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A Clark-Ocone type formula under change of measure for canonical Lévy processes

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Abstract

Suzuki ([10]) derived a Clark-Ocone type formula under change of measure (COCM) for Lévy processes with L^2-Lévy measure. In this paper, in order to simplify the description, we introduce it for canonical Lévy processes.

1 Introduction

Clark-Ocone (CO) formulae are explicitly martingale representations of random variables in terms of Malliavin derivatives. COCMs are Girsanov transformations versions of it. Suzuki ([10]) derived a COCM for Lévy processes with L^2-Lévy measure by using Malliavin calculus for Lévy processes based on [5]. Note that it is not depend on structure of probability space.

In this paper, we derive a COCM for canonical Lévy processes for the simplicity of the description. Since we can derive practical rules to compute Malliavin derivatives easily by using the weak derivative and the increment quotient operator on the canonical space (see e.g, [9]), we can simplify the description.

2 Malliavin Calculus for canonical Lévy processes

2.1 Setting

Throughout this paper, we consider Malliavin calculus for canonical Lévy processes, based on, [9]. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be the product of two canonical spaces; the usual canonical space $(\Omega_W, \mathcal{F}_W, \mathbb{P}_W)$ for a one-dimensional standard Brownian motion W and the canonical space $(\Omega_N, \mathcal{F}_N, \mathbb{P}_N)$ for a pure jump Lévy process with Lévy measure ν satisfying $\int_{\mathbb{R}_0} z^2\nu(dz) < \infty$, where $\mathbb{R}_0 := \mathbb{R}\setminus\{0\}$. For more details, see section 3 of [3] and section 4 of [9]. Let $\mathbb{F} = \{\mathcal{F}_t\}_{t\in[0,T]}$ be the canonical filtration, and X a Lévy process with Lévy measure ν, which is represented as $X_t = \sigma W_t + \int_0^t\int_{\mathbb{R}_0} z\widetilde{N}(ds,dz)$, where $\sigma \geq 0$. Here N is the Poisson random measure defined by $N(t,A) = \sum_{s\leq t}\mathbf{1}_A(\Delta X_s)$ for any $A \in \mathcal{B}(\mathbb{R}_0)$ and any $t \in [0,T]$, and $\widetilde{N}(dt,dz) := N(dt,dz) - \nu(dz)dt$, where $\Delta X_s := X_s - X_{s-}$.
We consider the finite measure q defined on $[0,T]\times\mathbb{R}$ by

$$q(E) = \sigma^2\int_{E(0)} dt\delta_0(dz) + \int_{E'} z^2 dt\nu(dz), \quad E \in \mathcal{B}([0,T]\times\mathbb{R}),$$

where $E(0) = \{(t,0) \in [0,T]\times\mathbb{R}; (t,0) \in E\}$ and $E' = E - E(0)$, and the random measure Q on $[0,T]\times\mathbb{R}$ by

$$Q(E) = \sigma\int_{E(0)} dW_t\delta_0(dz) + \int_{E'} z\tilde{N}(dt,dz), \quad E \in \mathcal{B}([0,T]\times\mathbb{R}).$$

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Let $L^2_{T,q,n}(\mathbb{R})$ denote the set of product measurable, deterministic functions $h : ([0,T] \times \mathbb{R})^n \to \mathbb{R}$ satisfying

$$\|h\|^2_{L^2_{T,q,n}} := \int_{([0,T]\times\mathbb{R})^n} |h((t_1,z_1),\cdots,(t_n,z_n))|^2 q(dt_1,dz_1)\cdots q(t_n,z_n) < \infty.$$

For $n \in \mathbb{N}$ and $h_n \in L^2_{T,q,n}(\mathbb{R})$, we denote

$$I_n(h_n) := \int_{([0,T]\times\mathbb{R})^n} h((t_1,z_1),\cdots,(t_n,z_n))Q(dt_1,dz_1)\cdots Q(dt_n,dz_n).$$

It is easy to see that $\mathbb{E}[I_0(h_0)] = h_0$ and $\mathbb{E}[I_n(h_n)] = 0$, for $n \geq 1$.
In this setting, we introduce the following chaos expansion (see Theorem 2 in [6], Section 2 of [9] and Section 3 of [3]).

