Research Report

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The TAO of TEX

by

Ihsakat Aredon

Ihsakat Aredon

Department of Mathematics
Faculty of Science and Technology
Keio University

Hiyoshi 3-14-1, Kohoku-ku
Yokohama, 223 Japan

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Faculty of Science and Technology
Keio University

Hiyoshi 3-14-1, Kohoku-ku, Yokohama, 223 Japan

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The TAO of TEX

A Gourmet Guide to Treasure Chest of Favorite Mathematician's Activities by Computer

Version 1.0

WITH NUMEROUS EXPLICIT ILLUSTRATIONS

By Ihsakat Aredon, Ph.D.

KSTS/RR-88/002
Faburuary 18, 1988

To Michael Spivak, Ph.D.

KSTS/RR-88/002
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序文

最近，数学者の世界にも技術革新のニューウエイブが押し寄せ，一大変革が起こりつつあるようだ。それは論文を清書するのに，ほんの数年前まで一世を風靡したＩ社製タイプライターが，マイコンのワープロに代わり，さらにクオリティの高いドキュメント・プロセッサの利用を彼らは要求しているからだ。

ところで，TeX という言葉を御存知の人も多いと思うのだが，これはスタンフォード大学，計算機科学科のD.E.Knuth 教授が，自分の著作を出版するために自ら設計し製作した文書清書システムである。Tex は，同系列の清書システムであるScribeと比較すると，非常に厄介なところもあるが，パブリック・ドメインで提供されていることもあり，近年，利用者が次第に増えている。特に，TeX の注目すべき点は，Scribeなどと比較すると，数式の取扱がかなり自由に行えることである。

現在，TeX には LaTeXとAMS-TeX という２つのサブセットがある。LaTeX は，TeX の複雑な書式をScribe風に書けるようにしたものである。AMS-TeX は，その入力方法に関する限り，あくまでplain TeX の入力形式を受け継いでいるものの，数式の取り扱いにおいては，沢山のマクロを持ち，数式入力が比較的簡単に行え，かつ高品質の出力が得られることで，数学の論文製作には恰好のしろものと言える。さらに，AMS などの学会に，TeX やAMS-TeX などの電子的な媒体を利用して論文を投稿すると，その論文が受理された場合には，そうでないものと比較して出版されるまでに20週間も短縮される利点がある。

このテキストは，一言でいうなれば，AMS-TeX を使って論文を製作するための入門書であり，かつ例題集である。特に，数式に関する例題を数多く掲載したつもりである(The Joy of TeXの例題の90% 以上を含んでいる) 。多分，細かい点を除けば，これ１冊で AMS-TeXを使用して論文を清書することがいとも簡単にできるのではないかと思う。お料理の本にたとえれば，３分間クッキングの本とでも言えるのではないか。ただし，ほんの１週間程度の日数でこのテキストを書き上げたので，説明不足の点も多いと思うのだか，ひらに御容赦あれ。

最後に，このテキストの製作にさいし，多くの助言をいただいた，慶應義塾大学情報科学研究所の大野義夫先生に一言感謝いたします。また，小生にこのような料理のテキストを書くきっかけを作って下さった慶應義塾大学理工学部数理科学科のＭ先生とAMS-TeX愛好者の皆さんにも一言お礼申しあげます。

2 15 1988　筆者

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○ はじめに

最初に，AMS-TeX の特徴について述べると次のようになる。AmS-TeX は，Plain Tex の上に構築されたマクロパッケージの１つであり，基本的には，Plain-TeX のコマンドがすべて使えるほかに，いろいろな拡張がおこなわれている。

(1) タイトル，著者名など，形式上の項目については，論理的な指定が可能であるが本文については，そうした配慮はあまりない。
(2) スタイルファイルの考え方を取り入れてはいるが，amspptしか現状では存在していない。
(3) 参考文献の取り扱いは，BiBTeXほどではないが，多少は考えられている。
(4) 数式に関してはいろいろと細かい指定ができる。
(5) 数式などの自動的な番号付けや，目次の自動生成と言う機能はない。
(6) 内容の相互参照の機能もない。
(7) ＼magnification を指定できるが，ロシア文字や特殊記号は，サイズが１通りしかない。

全体として，かなり人でに頼る部分が多く，自動車でいうなれば，完全なオートマ車ではなく，かといって完全なマュアル車でもない。ある程度，手動の楽しみが味わえる。

〔文書作成形式〕

env.tex（スタイルファイル）

```
\documentstyle{amsppt}
\nologo
\magnification=\magstep1
\pagewidth{4.8in}
\pageheight{7.4in}
%\hcorrection{-0.4in}
%\vcorrection{-0.4in}
\baselineskip=12pt
\abovedisplayskip=8pt
\belowdisplayskip=8pt
\parskip=5pt
\parindent=8mm
\document
\input  論文名.tex

\enddocument
```

〔手順〕

(1) 最初に，左記のスタイルファイルを作成するか，既に，AMS-TeX を使用している人からコピーさせてもらう。

(2) 下記の様式で論文を入力して，論文名.texなるファイルを作る。

(3) AMS-TeXを〔amstex　env.tex〕で起動してenv.dviファイルを作る。

(4) dvimpr env.dvi で imagen(レーザプリンタ)に出力する。

論文名.tex（file名）

```
\topmatter
\title .........  \endtitle
\author ........  \endauthor
\affil .........  \endaffil
\address  {......}
\date {......}
\thanks {......}
\keywords {......}
\subjclass  {......}
\endtopmatter
```

（\title から \subjclass まで：オプショナルで，指定する順序も自由。使用法は実例を参照すること。）

本　文

\Refs

参　考　文　献

［注］(1) \title ....\endtitleの間が2行以上にわたる時には，次のような形式で記述する。

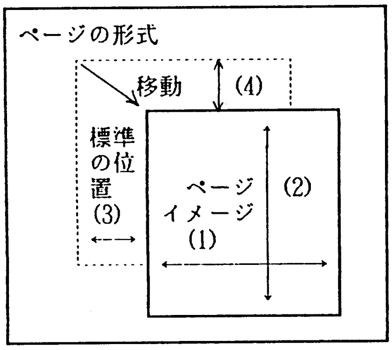
\title ..\\....\\..\endtitle

(2) 本文中には，次のものが書ける。

```
\heading ... \endheading
\subheading {......}
\proclaim .... \endproclaim
\demo {......} \enddemo
\roster        箇条書き
  \item .....
  \item .....\endroster
```

(3) \title や\authorの前に次の項目を書くこともできる。

```
\pretitle {..}       \predate  {..}
\preaffil {..}       \prepaper {..}
\preauthor {..}
\preabstract {..}
```



(1)\pagewidth {寸法}
(2)\pageheight {寸法}
(3)\hcorrection {寸法}
(4)\vcorrection {寸法}

The Tao of AMS-TeX

IHSAKAT AREDON

Department of Mathematics
Oick University

January 1, 2010

Abstract. This is a sample input file for AMS-TeX. Comparing it with the output it generates can show you how to produce a simple document of your own.

THE SILENT VOID

Something mysterious is formed, born in the silent void. Waiting alone and unmoving, it is at once still and yet in constant motion. It is the source of all document programs. I do know its name, so I will call it the Tao of TeX. Who gave the Tao? His name was Professor D.E.Knuth. He gave birth to plain TeX. Plain TeX gave birth to AMS-TeX and LaTeX. Each TeX has its purpose, however humble. Each TeX expresses the yin and yang of software. Each TeX has its place within the Tao. But do not write the document in plain TeX, if you can avoid it.

The wise man is told about Tao and follows it. The average man is told about Tao and searches fot it. The foolish man is told about Tao and laughs at it. If it were not for laughter, there would be no Tao.

The highest sounds are hardest to hear. Going forward is a way to retreat. Great talent shows itself late in the life. Even a perfect document still has bugs.

THE ANCIENT MASTERS

Thus spake the master programmer: "After three days without using TeX, life becomes meaningless." ...

....

REFERENCES

1. D.E. Knuth, "TeXbook," Addison-Wesley, 1984.

2. M.D.Spivak, "The Joy of TeX," Addison-Wesley, 1984.

3. L.Lamport, "A document preparation system LaTeX," Addison-Wesley, 1986.

4. G. James, "The Tao of Programming," Info Book, 1986.

Keywords. TeX, AMS-TeX, LaTeX

1-41-3 Ihsoyih Ukohok Amahokoy 322 Napaj

This research is partially supported by somebody.

```
%---------------------------
% A sample document for AMS-TeX
%---------------------------
%
\topmatter
\title The Tao of  AMS-\TeX \endtitle
\author Ihsakat Aredon \endauthor
\affil Department of Mathematics \\
       Oiek University \endaffil
\address{1-41-3 Ihsoyih Ukohok Amahokoy
        322 Napaj}
\date{ January 1, 2010}
\thanks{This research is partially supported
 by somebody.}
%\subjclass{  }
\keywords{\TeX, AMS-\TeX, La\TeX}
\abstract{This is a sample input file for AMS-\TeX.
   Comparing it with the output it generates can show
    you how to produce a simple document of your own. }
\endtopmatter
%
%---------------
% begin document
%---------------
\heading The Silent Void \endheading
Something mysterious is formed, born in the silent void.
Waiting alone and unmoving, it is at once still and yet
in constant motion. It is the source of all document
programs. I do know its name, so I will call it the
Tao of {\TeX}. Who gave the Tao? His name was
Professor D.E.Knuth. He gave birth to plain \TeX. Plain
{\TeX} gave birth to {AMS-\TeX} and La\TeX. Each {\TeX} has
its purpose, however humble. Each \TeX expresses the yin
and yang of software. Each {\TeX} has its place within the
Tao. But do not write the document in plain {\TeX}, if you can
avoid it.\par
The wise man is told about Tao and follows it. The average
man is told about Tao and searches fot it. The foolish man
is told about Tao and laughs at it.  If it were not for
laughter, there would be no Tao.\par
The highest sounds are hardest to hear. Going forward is a way
to retreat. Great talent shows itself late in the life.
Even a perfect document still has bugs.
\heading The Ancient Masters \endheading
Thus spake the master programmer: \lq\lq After three days without
using {\TeX}, life becomes meaningless." $\dots$
$$ \dots\dots\dots $$
%
%-----------------
% begin references
%-----------------
%
\Refs
 \ref \no 1\by D.E. Knuth \book \TeX book
   \publ Addison-Wesley \yr 1984 \endref
 \ref \no 2\by M.D.Spivak \book The Joy of \TeX
   \publ Addison-Wesley \yr 1984 \endref
 \ref \no 3\by L.Lamport \book A document
   preparation system La\TeX\publ Addison-Wesley
   \yr 1986\endref
\ref \no 4\by G. James\book The Tao of Programming
   \publ Info Book \yr 1986\endref
```

○ 文書のレイアウトの指定

(1) \par　文章の段落の区切りを示すコマンド

(2) \beginsection　新しい節の始まりと，その標題を指定するコマンド
[例] \beginsection　標題　\par

(3) \proclaim　定義とか定理などを印刷するコマンド
[例] \proclaim　見出し，本文　\endproclaim
この場合，見出しのあと，行がえをしないで同じ行に本文が続き，段落の終わりまで斜体 (Slanted)となる。

(4) \centerline　センタリング

(5) \item　箇条書きのためのコマンド
\itemitem　指定した1項目をさらに細分化したいときに利用するコマンド
この場合の字下げ幅は，\itemの倍である。

(6) \footnote　脚注のためのコマンド。これには，引数が2つあり，第1引数で脚注の記号を，第2引数で脚注の本文を指定する。
[例] \footnote {脚注の内容} ; 自動的に数字の脚注番号が生成される。
\footnote"${}↑*$ " {脚注の内容} : *脚注の内容
脚注番号を* に変える。(例題参照)

(7) 段落間の空白のコントロール
(i) \parskip　段落間すべてを一律に広げるコマンド
[例] \parskip =3 mm　のように長さを指定する。
(ii) 特定の場所だけを\parskip よりもさらに空けたい場合のために，次のコマンドがある。
(イ) \smallskip　3pt plus 1pt minus 1pt
「3pt を理想の間隔とするが，ページの下揃えをするために2pt まで縮めたり，4pt まで引き伸ばしてもよい。」ことを意味する。
(ロ) \medskip　6pt plus 2pt minus 2pt
(ハ) \bigskip　12pt plus 4pt minus 2pt
(ニ) \vskip　指定した長さ
[例] \vskip 1.0cm
\vskip 1.3cm plus 2mm minus 1.5mm

○ 字体の切り換えコマンド

コマンド	字体	例	コマンド	字体	例
\rm	ローマン	Roman	\tt	タイプライタ	`Typewriter`
\sl	斜体	*Slanted*	\bf	ボールド	**Bold**
\it	イタリック	*Italic*			

8. The Solution of Nonlinear Equations

8.2 Functional Iteration

1. Let $x = g(y)$ be the function inverse to $y = f(x)$.

(a) By using induction, show that we can write

$$g^{(k)}(y) = \frac{x_k}{(y')^{2k-1}}, \qquad k = 1,2,3,\dots$$

where x_k is a polynomial in $y', y'', \dots, y^k$ which satisfies the recurrence relation

$$X_{n+1} = \frac{dX_n}{dx}y' - (2n-1)X_n y'', \qquad X_1, \qquad n = 1,2,3,\dots$$

(b) Using this result to find explicit expression for $g^{(k)}(y)$ for $k = 1,2,3$.[1]

2.

(a) Show that the convergence of a functional iteration method of order 1 implies that the asymptotic error constant is less than equal 1.*

```
%-----------------------------------
%  A sample document for AMS-TeX
%-----------------------------------
%
\beginsection 8. The Solution of Nonlinear Equations \par
\beginsection 8.2 Functional Iteration \par
\item{\bf 1.} Let $x=g(y)$ be the function inverse to $y=f(x)$.
\par
\itemitem{$(a)$} By using induction, show that we can write
$$g^{(k)}(y) = \frac{x_k}{(y')^{2k-1}},\qquad k = 1,2,3,\dots$$
where $x_k$ is a polynomial in $y', y'',\dots,y^k$ which satisfies
the recurrence relation
$$ X_{n+1} = \frac{dX_n}{dx}y'-(2n-1)X_ny'', \qquad X_1,
\qquad n = 1,2,3, \dots $$
\itemitem{$(b)$} Using this result to
find explicit expression for
$g^{(k)}(y)$ for $k=1,2,3$.\footnote{Ostrowski(1973),pp.20-22.}\par
\item{\bf 2.}\par
\itemitem{$(a)$} Show that the convergence of a functional iteration
method of order 1 implies that the asymptotic error constant is
less than equal 1.\footnote"${}^*$"{See section 3.}\par
```

[1] Ostrowski(1973),pp.20-22.

*See section 3.

○ 行やページへの分割

<行の分割>

(1) ＼linebreak　　ここで行がえする。

(2) ＼newline　　ここで行がえし，前の行は右ぞろえしない。

(3) ＼nolinebreak　　ここで行がえしてはいけない。

(4) ＼allowlinebreak　　ここで行がえしてもよい。（ダッシュの前後など）

<数式の分割>

(1) ＼mathbreak　　数式のここで行がえする。

(2) ＼nomathbreak　　数式のここで行がえしてはならない。

(3) ＼allowmathbreak　　数式のここで行がえしてもよい。

<ページがえ>

(1) ＼pagebreak　　ここでページがえする。

(2) ＼newpage　　ここでページがえし，前のページの余白には空白をうめる。

(3) ＼nopagebreak　　ここでページがえしてはならない。

○ 図のためのスペース

<パラグラフ間（入らなければ次のページの先頭）>

(1) ＼midspace {寸法} ＼caption {..}

(2) ＼midspace {寸法} ＼caption＼captionwidth {寸法}　{..}

<ページの先頭（入らなければ次のページの先頭）>

(1) ＼topspace {寸法} ＼caption {..}

　＼topspace {寸法} ＼caption＼captionwidth {寸法}　{..}

〔寸法の種類〕

```
pt   point(the lines of this manual are 12pt apart)
pc   pica (1pc = 12pt)
in   inch (1 in = 72.27pt)
bp   big point (72bp = 1 in)
cm   centimeter (2.54cm = 1 in)
mm   millimeter (10mm = 1cm)
dd   didot point (1157dd = 1238pt)
cc   cicero (1 cc = 12dd)
sp   scaled point(65536sp = 1pt)
```

```
(例)  .3 in
      1pt
      + 29 pc
      -0.01in
      0cm
```

○ 数式(TeXt STYLE)

数式には，TeXt style(T-style)とDisply style(D-style) とがある。

TeXt style:	`$ .....$`	; 文章の中で使用する。
Display style:	`$$.....$$`	; 一行以上とる独立した数式に使用する。

(1) 指数と添え字

x^2	`$x^2$`	or `$x\sp$`
x^a	`$x^a$`	or `$x\sp$`
x^α	`$x^\alpha$`	or `$x\sp\alpha$`
x_2	`$x_2$`	or `$x\sb2$`
x_y	`$x_y$`	or `$x\sb y`
A_b^a	`$A_b^a$`	or `$A\sb b\sp2$`
$\Gamma_{y_b^a}^{z_c^d}$	`$\Gamma_{y^a_b}^{z_c^d}$`	

(2) ダッシュ(primary, secondaryなど)

f'	`$f^\prime$`
g'^2	`$g'{}^2$     % or $g^{\prime2}$`
f'_2	`$f_2^\prime$ % or $f_2'$`
$f'[g(x)]g'(x)$	`$f'[g(x)]g'(x)$`
$y'_1+y''_2+y'''_3$	`$y_1'+y_2''+y_3'''$`

(3) 根号

$\sqrt(2)$	`$\sqrt(2)$`
$\sqrt{x^3+\sqrt\alpha}$	`$\sqrt{x^3+\sqrt\alpha}$`
$\sqrt[3]{2}$	`$\root 3 \of 2$`
$\sqrt[n]{x^n+y^n}$	`$\root n\of {x^n+y^n}$`
$\sqrt[n+1]{a}$	`$\root n+1 \of a$`

(4) 二項演算子

$x + y - z$	`$x+y-z$`
$x + y * z$	`$x+y*z$`
$x * y/z$	`$x*y/z$`
$x = y > z$	`$x=y>z$`
$x := y$	`$x:=y$`
$x \le\ y \ne z$	`$x\le\ y\ne z$`

(5) 区切り記号

$f(x, y; z)$	`$f(x,y;z)$`
$f : A \to B$	`$f:A\to B$`
$f : A\colon \to B$	`$f:A\colon \to B$`

(6) アクセント記号

ò	(grave accent)	`` \`o ``
ó	(acute accent)	`\'o`
ô	(circumflex or "hat")	`\^o`
ö	(umlaut or dieresis)	`\"o`
õ	(tilde or "squiggle")	`\~o`
ŏ	(breve accent)	`\u o`
ǒ	(háček or "check")	`\v o`
ő	(long Hungarian umlaut)	`\H o`
ō	(macron or "bar")	`\B o`
o̲	(bar-under accent)	`\b o`
ȯ	(dot accent)	`\D o`
ọ	(dot-under accent)	`\d o`
o̧	(cedilla)	`\c o`

œ, Œ	`\oe, \OE`
æ, Æ	`\ae, \AE`
å, Å	`\aa, \AA`
ø, Ø	`\o, \O`
ł, Ł	`\l, \L`
ß	`\ss`

ACCENTS IN MATH MODE

$\hat a$	`$\hat a$`
$\check a$	`$\check a$`
$\tilde a$	`$\tilde a$`
$\acute a$	`$\acute a$`
$\grave a$	`$\grave a$`
$\dot a$	`$\dot a$`
$\ddot a$	`$\ddot a$`
$\dddot a$	`$\dddot a$`
$\ddddot a$	`$\ddddot a$`
$\breve a$	`$\breve a$`
$\bar a$	`$\bar a$`
$\vec a$	`$\vec a$`

