.756	0.139	0.744	-3.213	-0.096	-0.037	0.002	0.004	0.001	0.002
1.000	0.863	1.112	0.328	0.485	0.090	0.468	-0.770	-0.174	-0.091	-0.001	0.014	-0.001	0.018
1.500	0.547	0.348	0.052	0.179	0.031	0.165	-0.161	-0.120	-0.054	-0.008	0.041	0.006	0.022
2.000	0.263	0.093	0.008	0.051	0.008	0.046	-0.032	-0.032	-0.012	0.009	0.024	0.008	0.015
2.500	0.105	0.022	0.001	0.013	0.001	0.011	-0.006	-0.006	-0.003	0.007	0.014	0.003	0.010
3.000	0.038	0.005	0.000	0.003	0.000	0.002	-0.001	-0.001	-0.001	0.001	0.010	-0.001	0.005
4.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	-0.001	0.001
6.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

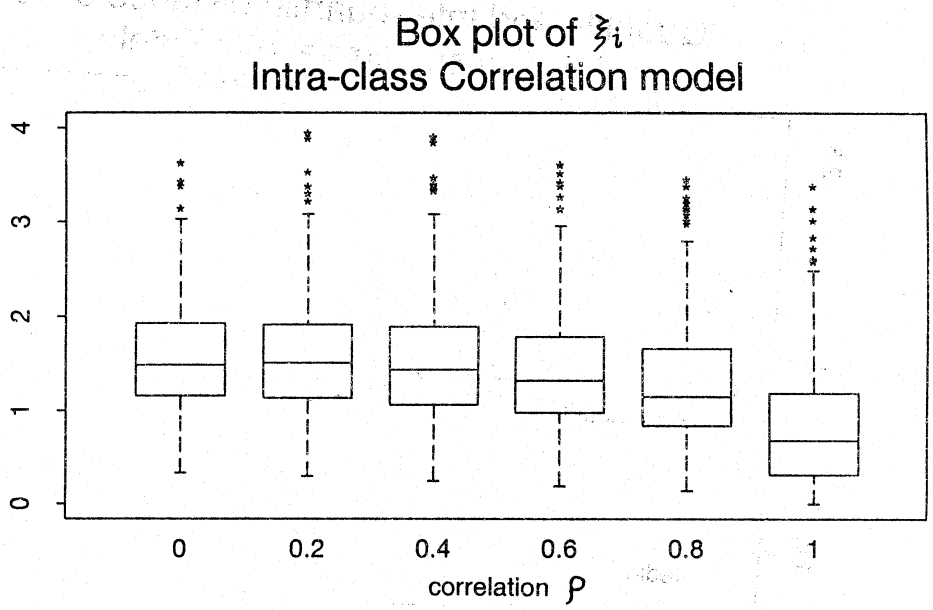

Figure 3.1

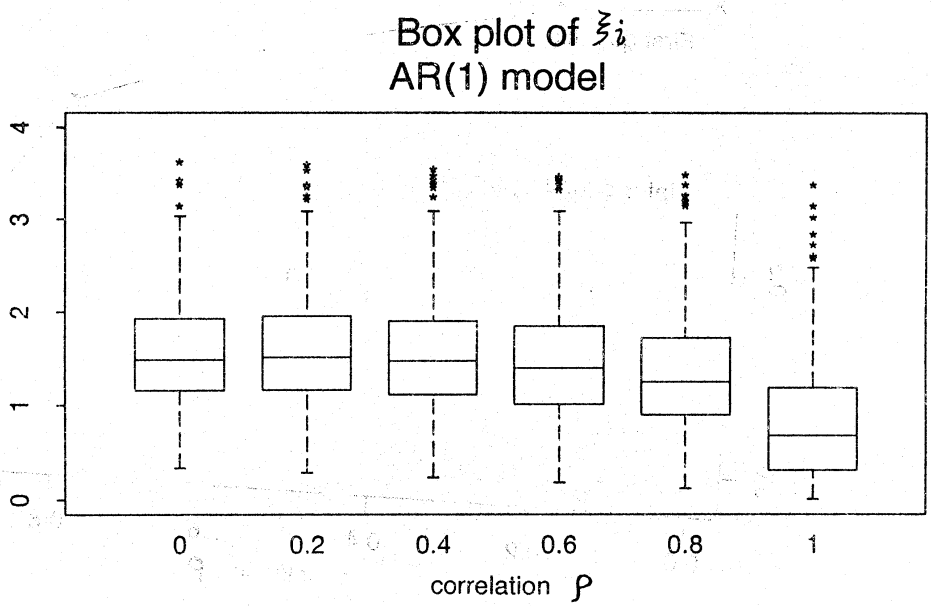

Figure 3.2

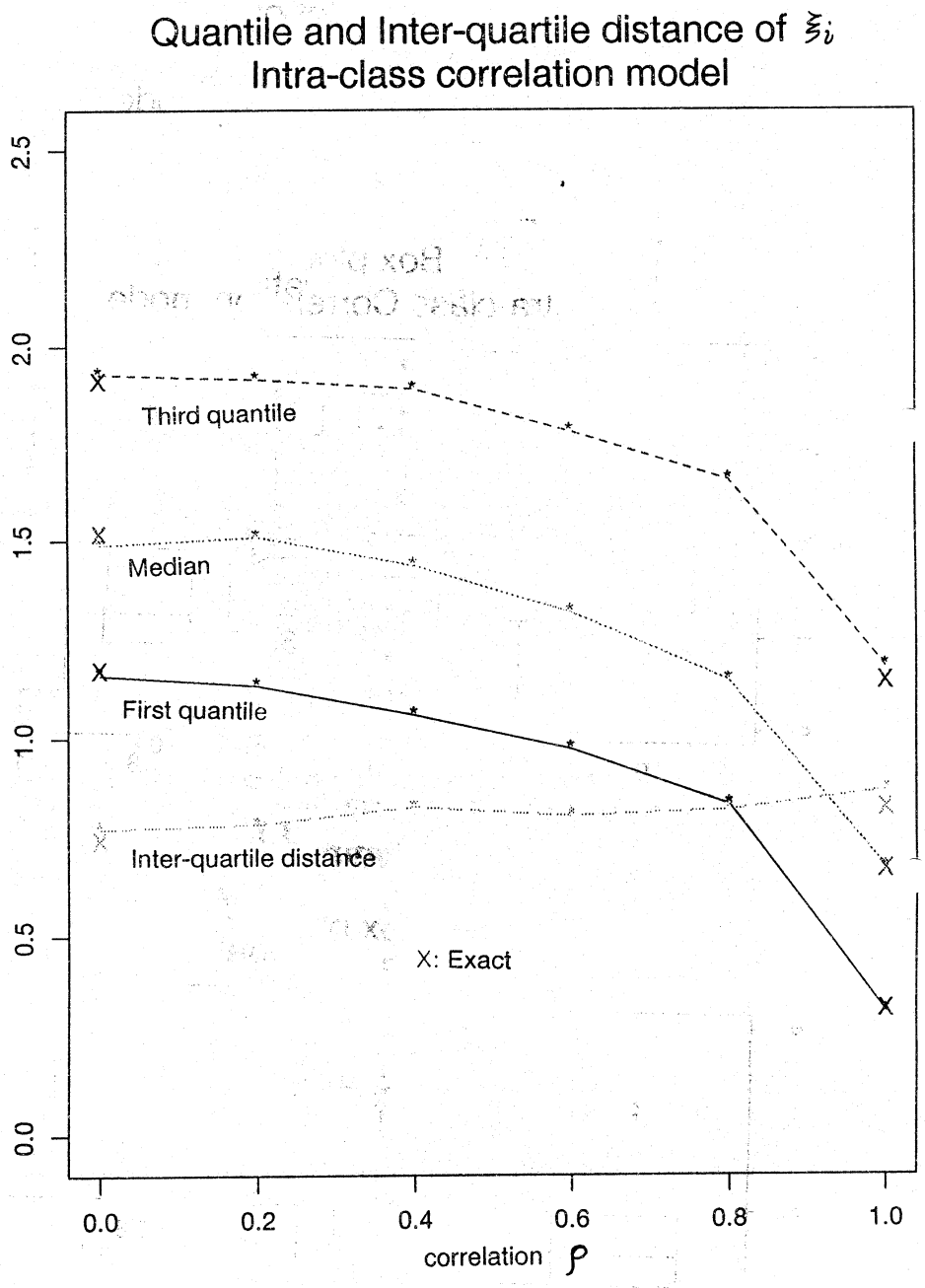
Quantile and Inter-quartile distance of ξ_i
Intra-class correlation model
2.5
2.0
1.5
1.0
0.5
0.0
Third quantile
Median
First quantile
Inter-quartile distance
X: Exact
0.0
0.2
0.4
0.6
0.8
1.0
correlation ρ