Theorem 2.1 *Any $\mathcal{F}$-measurable square integrable random variable F on the canonical space has a unique representation*

$$F = \sum_{n=0}^{\infty} I_n(h_n), \mathbb{P}\text{−a.s.}$$

with functions $h_n \in L^2_{T,q,n}(\mathbb{R})$ that are symmetric in the n pairs $(t_i,z_i), 1 \leq i \leq n$ and we have the isometry

$$\mathbb{E}[F^2] = \sum_{n=0}^{\infty} n!\|h_n\|^2_{L^2_{T,q,n}}.$$

Definition 2.2 *(1) For $\sigma \neq 0$, let $\mathbb{D}^{1,2}_0$ denote the set of $\mathcal{F}$ -measurable random variables $F \in L^2(\mathbb{P})$ with the representation $F = \sum_{n=0}^{\infty} I_n(f_n)$ satisfying*

$$\sum_{n=1}^{\infty} nn! \int_0^T \|f_n(\cdot,(t,0))\|^2_{L^2_{T,q,n-1}} \sigma^2 dt < \infty.$$

Then, for $F \in \mathbb{D}^{1,2}_0$, we can define

$$D_{t,0}F = \sum_{n=1}^{\infty} nI_{n-1}(f_n((t,0),\cdot)), \quad \text{valid for } q\text{−a.e. } (t,0) \in [0,T]\times\{0\}, \mathbb{P} - a.s.$$

(2) For $\nu \neq 0$, let $\mathbb{D}^{1,2}_1$ denote the set of $\mathcal{F}$ -measurable random variables $F \in L^2(\mathbb{P})$ with the representation $F = \sum_{n=0}^{\infty} I_n(f_n)$ satisfying

$$\sum_{n=1}^{\infty} nn! \int_0^T \int_{\mathbb{R}_0} \|f_n(\cdot,(t,z))\|^2_{L^2_{T,q,n-1}} z^2\nu(dz)dt < \infty.$$

Then, for $F \in \mathbb{D}^{1,2}_1$, we can define

$$D_{t,z}F = \sum_{n=1}^{\infty} nI_{n-1}(f_n((t,z),\cdot)), \quad \text{valid for } q\text{−a.e. } (t,z) \in [0,T]\times\mathbb{R}_0, \mathbb{P} - a.s.$$

(3) Let $\mathbb{D}^{1,2} = \mathbb{D}^{1,2}_0 \cap \mathbb{D}^{1,2}_1$. Then, for $F \in \mathbb{D}^{1,2}$, the Malliavin derivative $DF : \Omega\times[0,T]\times\mathbb{R} \to \mathbb{R}$ of a random variable F is a stochastic process defined by

$$D_{t,z}F := \sum_{n=1}^{\infty} nI_{n-1}(h_n((t,z),\cdot)), \quad \text{valid for } q\text{−a.e. } (t,z) \in [0,T]\times\mathbb{R}, \mathbb{P} - a.s.$$

(4) Let D^W be the classical Malliavin derivative with respect to the Brownian motion W and Dom D^W *be the domain of D^W (for more details see [8]). We define*

$$\mathbb{D}^W := \left\{ F \in L^2(\mathbb{P}); F(\cdot,\omega_N) \in \text{Dom } D^W \text{ for } \mathbb{P}^N\text{-a.e. } \omega_N \in \Omega_N \right\}.$$

(5) Let F be a random variable on $\Omega_W \times \Omega_N$. Then we define the increment quotient operator

$$\Psi_{t,z}F := \frac{F(\omega_W,\omega_N^{t,z}) - F(\omega_W,\omega_N)}{z}, z \neq 0,$$

where $\omega_N^{t,z}$ transforms a family $\omega_N = ((t_1,z_1),(t_2,z_2),\cdots) \in \Omega_N$ into a new family $\omega_N^{t,z} = ((t,z),(t_1,z_1),(t_2,z_2),\cdots) \in \Omega_N$, by adding a jump of size z at time t into the trajectory. Moreover, we denote

$$\mathbb{D}^J := \left\{ F \in L^2(\mathbb{P}); \mathbb{E}\left[\int_0^T \int_{\mathbb{R}_0} |\Psi_{t,z}F|^2 z^2 \nu(dz)dt\right] < \infty \right\}.$$