$\widehat x, \widetilde x$	`$\widehat x,\widetilde x$`
$\widehat{xy}, \widetilde{xy}$	`$\widehat{xy},\widetilde{xy}$`
$\widehat{xyz}, \widetilde{xyz}$	`$\widehat{xyz},\widetilde{xyz}$`

COMPOUND SYMBOLS

Mathematicians often like to make new symbols by setting things over or under a symbol, instead of as a superscript or subscript. $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX gives you `\overset` and `\underset` to accomplish this:

$\underset{X}{A}$	`$\underset X\to A$`
$\underset{\alpha\beta}{X}$	`$\underset\alpha\beta\to X$`
$\overset{\alpha\beta}{\longrightarrow}$	`$\overset\alpha\beta\to\longrightarrow$`
$\overset{\text{def}}{=}$	`$\overset\text{def}\to=$`
$\overset{s}{\underset{A}{X}}$	`$\overset s\to{\underset A\to X}$`

$\bigl(x-s(x)\bigr)\bigl(y-s(y)\bigr)$	`$\bigl(x-s(x)\bigr)\bigl(y-s(y)\bigr)$`
$\bigl[x-s[x]\bigr]\bigl[y-s[y]\bigr]$	`$\bigl[x-s[x]\bigr]\bigl[y-s[y]\bigr]$`
$\bigl\| \|x\|+\|y\| \bigr\|$	`$\bigl\| \|x\|+\|y\| \bigr\|$`
$\bigl\lfloor\sqrt A\bigr\rfloor$	`$\bigl\lfloor\sqrt A\bigr\rfloor$`

reverse slash: $\backslash$	`\backslash`
upward arrow: $\uparrow$	`\uparrow`
double upward arrow: $\Uparrow$	`\Uparrow`
downward arrow: $\downarrow$	`\downarrow`
double downward arrow: $\Downarrow$	`\Downarrow`
up-and-down arrow: $\updownarrow$	`\updownarrow`
double up-and-down arrow: $\Updownarrow$	`\Updownarrow`

○ 数式(Display Style)

Display Style の数式は，自動的に一行使用され，その行の中央に置かれ，TeXt Styleとは，印刷形式がことなる。

(1) 分数，連分数

$$z = \frac{x+y^2}{x-y^2} - 1$$

```
$$z=\frac{x+y^2}{x-y^2}-1$$
     or $$z= {{x+y^2} \over   {x-y^2}}-1$$
```

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4}}}}$$

```
$$a_0 + \cfracl \\
  a_1 + \cfracl \\
  a_2 + \cfrac 1 \\
  a_3 + \cfrac 1 \\
  a_4\endcfrac $$
```

$$\frac{x}{1+\frac{x}{2}}$$

```
$$\frac x{1+\frac x2}$$
```

$$\dfrac{x}{1+\dfrac{x}{2}}$$

```
$$\dfrac x{1 +\dfrac x2}$$
```

(2) 総和，積分　上下限の位置の設定)

$$\sum_{n=1}^{n} a_i$$ `$$\sum_{n=1}^n a_i$$`

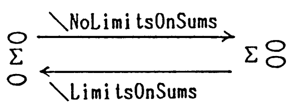
$$\iint$$ `$$\iint$$`

$$\int_a^b$$ `$$\int_a^b$$`

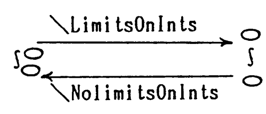
$$\int\cdots\int$$ `$$\idotsint$$`

総和，積分範囲の位置の指定を次のように指定することにより変更できる。

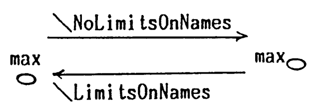
\sum



\int,\ointなど



\max,\min など



(3) 根号（自動的に適当な大きさの記号が選択される。）

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{3+x}}}}}}}}$$

```
$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+
   \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{3+x}}}}}}}}$$
```

(4) 関数名（次のような生成コマンドが用意されている。）

\arccos	\cot	\exp	(L) \lim	\sec
\arcsin	\coth	(L) \gcd	\ln	\sin
\arctan	\csc	\hom	\log	\sinh
\arg	\deg	(L) \inf	(L) \max	(L) \sup
\cos	(L) \det	\ker	(L) \min	\tan
\cosh	\dim	\lg	(L) \Pr	\tanh

$\sin^2 x + \cos^2 x = 1$	`$$\sin^2x+\cos^2x = 1$$`
$\log_2 x = (\log_2 e)(\log x)$	`$$\log_2x=(\log_2 e)(\log x)$$`
$\sin 2\theta = 2\sin\theta\cos\theta$	`$$\sin2\theta=2\sin\theta\cos\theta$$`

$\varliminf$	`$\varliminf$`
$\varlimsup$	`$\varlimsup$`
$\varinjlim$	`$\varinjlim$`
$\varprojlim$	`$\varprojlim$`

(5) スペーシング（より美しい印刷をするためには，微妙なスペーシングを自分で制御することが必要になることもある。）

`\,`	thin space (normally 1/6 of a quad)
`\>`	medium space (normally 2/9 of a quad)
`\;`	thick spce (normally 5/18 of a quad)
`\!`	negative thin space (normally -1/6 of a quad)

$\sqrt{\,\log x}$	`$\sqrt{\,\log x}$`
$[\,0,1)$	`$[\,0,1)$`
$\log n\,(\log\log n)^2$	`$\log n\,(\log\log n)^2$`
$x^2\!/2$	`$x^2\!/2$`
$n/\!\log n$	`$n/\!\log n$`
$\Gamma_{\!2}+\Delta^{\!2}$	`$\Gamma_{\!2}+\Delta^{\!2}$`
$R_i{}^j{}_{\!kl}$	`$R_i{}^j{}_{\!kl}$`
$\int_1^b\!\int_a^b$	`$$\int_1^b\!\int_a^b$$`

(6) 省略記号

\dots	AMS-TeX では，前後にあるものにより適当な量の空白を入れてくれ，テキストモードでも数式モードでも使用できる。数式モードでこれを使用すると，前後の文字やスタイルにより，\ldots または\cdots が自動選択される。

AMS-TeX では，次のコマンドも利用できる。

\dotsc	コンマやセミコロンの前
\dotsb	2項演算子の前
\dotsi	積分記号間
\dotsm	乗算記号間
\dotso	その他

(7) 集合

$\bigl(x\in A(n)\bigm\vert x\in B(n)\bigr)$	`$\bigl(x\in A(n)\bigm|x\in B(n)\bigr)$`
$\bigcup_n X_n\bigm\Vert\bigcap_n Y_n$	`$\bigcup_n X_n\bigm\|\bigcap_n Y_n$`

$\leq$ ("less than or equal")	\leq or \le
$\geq$ ("greater than or equal")	\geq or \ge
$\neq$ ("not equal")	\neq or \ne
$\notin$ ("not in")	\notin

(8) 場合分け

$$f(x)=\begin{cases} x+1, & \text{for } x>0 \\ x-1, & \text{for } x\le 0. \end{cases}$$

```
$$
f(x)=\cases x+1,&\text{for $x>0$}\\
    x-1,&\text{for $x\le0$}.\endcases
$$
```

(9) 数式の縦ぞろえ

● \align ..... \endalign

$$\begin{aligned} f(x) &= x^2 + 2 \\ g(x) &= f(x) + p(x) \\ h(x) &= p(x) + g(x) + q(x) \end{aligned}$$

```
$$\align
 f(x) & = x^2 +2 \\
 g(x) & = f(x) + p(x)\\
 h(x) & = p(x) +g(x) +q(x)
\endalign$$
```

$$\begin{aligned} &(1) \quad \max(f, g) = \frac{f + g + |f - g|}{2}, \\ &(2) \quad \max(f, -g) = \frac{f - g + |f + g|}{2}. \end{aligned}$$

```
$$\align
 \max(f,g)  &=\frac{f+g+|f-g|}2,  \tag1  \\
\max(f,-g)  &=\frac{f-g+|f+g|}2.  \tag2
\endalign$$
```

(イ) 数式の途中の\\でページがえしてもよい場合には，\\の後に
\allowdisplaybreak を入れる。

(ロ) 数式の途中の\\でページがえさせたい場合には，\\の後に，
\displaybreak を入れる。

(ハ) 数式の途中の何処の\\でページがえしてもよい場合には，
\align の前に\allowdisplaybreaksを入れる。

● \split.....\endsplit
$$ ....$$全体を１つにまとめて番号をつけたいときにこれを使う。

● \multline ..... \endmultline
縦ぞろえ不要

● \gather .....\endgather
縦ぞろえ不要で，各々の式のセンタリングのみを行う。個々に $$ で囲むよりも行間がつまる。

● ＼aligned .....＼endaligned
2列以上の同時縦ぞろえ

$$\left\{\begin{aligned} \alpha &= f(z) \\ \beta &= f(z^2) \\ \gamma &= f(z^3) \end{aligned}\right\} \qquad \left\{\begin{aligned} x &= \alpha^2 - \beta \\ y &= 2\gamma \end{aligned}\right\}.$$

```
$$
\left\{
\aligned \alpha&=f(z)\\ \beta&=f(z^2)\\
\gamma&=f(z^3)\endaligned
\right\}\qquad\left\{
\aligned x&=\alpha^2-\beta\\ y&=2\gamma\endaligned \right\}.
$$
```

● ＼alignat ......＼endalignat
何列にもまたがる縦ぞろえに使用する。各列の幅を調節するコマンドとして
＼topaligned, ＼botaligned, ＼xalignat, ＼xxalignat

(10) 行列 (＼matrix)

$$A = \begin{pmatrix} x-\lambda & 1 & 0 \\ 0 & x-\lambda & 1 \\ 0 & 0 & x-\lambda \end{pmatrix}.$$

```
$$
A=\left( \matrix
x-\lambda & 1 & 0\\
0 & x-\lambda & 1\\
0 & 0 & x-\lambda
\endmatrix \right).
$$
```

● ＼pmatrix ....＼endpmatrix　　( ) で囲った＼matrix
● ＼bmatrix ....＼endbmatrix　　[ ] で囲った＼matrix
● ＼vmatrix ....＼endvmatrix　　| | で囲った＼matrix
● ＼Vmatrix ....＼endVmatrix　　‖ ‖ で囲った＼matrix

各列の左寄せ，右寄せ，センタリングを＼formatで指定することがでいる。
行間の調節は，＼＼ の後に＼vspace {寸法} を入れて行う。
全体的な場合には，次の構文に従う。
$$ {＼spreadmatrixlines {寸法} ＼matrix .... ＼endmatrix} $$

(11) 数式の番号の指定

(2-5)　　$x = y + z^2$

```
$$x = y + z↑2 \tag2--5$$
```

数式番号の指定

数式番号の指定は，スタイルファイルによる。

(1) \TagsOnRight　スタイルファイルによらず，数式番号を数式の右端に印刷。

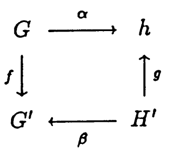
(2) \TagsOnleft　スタイルファイルによらず，数式番号を数式の左端に印刷。

(3) \TagsAsMath　\tag の後を数式モードとする。

(4) \TagsAsText　\tag の後をテキストモードとする。（デフォルト）

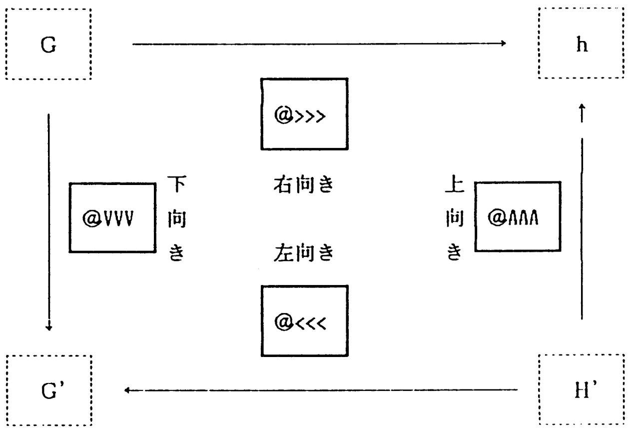
(5) \CenteredTagsOnSplits　\split した式の上下中央に数式番号をつける。

(12) Commutative Diagram
CD ...... \endCD




```
$$
\CD
G @>\alpha >> h\\
@VfVV          @AAgA \\
G' @<<\beta < H'
\endCD
$$
```

〔矢印の指定〕



(13) その他

$$\overbrace{x+\cdots+x}^{k \text{ times}}$$

```
$$\oversetbrace \text{$k$ times} \to
    {x+\dots+x}$$
```

$$\underbrace{x+y+z}_{>0}$$

```
$$\undersetbrace >\,0 \to{x+y+z}$$
```

$$\underline{4}$$

```
$$\underline 4$$
```

$$\underline{\underline{4+x}}$$

```
$$\underline{\underline{4+x}}$$
```

$$x^{\underline{n}+m}$$

```
$$x^{\underline n+m}$$
```

$$\overline{\overline{x^3+x^{x^3}}}$$

```
$$\overline{\overline{x^3}+x^{x^3}}$$
```

And you can put arrows of various sorts over a formula:

$$\overrightarrow{x+y}$$

```
$$\overrightarrow{x+y}$$
```

$$\overleftarrow{x-y}$$

```
$$\overleftarrow{x-y}$$
```

$$A^{\overleftrightarrow{x+y}}$$

```
$$A^{\overleftrightarrow{x+y}}$$
```

$$\underbrace{f\biggl(\frac{1}{n}\biggr)+\cdots+f\biggl(\frac{1}{n}\biggr)}_{n \text{ times}}=f\biggl(\underbrace{\frac{1}{n}+\cdots+\frac{1}{n}}_{n \text{ times}}\biggr)f(1)=c.$$

```
$$
\underbrace{f\biggl(\frac ln\biggr) +\dots +
   f\biggl(\frac ln\biggr)}_{\text{$n$ times}}=
f\biggl(\,\underbrace{\frac ln +\dots +
  \frac ln}_{\text{$n$ times}}\, \biggr)
f(1)=c.
$$
```

$$\sum_{x\in A}{}^{*} f(x)=\sum_{0\neq x\in A} f(x)$$

```
$$
\define\sumstar{\sideset \and^* \to\sum}
\sumstar_{x\in A}f(x)=\sum_{0\ne x\in A}f(x)
$$
```

○ 参考文献の書き方

● 参考文献の参照：本文の中で　By ＼cite {10, Theorem 4 } ..... と書くと，
By [10, Theorem 4] ...... の形式に変換される。

● 参考文献のコマンド ：各文献の順序は使用者が指定する。

＼Refs　見出しの印刷をする。
＼ref　1つの文献の始まり。以下の各項目は，オプショナルで，順序も自由である。
＼no　参考文献番号（＼key　キーワード）
＼by　著者（あるいは，＼manyby，＼bysame）
＼pages　ページ（1ページだけのときは＼page）
＼paper　論文のタイトル
＼yr　年
＼vol　巻
＼jour　雑誌名（本の中の場合には，＼inbook）
＼toappear　to appear
＼issue　special issue
＼endref　1つの文献の終わり

[注 1] 本の場合には，＼paper の代わりに＼bookを使う。この場合には，
＼publ　出版社
＼publaddr　出版社の住所
が利用できる。

[注 2] 説明用の ＼paperinfo　＼bookinfo，＼finalinfo，また，2 つ以上の文献を1つにまとめる ＼moreref がある。

○ コメント行

%　行にこのマークが現れた後の文は，コメント文となる。ただし，%マークがある行のみ有効となる。

＼comment
＼endcomment
このコマンドで囲まれた文章は，コメント文となる。

○ キーボードにない場合に利用できる入力記号

The following control sequences may be used if your keyboard lacks certain keys, or if they are inconvenient to type:

Use	*For*
`\lbrack`	[
`\lq`	'
`\rbrack`	]
`\rq`	'
`\sp`	^
`\sb`	_
`\tie`	~
`\vert`	|

If you need to use a control sequence for a particular key like ', then you will also need another control sequence to stand for the combination \':

Use	*For*
`\acuteaccent`	\'
`\graveaccent`	\`
`\hataccent`	\^
`\tildeaccent`	\~
`\underscore`	_
`\Vert`	\\|

Notice that the shorter names `\acute`, ..., `\tilde` are already used for accents in math mode.

The control sequences

`\lbrace`	\{
`\rbrace`	\}

may be used for printed curly braces. On foreign keyboards, using the extended ASCII coding, the curly braces themselves may actually be replaced by accented letters. In such situations the < and > keys might be used for grouping instead, with special control sequences like `\less` and `\greater` supplied for these printed symbols.

○ 数式モードで使える特殊記号

The math symbols + − = < > | / ( ) [ ] and * are available from the keyboard. \vert, \lbrack, \rbrack, \lbrace, \rbrace and \ast can be used instead of |, [,], \{, \} and *.

- **Lowercase Greek letters.**

α \alpha	β \beta	γ \gamma	δ \delta
ϵ \epsilon	ε \varepsilon	ζ \zeta	η \eta
θ \theta	ϑ \vartheta	ι \iota	κ \kappa
λ \lambda	μ \mu	ν \nu	ξ \xi
π \pi	ϖ \varpi	ρ \rho	ϱ \varrho
σ \sigma	ς \varsigma	τ \tau	υ \upsilon
ϕ \phi	φ \varphi	χ \chi	ψ \psi
ω \omega			

- **Uppercase Greek letters.**

Uppercase Greek letters come in the ordinary style, in a slanted variant, and in boldface:

Γ \Gamma	Δ \Delta	Θ \Theta	Λ \Lambda
Ξ \Xi	Π \Pi	Σ \Sigma	Υ \Upsilon
Φ \Phi	Ψ \Psi	Ω \Omega	

$\mathit{\Gamma}$ \varGamma	$\mathit{\Delta}$ \varDelta	$\mathit{\Theta}$ \varTheta	$\mathit{\Lambda}$ \varLambda
$\mathit{\Xi}$ \varXi	$\mathit{\Pi}$ \varPi	$\mathit{\Sigma}$ \varSigma	$\mathit{\Upsilon}$ \varUpsilon
$\mathit{\Phi}$ \varPhi	$\mathit{\Psi}$ \varPsi	$\mathit{\Omega}$ \varOmega	

$\boldsymbol{\Gamma}$ \boldGamma	$\boldsymbol{\Delta}$ \boldDelta	$\boldsymbol{\Theta}$ \boldTheta	$\boldsymbol{\Lambda}$ \boldLambda
$\boldsymbol{\Xi}$ \boldXi	$\boldsymbol{\Pi}$ \boldPi	$\boldsymbol{\Sigma}$ \boldSigma	$\boldsymbol{\Upsilon}$ \boldUpsilon
$\boldsymbol{\Phi}$ \boldPhi	$\boldsymbol{\Psi}$ \boldPsi	$\boldsymbol{\Omega}$ \boldOmega	

- **"Calligraphic" uppercase letters.**

The uppercase letters $\mathcal{A}, \ldots, \mathcal{Z}$ are obtained as \Cal A, ..., \Cal Z.