Figure 3.3

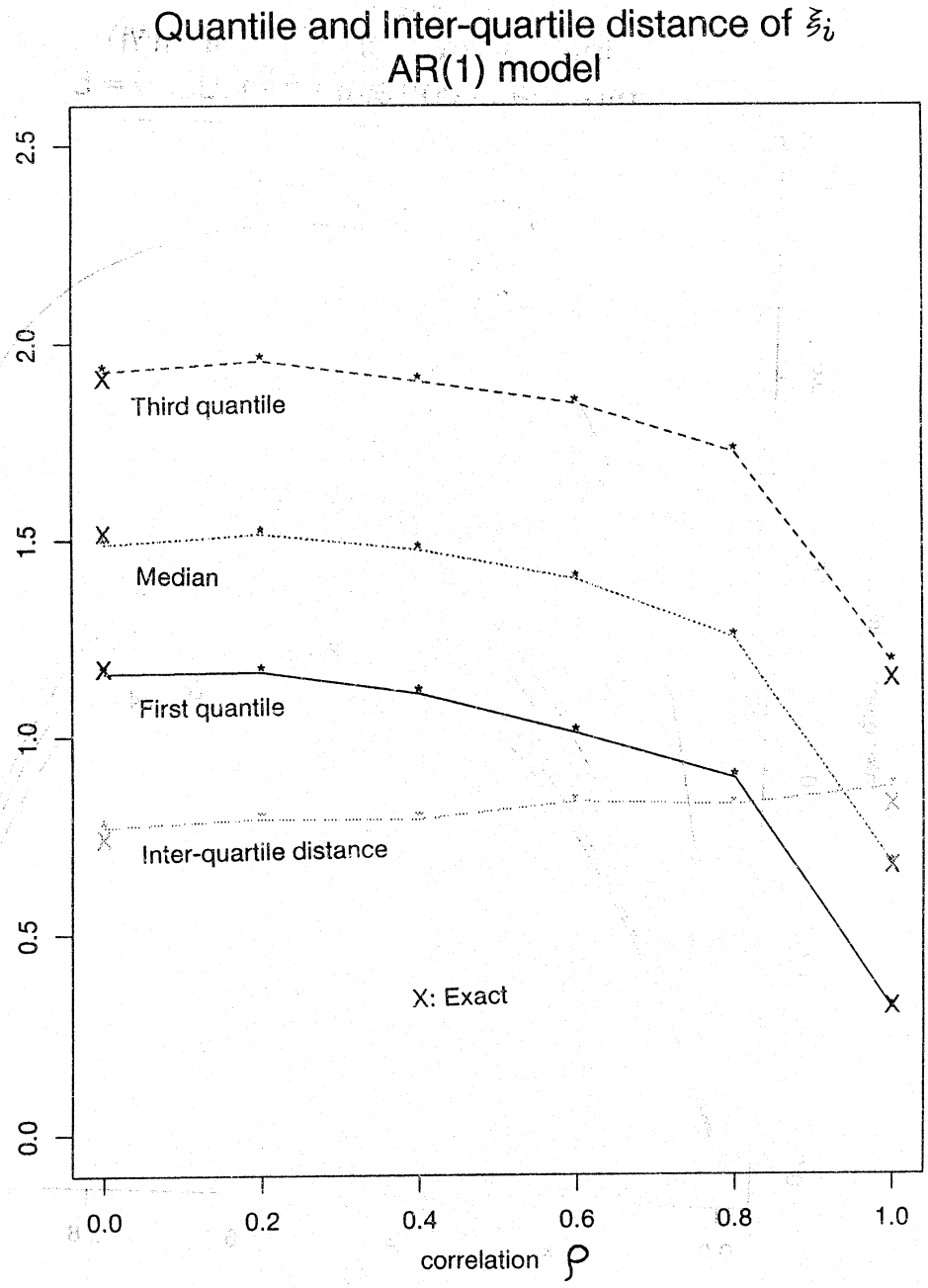
Quantile and Inter-quartile distance of ξ_i
AR(1) model
2.5
2.0
1.5
1.0
0.5
0.0
Third quantile
Median
First quantile
Inter-quartile distance
X: Exact
0.0
0.2
0.4
0.6
0.8
1.0
correlation ρ