By Propositions 2.6.1, 2.6.2 in [2] and result of [1] (see section 3.3), we can derive the following:

Proposition 2.3 *1. If $F \in \mathbb{D}^W$, then $F \in \mathbb{D}_0^{1,2}$ and $D_{t,0}F = \mathbf{1}_{\{\sigma>0\}}\sigma^{-1}D_t^W F(\cdot,\omega_N)(\omega_W)$ for q -a.e. $(t,z) \in [0,T] \times \{0\}, \mathbb{P}$ -a.s.*

2. *If $F \in \mathbb{D}^J$, then $F \in \mathbb{D}_1^{1,2}$ and $D_{t,z}F = \Psi_{t,z}F$ for q -a.e. $(t,z) \in [0,T] \times \mathbb{R}_0, \mathbb{P}$ -a.s.*
3. *$\mathbb{D}^{1,2} = \mathbb{D}^W \cap \mathbb{D}^J$ holds.*

Lemma 2.4 (Lemma 3.1 of [3]) *Let $F \in \mathbb{D}^{1,2}$. Then, for $0 \le t \le T$, $\mathbb{E}[F|\mathcal{F}_t] \in \mathbb{D}^{1,2}$ and*

$$D_{s,x}\mathbb{E}[F|\mathcal{F}_t] = \mathbb{E}[D_{s,x}F|\mathcal{F}_t]\mathbf{1}_{\{s\le t\}}, \text{ for } q\text{-a.e. } (s,x) \in [0,T] \times \mathbb{R},\ \mathbb{P}\text{-a.s.}$$

We next introduce a chain rule for the Malliavin derivatives.

Proposition 2.5 (Chain rule) *Let $\varphi \in C^1(\mathbb{R}^n;\mathbb{R})$, $F = (F_1,\cdots,F_n)$, where $F_1,\cdots,F_n \in \mathbb{D}^{1,2}$ and $\varphi(F) \in L^2(\mathbb{P})$.*

1. *If $\sum_{k=1}^n \frac{\partial}{\partial x_k}\varphi(F)D_{t,0}F_k \in L^2(\lambda \times \mathbb{P})$, then $\varphi(F) \in \mathbb{D}_0^{1,2}$ and*

$$D_{t,0}\varphi(F) = \sum_{k=1}^n \frac{\partial}{\partial x_k}\varphi(F)D_{t,0}F_k \quad \text{for } q\text{-a.e. } (t,z) \in [0,T] \times \{0\},\ \mathbb{P}\text{-a.s.} \tag{2.1}$$

2. *If $\frac{\varphi(F_1+zD_{t,z}F_1,\cdots,F_k+zD_{t,z}F_k)-\varphi(F_1,\cdots,F_k)}{z} \in L^2(z^2\nu(dz)dtd\mathbb{P})$, Then, $\varphi(F) \in \mathbb{D}_1^{1,2}$ and*

$$D_{t,z}\varphi(F) = \frac{\varphi(F_1+zD_{t,z}F_1,\cdots,F_k+zD_{t,z}F_k) - \varphi(F_1,\cdots,F_k)}{z} \quad \text{for } q\text{-a.e. } (t,z) \in [0,T] \times \mathbb{R}_0,\ \mathbb{P}\text{-a.s.} \tag{2.2}$$

Proof. (1) Equation (2.1) follows from Lemma A.1 in [7]. and Proposition 2.3.
(2) We next show (2.2). Since $F_1,\cdots,F_k \in \mathbb{D}^{1,2}$, Proposition 2.3 implies that

$$\begin{aligned}\Psi_{t,z}\varphi(F) &= \frac{\varphi(F_1(\omega_W,\omega_N^{t,z}),\cdots,F_k(\omega_W,\omega_N^{t,z})) - \varphi(F_1(\omega_W,\omega_N),\cdots,F_k(\omega_W,\omega_N))}{z} \\ &= \frac{\varphi(F_1 + z\cdot\frac{F_1(\omega_W,\omega_N^{t,z})-F_1}{z},\cdots,F_k + z\cdot\frac{F_k(\omega_W,\omega_N^{t,z})-F_k}{z}) - \varphi(F_K(\omega_W,\omega_N),\cdots,F_k(\omega_W,\omega_N))}{z} \\ &= \frac{\varphi(F_1+zD_{t,z}F_1,\cdots,F_k+zD_{t,z}F_k) - \varphi(F_1,\cdots,F_k)}{z}, z \neq 0.\end{aligned}$$