- **Binary operators.**

$\pm$ \pm	$\cap$ \cap	$\vee$ \vee, \lor
$\mp$ \mp	$\cup$ \cup	$\wedge$ \wedge, \land
$\setminus$ \setminus	$\uplus$ \uplus	$\oplus$ \oplus
$\cdot$ \cdot	$\sqcap$ \sqcap	$\ominus$ \ominus
$\times$ \times	$\sqcup$ \sqcup	$\otimes$ \otimes
$\ast$ \ast	$\triangleleft$ \triangleleft	$\oslash$ \oslash
$\star$ \star	$\triangleright$ \triangleright	$\odot$ \odot
$\diamond$ \diamond	$\wr$ \wr	$\dagger$ \dagger
$\circ$ \circ	$\bigcirc$ \bigcirc	$\ddagger$ \ddagger
$\bullet$ \bullet	$\bigtriangleup$ \bigtriangleup	$\amalg$ \amalg
$\div$ \div	$\bigtriangledown$ \bigtriangledown	& \and

Some mathematicians use the operator &, produced by \and, instead of $\wedge$. Notice that \dagger and \ddagger are used when † and ‡ function as binary operators.

• **Binary relations.**

$\leq$	`\leq, \le`	$\geq$	`\geq, \ge`	$\equiv$	`\equiv`
$\prec$	`\prec`	$\succ$	`\succ`	$\sim$	`\sim`
$\preceq$	`\preceq`	$\succeq$	`\succeq`	$\simeq$	`\simeq`
$\ll$	`\ll`	$\gg$	`\gg`	$\asymp$	`\asymp`
$\subset$	`\subset`	$\supset$	`\supset`	$\approx$	`\approx`
$\subseteq$	`\subseteq`	$\supseteq$	`\supseteq`	$\cong$	`\cong`
$\sqsubseteq$	`\sqsubseteq`	$\sqsupseteq$	`\sqsupseteq`	$\bowtie$	`\bowtie`
$\in$	`\in`	$\ni$	`\ni, \owns`	$\propto$	`\propto`
$\vdash$	`\vdash`	$\dashv$	`\dashv`	$\models$	`\models`
$\smile$	`\smile`	$\mid$	`\mid`	$\doteq$	`\doteq`
$\frown$	`\frown`	$\parallel$	`\parallel`	$\perp$	`\perp`
$\neq$	`\neq, \ne`	$\notin$	`\notin`		

`\mid` and `\parallel` are the same characters that you get with | and `\|`, but treated as binary relations, so that they get extra space around them.

> Many of these relations can be negated by putting `\not` before them. For example, `\not\subset` gives $\not\subset$. And `\ne` and `\neq` are simply abbreviations for `\not=`. But the positioning isn't always ideal, and, in particular, you should always use `\notin` for $\notin$, rather than `\not\in`.

• **Miscellaneous ordinary symbols.**

$\aleph$	`\aleph`	$\prime$	`\prime`	$\forall$	`\forall`
$\hbar$	`\hbar`	$\emptyset$	`\emptyset`	$\exists$	`\exists`
$\imath$	`\imath`	∇	`\nabla`	$\neg$	`\neg, \lnot`
$\jmath$	`\jmath`	$\surd$	`\surd`	$\flat$	`\flat`
ℓ	`\ell`	$\top$	`\top`	$\natural$	`\natural`
$\wp$	`\wp`	$\bot$	`\bot`	$\sharp$	`\sharp`
$\Re$	`\Re`	$\Vert$	`|, \Vert`	$\clubsuit$	`\clubsuit`
$\Im$	`\Im`	$\angle$	`\angle`	$\diamondsuit$	`\diamondsuit`
∂	`\partial`	$\triangle$	`\triangle`	$\heartsuit$	`\heartsuit`
∞	`\infty`	$\backslash$	`\backslash`	$\spadesuit$	`\spadesuit`
$\smallint$	`\smallint`	$\dag$	`\dag`	$\ddag$	`\ddag`
¶	`\P`	§	`\S`		

`\imath` and `\jmath` are for accenting: `$\hat\imath$` yields $\hat\imath$. `\backslash` (rather than `\setminus`) should be used for double cosets ($G\backslash H$), and to indicate that p divides n ($p\backslash n$). `\prime` is mainly used for superscripts and subscripts. The `\angle` symbol is built up from other pieces, and does not get smaller in subscripts and superscripts (see page 262). `\smallint` and `\surd` are seldom used. `\dag`, `\ddag`, `\P` and `\S` might be used for special effects; they change size correctly in subscripts and superscripts.

$\not<$	`\not<`	$\not>$	`\not>`	$\not=$	`\not=`
$\not\leq$	`\not\leq`	$\not\geq$	`\not\geq`	$\not\equiv$	`\not\equiv`
$\not\prec$	`\not\prec`	$\not\succ$	`\not\succ`	$\not\sim$	`\not\sim`
$\not\preceq$	`\not\preceq`	$\not\succeq$	`\not\succeq`	$\not\simeq$	`\not\simeq`
$\not\subset$	`\not\subset`	$\not\supset$	`\not\supset`	$\not\approx$	`\not\approx`
$\not\subseteq$	`\not\subseteq`	$\not\supseteq$	`\not\supseteq`	$\not\cong$	`\not\cong`
$\not\sqsubseteq$	`\not\sqsubseteq`	$\not\sqsupseteq$	`\not\sqsupseteq`	$\not\asymp$	`\not\asymp`

- **Arrows.**

$\leftarrow$	`\leftarrow, \gets`	$\longleftarrow$	`\longleftarrow`
$\Leftarrow$	`\Leftarrow`	$\Longleftarrow$	`\Longleftarrow`
$\rightarrow$	`\rightarrow, \to`	$\longrightarrow$	`\longrightarrow`
$\Rightarrow$	`\Rightarrow`	$\Longrightarrow$	`\Longrightarrow`
$\leftrightarrow$	`\leftrightarrow`	$\longleftrightarrow$	`\longleftrightarrow`
$\Leftrightarrow$	`\Leftrightarrow`	$\Longleftrightarrow$	`\Longleftrightarrow`
$\uparrow$	`\uparrow`	$\Uparrow$	`\Uparrow`
$\downarrow$	`\downarrow`	$\Downarrow$	`\Downarrow`
$\updownarrow$	`\updownarrow`	$\Updownarrow$	`\Updownarrow`
$\nearrow$	`\nearrow`	$\searrow$	`\searrow`
$\swarrow$	`\swarrow`	$\nwarrow$	`\nwarrow`
$\mapsto$	`\mapsto`	$\longmapsto$	`\longmapsto`
$\hookleftarrow$	`\hookleftarrow`	$\hookrightarrow$	`\hookrightarrow`
$\leftharpoonup$	`\leftharpoonup`	$\leftharpoondown$	`\leftharpoondown`
$\rightharpoonup$	`\rightharpoonup`	$\rightharpoondown$	`\rightharpoondown`
$\rightleftharpoons$	`\rightleftharpoons`		

The vertical arrows are "delimiters", like the others listed below, and change size when used after `\left` and `\right`. The control sequence `\iff` produces an arrow just like `\Longleftrightarrow`, except that there is more space around it. $\mathcal{A}_\mathcal{M}\mathcal{S}$-TEX also provides `\implies` and `\impliedby`, which are just like `\Longrightarrow` and `\Longleftarrow`, respectively, but again with more space around them.

- **Large operators like $\sum$.**

All large operators come in two sizes, with the larger used for `\dsize`.

$\textstyle\sum\displaystyle\sum$	`\sum`	$\textstyle\bigcap\displaystyle\bigcap$	`\bigcap`	$\textstyle\bigodot\displaystyle\bigodot$	`\bigodot`
$\textstyle\prod\displaystyle\prod$	`\prod`	$\textstyle\bigcup\displaystyle\bigcup$	`\bigcup`	$\textstyle\bigotimes\displaystyle\bigotimes$	`\bigotimes`
$\textstyle\coprod\displaystyle\coprod$	`\coprod`	$\textstyle\bigsqcup\displaystyle\bigsqcup$	`\bigsqcup`	$\textstyle\bigoplus\displaystyle\bigoplus$	`\bigoplus`
$\textstyle\bigvee\displaystyle\bigvee$	`\bigvee`	$\textstyle\biguplus\displaystyle\biguplus$	`\biguplus`	$\textstyle\bigwedge\displaystyle\bigwedge$	`\bigwedge`

- **Large operators like $\int$.**

$\textstyle\int$	$\displaystyle\int$	`\int`	$\textstyle\oint$	$\displaystyle\oint$	`\oint`
$\textstyle\iint$	$\displaystyle\iint$	`\iint`	$\textstyle\iiint$	$\displaystyle\iiint$	`\iiint`
$\textstyle\iiiint$	$\displaystyle\iiiint$	`\iiiint`	$\textstyle\int\cdots\int$	$\displaystyle\int\cdots\int$	`\idotsint`

- **Delimiters.**

The following symbols are recognized as "delimiters".

$($ `(`	$[$ `[, \lbrack`	$\{$ `\{, \lbrace`
$)$ `)`	$]$ `], \rbrack`	$\}$ `\}, \rbrace`
$\lfloor$ `\lfloor`	$\lceil$ `\lceil`	$\langle$ `\langle`
$\rfloor$ `\rfloor`	$\rceil$ `\rceil`	$\rangle$ `\rangle`
$\vert$ `|, \vert`	$\Vert$ `|, \Vert`	$/$ `/`
$\backslash$ `\backslash`		

All the up and down arrows can also be used as delimiters. Moreover, < and > can be used instead of `\langle` and `\rangle` after `\left` and `\right`. And, of course, there is also '.' for an "empty" delimiter after `\left` and `\right`.

○参考文献

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A NOTE ON CALDERÓ N-ZYGMUND SINGULAR INTEGRAL CONVOLUTION OPERATORS

JOAQIUM BRUNA AND BORIS KORENBLUM

The purpose of this note is to show that the notation of weak maximal function introduction in [1] (see also [4], where a similar notation is considered) can be used to obtain some new information on the Calderón-Zygmund singular integral convolution operator.

We will follow the notation of [3]. Let K be a kernel in R^n of class C^1 outside the origin satisfying

$$|K(x)| \le C|x|^{-n}, \tag{1}$$
$$|\nabla K(x)| \le C|x|^{-n-1}, \tag{2}$$

For $\epsilon > 0$ and $f \in L^p(R^n), 1 \le p < \infty$, set

$$T_\epsilon(f)(x) = \int_{|y|\ge\epsilon} f(x-y)K(y)\,dy$$

and

$$T(f)(x) = \lim_{\epsilon\to 0} T_\epsilon(x), \qquad T^*(f)(x) = \sup_{\epsilon>0} |T_\epsilon(f)(x)|.$$

We will assume that K satisfies the usual properties ensuring that the mapping $f \mapsto T^*(f)$ is of weak type (1,1) and that $T(f)(x)$ makes sense for a.e. x.

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\topmatter
\title A NOTE ON CALDER\'O N-ZYGMUND SINGULAR \\
      INTEGRAL CONVOLUTION OPERATORS \endtitle
\author JOAQIUM BRUNA AND BORIS KORENBLUM \endauthor
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\ref \no 1 \by J.Bruna  and B.Korenblum \paper On Kolomogorov's
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  \yr 1980 \pages 249--254 \endref
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On a polynomial lemma of Andrievskii*

Dieter Gaier

Hern Günter Picket zum 70. Geburtstag gewidmet

1. Introduction. In the study of the convergence of Bieberbach polynomials to the conformal map of region G onto the unit disc D the following lemma plays a crucial role.

LEMMA A (ANDRIEVSKII[1]). *Let G be a region bounded by a quasiconformal Jordan curve F, and let $0 \in G$. Then for every polynomial P of degree $n \geq 2$ with $P(0) = 0$, we have*

$$\max_{z\in G} |P(z)| \geq c(G) \cdot \sqrt{\log n} \cdot [\iint_G |P'(z)|^2 \, db_z]^{1/2}, \tag{1.1}$$

where $c(G)$ depends on G only.

This lemma allows one to transform estimates of norms

$$\|f\|_{L_1^2(G)} := [\iint_G |P'(z)|^2 \, db_z]^{1/2}, \tag{1.2}$$

into estimates in the maximum norm. We first make two comments on (1.1).

a) The estimate (1.1) with $\sqrt{\log n}$ replaces by $n^\epsilon(\epsilon > 0)$ was given by Simonenko[7], Lemma 22. The factor $\sqrt{\log n}$, however, cannot be improved even in the special case $G = D$. To see this, consider

$$P(z) = z + \frac{z^2}{2} + \cdots + \frac{z^n}{n}.$$

The embedding $W_1^2(G) \to C(\overline{G})$ was recently investigated by Kulikov[3].

b) An estimate of the norm (1.1) is not possible if Γ is an arbitrary Jordan curve. To see this, consider a region G of the form

$$G = \left\{ z = re^{i\psi} : 0 < r < 1 - c\psi^\beta ; 0 < \psi < \frac{\pi}{2} \right\} \cup N_0,$$

with $\beta \in (0, 1)$ and a neighborhood N_0 of 0, and $P(z) = z^n$. Here the left hand side of (1.1) is 1, whereas the right hand side will tend to zero for $n \to \infty$.

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*This paper is reprinted from Archiv der Mathematik Vol.49,p.109 (1987).

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\centerline{{\bf On a polynomial lemma of Andrievskii}\footnote"${}^*$"
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(1987).}}\rm
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\centerline{Dieter Gaier}
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\centerline {\it Hern G\"unter Picket zum 70. Geburtstag gewidmet}\par
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In the study of the convergence of Bieberbach polynomials to the
conformal map of region $G$ onto the unit disc $D$ the following lemma
plays a crucial role.\par
\proclaim{Lemma A (Andrievskii[1])} Let $G$ be a region bounded by a
quasiconformal Jordan curve $F$, and let $0 \in G$. Then for every
polynomial $P$ of degree $n\ge 2$ with $P(0)=0$, we have
$$
\max_{z\in G}|P(z)|\ge c(G)\cdot\sqrt{\log n}\cdot
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$$
where $c(G)$ depends on $G$ only.
\endproclaim
This lemma allows one to transform estimates of norms
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$$\dots\dots\dots$$
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A Quasilinear Parabolic System Arising in Modelling of Catalytic Reactors*

AVNER FRIEDMAN
ATHANASSIOS E.TZAVARAS
Department of Mathematics, Purdue University

Abstract. A system of four quasilinear parabolic equations arising in modelling of catalytic reactors is studied; the system is coupled in a nonstandard way. We prove that the system has unique global solution. The asymptotic behavior (as $t \to \infty$) is studied. ©1987 Academic Press, Inc.

0. Introduction

In this paper we consider a semilinear parabolic systems which arises in modelling of a catalytic reactor with a fixed bed [2]. The relevant physical problem is the following: ...

....

We denote by $u(x,t)$ and $v(x,t)$ the concentration of the reactant in the fluid and the temperature of the fluid, respectively. We also denote by $u''(x',x,t)$ and $v'(x',x,t)$ the corresponding concentration and temperature in the solid catalyst pellet. Then the equations modelling the above interaction process are (see [2])

$$\frac{\partial u}{\partial t} = \nabla\cdot(\alpha(u)\nabla u) - V_1\cdot\nabla u - \int_{\partial\Omega'}\beta_1(u-u') \quad \text{in } \Omega, \tag{0.1}$$

$$\frac{\partial v}{\partial t} = \nabla\cdot(\beta(u)\nabla v) - V_1\cdot\nabla v - \int_{\partial\Omega'}\beta_1(v-v') \quad \text{in } \Omega, \tag{0.2}$$

$$\frac{\partial u'}{\partial t} = \nabla\cdot(\alpha'(u')\nabla' u') - r(u')\phi(v') \quad \text{in } \Omega', \tag{0.3}$$

$$\frac{\partial v'}{\partial t} = \nabla\cdot(\beta'(v')\nabla' v') + r(u')\phi(u') \quad \text{in } \Omega' \quad (\gamma > 0); \tag{0.4}$$

where (0.1), (0.2) describe the balance of mass and energy of the reactant ....

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1. H. AMANN, *Existence and stability of the solution for semilinear parabolic systems and applications to some diffusion reaction equations*, Proc. Roy. Soc. Edinburgh Sect. A 81 (1978), 35–47.

....

West Lafayette, Indiana 47907

*This work is partially supported by the national Science Foundation under Grants DMS-8420896 and DMS-8501397. We would like to express our gratitude to Professor R. Ramkrishna for bringing to our attention the model studied in this paper.

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\topmatter
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       in Modelling of Catalytic Reactors$^*$ \endtitle
\author AVNER FRIEDMAN\\
        ATHANASSIOS E.TZAVARAS \endauthor
\affil Department of Mathematics,
        Purdue University \endaffil
\address{West Lafayette, Indiana 47907}
\thanks{\indent
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\frac{\partial v}{\partial t} & =
     \nabla\cdot(\beta(u)\nabla v) - V_1\cdot \nabla v -
      \int_{\partial\Omega'}\beta_1(v-v') &&\quad \text{in}\;\Omega,\tag0.2\\
\frac{\partial u'}{\partial t} & =
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     \; \Omega',\tag0.3\\
\frac{\partial v'}{\partial t} & =
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     \quad (\gamma >0);\tag0.4
\endalignat$$
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    reaction equations \jour Proc. Roy. Soc. Edinburgh Sect. A \vol 81
    \pages 35--47 \yr 1978 \endref
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Fast Parallel Algorithms for the Moore-Penrose Pseudo-inverse Solution to Large Sparse Consistent Systems

MAURICE W. BENSON*
PAUL O. FREDERICSON**

ABSTRACT. The idea of an approximate pseudo-inverse (API) to a singular linear operator A is reviewed and iterative algorithms employing APIs for finding the Moore-Penrose pseudo-inverse solution $x^+ = A^+y$ to the singular system $Ax = y$ are examined. ...

1. Introduction

We present a method for constructing approximate pseudo-inverse (APIs) for certain large sparse underdetermined linear systems and demonstrates our approach on the free boundary spline interpolation problem. Using a 32 node hypercube, we have solved such problems with 2^{19} unknowns.

We begin with a review of the iterative solution of $Ax = y$ using API where $A : X \to Y$, where X and Y are Hilbert spaces. We will employ the iterative algorithm

$$r^n = y - Ax^n, \quad x^{n+1} = x^n + Zr^n \tag{1}$$

to construct the Moore-Penrose pseudo-inverse solution $x^+ = A^+y$. First we give sufficient conditions for convergence. Let R and N denote range and null space of linear operator. We use

DEFINITION 1. *The linear operator $Z : Y \to X$ is an ϵ-approximate inverse ($\epsilon - API$) of the linear operator $A : X \to Y$ if $\epsilon < 1$ and*

$$\|(Z - ZAZ)v\| \le \epsilon\|Zv\| \quad \forall y \in Y, \quad N(Z) \perp R(A), \quad R(Z) \perp N(A) \tag{2}$$

Keywords. Moore-Penrose, pseudo-inverse, parallel computing, hypercube multiprocessor, partial differential equations

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*Chr.Michelsen Institute, Bergen Norway. Permanent address Department of Mathematical Sciences, Lakehead University, Thunder Bay, Canada. This work was supported in the part by the National Sciences and Engineering Research Council of Canada though grant A5031.
**Chr. Michelsen Institute, Bergen Norway. Permanent address Los Alamos national Laboratory, Los Alamos New Mexico. This work was supported in the part by Norges Teknisk-Naturvitenskapelige Forskingsråd through grant ED0228.18233.

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      Solution to Large Sparse Consistent Systems \endtitle
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\author Maurice W. Benson \footnote"$^(*)$"
        {Chr.Michelsen Institute, Bergen Norway. Permanent
         address Department of Mathematical Sciences,
         Lakehead University, Thunder Bay, Canada. This work
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         and Engineering Research Council of Canada though
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        {Chr. Michelsen Institute, Bergen Norway. Permanent
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\endauthor
\keywords{Moore-Penrose, pseudo-inverse, parallel computing,
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We will employ the iterative algorithm
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  r^n = y - Ax^n, \quad x^{n+1} = x^n + Zr^n \tag1
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\quad N(Z) \perp R(A),\quad R(Z) \perp N(A) \tag2
$$
\endproclaim
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ON A ZETA FUNCTION ASSOCIATED TO TERNARY ZERO FORMS

BARRY A.CIPRA

Abstract. We rederive a relation due to Eie between a zeta function for ternary quadratic forms and the Riemann zeta function, correcting a factor corresponding to the prime $p = 2$.