Figure 3.4

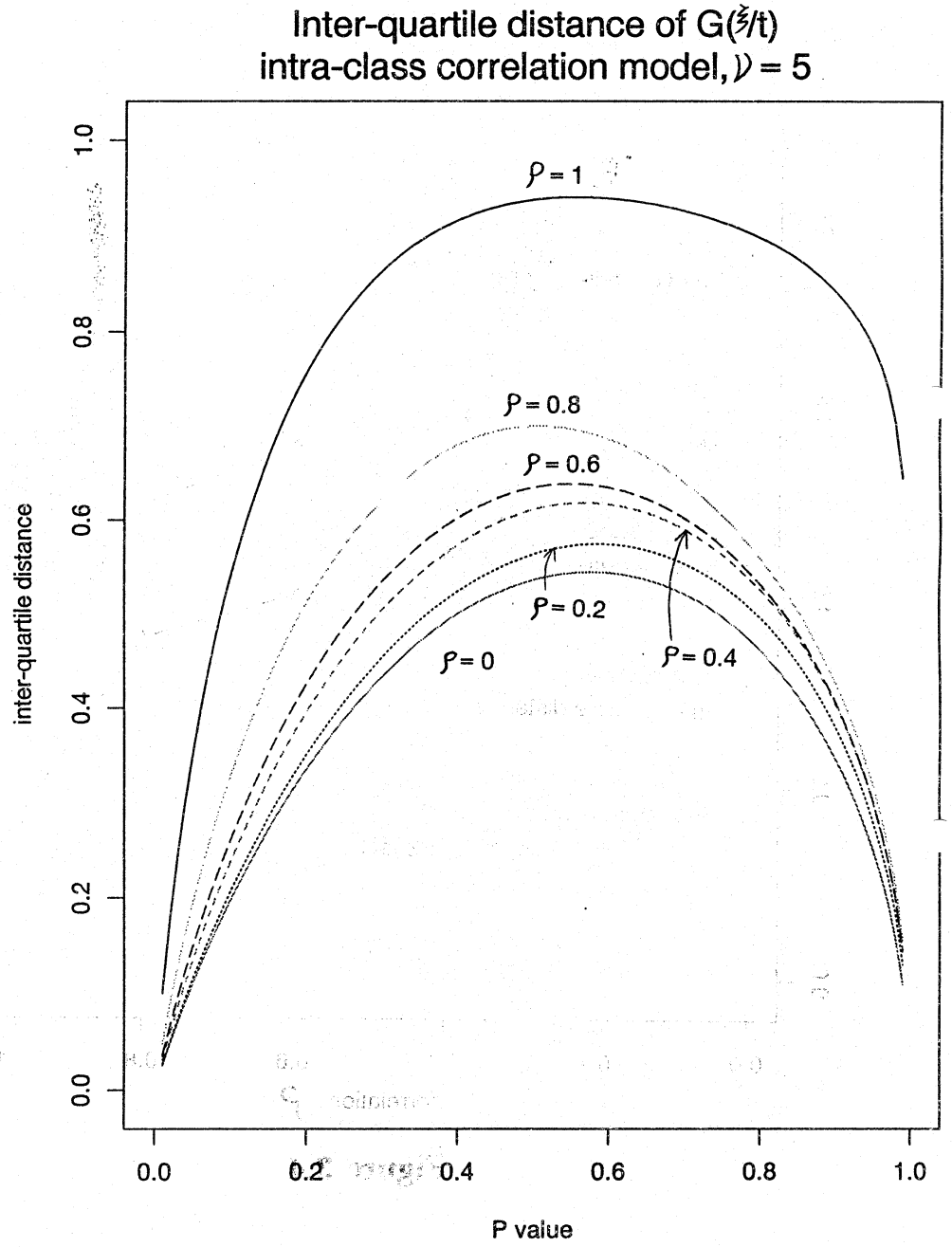
Inter-quartile distance of G(ξ/t)
intra-class correlation model, ν = 5
ρ = 1
ρ = 0.8
ρ = 0.6
ρ = 0.2
ρ = 0
ρ = 0.4
inter-quartile distance
0.0
0.2
0.4
0.6
0.8
1.0
P value