Moreover, from $\frac{\varphi(F_1+zD_{t,z}F_1,\cdots,F_k+zD_{t,z}F_k)-\varphi(F_1,\cdots,F_k)}{z} \in L^2(z^2\nu(dz)dtd\mathbb{P})$, we have $\Psi_{t,z}\varphi(F) = D_{t,z}\varphi(F)$ and $\varphi(F) \in \mathbb{D}^J$ by Proposition 2.3. □

If we take $\varphi(x,y) = xy$, then, we can derive the following product rule.

Corollary 2.6 *Let $F_1, F_2 \in \mathbb{D}^{1,2}$ and $F_1F_2 \in L^2(\mathbb{P})$. Moreover, assume that $F_1D_{t,z}F_2 + F_2D_{t,z}F_1 + zD_{t,z}F_1 \cdot D_{t,z}F_2 \in L^2(q \times \mathbb{P})$. Then $F_1F_2 \in \mathbb{D}^{1,2}$ and*

$$D_{t,z}F_1F_2 = F_1D_{t,z}F_2 + F_2D_{t,z}F_1 + zD_{t,z}F_1 \cdot D_{t,z}F_2 \quad q\text{-a.e. } (t,z) \in [0,T] \times \mathbb{R}, \mathbb{P}-a.s. \tag{2.3}$$

2.2 Commutation of integration and the Malliavin differentiability

Definition 2.7 *(1) Let $\mathbb{L}^{1,2}$ denote the space of product measurable and $\mathbb{F}$ -adapted processes $G : \Omega \times [0,T] \times \mathbb{R} \to \mathbb{R}$ satisfying*

$$\mathbb{E}\left[\int_{[0,T]\times\mathbb{R}} |G_{s,x}|^2 q(ds,dx)\right] < \infty,$$

$G_{s,x} \in \mathbb{D}^{1,2}$, q-a.e. $(s,x) \in [0,T] \times \mathbb{R}$ and

$$\mathbb{E}\left[\int_{([0,T]\times\mathbb{R})^2} |D_{t,z}G_{s,x}|^2 q(ds,dx)q(dt,dz)\right] < \infty.$$

(2) $\mathbb{L}_0^{1,2}$ denotes the space of $G : [0,T] \times \Omega \to \mathbb{R}$ satisfying

1. *$G_s \in \mathbb{D}^{1,2}$ for a.e. $s \in [0,T]$,*
2. *$E\left[\int_{[0,T]} |G_s|^2 ds\right] < \infty$,*
3. *$E\left[\int_{[0,T]\times\mathbb{R}} \int_0^T |D_{t,z}G_s|^2 ds q(dt,dz)\right] < \infty$.*

(3) $\mathbb{L}_1^{1,2}$ is defined as the space of $G : [0,T] \times \mathbb{R}_0 \times \Omega \to \mathbb{R}$ such that

1. *$G_{s,x} \in \mathbb{D}^{1,2}$ for q-a.e. $(s,x) \in [0,T] \times \mathbb{R}$,*
2. *$E\left[\int_{[0,T]\times\mathbb{R}_0} |G_{s,x}|^2 \nu(dx)ds\right] < \infty$,*
3. *$E\left[\int_{[0,T]\times\mathbb{R}} \int_{[0,T]\times\mathbb{R}_0} |D_{t,z}G_{s,x}|^2 \nu(dx)ds q(dt,dz)\right] < \infty$.*

(4) $\tilde{\mathbb{L}}_1^{1,2}$ is defined as the space of $G \in \mathbb{L}_1^{1,2}$ such that

1. *$E\left[\left(\int_{[0,T]\times\mathbb{R}_0} |G_{s,x}|\nu(dx)ds\right)^2\right] < \infty$,*
2. *$E\left[\int_{[0,T]\times\mathbb{R}} \left(\int_{[0,T]\times\mathbb{R}_0} |D_{t,z}G_{s,x}|\nu(dx)ds\right)^2 q(dt,dz)\right] < \infty$.*

We next discuss the commutation relation of the stochastic integral with the Malliavin derivative.