1. Introduction. A recent paper by Eie[1] obtained a relationship between a zeta function associated to ternary quadratic forms and the ordinary Riemann zeta function. However, some technical errors in the derivation resulted in an incorrect and awkward factor corresponding to the prime $p = 2$. In view of potential applications of the result to dimension formulas for spaces of Siegel modular forms, it seems worthwhile to establish the correct formula, which do in Theorem A. The $p = 2$ factor appears in a similar, more elegant form.

2. Definitions and statement of Theorem A. We follow the notation in [1]. Let s_{13} and s_2 be nonzero integers and define

$$\Delta(s_{13}, s_2) = \left\{ S = \begin{pmatrix} 0 & 0 & s_{13} \\ 0 & s_2 & s_{23} \\ s_{13} & s_{23} & s_3 \end{pmatrix} \mid s_{23}, s_3 \in Z \right\}.$$

(Observe that every ternary zero form is equivalent to one given by such a matrix.)

Let

$$P = \left\{ U = \begin{pmatrix} 1 & u & v \\ 0 & 1 & w \\ 0 & 0 & 1 \end{pmatrix} \mid u, v, w \in Z \right\}.$$

Then P acts on $\Delta(s_{13}, s_2)$ by $S \to {}^tUSU$. Let $\mu(s_{12}, s_2)$ be the number of distinct "orbit" formed under the action of $P : \mu(s_{13}, s_2) = |\Delta(s_{13}, s_2)/P|$. Define the zeta function

$$\zeta(t) = \sum_{s_2 \neq 0} \sum_{s_{13}}^{\infty} \frac{\mu(s_{13}, s^2)}{|s_2 s_{13}^2|^t}$$

(Note that $|s_2 s_{13}^2| = |\det S|$.) We shall prove ...

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DEPARTMENT OF MATHEMATICS, ST. OLAF COLLEGE, NORTHFIELD MINNESOTA 55057

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\author BARRY A.CIPRA \endauthor
\address {\indent DEPARTMENT OF MATHEMATICS, ST. OLAF COLLEGE,
         NORTHFIELD MINNESOTA 55057}
\thanks{\indent Received by the
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   \paper A zeta function associated with zero ternary forms
   \jour Proc. Amer. Math. Soc.
   \vol 94 \pages 387--392
   \yr 1985
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OBLIQUE PROCRUSTES ROTATIONS IN FACTOR ANALYSIS*

FRACLIN T. LUK†

Abstract. This paper concerns the oblique rotation of a factor matrix so as to be a least squares fit to a target matrix. An iterative computing procedure is presented.

1. Introduction An important problem in factor analysis is the so-called Procrustes problem(cf. Harman[7,§15.5]). It addresses the extent to which a given body of data can be described in the terms of a prescribed factor pattern. Let A be the given $p \times m$ factor matrix and B the prescribed $p \times m$ factor pattern. Suppose that X is a nonsingular $m \times m$ transformation matrix and that $Z = AX$. We want to find X so as to minimize the least squares criterion

$$\sum_{j=1}^{m} \left\{ \sum_{i=1}^{p} w_{ij}(z_{ij} - b_{ij})^2 \right\},$$

where $Z = (z_{ij})$, $B = (b_{ij})$ and w_{ij} are some fixed arbitrary nonnegative weights (usually equal to one or zero). ...

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4. R. FLETCHER AND M.J.D.POWELL, *A rapidly convergent descent method for minimization*, Comput. J. 2 (1963), 163-168.

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†Department of Computer Science, Cornell University, Ithaca, New York 14853

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%  A sample document for A   eX
%--------------------------------------------------
% This paper is reprinted from SIAM J. Sci. Stat.
% Comp. Vol.5 No.4
%--------------------------------------------------
\topmatter
\title OBLIQUE PROCRUSTES ROTATIONS IN FACTOR ANALYSIS$^*$ \endtitle
\author FRACLIN T. LUK$^\dagger$ \endauthor
\thanks{\indent
        ${}^*$Received by the editors February 1, 1982, and in revised
        from February 15, 1983. This research was supported in part by
        the U.S. Army Research Office under grant DAAG 29-79-C0124.\newline
        \indent
        ${}^\dagger$Department of Computer Science, Cornell University,
        Ithaca, New York 14853}
\abstract{This paper concerns the oblique rotation of a factor matrix so
          as to be a least squares fit to a target matrix. An iterative
          computing procedure is presented.}
\endtopmatter
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% begin document
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{\bf 1. Introduction} An important problem in factor analysis is the so-called
Procrustes problem(cf. Harman[7,\S 15.5]). It addresses the extent to which
a given body of data can be described in the terms of a prescribed
factor pattern. Let $A$ be the given $p \times m$ factor matrix and $B$ the
prescribed $p\times m$ factor pattern. Suppose that $X$ is a nonsingular
$m\times m$ transformation matrix and that $Z=AX$. We want to find $X$ so as
to minimize the least squares criterion
$$
\sum_{j=1}^m\left\{ \sum_{i=1}^p w_{ij}(z_{ij}-b_{ij})^2\right\},
$$
where $Z=(z_{ij}), B=(b_{ij})$ and $w_{ij}$ are some fixed arbitrary nonnegative
weights (usually equal to one or zero). $\dots$ \par
%
%--------------------
%  begin references
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%
\Refs
\ref \no 1 \by R.P. BRENT \book Algorithms for Minimization
    Without Derivatives \publ Prentice-Hall \publaddr Englewood Cliffs, NJ
    \yr 1973 \endref
\ref \no 2 \by M.W. BROWNE \paper Orthogonal rotation to a partially
    specified target \jour Br. J. Math. Statist. Psychol. \yr 1972 \vol 25
    \pages 115--120 \endref
\ref \no 3  \by {---------} \paper Oblique rotation
    to a partially specified target
    \jour Br. J. Math. Statist. Psychol. \vol 25
    \yr 1972 \pages 207--212\endref
\ref \no 4 \by R. FLETCHER AND M.J.D.POWELL \paper A rapidly convergent
     descent method for minimization \jour Comput. J. \vol 2 \yr 1963
     \pages 163--168 \endref
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KSTS/RR-88/002
February 18, 1988

On a theorem of Goldschmidt

BY RICHARD WEISS*

1. INTRODUCTION

In 1980, Goldschmidt proved the following remarkable theorem about trivalent graphs:

(1.1) THEOREM [4]. *Let Γ be a connected, trivalent graph, let $\{\alpha, \beta\}$ be an edge of Γ and let G be a subgroup of aut(Γ) acting transitively on the edge set of Γ such that $|G_\alpha| < \infty$. Then $G_{\alpha\beta}$ divides 2^7.*

(In fact, Goldschmidt showed that the amalgam (G_α, G_β) must belong to one of fifteen isomorphism types.) This theorem was inspired by a result [7] of Tutte from 1947 where the additional assumption was made that G acts transitively on the vertex set of Γ. Goldschmidt's result and various generalizations and modifications of the methods he used to prove it have had a profound influence on the recent theory of the geometries associated with finite groups and their role in the classification of finite simple groups. The original proof has been considerably shortened ans simplified by Delgado and Stellmacher in [2,(3.10) and (6.5)]. Like the original proof, however, their proof makes frequent use of the pushing-up result for $L_2(2)$ of Baumann[1] and Niles[5] (see also [3]). Using the sorts of graph theoretical arguments Goldschmidt used, Stellmacher has recently given a greatly simplified proof of this result [6].Nevertheless, the pushing-up approach to Goldschmidt's result involves certain inherent difficulties. In this note, we give a direct and very simple proof of (1.1) which does not use pushing-up (but which does, of course, make frequent use of ideas distilled from [2], [4] and [6]).

For each vertex α, let

$$Q_\alpha = \bigcup_{\beta \in \Gamma(\alpha)} G_{\alpha\beta},$$

where $\Gamma(\alpha)$ denotes the set of vertices adjacent to α. ...

*Research partially supported by NSF Grants DMS-850192 and DM-8610730(1)

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%--------------------------------------------
%  A sample document for AMS-TeX
%--------------------------------------------
% This paper is reprinted f    Annals of Math.
% Vol.126, No.2.
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\topmatter
\title On a theorem of Goldschmidt \endtitle
\author By Richard Weiss$^*$ \endauthor
\thanks{${}^*$Research partially supported by NSF Grants
        DMS-850192 and DM-8610730(1)}
\endtopmatter
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% begin document
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%
\heading 1. Introduction \endheading
In 1980, Goldschmidt proved the following remarkable theorem about
trivalent graphs:
\proclaim{(1.1) Theorem [4]} Let $\Gamma$ be a connected, trivalent graph,
let $\{ \alpha,\beta \}$ be an edge of $\Gamma$ and let $G$ be a subgroup
of aut($\Gamma$) acting transitively on the edge set of $\Gamma$ such
 that $|G_\alpha| < \infty$. Then $G_{\alpha\beta}$ divides $2^7$.
\endproclaim
(In fact, Goldschmidt showed that the amalgam $(G_\alpha, G_\beta)$
must belong to one of fifteen isomorphism types.) This theorem was
inspired by a result [7] of Tutte from 1947 where the additional
assumption was made that $G$ acts transitively on the vertex set of
$\Gamma$. Goldschmidt's result and various generalizations and
modifications of the methods he used to prove it have had a profound
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The original proof has been considerably shortened ans simplified by
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result for $L_2(2)$ of Baumann[1] and Niles[5] (see also [3]).
Using the sorts of graph theoretical arguments Goldschmidt used,
Stellmacher has recently given a greatly simplified proof of this
result [6].Nevertheless, the pushing-up approach to Goldschmidt's
result involves certain inherent difficulties. In this note, we give
a direct and very simple proof of (1.1) which does not
use pushing-up (but which does, of course, make frequent use of ideas
distilled from [2], [4] and [6]).

For each vertex $\alpha$, let
$$
Q_\alpha = \bigcup_{\beta \in \Gamma(\alpha)} G_{\alpha\beta},
$$
where $\Gamma(\alpha)$ denotes the set of vertices adjacent to $\alpha$. $\dots$
```

Chapter 1.

Binomial Identities

1.1 Summary of Useful Identities

So that the identities themselves do not become buried on an obscure page, we summarize them immediately:

$$\begin{aligned}
(x+y)^n &= \sum_k \binom{n}{k} x^k y_{n-k}, && \begin{matrix}\text{integer } n \text{ or}\\ n \text{ real and } |x/y|<1\end{matrix} && (1.1)\\
\binom{r}{k} &= \binom{r-1}{k} + \binom{r-1}{k-1}, && \begin{matrix}\text{real } r\\ \text{integer } k\end{matrix} && (1.2)\\
\binom{n}{k} &= \binom{n}{n-k}, && \begin{matrix}\text{integer } n\ge 0\\ \text{integer } k\end{matrix} && (1.3)\\
\binom{r}{k} &= \frac{r}{k}\binom{r-1}{k-1}, && \begin{matrix}\text{real } r\\ \text{integer } k\ne 0\end{matrix} && (1.4)\\
\sum_{k=0}^{n}\binom{r+k}{k} &= \binom{r+n+1}{n}, && \begin{matrix}\text{real } r\\ \text{integer } n\ge 0\end{matrix} && (1.5)\\
\sum_{k=0}^{n}\binom{k}{m} &= \binom{n+1}{m+1}, && \text{integer } m,n\ge 0 && (1.6)\\
\binom{-r}{k} &= (-1)^k\binom{r+k-1}{k}, && \begin{matrix}\text{real } r\\ \text{integer } k\end{matrix} && (1.7)\\
\binom{r}{m}\binom{m}{k} &= \binom{r}{k}\binom{r-k}{m-k}, && \begin{matrix}\text{real } r\\ \text{integer } m,k\end{matrix} && (1.8)\\
\sum_k\binom{r}{k}\binom{s}{n-k} &= \binom{r+s}{n}, && \text{integer } n && (1.9)\\
\sum_k\binom{r}{k}\binom{s}{n-k} &= \binom{r+s}{r+n}, && \begin{matrix}\text{integr } n\\ \text{integer } r\ge 0\end{matrix} && (1.10)\\
\sum_k\binom{r}{k}\binom{s+k}{n}(-1)^k &= (-1)^r\binom{s}{n-r}, && \begin{matrix}\text{integr } n\\ \text{integr } r\ge 0\end{matrix} && (1.11)\\
\sum_{k=0}^{r}\binom{r-k}{m}\binom{s+k}{n} &= \binom{r+s+1}{m+n+1}, && \begin{matrix}\text{integer } m,n,r,s\ge 0\\ n\ge s\end{matrix} && (1.12)
\end{aligned}$$

One particularly confusing aspect of binomial coefficients is the ease with which a familiar formula can be rendered unrecognizable by a few transformations. Because of their chameleon chapter there is no substitute for practice of manipulations with binomial coefficients. The reader is referred to Section **1.2.6** of [Knuth 1] for an explanation of the formulas above and for a useful collection of exercises. A large catalog of sums of binomial coefficients, arranged according to the number of terms in the numerator and denominator of the summand, appears in [Gould **72**].

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%-----------------------------------------------
% A sample   cument for AMS-TeX
%---------    ----------------------------------
% Mathemati.. for the Analysis of Algorithms
% {D.H.Greene and D.E.Knouth}, pp5-6.
%-----------------------------------------------
%
\beginsection  Chapter 1.\par
\vskip 0.5cm
\noindent{\bf  Binomial Identities}
\beginsection 1.1 Summary of Useful Identities \par
So that the identities themselves do not become buried on an obscure page,
we summarize them immediately:
$$\alignat2\TagsOnRight
(x+y)^n &= \sum_k \binom nk x^ky_{n-k}, &&\quad
              \matrix
                \text{integer $n$ or} \\
                \text{$n$ real and $|x/y|<1$}
              \endmatrix\tag1.1 \\
\binom rk &= \binom {r-1}k +\binom {r-1}{k-1}, &&\quad
              \matrix
                \text{real $r$}\\
                \text{integer $k$}
              \endmatrix\tag1.2 \\
\binom nk &= \binom n{n-k}, &&\quad
              \matrix
                \text{integer $n\ge0$} \\
                \text{integer $k$}
              \endmatrix\tag1.3 \\
r\choose k &= \frac rk \binom {r-1}{k-1}, &&\quad
              \matrix
                \text{real $r$}\\
                \text{integer $k\neq0$}
              \endmatrix\tag1.4\\
\sum_{k=0}^n \binom {r+k}k & = {{r+n+1}\choose n}, &&\quad
              \matrix
                \text{real $r$}\\
                \text{integer $n\ge0$}
              \endmatrix\tag1.5\\
\sum_{k=0}^n {k\choose m} &=\binom{n+1}{m+1}, &&\quad
                \text{integer $m,n\ge0$} \tag1.6\\
{-r}\choose k &= (-1)^k \binom {r+k-1}k, &&\quad
              \matrix
                 \text{real $r$}\\
                 \text{integer $k$}
              \endmatrix\tag1.7\\
\binom rm \binom mk &= \binom rk \binom{r-k}{m-k}, &&\quad
              \matrix
                 \text{real $r$} \\
                 \text{integer $m,k$}
              \endmatrix\tag1.8\\
\sum_k\binom rk\binom s{n-k} &= \binom{r+s}n , &&\quad
                 \text{integer $n$}\tag1.9\\
\sum_k\binom rk\binom s{n-k} &= \binom {r+s}{r+n}, &&\quad
               \matrix
                  \text{integr $n$}\\
                  \text{integer $r\ge0$}
               \endmatrix\tag1.10\\
\sum_k\binom rk\binom{s+k}n (-1)^k &= (-1)^r\binom s{n-r}, &&\quad
               \matrix
                   \text{integr $n$} \\
                   \text{integr $r\ge0$}
                \endmatrix\tag1.11\\
\sum_{k=0}^r \binom{r-k}m\binom{s+k}n &=\binom {r+s+1}{m+n+1},&&\quad
                \matrix
                   \text{integer $m,n,r,s\ge0$}\\
                   n\ge s
                \endmatrix\tag1.12
\endalignat$$
One particularly confusing aspect of binomial coefficients is the
ease with which a familiar formula can be rendered unrecognizable by
a few transformations. Because of their chameleon chapter there is no
substitute for practice of manipulations with binomial coefficients.
The reader is referred to Section {\bf 1.2.6} of [Knuth 1] for an explanation
of the formulas above and for a useful collection of exercises. A large
catalog of sums of binomial coefficients, arranged according to the
number of terms in the numerator and denominator of the summand, appears
in [Gould {\bf 72}].
```

At this point our estimate of p_n is good to begin dissecting the sum in equation (**4.34**). We wish to introduce more than an O-term in the asymptotic for p_n, so we remove the document part of the series in a form that is easy to sum:

$$\begin{aligned} -2np_n &= \sum_{0\le k<n} \frac{p_k}{n} + \sum_{0\le k<n} p_k\left(\frac{1}{n-k} - \frac{1}{n}\right) \\ &= \frac{1}{n}\sum_{k\ge 0} p_k - \frac{1}{n}\sum_{k\ge n} p_k + \frac{1}{n}\sum_{0\le k<n} p_k\left(\frac{k}{n-k}\right) \\ &= \frac{1}{n}p(1) - \frac{1}{n}\sum_{k\ge n} O\left(\frac{\log k}{k}\right)^2 + \frac{1}{n}\sum_{0\le <n} O\left(\frac{(\log k)^2}{k(n-k)}\right) \\ &= \frac{1}{n}e^{-\pi^2/12} + O\left(\frac{(\log n)^3}{n^2}\right) \end{aligned} \tag{4.38}$$

In the last step we computed $p(1)$ by summing the infinite series

$$\sum_{k\ge 1}\frac{1}{k^2} = \zeta(2) = \frac{\pi^2}{6}. \tag{4.39}$$

We estimated the second term, $\sum_{k\ge n} O(\frac{\log n}{k})^2$, with its integral counterpart

$$\int_n^\infty \left(\frac{\log x}{x}\right)^2 dx = O\left(\frac{(\log n)^2}{n}\right). \tag{4.40}$$

And we computed the last sum with partial fractions:

$$\begin{aligned} \sum_{0\le k<n} O\left(\frac{(\log k)^2}{k(k-n)}\right) &= O\left((\log n)^2 \sum \frac{1}{k(n-k)}\right) \\ &= O\left(\frac{(\log n)^2}{n} \sum\left(\frac{1}{k} + \frac{1}{n-k}\right)\right) \\ &= O\left(\frac{(\log n)^3}{n}\right). \end{aligned} \tag{4.41}$$

Returning to equation (**4.38**) we have a refined estimate of p_n,

$$p_n = \frac{-e^{-\pi^2/12}}{2n^2} + O\left(\frac{\log n}{n}\right)^3. \tag{4.42}$$

Although the details are omitted, the expression for p_n can be bootstrapped through another iteration to obtain

$$p_n = \frac{-e^{\pi^2/12}}{2n^2} + O\left(\frac{\log n}{n^3}\right). \tag{4.43}$$

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%-----------------------------------------------
% A sample document for /   TeX
%-------------------------   ------------------
% Mathematics for the Analysis of Algorithms
% [D.H.Greene and D.E.Knouth], pp.55-56.
%-----------------------------------------------
%
\TagsOnRight\par