Figure 3.5

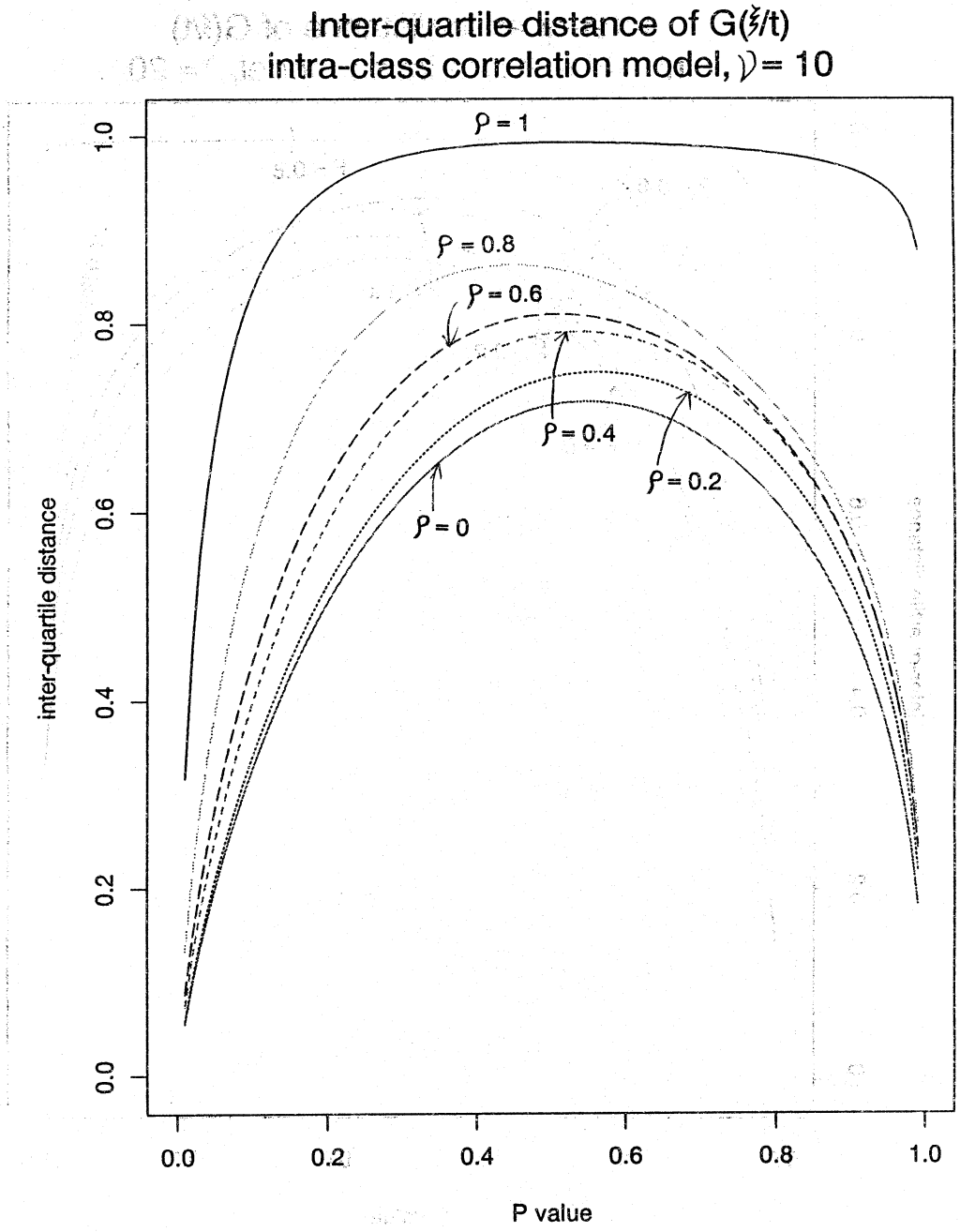
Inter-quartile distance of G(ξ/t)
intra-class correlation model, ν= 10
ρ = 1
ρ = 0.8
ρ = 0.6
ρ = 0.4
ρ = 0.2
ρ = 0
inter-quartile distance
P value
0.0
0.2
0.4
0.6
0.8
1.0

Figure 3.6

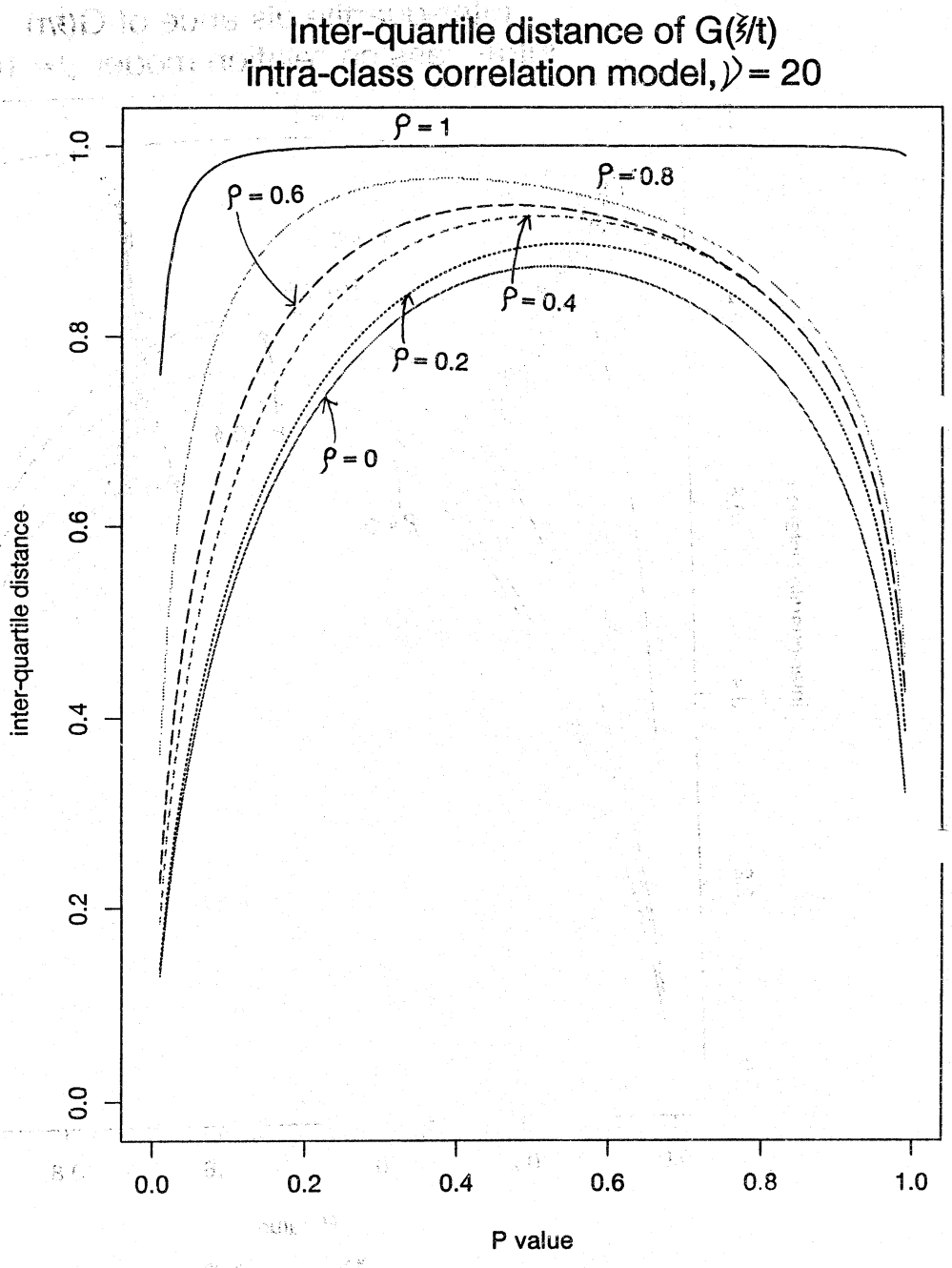
Inter-quartile distance of G(ξ/t)
intra-class correlation model, ρ = 20
ρ = 1
ρ = 0.8
ρ = 0.6
ρ = 0.4
ρ = 0.2
ρ = 0
inter-quartile distance
0.0
0.2
0.4
0.6
0.8
1.0
P value