Proposition 2.8 (Lemma 3.3 of [3]) *Let $G : \Omega \times [0,T] \times \mathbb{R} \to \mathbb{R}$ be a predictable process with*

$$\mathbb{E}\left[\int_{[0,T]\times\mathbb{R}} |G_{s,x}|^2 q(ds,dx)\right] < \infty.$$

Then

$$G \in \mathbb{L}^{1,2} \text{ if and only if } \int_{[0,T]\times\mathbb{R}} G_{s,x} Q(ds,dx) \in \mathbb{D}^{1,2}.$$

Furthermore, if $\int_{[0,T]\times\mathbb{R}} G_{s,x} Q(ds,dx) \in \mathbb{D}^{1,2}$*, then, for* q *-a.e.* $(t,z) \in [0,T]\times\mathbb{R}$*, we have*

$$D_{t,z} \int_{[0,T]\times\mathbb{R}} G_{s,x} Q(ds,dx) = G_{t,z} + \int_{[0,T]\times\mathbb{R}} D_{t,z} G_{s,x} Q(ds,dx), \quad \mathbb{P}-\text{a.s.},$$

and $\int_{[0,T]\times\mathbb{R}} D_{t,z} G_{s,x} Q(ds,dx)$ *is a stochastic integral in Itô sense.*

Next proposition provides commutation of the Lebesgue integration and the Malliavin differentiability.

Proposition 2.9 (Lemma 3.2 of [3]) *Assume that* $G : \Omega \times [0,T]\times\mathbb{R} \to \mathbb{R}$ *is a product measurable and* $\mathbb{F}$ *-adapted process,* η *on* $[0,T]\times\mathbb{R}$ *a finite measure, so that conditions*

$$\begin{aligned}
&\mathbb{E}\left[\int_{[0,T]\times\mathbb{R}} |G_{s,x}|^2 \eta(ds,dx)\right] < \infty, \\
&G_{s,x} \in \mathbb{D}^{1,2}, \quad \text{for } \eta-\text{a.e. } (s,x) \in [0,T]\times\mathbb{R}, \\
&\mathbb{E}\left[\int_{([0,T]\times\mathbb{R})^2} |D_{t,z} G_{s,x}|^2 \eta(ds,dx) q(dt,dz)\right] < \infty
\end{aligned}$$

are satisfied. Then we have

$$\int_{[0,T]\times\mathbb{R}} G_{s,x} \eta(ds,dx) \in \mathbb{D}^{1,2}$$

and the differentiation rule

$$D_{t,z} \int_{[0,T]\times\mathbb{R}} G_{s,x} \eta(ds,dx) = \int_{[0,T]\times\mathbb{R}} D_{t,z} G_{s,x} \eta(ds,dx)$$

holds for q *-a.e.* $(t,z) \in [0,T]\times\mathbb{R}, \mathbb{P}$ *-a.s.*

By using σ-finiteness of ν and Proposition 2.9, we can show the following proposition.

Proposition 2.10 (Proposition 3.5 in [10]) *Let* $G \in \tilde{\mathbb{L}}_1^{1,2}$*. Then,*

$$\int_{[0,T]\times\mathbb{R}_0} G_{s,x} \nu(dx) ds \in \mathbb{D}^{1,2}$$

and the differentiation rule

$$D_{t,z} \int_{[0,T]\times\mathbb{R}_0} G_{s,x} \nu(dx) ds = \int_{[0,T]\times\mathbb{R}_0} D_{t,z} G_{s,x} \nu(dx) ds$$

holds for q *-a.e.* $(t,z) \in [0,T]\times\mathbb{R}, \mathbb{P}$ *-a.s.*

2.3 Clark-Ocone type formula for canonical Lévy functionals

We next present an explicit form of the martingale representation formula by using Malliavin calculus.