At this point our estimate of $p_n$ is good to begin dissecting the
sum in equation ({\bf 4.34}). We wish to introduce more than an
$O$-term in the asymptotic for $p_n$, so we remove the document
part of the series in a form that is easy to sum:
$$\split
-2np_n &= \sum_{0\le k<n} \frac{p_k}n + \sum_{0\le k<n}
      p_k\left(\frac1{n-k} -\frac1n\right) \\
     &= \frac 1n \sum_{k\ge 0}p_k - \frac1n \sum_{k\ge n}p_k
        + \frac1n \sum_{0\le k<n}p_k\left(\frac k{n-k}\right) \\
     &= \frac1n p(1) -\frac1n\sum_{k\ge n}O\left(\frac{\log k}k\right)^2
        +\frac1n\sum_{0\le <n}O\left(\frac{(\log k)^2}{k(n-k)}\right)\\
     &=\frac1n e^{-\pi^2/12}+ O\left(\frac{(\log n)^3}{n^2}\right)
\endsplit\tag 4.38
$$
In the last step we computed $p(1)$ by summing the infinite series
$$
\sum_{k\ge 1}\frac1{k^2} = \zeta(2) = \frac{\pi^2}6.\tag 4.39
$$
We estimated the second term, $\sum_{k\ge n}O(\frac{\log n}k)^2$, with
its integral counterpart
$$
\int_n^\infty \left( \frac{\log x}x \right)^2\,dx = O \left(\frac{(\log n)^2}n
\right).\tag 4.40
$$
And we computed the last sum with partial fractions:
$$\split
\sum_{0\le k<n}O\left(\frac{(\log k)^2}{k(k-n)}\right)
        &=O\left( (\log n)^2\sum\frac1{k(n-k)}\right)\\
        &=O\left(\frac{(\log n)^2}n\sum\left(\frac1k+
           \frac1{n-k}\right)\right)\\
        &=O\left(\frac{(\log n)^3}n\right).
\endsplit\tag4.41
$$
Returning to equation ({\bf 4.38}) we have a refined estimate of $p_n$,
$$
p_n = \dfrac{-e^{-\pi^2/12}}{2n^2} + O\left(\frac{\log n}n\right)^3.\tag4.42
$$
Although the details are omitted, the expression for $p_n$ can be
bootstrapped through another iteration to obtain
$$
p_n=\dfrac{-e^{\pi^2/12}}{2n^2}+ O\left(\frac{\log n}{n^3}\right). \tag4.43
$$
```

To determine the solution having this transformation, we first expand $1/(1+e^{-2s\frac{l}{v}})$ in a series of ascending powers of $e^{-2s\frac{l}{v}}$, and so obtain

$$\bar{e}(x,s) = f(s)[e^{-s\frac{x}{v}} + e^{-\frac{s}{v}(2l-x)} - e^{-\frac{s}{v}(2l+x)} - e^{-\frac{s}{v}(4l-x)} + e^{-\frac{s}{v}(4l+x)} + \dots]. \tag{257}$$

Again making use of formula T9, page 74, we obtain $e(x,t)$ in the form

$$\begin{aligned} e(x,t) = & \left\{ \begin{matrix} f\left(t-\dfrac{x}{v}\right) & \text{when } t > \dfrac{x}{v} \\ 0 & \text{when } t < \dfrac{x}{v} \end{matrix} \right\} \\ & + \left\{ \begin{matrix} f\left(t-2T+\dfrac{x}{v}\right) & \text{when } t > 2T-\dfrac{x}{v} \\ 0 & \text{when } t < 2T-\dfrac{x}{v} \end{matrix} \right\} \\ & - \left\{ \begin{matrix} f\left(t-2T-\dfrac{x}{v}\right) & \text{when } t > 2T-\dfrac{x}{v} \\ 0 & \text{when } t < 2T-\dfrac{x}{v} \end{matrix} \right\} \\ & - \left\{ \begin{matrix} f\left(t-4T+\dfrac{x}{v}\right) & \text{when } t > 4T-\dfrac{x}{v} \\ 0 & \text{when } t < 4T-\dfrac{x}{v} \end{matrix} \right\} \\ & - \left\{ \begin{matrix} f\left(t-4T-\dfrac{x}{v}\right) & \text{when } t > 4T-\dfrac{x}{v} \\ 0 & \text{when } t < 4T-\dfrac{x}{v} \end{matrix} \right\} + \cdots \end{aligned} \tag{258}$$

where we have written

$$T = \frac{l}{v} = \sqrt{LC}\, l, \tag{259}$$

so that T is the time required for a wave to travel the length of the line. ...

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% A sample document for AMS-TeX
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% F.B. Hildebrand, Advanced Calculus for
% Applications, Prentice-Hall, p.468.
%------------------------------------------------
%
\TagsOnRight

To determine the solution having this transformation,
we first expand $1/(1+e^{-2s\frac lv})$ in a series
of ascending powers of $e^{-2s\frac lv}$, and so obtain
$$\multline
\overline e(x,s) = f(s)[e^{-s\frac xv} +
   e^{-\frac sv (2l-x)} - e^{-\frac sv (2t+x)}
  -e^{-\frac sv (4l-x)} \\
  + e^{-\frac sv (4l+x)} +\dots ].
\endmultline\tag257$$
Again making use of formula T9, page 74, we obtain $e(x,t)$
in the form
$$
\align
e(x,t) =\quad &\left\{
 \matrix \format\l&\quad\l\\
  f\left( t -\dfrac xv \right)& \text{when $ t > \dfrac xv$}\\
  0                           & \text{when $ t < \dfrac xv$}\\
 \endmatrix\right\}\\
      +&\left\{
 \matrix\format\l&\quad\l\\
  f\left( t - 2T + \dfrac xv \right) & \text{when $t>2T-\dfrac xv$}\\
  0                                  & \text{when $t<2T-\dfrac xv$}\\
 \endmatrix\right\}\\
      -&\left\{
 \matrix\format \l &\quad\l\\
  f\left( t - 2T - \dfrac xv\right) & \text{when $t>2T-\dfrac xv$}\\
  0                                 & \text{when $t<2T-\dfrac xv$}\\
 \endmatrix\right\}\\
      -&\left\{
 \matrix\format \l &\quad\l\\
  f\left( t - 4T + \dfrac xv\right) & \text{when $t>4T-\dfrac xv$}\\
  0                                 & \text{when $t<4T-\dfrac xv$}\\
 \endmatrix\right\}\\
      -&\left\{
 \matrix\format \l &\quad\l\\
  f\left( t - 4T - \dfrac xv\right) & \text{when $t>4T-\dfrac xv$}\\
  0                                 & \text{when $t<4T-\dfrac xv$}
 \endmatrix\right\} +\cdots\tag258
\endalign$$
where we have written
$$
T = \dfrac lv = \sqrt{LC} \, l,\tag259
$$
so that $T$ is the time required for a wave to travel the length
of the line. $\dots$
```

数式に関する例題

<<< インデックス >>>

x^2	`$x\sp2$`
x^a	`$x\sp a$`
x^α	`$x\sp\alpha$`
2^x	`$2\sp x$`
——	
x^2	`$x^2$`
x^y	`$x^y$`
——	
x_2	`$x\sb2$`
x_y	`$x\sb y$`
——	
x_2	`$x_2$`
x_y	`$x_y$`
——	
x^2y^2	`$x^2y^2$`
x^2y^2	`$x\sp2y\sp2$`
x^2y^2	`$x^ 2y ^2$`
x_2y_2	`$x_2y_2$`
${}_2F_3$	`$_2F_3$`
${}_2F_3$	`${}_2F_3$`
——	
$z=x^{2y}$	`$z=x^{2y}$`
$x_{y+z}=w$	`$x_{y+z}=w$`
2^{32}	`$2^{32}$`
x_{10}	`$x_{10}$`
$x^{\{3y\}}$	`$x^{\{3y\}}$`
——	
$(x+1)^3$	`$(x+1)^3$`

$(x^2)^3$	`$(x^2)^3$`
$[x^2]^3$	`$[x^2]^3$`
$\{x^2\}^{3y}$	`$\{x^2\}^{3y}$`
—	
$\left(x+1\right)^3$	`${(x+1)}^3$`
$\left(x^2\right)^3$	`${(x^2)}^3$`
$\left[x^2\right]^3$	`${[x^2]}^3$`
$\left\{x^2\right\}^{3y}$	`${ \{ x^2 \} }^ {3y}$`
$\left(\left(x^2\right)^3\right)^4$	`${({(x^2)}^3)}^4$`
—	
a^{b^c}	`$a^{b^c}$`
$2^{(2^x)}$	`$2^{(2^x)}$`
$2^{2^{2^x}}$	`$2^{2^{2^x}}$`
$2^{(a+b)^2}$	`$2^{(a+b)^2}$`
x_{y_2}	`$x_{y_2}$`
x_{y^2}	`$x_{y^2}$`
—	
$({\scriptscriptstyle x+}{\scriptstyle y+z})^{\textstyle z+\mathbf{a}}$	`$(\sssize x+ \ssize{y+z})^{\tsize{z+\bold a}}$`
—	
A^a_b	`$A^a_b$`
A_b^a	`$A_b^a$`
$x^{3141516}_{92}+\pi$	`$x^{3141516}_{92} + \pi$`
$\Gamma_{y^a_b}^{z_x^d}$	`$\Gamma_{y^a_b}^{z_x^d}$`
—	
$A^a{}_b$	`${A^a}_b$` or `$A^a{}_b$`
$x_i{}^2$	`${x_i}^2$` or `$x_i{}^2$`
$R_i{}^j{}_{ki}$	`$R_i{}^j{}_{ki}$`
—	
$y_1'+y_2'''$	`$y_1^\prime + y^{\prime\prime\prime}_2$` .
—	
$f^*(x)\cap f_*(\mu)$	`$f^\ast(x)\cap f_\ast(\mu)$`
R^{iii}_{mmm}	`$R^{iii}_{mmm}$`
$R^i_m{}^i_m{}^i_m$	`$R^i_m{}^i_m{}^i_m$`

$\frac{x+y^2}{k+1}$	`$$ \frac{x+y^2}{k+1}$$`
$x+\frac{y^2}{k}+1$	`$$ x+ \frac{y^2} k  +1 $$`
$x+\frac{y^2}{k+1}$	`$$x+\frac{y^2}{k+1}$$`
$x+y^{\frac{2}{k+1}}$	`$$x+y^{\frac 2 {k+1}}$$`
$x=\frac{y^2}{k+1}$	`$$x=\frac{y^2}{k+1}$$`
$\frac{n}{2}+\frac{n}{2}$	`$${\frac n 2} + {\frac n 2 }$$`
$\dfrac{a+1}{b}\bigg/\dfrac{c+1}{d}$	`$$\dfrac{a+1}b\bigg/\dfrac{c+1}d$$`
${n \atop k}$	`$$n\atop k$$`
$\displaystyle{n \choose k}$	`$$n\choose k $$`
${n \choose k}$	`$n\choose k$`
${n+1 \atop k^2}$	`$$n+1\atop k^2$$`

$n+\binom{1}{k}^2$	`$$n+{1\choose k}^2$$`
$\dfrac{n}{2+\frac{n}{2}}$	`$$\frac n {2+{\frac n2}}$$`
$\dfrac{\frac{n}{2}+n}{2}$	`$$\frac{{\frac n 2 } + n} 2$$`
$\dbinom{n}{\frac{k}{2}}$	`$$n\choose{\frac k 2}$$`
$\dfrac{\binom{n}{k}}{2}$	`$$\frac{{n\choose k}} 2$$`
$\dfrac{\binom{n}{k}}{2}$	`$${n\choose k} \over 2 $$`
$\dbinom{n}{k/2}$	`$$n\choose k/2$$`
$\dbinom{n}{\frac{1}{2}k}$	`$$n\choose{\frac1 2} k$$`
$\dfrac{\dbinom{n}{k}}{2}$	`$$\frac {\dsize{n\choose k}} 2$$`
$\dfrac{\frac{\mathbf{n}}{\mathbf{2}}+\mathbf{n}}{2}$	`$${\frac{\bold{\frac n2} + \bold{n}} 2}$$`
$\dfrac{\dfrac{n}{2}+n}{2}$	`$$\frac {\dsize{\frac n2}+n} 2$$`

$$\cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3}}}$$

```
$$\dfrac1{a_1+{\dfrac 1{a_2+\dfrac1{a_3}}}}$$
```

$$\cfrac{n_1}{d_1 + \cfrac{n_2}{d_2}}$$

```
$$\cfrac n_1\\
d_1+\cfrac n_2\\
d_2
\endcfrac$$
```

$$\cfrac{f(n,1)}{d_1 + \cfrac{n_2}{d_2 - \cfrac{1}{d_3}}}$$

```
$$\cfrac {f(n,1)}\\
d_1+\cfrac n_2\\
d_2-\cfrac 1\\
d_3
\endcfrac$$
```

$$\cfrac[l]{n_1}{d_1 + \cfrac[l]{n_2}{d_2 - \cfrac{1}{d+3}}}$$

```
$$\lcfrac n_1\\
d_1+\lcfrac n_2\\
d_2-\cfrac 1\\
{d+3}
\endcfrac$$
```

$\underline{3}$	`$$\underline 3$$`
$\underline{3.1456}$	`$$\underline {3.1456}$$`
$\underline{\underline{4+x}}$	`$$\underline {\underline{4+x}}$$`
$x^{\underline{n}+m}$	`$$x^{\underline n+m}$$`
$\overline{\overline{x^6}+y^{y^5}}$	`$$\overline{\overline{x^6}+y^{y^5}}$$`
$\overleftarrow{x-y}$	`$$\overleftarrow {x-y}$$`
$\overrightarrow{x+y}$	`$$\overrightarrow {x+y}$$`
$A^{\overleftrightarrow{x+y}}$	`$$A^{\overleftrightarrow {x+y}} $$`
$\sqrt{3}$	`$$\sqrt 3$$`
$\sqrt{x+3}$	`$$\sqrt{x+3}$$`
$\sqrt{1-x^2}$	`$$\sqrt{1-x^2}$$`
$\sqrt{\frac{a}{b}}$	`$$\sqrt{\frac a b}$$`

$$\sqrt{1+\sqrt{2+\sqrt{3+\sqrt{4+\sqrt{5+\sqrt{6+\sqrt{7+x}}}}}}}$$

```
$$\sqrt{1+\sqrt{2+\sqrt{3+\sqrt{
 4+\sqrt{5+\sqrt{6+\sqrt{7+x}}}}}}}$$
```

$$\left(\; formula \; \right)$$

```
$$\left( \; formula \; \right)$$
```

$$\left[\; formula \; \right]$$

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$$\left[ \; formula \; \right]$$
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$$\rangle$$

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$$\rangle$$
```

$$1+\left({\frac 1 {1-x^2}}\right)^3$$

```
$$1+\left({\frac 1 {1-x^2} }\right)^3$$
```

$$x=\left[{y^3 \atop k+3}\right]$$

```
$$x=\left[y^3\atop k+3\right]$$
```

$$\left[\frac{a+b}{2}\right]$$

```
$$\left[\frac{a+b}2\right]$$
```

$$f(x) = \frac{1}{\left[\dfrac{1}{x}\right]}$$

```
$$f(x)  = \frac1{\left[\dfrac 1x\right]}$$
```

$$\|f\|_p = \left\{\int_0^1 |f|^p\right\}^{\frac{1}{p}}$$

```
$$
\|f\|_p=\left\{\int_0^1
    |f|^p\right\}^{\frac 1p}$$
```

$$ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) = a\left[\left(x + \frac{b}{2a}\right)^2 + \left(\frac{c}{a} - \frac{b^2}{4a}\right)\right].$$

```
$$
ax^2+bx+c = a\left( x^2+\frac bax +\frac ca \right)
=a\left[\left(x+\frac b{2a}\right)^2 +
 \left(\frac ca - \frac {b^2}{4a} \right)\right].$$
```

$$\frac{d}{dt}\langle V, W\rangle = \left\langle\frac{DV}{dt}, W\right\rangle + \left\langle V, \frac{DW}{dt}\right\rangle.$$

```
$$
\frac d{dt}\langle V, W \rangle
=\left<\frac {DV}{dt},W\right> +
\left<V,\frac {DW}{dt} \right>.
$$
```

$$\biggl(\sum_{k=1}^n A_k\biggr)^2$$

```
$$\biggl(\sum_{k=1}^n A_k \biggr)^2$$
```

$$\biggl(\sum_{i=1}^n x_i y_i\biggr)^2 = \sum_{i=1}^n (x_i y_i)^2 + \sum_{i \ne j} x_i y_i x_j y_j$$

```
$$\biggl(\sum_{i=1}^n x_i y_i \biggr)^2
= \sum_{i=1}^n(x_iy_i)^2 +
  \sum_{i \ne j} x_iy_ix_jy_j
$$
```

$$x+y\in\left(\frac{a}{b},\frac{c}{d}\right]$$

```
$$x+y\in\left({\frac a b},
          {\frac c d} \right]$$
```

$$x+y\in\left]\frac{a}{b},\frac{c}{d}\right[$$

```
$$x+y\in\left]{\frac a b},
          {\frac c d} \right[$$
```

$$1+\left\{\frac{1}{1-x^2}\right\}^3$$

```
$$1+\left\{\frac 1 {1-x^2} \right\}^3$$
```

$$\left\|\frac{x}{a}\right\|=\frac{\|x\|}{|a|}$$

```
$$\left\| \frac x a \right\|
  ={\frac {\|x\|} {|a|}}$$
```

$$\left.\frac{dy}{dx}\right|_{x=a}$$

```
$$\left. \frac {dy} {dx} \right|_{x=a}$$
```

$$\left.\frac{c+1}{d}\right/x^2$$

```
$$\left. \frac {c+1} d \right/x^2$$
```

$$x^2\left/\frac{c+1}{d}\right.$$

```
$$x^2\left/ \frac {c+1} d \right.$$
```

$$\left[\left.\frac{dy}{dx}\right|_{x=a}\right]^2$$

```
$$\left[\left. \frac {dy} {dx}
        \right|_{x=a}\right]^2$$
```

$$x+\frac{B}{2A}=\pm\sqrt{\frac{-C}{A}}\left(y+\frac{D}{2C}\right).$$

```
$$
x+\frac B{2A} =\pm\sqrt{\frac{-C}A}
\left(y+\frac D{2C} \right).
$$
```

$$F_n=\frac{\left(\dfrac{1+\sqrt{5}}{2}\right)^2-\left(\dfrac{1-\sqrt{5}}{2}\right)^2}{\sqrt{5}}$$

```
$$
F_n  =\frac {\left(\dfrac{1+\sqrt 5}2\right)^2
-\left(\dfrac{1-\sqrt 5}2\right)^2} {\sqrt 5}
$$
```

$$y = f(x + \mathit{constant})$$

```
$$y = f(x + constant) $$
```

$$y = f(x + \text{constant})$$

```
$$y = f(x + \text{constant})$$
```

$$F_n = F_{n-1} + F_{n-2}, \quad n > 1.$$

```
$$F_n = F_ {n-1} + F_{n-2},
  \quad n >1.$$
```

$$F_n = F_{n-1} + F_{n-2}, \qquad n > 1.$$

```
$$F_n = F_ {n-1} + F_{n-2},
  \qquad n >1.$$
```

$$F_n = F_{n-1} + F_{n-2} \qquad \text{for } \textsl{every } n > 1$$

```
$$F_n = F_{n-1}+F_{n-2}\qquad
  \text{for {\sl every} } n >1$$
```

$$\sin 2\theta = 2\sin\theta\cos\theta$$