Figure 3.7

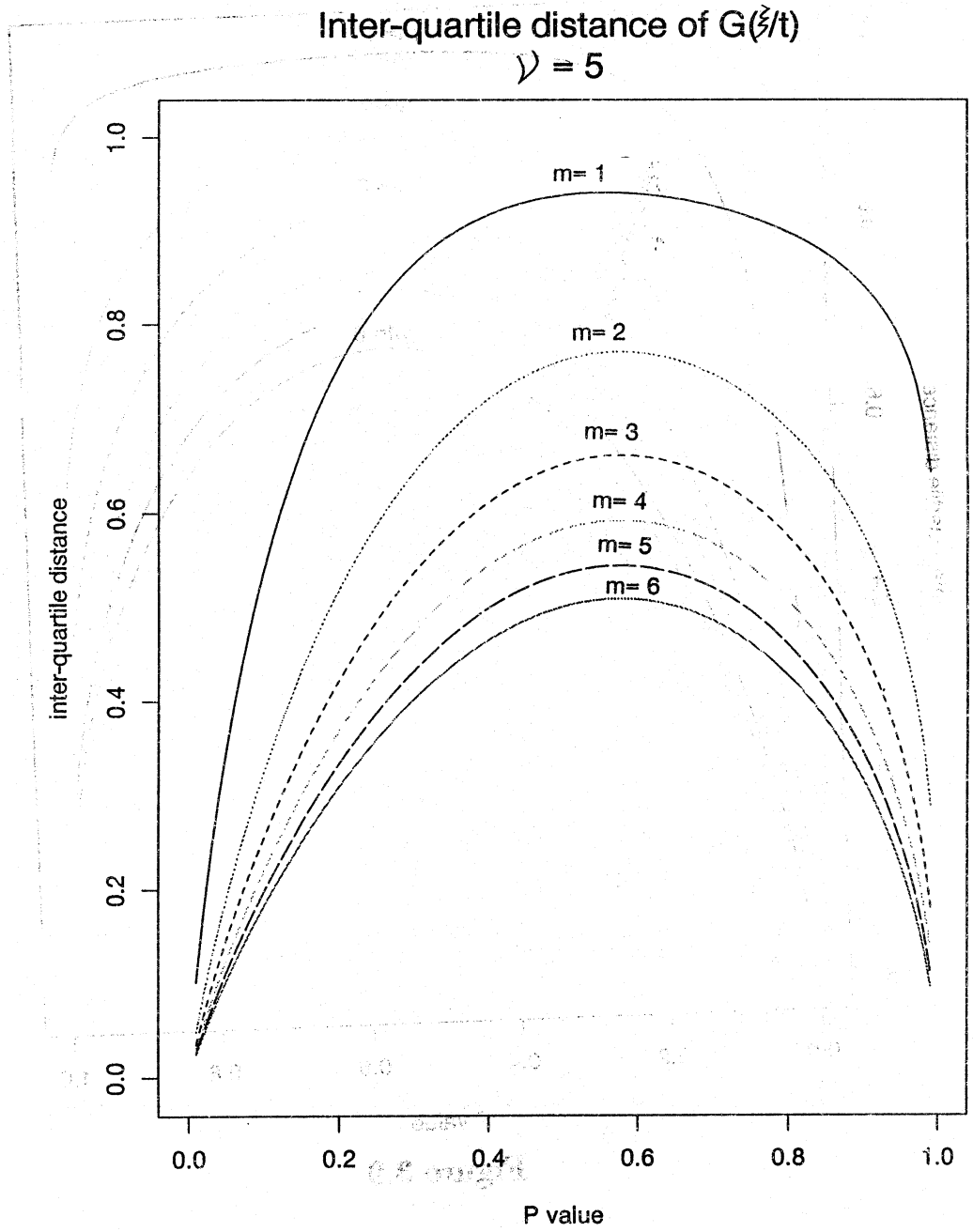
Inter-quartile distance of G(ξ/t)
ν = 5
m= 1
m= 2
m= 3
m= 4
m= 5
m= 6
inter-quartile distance
P value
0.0
0.2
0.4
0.6
0.8
1.0