Proposition 2.11 (Clark-Ocone type formula for canonical Lévy functionals) *Let* $F \in \mathbb{D}^{1,2}$*. Then*

$$\begin{aligned}
F &= \mathbb{E}[F] + \int_{[0,T]\times\mathbb{R}} \mathbb{E}[D_{t,z}F|\mathcal{F}_{t-}] Q(dt,dz) \\
&= \mathbb{E}[F] + \sigma \int_0^T \mathbb{E}[D_{t,0}F|\mathcal{F}_{t-}] dW_t + \int_0^T \int_{\mathbb{R}_0} \mathbb{E}[D_{t,z}F|\mathcal{F}_{t-}] z \tilde{N}(dt,dz). \qquad (2.4)
\end{aligned}$$

Remark 2.12 *We can show the same step as Theorems 4.1, 12. 16 and 12.20 in [4] and note that representation in Theorem 12. 20 in [4] is different from (2.4). In Theorem 2.11, we rewrite it to (2.4) more precisely and to fits our framework.*

2.4 Girsanov theorem for Lévy processes

We recall the Girsanov theorem for Lévy processes (see, e.g., Theorem 12.21 of [4]).

Theorem 2.13 *Let $\theta_{s,x} < 1, s \in [0,T], x \in \mathbb{R}_0$ and $u_s, s \in [0,T]$, be predictable processes such that*

$$\int_0^T \int_{\mathbb{R}_0} \{|\log(1-\theta_{s,x})|^2 + \theta_{s,x}^2\}\nu(dx)ds < \infty, \text{ a.s.},$$
$$\int_0^T u_s^2 ds < \infty, \text{ a.s.}$$

Moreover we denote

$$Z(t) := \exp\left(-\int_0^t u_s dW_s - \frac{1}{2}\int_0^t u_s^2 ds + \int_0^t \int_{\mathbb{R}_0} \log(1-\theta_{s,x})\tilde{N}(ds,dx) + \int_0^t \int_{\mathbb{R}_0} (\log(1-\theta_{s,x}) + \theta_{s,x})\nu(dx)ds\right), t \in [0,T].$$

Define a measure $\mathbb{Q}$ on $\mathcal{F}_T$ by

$$d\mathbb{Q}(\omega) = Z_T(\omega)d\mathbb{P}(\omega),$$

and we assume that $Z(T)$ satisfies the Novikov condition, that is,

$$\mathbb{E}\left[\exp\left(\frac{1}{2}\int_0^T u_s^2 ds + \int_0^T \int_{\mathbb{R}_0} \{(1-\theta_{s,x})\log(1-\theta_{s,x}) + \theta_{s,x}\}\nu(dx)ds\right)\right] < \infty.$$

Then $\mathbb{E}[Z_T] = 1$ and hence $\mathbb{Q}$ is a probability measure on $\mathcal{F}_T$. Furthermore if we denote

$$\tilde{N}^{\mathbb{Q}}(dt,dx) := \theta_{t,z}\nu(dx)dt + \tilde{N}(dt,dx)$$

and

$$dW_t^{\mathbb{Q}} := u_t dt + dW_t,$$

then $\tilde{N}^{\mathbb{Q}}(\cdot,\cdot)$ and $W^{\mathbb{Q}}_\cdot$ are the compensated Poisson random measure of $N(\cdot,\cdot)$ and a standard Brownian motion under $\mathbb{Q}$, respectively.

3 A Clark-Ocone type formula under change of measure for canonical Lévy processes

In this section, we introduce a Clark-Ocone type formula under change of measure for canonical Lévy processes. Throughout this section, under the same setting as Theorem 2.13, we assume the following.