```
$$\sin2\theta = 2\sin\theta\cos\theta$$
```

$$\sin(2x) = 2\sin x\cos x$$

```
$$\sin(2x) = 2\sin x \cos x$$
```

$$e^{\log x} = x$$

```
$$e^{\, \log x} = x$$
```

$$\sin^2 x + \cos^2 x = 1$$

```
$$\sin^2x + \cos^2x = 1$$
```

$$\log_2 x = (\log_2 e)(\log x)$$

```
$$\log_2x=(\log_2 e)(\log x)$$
```

$$\max_{1<n<m} \log_2 P_n$$

```
$$\max_{1<n<m}\log_2P_n$$
```

$$\lim_{x\to 0} \frac{\sin x}{x}$$

```
$$\lim_{x\to 0}{\frac {\sin x} x}$$
```

$$\frac{\max_{1<n<m} \log_2 P_n}{\lim_{x\to 0} \frac{\sin x}{x}}$$

```
$$\frac {\max_{1<n<m}\log_2P_n}
        {\lim_{x\to 0}
     {\frac {\sin x}x}} $$
```

$\dfrac{\max\limits_{1<n<m} \log_2 P_n}{\lim\limits_{x\to 0} \dfrac{\sin x}{x}}$	`$$\frac {\dsize{\max_{1<n<m}\log_2P_n}} {\dsize{\lim_{x\to 0} {\frac {\sin x} x}}}$$`
$\operatorname{Log}_2 x$	`$$\operatorname{Log}_2x$$`
e^{Log}	`$$e^{\, \operatorname{Log}}$$`
$2^{\operatorname{Log}_2 x}$	`$$\def\Log{\operatorname{Log}}2^{\,\Log_2 x}$$`
$\operatorname*{Res}_{x=0} \dfrac{f(x)}{x}$	`$$\operatornamewithlimits{Res} _{x=0}{\frac{f(x)} x}$$`

$\sum x_n$	`$\sum x_n$`
$\displaystyle\sum x_n$	`$$\sum x_n$$`
$\displaystyle\sum_{n=1}^{m} x_n$	`$$\sum_{n=1}^m x_n$$`
$\displaystyle\sum_{i=1}^{n} a_i$	`$$\sum_{i=1}^n a_i$$`
$\displaystyle\frac{\sum_{n=1}^{m} x_n}{\sum_{n=1}^{m} y_n}$	`$$\frac {\sum_{n=1}^m  x_n} {\sum_{n=1}^m y_n}$$`
$\displaystyle\left(\frac{A}{B}\right)^{\sum_{n=1}^{m} x_n}$	`$$\left(\frac A B\right)^{\sum^m_{n=1}x_n}$$`
$\displaystyle\sum_{0<i<m \atop 0<j<n} P(i,j)$	`$$\sum_{0<i<m\atop 0<j<n} P(i,j)$$`
$\displaystyle\sum_{0<i<m \atop 0<j<n} P(i,j)$	`$$\sum_{\ssize{0<i<m}\atop\ssize{0<j<n}} P(i,j)$$`
$\displaystyle\int_{-\infty}^{+\infty}$	`$$\int_{-\infty}^{+\infty}$$`
$\displaystyle\int\int$	`$$\int\int$$`
$\displaystyle\iint$	`$$\iint$$`
$\displaystyle\iiint$	`$$\iiint$$`
the symbol $\int_{-\infty}^{+\infty}$	`the symbol $\int_{-\infty}^{+\infty}$`

$$\int \cdots \int$$

```
$$\idotsint$$
```

$$\iint_M$$

```
$$\iint_M$$
```

$$\int_a^b f(x)\,dx$$

```
$$\int_a^b f(x) \, dx$$
```

$$\left(\sum_{i=1}^N a_i\right)^2$$

```
$$\left(\sum_{i=1}^N a_i\right)^2$$
```

$$\left(\sqrt{\frac A B}+\sum_{i=1}^N a_i\right)^2$$

```
$$\left(\sqrt{\frac A B}
     +\sum_{i=1}^N a_i \right)^2$$
```

$$f\left(\sum_{i=1}^N a_i\right)$$

```
$$f\left(\sum_{i=1}^N a_i \right)$$
```

$$\left(\sqrt{\frac A B}+\sum_{i=1}^N a_i\right)^2$$

```
$$\left(\sqrt{\frac A B}
   +\sum_{i=1}^N a_i \right)^2$$
```

$$\sqrt{\sum_{i=1}^N a^i}$$

```
$$\sqrt{\sum_{i=1}^N a^i}$$
```

$$\sqrt{\sum_{0<i<N} a_i}$$

```
$$\sqrt{\sum_{0<i<N} a_i}$$
```

$$\frac{1+\sum_{i=1}^N a_i}{1+\sum_{j=1}^M b_j}$$

```
$$\frac {1+\sum_{i=1}^N a_i}
         {1+\sum_{j=1}^M b_j}$$
```

$$\frac{\displaystyle 1+\sum_{i=1}^N a_i}{\displaystyle 1+\sum_{j=1}^M b_j}$$

```
$$\frac {\dsize{1+\sum_{i=1}^N a_i}}
      {\dsize{1+\sum_{j=1}^M b_j}}$$
```

$$\pi(n) = \sum m = 2^n \left\lfloor \left(\sum_{m=2}^{m-1} \lfloor (m/k)/\lceil m/k \rceil \rfloor \right)^{-1} \right\rfloor$$

```
$$
\pi(n)\,=\, \sum{m=2}^n\left\lfloor\biggl(\,
\sum_{m=2}^{m-1}\lfloor(m/k)/\lceil m/k
\rceil\rfloor\bigr)^{-1}\right\rfloor
$$
```

$$m^* \left(A \cap \left[\bigcup_{i=1}^{n} E_i \right] \right) = \sum_{i=1}^{n} m^*(A \cup E_i)$$

```
$$
m^*\left(A\cap\biggl[\,\bigcup_{i=1}^n
E_i\biggr]\right)=\sum_{i=1}^n m^*(A\cup E_i)
$$
```

$$\left(\sqrt{\frac{A}{B}} + \sum_{i=1}^{N} a_i \right)^2$$

```
$$
\left(\sqrt{\frac AB}+ \sum_{i=1}^Na_i\right)^2
$$
```

$$f \left(\sum_{i=1}^{n} p_i x_i \right) \le \sum_{i=1}^{n} p_i f(x_i)$$

```
$$
f\left(\sum_{i=1}^np_ix_i\right)
\le
\sum_{i=1}^np_if(x_i)
$$
```

$$\sqrt[\alpha+\beta]{1\frac{a}{b}}$$

```
$$
\root\alpha+\beta\of{1\frac ab}
$$
```

$$\sqrt[\alpha+\beta]{1+\frac{a}{b}}$$

```
$$\root\uproot 3\leftroot{-2}
  \alpha+\beta \of{1+\frac ab}
$$
```

$$(f \cdot g)^{(n+1)}(a) = \sum_{k=0}^{n+1} \binom{n+1}{k} f^{(k)}(a) g^{(n+1-k)}(a)$$

```
$$
(f\cdot g)^{(n+1)}(a)=\,
\sum_{k=0}^{n+1}\binom{n+1}k
  f^{(k)}(a)g^{(n+1-k)}(a)
$$
```

$$0 = (1 + (-1))^n = \sum_{j=0}^{n} (-1)^j \binom{n}{j}$$

```
$$
0 = (1 + (-1))^n
  = \sum_{j=0}^{n}(-1)^j\binom nj
$$
```

$$\sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} y_i^2 = \sum_{i=1}^{n} x_i^2 y_i^2 + \sum_{i \neq j}^{n} x_i^2 y_j^2$$

```
$$
\sum_{i=1}^n x_i^2 \cdot
  \sum_{i=1}^n y_i^2 \,=
  \sum_{i=1}^n x_i^2 y_i^2 +
    \sum_{i\neq j}^n x_i^2 y_j^2
$$
```

$$\sum_{i=1}^{p} \sum_{j=1}^{q} \sum_{k=1}^{r} a_{ij} b_{jk}^2 c_{ki}$$

```
$$
\sum_{i=1}^p\sum_{j=1}^q\sum_{k=1}^r
 a_{ij} b_{jk}^2 c_{ki}
$$
```

$$\Sigma^2 : [X, S_0(\infty)] \to [\Sigma^2 X, S_0(\infty)]$$

```
$$
\Sigma^2:[X,S_0(\infty)]
\to[\Sigma^2X,S_0(\infty)]
$$
```

$$\bigcup_{n=1}^{m} (A_m \cup B_n)$$

```
$$ \bigcup_{n=1}^m (A_m \cup B_n) $$
```

$$\bigcup_n X_n \bigm\| \bigcap_n Y_n$$

```
$$ \bigcup_n X_n \bigm\| \bigcap_n Y_n$$
```

$$X \setminus \bigcup_{i \in I} A_i = \bigcap_{i \in I} X \setminus A_i$$

```
$$X \setminus \bigcup_{i\in I}A_i =
   \bigcap_{i\in I}X\setminus A_i
$$
```

$$\sum_{\substack{0\le i\le m \\ 0<j<n}} P(i,j)$$

```
$$\sum \Sb 0\le i\le m \\
          0 < j < n \endSb P(i,j)
$$
```

$$\sum_{\substack{0\le i\le p \\ 1\le j\le q \\ 1\le k\le r}} a_{ij}b_{jk}^2c_{ki}$$

```
$$\sum \Sb 0\le i\le p \\
          1\le j\le q \\
          1\le k\le r \endSb
   a_{ij}b_{jk}^2c_{ki}
$$
```

$$\lim_{x\to 0}\frac{\sin^2 ax}{\sin^2 bx} = \left(\frac{a}{b}\right)^2$$

```
$$
\lim_{x\to0}\frac{\sin^2ax}{\sin^2bx}
 = \left(\frac ab\right)^2
$$
```

$$\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{x\to\infty}\frac{\sin\frac{1}{x}}{\dfrac{1}{x}} = \lim_{x\to 0+}\frac{\sin x}{x}$$

```
$$\lim_{x\to\infty}x\sin\frac 1x \,=\,
  \lim_{x\to\infty}\dfrac{\sin\frac 1x}
       {\dfrac 1x}\,=\,
  \lim_{x\to 0^+}\frac{\sin x}x
$$
```

$$\frac{1}{2} + \cos x + \cos 2x + \cdots + \cos nx = \frac{\sin(n+\frac{1}{2})x}{2\sin\dfrac{x}{2}}$$

```
$$
\frac 12 +\cos x +\cos 2x +\cdots + \cos nx
   \, = \, \frac {\sin[(n+\frac 12 )x]}
   {2\sin{\dfrac x2}}
$$
```

$$(\log\circ f)' = f'/f$$

```
$$
(\log\circ f)' \, = \, f'/f
$$
```

$$\lim_{x\to 0+} x(\log x)^n = 0$$

```
$$
\lim_{x\to 0^+} x(\log x)^n = 0
$$
```

$$\lim_{h\to 0^+} \int_{-1}^{1} \frac{h}{h^2+x^2} \,=\, \lim_{h\to 0^+} \arctan\frac{x}{h}\right|_{-1}^{1} \,=\, \pi$$

```
$$
\lim_{h\to 0^+} \int_{-1}^{1}
\frac h{h^2+x^2} \,=\,\left.
\lim_{h\to 0^+} \arctan\frac xh\right|_{-1}^1
\,=\, \pi
$$
```

$$\arctan\tfrac 12 + \arctan\tfrac13 \,=\, \arctan\left(\frac{\frac 12 - \tfrac 13}{1-\tfrac 16}\right) \,=\, \frac{\pi}{4}$$

```
$$
\arctan\tfrac 12 + \arctan\tfrac13 \,=\,
\arctan\left(\frac{\frac 12 - \tfrac 13}
{1-\tfrac 16}\right) \,=\,
\frac {\pi}4
$$
```

$$l-m = \lim_{\substack{n\to\infty\\ \text{n even}}} \frac{2-a_n^2}{1+a_n} \,=\, \frac{2-m^2}{1+m}$$

```
$$
1-m =
\lim\Sb n\to\infty\\
        \text{n even}\endSb
\frac{2-a_n^2}{1+a_n} \,=\,
\frac{2-m^2}{1+m}
$$
```

$$\delta = \min\left(\sin^2\left(\frac{[\min(1,\epsilon/10)]^2}{9}\right) + \min(1,\epsilon/10), [min(1,\epsilon/6]^2\right)$$

```
$$
\delta = \min\left(\sin^2\left(
\frac{[\min(1,\varepsilon/10)]^2}9\right)+
\min(1,\varepsilon/10),[min(1,\varepsilon/6]^2
\right)
$$
```

$$\int\limits_{k\pi+\pi/2-\delta}^{k\pi+\pi/2+\delta} \left|\frac{\sin x}{x}\right| dx \ge \frac{\delta}{k\pi+\pi/2}$$

```
$$
\int\limits_{k\pi+\pi/2-\delta}^{k\pi+\pi/2+\delta}
\left|\frac{\sin x}x \right|\,dx
\ge
\frac\delta{k\pi+\pi/2}
$$
```

$$\lim_{x\to\infty}\frac{\int_x^{x+\frac{\log x}{2x}} e^{t^2}\,dt}{e^{x^2}} = \lim_{x\to\infty}\frac{e^{\left(x+\frac{\log x}{2x}\right)^2}-e^{x^2}}{2xe^{x^2}} = \frac{1}{2}$$

```
$$
\lim_{x\to\infty}\dfrac{\int_x^{x+\frac{\log x}{2x}}
e^{t^2}\,dt}{e^{x^2}}\,=\,
\lim_{x\to\infty}\frac{ e^{\left( x+\frac{\log x}{2x}\right)^2}
-e^{x^2}}{2xe^{x^2}} \,=\, \frac12
$$
```

$$\begin{aligned} x &= y = z \\ x^2 &= y^2 \\ x^3 &= y^3 \end{aligned}$$

```
$$\align x & = y = z \\
         x^2 & = y^2 \\
         x^3 & =y^3 \endalign$$
```

$$\begin{aligned} Z = (X+Y)&(A+B+C+D+E+F) \\ &\qquad + G+H+K+L+M+N) \end{aligned}$$

```
$$\align Z = (X+Y)
     & (A+B+C+D+E+F)\\
     & \qquad + G+H+K+L+M+N) \endalign$$
```

$$\begin{aligned} (a+b)(a+b) &= a^2 + 2ab + b^2, \\ (a-b)(a-b) &= (a+b)a - (a+b)b \\ &= a^2 + ab - ab + b^2 \\ &= a^2 - b^2. \end{aligned}$$

```
$$\align (a+b)(a+b)  & = a^2 + 2ab + b^2, \\
         (a-b)(a-b)  & = (a+b)a - (a+b)b \\
                     & = a^2 + ab -ab +b^2 \\
                     & = a^2 -b^2. \endalign$$
```

$$\left\{ \begin{aligned} a &= b \\ a^2 &= b^2 \end{aligned} \right.$$

```
$$\left\{ \aligned a &= b \\
   a^2 & = b^2 \endaligned\right.$$
```

$$\left\{\begin{aligned} a &= b \\ a^2 &= b^2 \end{aligned}\right. \qquad \left\{\begin{aligned} c &= d \\ c^2 &= d^2 \\ c^3 &= d^3 \end{aligned}\right.$$

```
$$\left\{ \aligned a &= b \\
  a^2 & = b^2 \endaligned\right.
   \qquad\left\{\aligned c &=d \\
  c^2 & = d^2 \\
  c^3 & = d^3 \endaligned\right.$$
```

$$f(x) = \begin{cases} x+1, & \text{for } x > 0 \\ x-1, & \text{for } x \le 0. \end{cases}$$

```
$$f(x)=
\cases x+1, &\text{for $x>0$}\\
       x-1, &\text{for $x\le 0$}.
\endcases$$
```

$$\begin{aligned} a &= b = c \qquad (1) \\ a^2 &= b^2 \\ a^3 &= c^3 \qquad (2) \end{aligned}$$

```
$$\align a&=b=c \tag 1 \\
       a^2& = b^2 \\
       a^3& = c^3 \tag2\endalign$$
```

$$\begin{aligned} \max(f,g) &= \frac{f+g+|f-g|}{2}, \qquad (1) \\ \max(f,-g) &= \frac{f-g+|f+g|}{2}. \qquad (2) \end{aligned}$$

```
$$\align
\max(f,g)  & = \frac{f+g+|f-g|}2, \tag1 \\
\max(f,-g) & = \frac{f-g+|f+g|}2. \tag2
\endalign$$
```

$$Q^l = Q_1\biggl\{\sum_k(-1)^k(PQ_1 - I)^k\biggr\} \tag{1_l}$$

$$Q^r = \biggl\{\sum_k(-1)^k(Q_1P - I)^k\biggr\} \tag{1_r}$$

```
$$\align
Q^l\, & = Q_1\biggl\{\sum_k(-1)^k
    (PQ_1-I)^k\biggr\} \tag1${}_l$ \\
Q^r\, & = \biggl\{\sum_k(-1)^k
    (Q_1P-I)^k\biggr\} \tag1${}_r$
\endalign$$
```

$$\alpha_4 = \sqrt{\frac12}$$

$$\alpha_8 = \sqrt{\frac12 + \frac12\sqrt{\frac12}}$$

$$\alpha_{16} = \sqrt{\frac12 + \frac12\sqrt{\frac12 + \frac12\sqrt{\frac12}}}$$

etc.