Figure 3.8

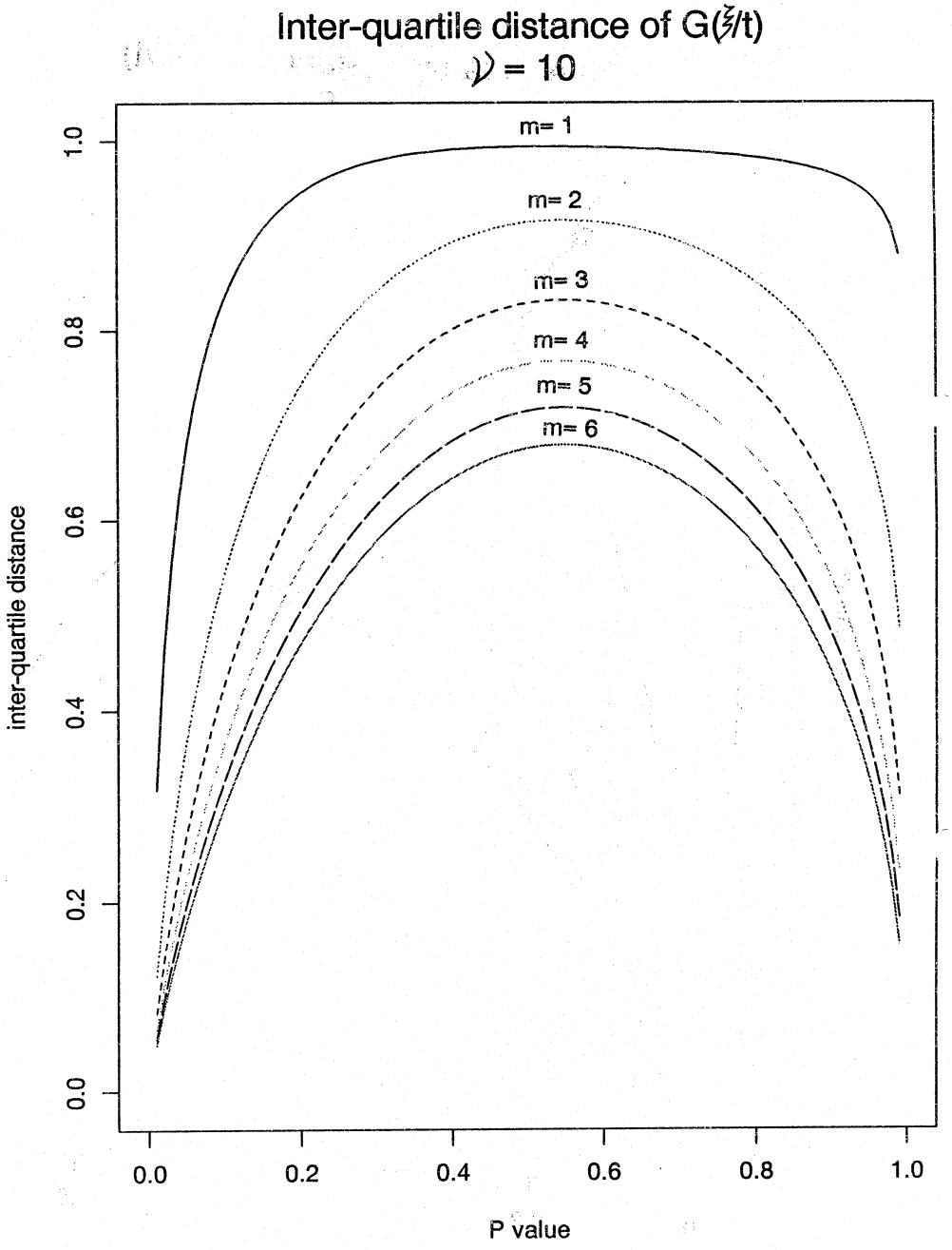
Inter-quartile distance of G(ξ/t)
ν = 10
m= 1
m= 2
m= 3
m= 4
m= 5
m= 6
inter-quartile distance
P value
0.0
0.2
0.4
0.6
0.8
1.0