Assumption 3.1 *(1) $u, u^2 \in \mathbb{L}_0^{1,2}$; and $2u_s D_{t,z}u_s + z(D_{t,z}u_s)^2 \in L^2(q \times \mathbb{P})$ for a.e. $s \in [0,T]$.*
(2) $\theta + \log(1-\theta) \in \tilde{\mathbb{L}}_1^{1,2}$, and $\log(1-\theta) \in \mathbb{L}_1^{1,2}$
(3) For q-a.e. $(s,x) \in [0,T] \times \mathbb{R}_0$, there is an $\varepsilon_{s,x} \in (0,1)$ such that $\theta_{s,x} < 1 - \varepsilon_{s,x}$.
(4) $Z_T \in L^2(\mathbb{P})$; and $Z_T\{D_{t,0}\log Z_T \mathbf{1}_{\{0\}}(z) + \frac{e^{zD_{t,z}\log Z_T}-1}{z}\mathbf{1}_{\mathbb{R}_0}(z)\} \in L^2(q \times \mathbb{P})$.
(5) $F \in \mathbb{D}^{1,2}$ with $FZ_T \in L^2(\mathbb{P})$; and $Z_T D_{t,z}F + FD_{t,z}Z_T + zD_{t,z}F \cdot D_{t,z}Z_T \in L^2(q \times \mathbb{P})$.
(6) $F\tilde{H}_{t,z}, \tilde{H}_{t,z}D_{t,z}F \in L^1(\mathbb{Q})$, (t,z) -a.e. where $\tilde{H}_{t,z} = \exp(zD_{t,z}\log Z_T - \log(1-\theta_{t,z}))$

Remark 3.2 *The statement of the main theorem in [10] includes error. Hence, we correct and revise it sophisticatedly.*

To show the main theorem, we need the following:

Lemma 3.3 *We have*

$$D_{t,0}Z_T = Z_T\left[-\sigma^{-1}u_t - \int_0^T D_{t,0}u_s dW_s^{\mathbb{Q}} - \int_0^T\int_{\mathbb{R}_0}\frac{D_{t,0}\theta_{s,x}}{1-\theta_{s,x}}\tilde{N}^{\mathbb{Q}}(ds,dx)\right] \tag{3.5}$$

for q-a.e. $(t,z)\in[0,T]\times\{0\}$, $\mathbb{P}$-a.s. *and*

$$D_{t,z}Z_T = z^{-1}Z_T[\exp(zD_{t,z}\log Z_T)-1] \quad \text{for } q\text{−a.e. } (t,z)\in[0,T]\times\mathbb{R}_0,\ \mathbb{P}\text{−a.s.}, \tag{3.6}$$

where

$$\begin{aligned} D_{t,z}\log Z_T = &-\int_0^T D_{t,z}u_s dW_s^{\mathbb{Q}} - \frac{1}{2}\int_0^T z(D_{t,z}u_s)^2 ds \\ &+\int_0^T\int_{\mathbb{R}_0}\left((1-\theta_{s,x})D_{t,z}\log(1-\theta_{s,x}) + D_{t,z}\theta_{s,x}\right)\nu(dx)ds \\ &+\int_0^T\int_{\mathbb{R}_0}D_{t,z}\log(1-\theta_{s,x})\tilde{N}^{\mathbb{Q}}(ds,dx) + z^{-1}\log(1-\theta_{t,z}) \end{aligned} \tag{3.7}$$

for q-a.e. $(t,z)\in[0,T]\times\mathbb{R}_0$, $\mathbb{P}$-a.s.

Proof. By conditions (1), (2) and (3) in Assumption 3.1, Propositions 2.8, 2.9 and 2.10 imply $\log Z_T\in\mathbb{D}^{1,2}$. Moreover, from (4) in Assumption 3.1, Proposition 2.5 leads to $Z_T\in\mathbb{D}^{1,2}$,

$$\begin{aligned} D_{t,0}Z_T = Z_T\Bigg[&-D_{t,0}\int_0^T u_s dW_s - \frac{1}{2}D_{t,0}\int_0^T u_s^2 ds \\ &+D_{t,0}\int_0^T\int_{\mathbb{R}_0}\log(1-\theta_{s,x})\tilde{N}(ds,dx) \\ &+D_{t,0}\int_0^T\int_{\mathbb{R}_0}(\log(1-\theta_{s,x})+\theta_{s,x})\nu(dx)ds\Bigg]. \end{aligned} \tag{3.8}$$

and

$$D_{t,z}Z_T = \frac{\exp(\log Z_T + zD_{t,z}\log Z_T) - Z_T}{z} = z^{-1}Z_T[\exp(zD_{t,z}\log Z_T)-1].$$