```
$$\align
  \alpha_4 & = \sqrt{\frac12}\\
  \alpha_8 & = \sqrt{\frac12
     + \frac12 \sqrt{\frac12}}\\
  \alpha_{16} & = \sqrt{\frac12
     + \dfrac12\sqrt{\frac12
       +\frac12\sqrt{\frac12}}} \\
  \text{etc.}&
  \endalign$$
```

$$\begin{aligned} a&=b\\ c&=d+e\\ f+g&=h+k\\ l&=m+n \end{aligned}\tag{1}$$

```
$$\aligned a&=b \\
          c &=d+e \\
        f+g&=h+k\\
          l&=m+n
\endaligned\tag1$$
```

$$\left\{\begin{aligned} \alpha&=f(z)\\ \beta&=f(z^2)\\ \gamma&=f(z^3) \end{aligned}\right\}\qquad\left\{\begin{aligned} x&=\alpha^2-\beta\\ y&=\gamma \end{aligned}\right\}.$$

```
$$
\left\{
 \aligned
  \alpha & = f(z)\\
  \beta  & = f(z^2)\\
  \gamma & = f(z^3)
 \endaligned
\right\}\qquad\left\{
 \aligned
   x & = \alpha^2-\beta\\
   y & = \gamma
\endaligned\right\}.
$$
```

$$k_1, k_2 \,= H\pm\sqrt{H^2-K}\qquad\text{where}\qquad\left\{\begin{aligned} K&=\frac{eg-f^2}{EG-F^2}\\ H&=\frac{Eg-2Ff+Ge}{2(EG-F^2)}. \end{aligned}\right.$$

```
$$
k_1, k_2 \,= H \pm\sqrt{H^2-K}
\qquad\text{where}\qquad
\left\{
 \aligned
  K & = \frac{eg-f^2}{EG-F^2} \\
  H & = \frac{Eg-2Ff+Ge}{2(EG-F^2)}.
 \endaligned
\right.
$$
```

$$\begin{aligned} K &= \frac{cg-f^2}{EG-F^2} \\ H &= \frac{Eg-2Ff+Ge}{2(EG-F^2)}. \end{aligned} \tag{22}$$

```
$$\aligned
  K & = \frac{eg-f^2}{EG-F^2} \\
  H & = \frac{Eg-2Ff+Ge}{2(EG-F^2)}.
\endaligned\tag 22$$
```

$$\begin{multline} A+B+C+D+E+F+G+H+I \\ +J+K+L+M+N+O+P+Q+R \\ =S+T+U+V+W+X+Y+Z \end{multline}$$

```
$$\multline
  A+B+C+D+E+F+G+H+I\\
    +J+K+L+M+N+O+P+Q+R\\
      = S+T+U+V+W+X+Y+Z
\endmultline$$
```

$$\begin{multline} A+B+C+D+E+F+G+H+I+J+K+L+M+N \\ +A'+B'+C'+D'+E'+F'+G'+H' \\ +A_1'+B_1'+C_1'+D_1'+E_1'+F_1' \\ =P+Q+R+S+T+U+V+W \end{multline} \tag{1}$$

```
$$\multline
  A+B+C+D+E+F+G+H+I+J+K+L+M+N\\
   +A^\prime +B^\prime+C^\prime
     +D^\prime +E^\prime +F^\prime
       +G^\prime +H^\prime \\
  +A_1^\prime + B_1^\prime
   +C_1^\prime +D_1^\prime
    +E_1^\prime +F_1^\prime \\
  =P+Q+R+S+T+U+V+W
\endmultline\tag1$$
```

$$\begin{gathered} a = b + c \\ d = e \\ f + g = h \end{gathered}$$

```
$$
\gather a = b+c\\
        d = e \\
        f+g=h\endgather
$$
```

$$g = \det(g_{ij}) \tag{3-2}$$

$$g^{kl} = (k, l) \text{ entry of the inverse matrix of } (g_{ij}) \tag{3-3}$$

```
$$\gather
g=\det(g_{ij})\tag3-2\\
\text{$g^{kl}=(k,l)$ entry of
the inverse matrix of $(g_{ij})$}\tag3-3
\endgather$$
```

$$\begin{gathered} a + b = c \\ f(a) + f(b) = f(c) \\ \alpha = \beta + \delta \\ \alpha' = \beta' + \delta' \\ A + B = C + D + E \end{gathered}$$

```
$$
\gather
  a+b=c \\
  f(a)+f(b) =f(c) \\
   {\align
     \alpha  &=\beta +\delta \\
     \alpha' &=\beta' +\delta'
    \endalign} \\
  A+B = C +D + E
\endgather
$$
```

We have $(a+bi)^2=\alpha+\beta i$ if and only if

$$\begin{gathered} a^2-b^2=\alpha \\ 2ab=\beta \tag{*} \end{gathered}$$

which can be solved to give

$$\left.\begin{aligned} a &= \sqrt{2\alpha+2\sqrt{\alpha^2+\beta^2}} \\ b &= \frac{\beta}{2\sqrt{2\alpha+2\sqrt{\alpha^2+\beta^2}}} \end{aligned}\right\} \qquad\text{or}\qquad \left\{\begin{aligned} a &= -\sqrt{2\alpha+2\sqrt{\alpha^2+\beta^2}} \\ b &= \frac{-\beta}{2\sqrt{2\alpha+2\sqrt{\alpha^2+\beta^2}}} \end{aligned}\right.$$

```
We have $(a+bi)^2=\alpha +\beta i$
if and only if
$$\gather
    a^2 -b^2 = \alpha \\
    2ab = \beta \tag${}^*$
  \endgather$$
which can be solved to give
$$\left.
   \aligned
    a & = \sqrt{2\alpha
       +2\sqrt{\alpha^2+\beta^2}} \\
    b & =\frac{\beta}{2\sqrt{2\alpha
       +2\sqrt{\alpha^2+\beta^2}}}
   \endaligned
  \right\}
\qquad\text{or}\qquad
\left\{
 \aligned
   a & = -\sqrt{2\alpha
      +2\sqrt{\alpha^2+\beta^2}} \\
   b & = \frac{-\beta}{2\sqrt{2\alpha
      +2\sqrt{\alpha^2+\beta^2}}}
 \endaligned
\right.$$
```

$$\begin{aligned}
(a+b)^{n+1} &= (a+b)(a+b)^n = (a+b)\sum_{j=0}^n \binom{n}{j} a^{n+1-j}b^j \\
&= \sum_{j=0}^n \binom{n}{j} a^{n+1-j}b^j + \sum_{j=1}^n \binom{n}{j-1} a^{n-j}b^j \\
&= \sum_{j=0}^n \binom{n+1}{j} a^{n+1-j}b^j.
\end{aligned}$$

```
$$\align
(a+b)^{n+1} \,
& =\,(a+b)(a+b)^n
    \,=\,(a+b)\sum_{j=0}^n
    \binom nj a^{n+1-j}b^j \\
& =\, \sum_{j=0}^n\binom nj
    a^{n+1-j}b^j\,+\, \sum_{j=1}^n
     \binom n{j-1} a^{n-j}b^j \\
& =\, \sum_{j=0}^n \binom{n+1}j
    a^{n+1-j}b^j.
\endalign$$
```

$$\begin{aligned}
(a+b)^{n+1} &= (a+b)(a+b)^n = (a+b)\sum_{j=0}^n \binom{n}{j} a^{n+1-j}b^j \\
&= \sum_{j=0}^n \binom{n}{j} a^{n+1-j}b^j + \sum_{j=1}^n \binom{n}{j-1} a^{n-j}b^j \\
&= \sum_{j=0}^n \binom{n+1}{j} a^{n+1-j}b^j.
\end{aligned} \tag{1-2}$$

```
$$\split
(a+b)^{n+1} \,
& =\,(a+b)(a+b)^n
    \,=\,(a+b)\sum_{j=0}^n
   \binom nj a^{n+1-j}b^j \\
& =\, \sum_{j=0}^n\binom nj
    a^{n+1-j}b^j\,+\,
      \sum_{j=1}^n\binom n{j-1}
        a^{n-j}b^j \\
& =\, \sum_{j=0}^n
    \binom{n+1}j a^{n+1-j}b^j.
\endsplit\tag1--2$$
```

We have

$$X = (-1)^{i+j-k/3+*[\alpha,\beta]} Z_1 + (-1)^{\alpha/\beta-*[i+j/2,i+k/3]} Z_2$$

which by properties (a)–(b) of $*$, together with commutativity of the ring,

$$= \alpha Z_1 + \beta Z_2,$$

which is the desired formula.

```
We have
$$
\align
X & = (-1)^{i+j-k/3+*[\alpha,\beta]}Z_1
   +(-1)^{\alpha/\beta-*[i+j/2,i+k/3]}Z_2 \\
\intertext{which by properties (a)--(b) of $*$,
 together with commutativity of the ring,}
  & =\alpha Z_1+\beta Z_2,
\endalign
$$
which is the desired formula. \par
```

$$\begin{aligned}(f\circ g)'''(x) &= \bigl[f'''(g(x))\cdot g'(x)^3 + 2f''(g(x))\cdot g'(x)g''(x)\bigr]\\ &\qquad + \bigl[f''(g(x))\cdot g'(x)g''(x) + f'(g(x))\cdot g'''(x)\bigr]\end{aligned}$$

```
$$\split
(f\circ g)'''(x) \,
& = \bigl[f'''(g(x))\cdot g'(x)^3+
    2f''(g(x))\cdot g'(x)g''(x)\bigr] \\
&\qquad +\bigl[f''(g(x))\cdot g'(x)g''(x)
    + f'(g(x))\cdot g'''(x)\bigr]
\endsplit$$
```

$$\begin{aligned}\Delta &= [a+b+c]^n(a_{11}+b_{11}+c_{11}\\ &\qquad\qquad + a_{12}+b_{12}+c_{12}+a_{22}+b_{22}+c_{22}).\end{aligned}$$

```
$$\split
\Delta \,
&= [a+b+c]^n(a_{11}+b_{11}+c_{11} \\
&\qquad \qquad + a_{12}+b_{12}
   +c_{12}+a_{22}+b_{22}+c_{22}).
\endsplit$$
```

$$\begin{multline}\int_a^b\biggl\{ \int_a^b[f(x)^2 g(y)^2 +f(x)^2g(x)^2-2f(x)g(x)f(y)g(y)\, dx\biggr\}\,dy \\ = \int_a^b\biggl\{g(y)^2\int_a^b f^2+f(y)^2 \int_a^b g^2-2f(y)g(y)\int_a^b fg \biggr\}\, dy\end{multline}$$

```
$$\multline
\int_a^b\biggl\{ \int_a^b[f(x)^2 g(y)^2
 +f(x)^2g(x)^2-2f(x)g(x)f(y)g(y)\,
  dx\biggr\}\,dy \\
 = \int_a^b\biggl\{g(y)^2\int_a^b f^2+f(y)^2
   \int_a^b g^2-2f(y)g(y)\int_a^b fg \biggr\}\,
   dy \endmultline$$
```

$$\begin{multline}\int_a^b\biggl\{ \int_a^b[f(x)^2 g(y)^2 +f(x)^2g(x)^2-2f(x)g(x)f(y)g(y)\, dx\biggr\}\,dy \\ = \int_a^b\biggl\{g(y)^2\int_a^b f^2+f(y)^2 \int_a^b g^2-2f(y)g(y)\int_a^b fg \biggr\}\, dy \tag{15}\end{multline}$$

```
$$\multline
\int_a^b\biggl\{ \int_a^b[f(x)^2 g(y)^2
 +f(x)^2g(x)^2-2f(x)g(x)f(y)g(y)\,
  dx\biggr\}\,dy \\
 = \int_a^b\biggl\{g(y)^2\int_a^b f^2+f(y)^2
   \int_a^b g^2-2f(y)g(y)\int_a^b fg \biggr\}\,
   dy \endmultline\tag 15$$
```

$$f^{(k)}(x) = e^{-1/x^2}[\sum_{i=1}^{3k}\frac{a_i}{x^i}\sin\frac{1}{x} + \sum_{i=1}^{3k}\frac{b_i}{x^i}\cos\frac{1}{x}]$$

for some numbers $a_1, \dots, a_{3k}, b_1, \dots, b_{3k}$.

```
$$\multline
f^{(k)}(x)\, =\,
e^{-1/x^2}\bigl[
 \sum_{i=1}^{3k}\frac{a_i}{x^i} \sin\frac 1x
 +\sum_{i=1}^{3k}\frac{b_i}{x^i} \cos\frac 1x
 \bigr]\\
\text{for some numbers
  $a_1,\dots,a_{3k},b_1,\dots,b_{3k}.$}
\endmultline$$
```

$$f^{(k)}(x) = ax^{m-k}\sin\frac{1}{x} + \sum_{l=k+1}^{2k-1}\left(a_l x^{m-1}\sin\frac{1}{x} + b_l x^{m-1}\cos\frac{1}{x}\right) \pm \begin{cases} x^{m-2k}\sin\dfrac{1}{x}, & k \text{ even} \\ x^{m-2k}\cos\dfrac{1}{x}, & k \text{ odd.} \end{cases}$$

```
$$\multline
f^{(k)}(x)\,=\, ax^{m-k}\sin\frac 1x \\
   +\sum_{l=k+1}^{2k-1}\bigl(
   a_lx^{m-1}\sin\frac 1x
   + b_lx^{m-1}\cos\frac 1x \bigr)\pm
\cases x^{m-2k}\sin\dfrac 1x,&\text{$k$ even}\\
       x^{m-2k}\cos\dfrac 1x,&\text{$k$ odd.}
\endcases\endmultline$$
```

$$\begin{aligned} m' &= m_1 + 2n_1 = 3m + 4n, \\ n' &= m_1 + n_1 = 2m + 3n. \end{aligned}$$

```
$$
\alignat 2
m' & = m_1 +2n_1 && =3m+4n,\\
n' &=m_1+\hphantom2n_1&&=2m+3n.
\endalignat$$
```

$$\begin{alignedat}{2}
x &= y &&\qquad\text{by (1)} \\
x' &= y' &&\qquad\text{by (2)} \\
x+x' &= y+y' &&\qquad\text{by Axiom 1.}
\end{alignedat}$$

```
$$
\alignat 2
x & =y && \qquad\text{ by (1)} \\
x'& =y' &&\qquad\text{ by (2)} \\
x+x' & = y +y' &&\qquad\text{by Axiom 1.}
\endalignat
$$
```

$$\begin{alignedat}{2}
f(x) &= \sum_{n=1}^\infty a_nx^{n-1} &&= 1+x+2x^2+3x^3+\dotsb, \\
xf(x) &= \sum_{n=1}^\infty a_nx^n &&= \hphantom{1+{}}x+\hphantom{2}x^2+2x^3+\dotsb, \\
x^2f(x) &= \sum_{n=1}^\infty a_nx^{n+1} &&= \hphantom{1+x+2}x^2+\hphantom{2}x^3+\dotsb.
\end{alignedat}$$

```
$$
\alignat 2
f(x) & = \sum_{n=1}^\infty a_nx^{n-1}
        && = 1+x+2x^2+3x^3+\dotsb, \\
xf(x) & =\sum_{n=1}^\infty a_nx^n
        && = \hphantom{1+{}}x+\hphantom2x^2
                +2x^3 +\dotsb,\\
x^2f(x) & =\sum_{n=1}^\infty a_nx^{n+1}
        && = \hphantom{1+x+2}x^2
                +\hphantom2x^3+\dotsb.
\endalignat$$
```

$$
\begin{aligned}
&(23) \quad V_i = v_i - q_i v_i, \qquad X_i = x_i - q_i x_j, \qquad U_i = u_i, \qquad \text{for } i \ne j;\\
&(24) \quad V_i v_j, \qquad X_j = x_j, \qquad U_j = u_j + \sum_{i\ne j} q_i u_1.
\end{aligned}
$$

```
$$
\alignat 3
V_i & =v_i-q_iv_i, & \qquad X_i & =x_i-q_ix_j,
 & \qquad U_i & =u_i, \qquad\text{for $i\ne j$;}
 \tag 23 \\
V_i & v_j,              & \qquad X_j & =x_j,
 & \qquad U_j & =u_j + \sum_{i\ne j}q_iu_l.\tag 24
\endalignat
$$
```

$$
\begin{aligned}
G(z) = e^{\ln G(z)} = \exp\left(\sum_{k\ge1}\frac{S_kz^k}{k}\right) &= \prod_{k\ge1} e^{S_kz^k/k}\\
&= \left(1+S_1z+\frac{S_1^2z^2}{2!}+\cdots\right)\left(1+\frac{S_2z^2}{2}+\frac{S_2^2z^4}{2^2\cdot 2!}+\cdots\right)\cdots\\
&= \sum_{m\ge0}\left(\sum_{\substack{k_1,k_2,\dots,k_m\ge0\\ k_1+2k_2+\cdots+mk_m=m}} \frac{S_1^{k_1}}{1^{k_1}k_1!}\frac{S_2^{k_2}}{2^{k_2}k_2!}\cdots\frac{S_m^{k_m}}{m^{k_m}k_m!}\right)z^m
\end{aligned}
$$

```
$$
\align
G(z) & = e^{\ln G(z)} = \exp\left(
          \sum_{k\gel}\frac{S_kz^k}k
          \right)=\prod_{k\gel}e^{S_kz^k/k}\\
     & = \left( 1 + S_1z +\frac{S_1^2z^2}{2!}+
          \dotsb\right)\left(1+\frac{S_2z^2}2+
           \frac{S_2^2z^4}{2^2\cdot2!}+\dotsb
            \right)\dotsm \\
     & = \sum_{m\ge0}\left(\sum\Sb
                k_1,k_2,\dots,k_m\ge0 \\
                k_1+2k_2+\dots+mk_m=m \endSb
          \frac{S_1^{k_1}}{1^{k_1}k_1!}
                \frac{S_2^{k_2}}{2^{k_2}k_2!}\dotsm
        \frac{S_m^{k_m}}{m^{k_m}k_m!}\right) z^m
\endalign
$$
```

$$
\begin{aligned}
a_n &= a_{n-1} + a_{n-2} \\
&= \frac{\left(\dfrac{1+\sqrt5}{2}\right)^{n-2} - \left(\dfrac{1-\sqrt5}{2}\right)^{n-2} + \left(\dfrac{1+\sqrt5}{2}\right)^{n-1} - \left(\dfrac{1-\sqrt5}{2}\right)^{n-1}}{\sqrt5} \\
&= \frac{\left(\dfrac{1+\sqrt5}{2}\right)^{n-2} \left(1+\dfrac{1+\sqrt5}{2}\right) - \left(\dfrac{1-\sqrt5}{2}\right)^{n-2} \left(1+\dfrac{1-\sqrt5}{2}\right)}{\sqrt5} \\
&= \frac{\left(\dfrac{1+\sqrt5}{2}\right)^{n-2} \left(\dfrac{1+\sqrt5}{2}\right)^{2} - \left(\dfrac{1-\sqrt5}{2}\right)^{n-2} \left(\dfrac{1-\sqrt5}{2}\right)^{2}}{\sqrt5} \\
&= \frac{\left(\dfrac{1+\sqrt5}{2}\right)^{n} - \left(\dfrac{1-\sqrt5}{2}\right)^{n}}{\sqrt5}.
\end{aligned}
$$

```
$$
\define\1{\left( \dfrac{1+\sqrt 5}2 \right)}
\define\2{\left( \dfrac{1-\sqrt 5}2 \right)}
\define\5{\sqrt5}
\align
a_n & = a_{n-1} + a_{n-2} \\
    & = \frac{\1^{n-2} - \2^{n-2}
        + \1^{n-1} - \2^{n-1}} \5 \\
    & = \frac{\1^{n-2} \left( 1 +\dfrac{ 1+ \5 }2 \right)
        - \2^{n-2} \left( 1+ \dfrac{1- \5 }2 \right)} \5 \\
    & = \frac{\1^{n-2} \1^2 - \2^{n-2} \2^2} \5 \\
    & = \frac{\1^n - \2^n} \5.
\endalign
$$
```

$x_1x_2\dots x_n$

```
${x_1x_2\dots}x_n$
```

$$\int f_1(x)\, dx\dots\int f_n(x)\, dx$$

```
$$\int f_1(x)\, dx\dots{\int f_n(x)\, dx}$$
```

$[-x] = [+x]$

```
$[-x] =[+x]$
```

$\{-x\} = \{+x\}$

```
$\{-x\} =\{+x\}$
```

$\lfloor -x\rfloor = \lfloor +x\rfloor$

```
$\lfloor -x\rfloor =\lfloor +x\rfloor$
```

$|-x| = |+x|$

```
$|-x| = |+x|$
```

$\left|-x\right| = \left|+x\right|$

```
$\left|-x\right| = \left|+x\right|$
```

$$\left|-\frac a b\right| = \left|\frac a b\right|$$

```
$$\left|-{\frac a b}\right| = \left|
   \frac a b\right|$$
```

$\left\|-x\right\| = \left\|+x\right\|$

```
$\left\|-x\right\| = \left\|+x\right\|$
```

$\dot x$

```
$\dot x$
```

$\ddot x$

```
$\ddot x$
```

$\dddot x$

```
$\dddot x$
```

$\ddddot x$

```
$\ddddot x$
```

$$\boxed{x+y=z}$$

```
$$\boxed{x+y=z}$$
```

$$\boxed{\boxed{x+y=z}}\qquad\text{is true.}$$

```
$$\boxed{\boxed{x+y=z}}\qquad\text{is true.}$$
```

$$\boxed{x+y=z}\tag{3–1}$$

```
$$\boxed{x+y=z}\tag3--1$$
```

$$\begin{matrix} x & 1 & .1 \\ x+y & 11 & .11 \\ x+y+z & 111 & .111 \end{matrix}$$

```
$$\matrix
     x  &  1        &  .1 \\
   x+y  &  11       &  .11 \\
x+y+z  & 111       &  .111
\endmatrix$$
```

$$\begin{array}{clr} x & .1 & 1 \\ x+y & .11 & 11 \\ x+y+z & .111 & 111 \end{array}$$