Figure 3.9

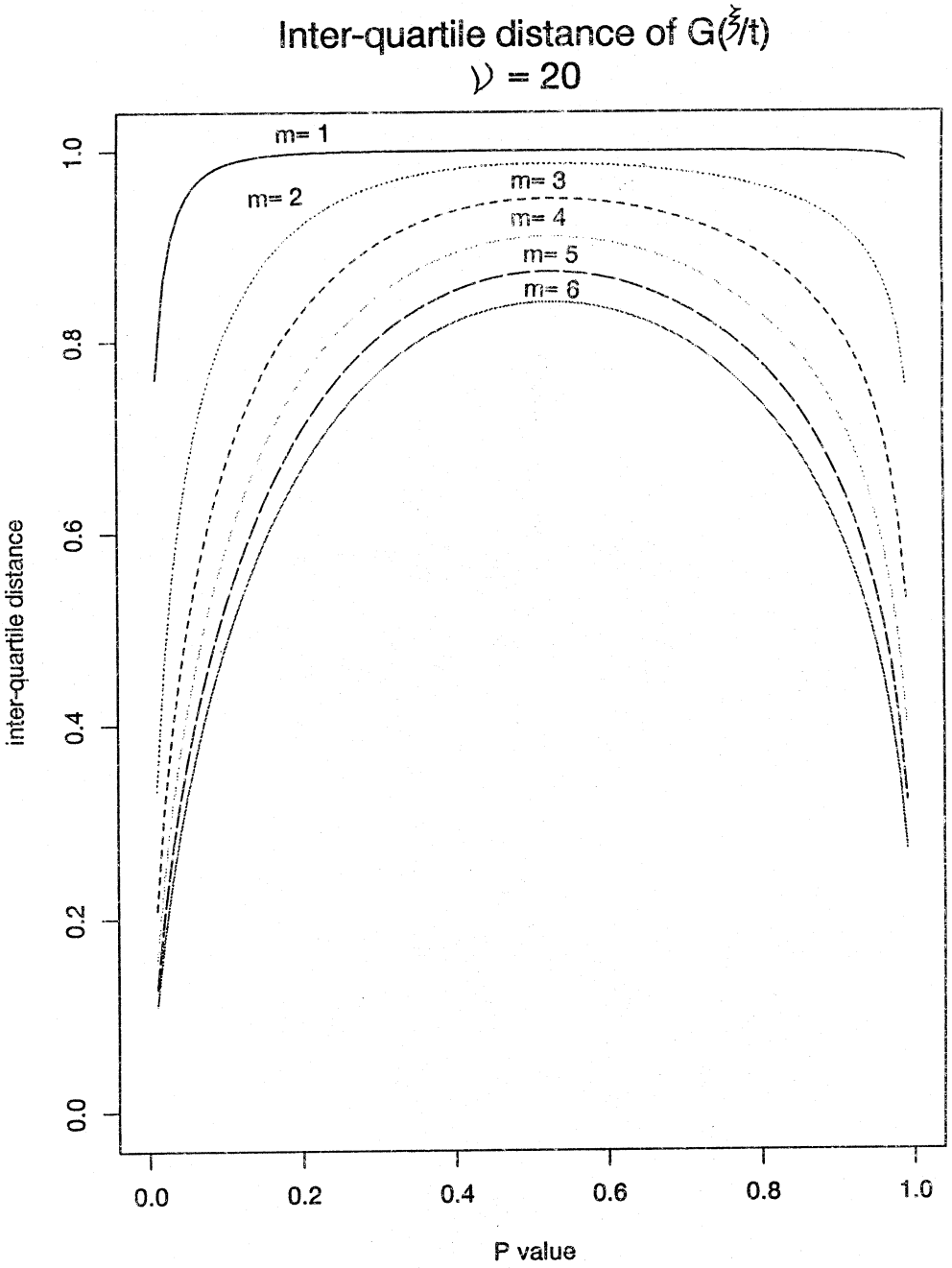
Inter-quartile distance of G(ξ/t)
ν = 20
m= 1
m= 2
m= 3
m= 4
m= 5
m= 6
inter-quartile distance
0.0
0.2
0.4
0.6
0.8
1.0
P value

Figure 3.10