```
$$\matrix
\format\c&\quad\l&\quad\r\\
x & .1 & 1 \\
x+y & .11 & 11 \\
x+y+z & .111 & 111 \\
\endmatrix$$
```

$$\left(\begin{matrix} x & 1 & .1 \\ x+y & 11 & .11 \\ x+y+z & 111 & .111 \end{matrix} \right)$$

```
$$\left( \matrix
     x  &  1        &  .1 \\
   x+y  &  11       &  .11 \\
x+y+z  & 111       &  .111
\endmatrix \right)$$
```

$$\begin{pmatrix} x & 1 & .1 \\ x+y & 11 & .11 \\ x+y+z & 111 & .111 \end{pmatrix}$$

```
$$\pmatrix
     x  &  1        &  .1 \\
   x+y  &  11       &  .11 \\
x+y+z  & 111       &  .111
\endpmatrix$$
```

$$\begin{bmatrix} x & 1 & .1 \\ x+y & 11 & .11 \\ x+y+z & 111 & .111 \end{bmatrix}$$

```
$$\bmatrix
     x  &  1        &  .1 \\
   x+y  &  11       &  .11 \\
x+y+z  & 111       &  .111
\endbmatrix$$
```

$$\begin{vmatrix} x & 1 & .1 \\ x+y & 11 & .11 \\ x+y+z & 111 & .111 \end{vmatrix}$$

```
$$\vmatrix
     x  &  1        &  .1 \\
   x+y  &  11       &  .11 \\
x+y+z  & 111       &  .111
\endvmatrix$$
```

$$\begin{Vmatrix} x & 1 & .1 \\ x+y & 11 & .11 \\ x+y+z & 111 & .111 \end{Vmatrix}$$

```
$$\Vmatrix
     x  &  1        &  .1 \\
   x+y  &  11       &  .11 \\
x+y+z  & 111       &  .111
\endVmatrix$$
```

$$\begin{array}{r@{}l} 3&.14159\\ 2&.71828\\ 1&.61808\\ &.57701 \end{array}$$

```
$$\matrix
  \format\r&\l\\
   3&.14159\\
   2&.71828\\
   1&.61808\\
    &.57701
  \endmatrix$$
```

$$\begin{array}{l} 3.14159\\ 2.71828\\ 1.61808\\ \hphantom{0}.57 \end{array}$$

```
$$
 \define\dwidth{\hphantom0}
 \matrix
   \format\l\\
   3.14159\\
   2.71828\\
   1.61808\\
   \dwidth .57
 \endmatrix$$
```

$$\begin{pmatrix} 0 \\ 0 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$$

```
$$\pmatrix
0\\
0 & 1\\
0 & 1 & 2\\
0 & 1 & 2 & 3\\
0 & 1 & 2 & 3 & 4\\
0 & 1 & 2 & 3\\
0 & 1 & 2
\endpmatrix$$
```

$$A = \begin{pmatrix} x-\lambda & 1 & 0 \\ 0 & x-\lambda & 1 \\ 0 & 0 & x-\lambda \end{pmatrix}$$

```
$$A=
\pmatrix
x-\lambda & 1 & 0 \\
0 & x-\lambda & 1 \\
0 & 0 & x-\lambda
\endpmatrix$$
```

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

```
$$\pmatrix
\format\r&\quad\r\\
\cos\theta &\sin\theta \\
-\sin\theta &\cos\theta \endpmatrix$$
```

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

```
$$\pmatrix
a_{11} & a_{12} & \hdots & a_{1n} \\
a_{21} & a_{22} & \hdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \hdots & a_{mn}
\endpmatrix$$
```

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \hdotsfor{4} \\ \hdotsfor{4} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

```
$$\pmatrix
a_{11} & a_{12} & \hdots & a_{1n} \\
a_{21} & a_{22} & \hdots & a_{2n} \\
\hdotsfor 4  \\
\hdotsfor 4  \\
a_{m1} & a_{m2} & \hdots & a_{mn}
\endpmatrix$$
```

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & \hdotsfor{3} \\ \hdotsfor{4} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

```
$$\pmatrix
a_{11} & a_{12} & \hdots & a_{1n} \\
a_{21} & a_{22} & \hdots & a_{2n} \\
a_{31} & \hdotsfor 3 \\
\hdotsfor 4  \\
a_{m1} & a_{m2} & \hdots & a_{mn}
\endpmatrix$$
```

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & \hdotsfor{3} \\ \hdotsfor{4} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

```
$$\pmatrix
a_{11} & a_{12} & \hdots & a_{1n} \\
a_{21} & a_{22} & \hdots & a_{2n} \\
a_{31} & \innerhdotsfor 3\after \quad \\
\hdotsfor 4  \\
a_{m1} & a_{m2} & \hdots & a_{mn}
\endpmatrix$$
```

$$\begin{pmatrix} a_{11} & & \\ & \ddots & \\ & & a_{nn} \end{pmatrix}$$

```
$$\pmatrix
a_{11} & & \\
& \ddots & \\
&& a_{nn}
\endpmatrix$$
```

$$A = \begin{pmatrix} b_1 & c_1 & & 0 \\ a_2 & b_2 & c_2 & \\ & \ldots & \ldots & \\ & & \ldots & \ldots \\ 0 & & a_n & b_n \end{pmatrix}$$

```
\font\b=cmr10 scaled \magstep4
\def\bigzerol{\smash{\hbox{\b 0 }}}
\def\bigzerou{\smash{\lower1.7ex\hbox{\b 0 }}}
$$
A=\pmatrix
   b_1     & c_1   &        &\bigzerou \\
   a_2     & b_2   & c_2  &          \\
           & \ldots& \ldots &          \\
           &       & \ldots &\ldots   \\
 \bigzerol &       & a_n  &  b_n      \\
 \endpmatrix
$$
```

$$\begin{aligned}\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} &= \begin{pmatrix} a\cdot 0+b\cdot 1 & a\cdot 1+b\cdot 0 \\ c\cdot 0+d\cdot 1 & c\cdot 1+d\cdot 0 \end{pmatrix} \\ &= \begin{pmatrix} b & a \\ d & c \end{pmatrix}\end{aligned}$$

```
$$\align
{{\pmatrix
a & b \\ c & d \endpmatrix}
\cdot
{\pmatrix
0 & 1 \\ 1 & 0 \endpmatrix} }
& = {\pmatrix
a\cdot 0+b\cdot 1 & a\cdot 1+b\cdot0 \\
c\cdot 0+d\cdot 1 & c\cdot 1+d\cdot0\endpmatrix} \\
& ={\pmatrix b & a \\ d & c \endpmatrix}
\endalign$$
```

$$det\begin{vmatrix} c_0 & c_1 & c_2 & \cdots & c_n \\ c_1 & c_2 & c_3 & \cdots & c_{n+1} \\ c_2 & c_3 & c_4 & \cdots & c_{n+2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_n & c_{n+1} & c_{n+2} & \cdots & c_{2n} \end{vmatrix} > 0.$$

```
$$ det{\vmatrix
\format\l\quad&\l\quad&\l\quad&\l\quad&\l\\
c_0 & c_1 & c_2 & \hdots & c_n \\
c_1 & c_2 & c_3 & \hdots & c_{n+1} \\
c_2 & c_3 & c_4 & \hdots & c_{n+2} \\
\vdots & \vdots & \vdots & \vdots &\vdots \\
c_n & c_{n+1} & c_{n+2} & \hdots & c_{2n}
\endvmatrix} >0.$$
```

$$\omega = \begin{pmatrix} 0 & \omega_{12} & 0 & & & \cdots & 0 \\ -\omega_{12} & 0 & \omega_{23} & 0 & & \cdots & 0 \\ 0 & -\omega_{23} & 0 & \omega_{34} & 0 & \cdots & 0 \\ \vdots & & & & & \ddots & \vdots \\ 0 & & \cdots & & & 0 & \omega_{n-1,n} \\ 0 & & \cdots & & 0 & -\omega_{n-1,n} & 0 \end{pmatrix}$$

```
$$
\omega =\pmatrix
0&\omega_{12} & 0 & & &\hdots & 0 \\
-\omega_{12} & 0 &\omega_{23} & 0 & &\hdots & 0 \\
0 & -\omega_{23}& 0  &\omega_{34} & 0 & \hdots & 0 \\
\vdots &  & & & & \ddots & \vdots \\
0 & & \hdots & & & 0 & \omega_{n-1,n} \\
0 & & \hdots &  & 0& -\omega_{n-1,n}&  0 \endpmatrix$$
```

$$B_{(1)}(t) = \begin{pmatrix} \partial_x\partial_\xi H & \partial_\xi\partial_\xi H & \vec{\partial}_\theta\partial_\xi H & \vec{\partial}_\pi\partial_\xi H \\ -\partial_x\partial_x H & -\partial_\xi\partial_x H & -\vec{\partial}_\theta\partial_x H & -\partial_\pi\partial_x H \\ -\partial_x\vec{\partial}_\pi H & -\partial_\xi\vec{\partial}_\pi H & -\vec{\partial}_\theta\vec{\partial}_\pi H & -\vec{\partial}_\pi\vec{\partial}_\pi H \\ -\partial_x\vec{\partial}_\theta H & -\partial_\xi\vec{\partial}_\theta H & -\vec{\partial}_x\theta\vec{\partial}_\theta H & -\vec{\partial}_\pi\vec{\partial}_\theta H \end{pmatrix}$$

```
$$
\define\rpartial{\vec{\partial}}
B_{(1)}(t) =
\pmatrix
\quad\enskip
    \partial_x\partial_{\xi}H
  &\quad\enskip
    \partial_{\xi}\partial_{\xi}H
  &\quad\enskip
    \rpartial_{\theta}\partial_{\xi}H
  &\quad\enskip
    \rpartial_{\pi}\partial_{\xi}H \\
-\partial_x\partial_xH
  & -\partial_{\xi}\partial_xH&
-\rpartial_{\theta}\partial_xH
  &-\partial_{\pi}\partial_xH\\
-\partial_x\rpartial_{\pi}H
  &-\partial_{\xi}\rpartial_{\pi}H
  &-\rpartial_{\theta}\rpartial_{\pi}H
  &-\rpartial_{\pi}\rpartial_{\pi}H\\
-\partial_x\rpartial_{\theta}H
  &\partial_{\xi}\rpartial_{\theta}H
  &-\rpartial_x{\theta}\rpartial_{\theta}H
  &-\rpartial_{\pi}\rpartial_{\theta}H
\endpmatrix
$$
```

$$A = \left[\begin{array}{ccc|c} a_{11} & \ldots & a_{1,n-1} & a_{1n} \\ a_{21} & \ldots & a_{2,n-1} & a_{1n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m-1,1} & \ldots & a_{m-1,n-1} & a_{m-1,n} \\ \hline a_{m1} & \ldots & a_{m,n-1} & a_{mn} \end{array} \right]$$

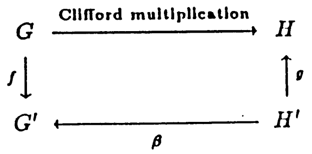
```
$$
A=
\left[
 \vcenter{\hbox{\vbox{
 \tabskip = 0pt \offinterlineskip
 \halign{
 \hfil #\hfil\quad & \hfil #\hfil\quad & \hfil #\hfil\quad
      &#& \quad\hfil  #\hfil \cr
 \strut $a_{11}$ & $\ldots$ &$a_{1,n-1}$ & \vrule & $a_{1n}$ \cr
 \strut $a_{21}$ & $\ldots$ &$a_{2,n-1}$ & \vrule & $a_{1n}$ \cr
 \strut $\ldots$ & $\ldots$ &$\ldots$ & \vrule & $\ldots$ \cr
 \strut $a_{m-1,1}$&$\ldots$&$a_{m-1,n-1}$ & \vrule & $a_{m-1,n}$ \cr
 \vphantom{,}    &          &          & \vrule &        \cr
 \noalign{\hrule}
 \vphantom{,}    &          &          &        &        \cr
 \strut $a_{m1}$& $\ldots$ & $a_{m,n-1}$ &    & $a_{mn}$ \cr
}}}}\right]
$$
```

$$\begin{CD} G @>\alpha>> H \\ G' @<<\beta< H' \end{CD}$$

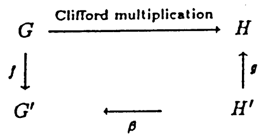
```
$$
\CD
G       @>\alpha >>      H\\
G'      @<<\beta<        H'
\endCD
$$
```

$$\begin{CD} G @>\alpha>> H \\ @. @VVgV \\ @. H' \end{CD}$$

```
$$
\CD
G  @>\alpha >> H \\
@.             @VVgV \\
      @.       H'
\endCD
$$
```




```
$$
\define\bottomarrow{@<<\pretend\beta\haswidth
          {\text{Clifford multiplication}}<}
\CD
G        @>\text{Clifford multiplication}>>        H\\
@VfVV                   @AAgA  \\
G'           \bottomarrow       H'
\endCD
$$
```




```
$$
\CD
G  @>\text{Clifford multiplication}>>  H \\
@VfVV                                 AAgA \\
G'          @<<\beta<                   H'
\endCD$$
\midspace{5cm}
```

Year	World Population
8000 B.C.	5,000,000
50 A.D.	200,000,000
1650 A.D.	500,000,000
1850 A.D.	1,000,000,000
1945 A.D.	2,300,000,000
1980 A.D.	4,400,000,000

```
\vbox{\offinterlineskip
\hrule
\halign{&\vrule#&
  \strut\quad\hfil#\quad\cr
height2pt&\omit&&\omit&\cr
&Year\hfil&&World Population&\cr
height2pt&\omit&&\omit&\cr
\noalign{\hrule}
height2pt&\omit&&\omit&\cr
&8000\BC&&5,000,000&\cr
&50\AD&&200,000,000&\cr
&1650\AD&&500,000,000&\cr
&1850\AD&&1,000,000,000&\cr
&1945\AD&&2,300,000,000&\cr
&1980\AD&&4,400,000,000&\cr
height2pt&\omit&&\omit&\cr}
\hrule}
```

AT&T Common Stock		
Year	Price	Dividend
1971	41-54	\$2.60
2	41-54	2.70
3	46-55	2.87
4	40-53	3.24
5	45-52	3.40
6	51-59	.95*

* (first quarter only)

AT&T Common Stock		
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5	45-52	3.40
6	51-59	.95*

* (first quarter only)

```
\vbox{\tabskip=0pt \offinterlineskip
\def\tablerule{\noalign{\hrule}}
\halign to(dimen){\strut#& \vrule#\tabskip=1em plus2em&
  \hfil#& \vrule#& \hfil#\hfil& \vrule#&
  \hfil#& \vrule#\tabskip=0pt\cr\tablerule
&&\multispan5\hfil AT\&T Common Stock\hfil&\cr\tablerule
&&\omit\hidewidth Year\hidewidth&&
 \omit\hidewidth Price\hidewidth&&
 \omit\hidewidth Dividend\hidewidth&\cr\tablerule
&&1971&&41--54&&\$2.60&\cr\tablerule
&&   2&&41--54&&2.70&\cr\tablerule
&&   3&&46--55&&2.87&\cr\tablerule
&&   4&&40--53&&3.24&\cr\tablerule
&&   5&&45--52&&3.40&\cr\tablerule
&&   6&&51--59&&.95\rlap*&\cr\tablerule \noalign{\smallskip}
&\multispan7* (first quarter only)\hfil\cr}}
```

$\Gamma(n) = (n-1)! \qquad$ when n is an integer

```
\TagsOnRight
$$\Gamma(n) =(n-1)! \qquad \text{when $n$ is an integer}$$
```

$$J_{-n}(x) = \sum_{k=n}^{\infty} \frac{(-1)^k(\frac{x}{2})^{2k-n}}{(k!)(k-n)!}$$

or, replacing the index k by $k+n$,

$$= \sum_{k=0}^{\infty} \frac{(-1)^{k+n}(\frac{x}{2})^{2k+n}}{(k!)(k+n)!}.$$

```
$$
\align
J_{-n}(x) &= \sum_{k=n}^\infty
   \dfrac{(-1)^k(\dfrac x2)^{2k-n}}{(k!)(k-n)!}\\
\intertext{or, replacing the index $k$ by $k+n$,}
          &= \sum_{k=0}^\infty
   \dfrac{(-1)^{k+n}(\dfrac x2)^{2k+n}}{(k!)(k+n)!}.
\endalign
$$
```

$$\begin{aligned} \sqrt{k+1} - \sqrt{k} &= f(k+1) - f(k) \\ &= \frac{1}{2\sqrt{x}} \qquad \text{for some x in $(k, k+1)$, by the mean Value Theorem} \\ &< \frac{1}{2\sqrt{k}}. \end{aligned}$$

```
$$
\align
\sqrt{k+1}-\sqrt k & = f(k+1)-f(k)\\
     & = \frac 1{2\sqrt x}\qquad
       \foldedtext{for some $x$ in $(k,k+1)$,
         by the mean Value Theorem}\\
     & < \frac 1{2\sqrt k}.
\endalign
$$
```

From (108) becomes

$$\frac{d}{dx}y_p(\alpha x) = \begin{cases} \alpha y_{p-1}(\alpha x) - \dfrac{p}{x}y_p(\alpha x), & (y = J, Y, I, H^{(1)}, H^{(2)}) \\ -\alpha y_{p-1}(\alpha x) - \dfrac{p}{x}y_p(\alpha x), & (y = K). \end{cases} \tag{110}$$

whereas (109) becomes

$$\frac{d}{dx}y_p(\alpha x) = \begin{cases} \alpha y_{p+1}(\alpha x) + \dfrac{p}{x}y_p(\alpha x), & (y = J, Y, K, H^{(1)}, H^{(2)}) \\ -\alpha y_{p+1}(\alpha x) + \dfrac{p}{x}y_p(\alpha x), & (y = I). \end{cases} \tag{111}$$

```
From (108) becomes
$$\align
\frac d{dx} y_p(\alpha x) &=
 \left\{\matrix\format\l&\quad\l\\
     \alpha y_{p-1}(\alpha x) -\dfrac px y_p(\alpha x),
      & (y= J, Y, I, H^{(1)}, H^{(2)}) \\
    -\alpha y_{p-1}(\alpha x) -\dfrac px y_p(\alpha x),
      & (y=K). \\
        \endmatrix\right.\tag110\\
\intertext{whereas (109) becomes }
\frac d{dx} y_p(\alpha x) &=
 \left\{\matrix\format\l &\quad\l\\
     \alpha y_{p+1}(\alpha x) +\dfrac px y_p(\alpha x),
      & (y= J, Y, K, H^{(1)}, H^{(2)}) \\
    -\alpha y_{p+1}(\alpha x) +\dfrac px y_p(\alpha x),
      & (y=I).
       \endmatrix\right.\tag111
\endalign
$$
```

From (108) becomes

```
$$\align
\frac d{dx} y_p(\alpha x) &=
 \cases
     \alpha y_{p-1}(\alpha x) -\dfrac px y_p(\alpha x),
      & (y= J, Y, I, H^{(1)}, H^{(2)}) \\
    -\alpha y_{p-1}(\alpha x) -\dfrac px y_p(\alpha x),
      & (y=K). \\
        \endcases\tag110\\
\intertext{whereas (109) becomes }
\frac d{dx} y_p(\alpha x) &=
 \cases
     \alpha y_{p+1}(\alpha x) +\dfrac px y_p(\alpha x),
      & (y= J, Y, K, H^{(1)}, H^{(2)}) \\
    -\alpha y_{p+1}(\alpha x) +\dfrac px y_p(\alpha x),
      & (y=I).
       \endcases\tag111
\endalign
$$
```

<<< インデックス >>>