We next calculate right side of (3.8). From assumption (1) in Assumption 3.1, Proposition 2.9 implies

$$D_{t,0}\int_0^T u_s^2 ds = \int_0^T D_{t,0}u_s^2 ds \tag{3.9}$$

and by Proposition 2.10,

$$D_{t,0}\int_0^T\int_{\mathbb{R}_0}(\log(1-\theta_{s,x})+\theta_{s,x})\nu(dx)ds = \int_0^T\int_{\mathbb{R}_0}(D_{t,0}\log(1-\theta_{s,x})+D_{t,0}\theta_{s,x})\nu(dx)ds. \tag{3.10}$$

Since condition (1) in Assumption 3.1 holds, by Corollary 2.6, we have

$$D_{t,0}u_s^2 = 2u_s D_{t,0}u_s. \tag{3.11}$$

We calculate $D_{t,0}\log(1-\theta_{s,x})$. From (3) in Assumption 3.1, we have $\theta_{s,x}<1-\varepsilon_{s,x}$. We fix $(s,x)\in[0,T]\times\mathbb{R}_0$. We denote

$$l_{s,x}(y) = -\varepsilon_{s,x}^{-1}y + \varepsilon_{s,x}^{-1} - 1 + \log\varepsilon_{s,x}$$

and

$$g_{s,x}(y) = \begin{cases} \log(1-y), & y < 1-\varepsilon_{s,x} \\ l_{s,x}(y), & y \geq 1-\varepsilon_{s,x} \end{cases}.$$

Then, $g_{s,x} \in C^1(\mathbb{R})$ and

$$\log(1-\theta_{s,x}) = g_{s,x}(\theta_{s,x}).$$

Moreover, we have $\left|\frac{D_{t,0}\theta_{s,x}}{1-\theta_{s,x}}\right| < \varepsilon_{s,x}^{-1}|D_{t,0}\theta_{s,x}| \in L^2(\lambda \times \mathbb{P})$ by $\frac{1}{1-\theta_{s,x}} < \varepsilon_{s,x}^{-1}$ and $\theta_{s,x} \in \mathbb{D}^{1,2}$. Hence, Proposition 2.5 implies that $\log(1-\theta_{s,x}) \in \mathbb{D}_0^{1,2}$ and

$$D_{t,0}\log(1-\theta_{s,x}) = D_{t,0}g_{s,x}(\theta_{s,x}) = g'_{s,x}(\theta_{s,x})D_{t,0}\theta_{s,x} = -\frac{D_{t,0}\theta_{s,x}}{1-\theta_{s,x}}.$$

From condition (1), (2) in Assumption 3.1, Proposition 2.8 implies

$$D_{t,0}\int_0^T u_s dW_s = \sigma^{-1}u_t + \int_0^T D_{t,0}u_s dW_s \tag{3.12}$$

and

$$D_{t,0}\int_0^T\int_{\mathbb{R}_0}\log(1-\theta_{s,x})\tilde{N}(ds,dx) = \int_0^T\int_{\mathbb{R}_0}D_{t,0}\log(1-\theta_{s,x})\tilde{N}(ds,dx). \tag{3.13}$$

Hence, by (3.8) - (3.13), we obtain

$$\begin{aligned} D_{t,0}Z_T = Z_T\Bigg[&-\sigma^{-1}u_t - \int_0^T D_{t,0}u_s dW_s - \int_0^T u_s D_{t,0}u_s ds \\ &- \int_0^T\int_{\mathbb{R}_0}\frac{D_{t,0}\theta_{s,x}}{1-\theta_{s,x}}\tilde{N}(ds,dx) + \int_0^T\int_{\mathbb{R}_0}\left(-\frac{D_{t,0}\theta_{s,x}}{1-\theta_{s,x}} + D_{t,0}\theta_{s,x}\right)\nu(dx)ds\Bigg] \\ = Z_T\Bigg[&-\sigma^{-1}u_t - \int_0^T D_{t,0}u_s dW_s^{\mathbb{Q}} - \int_0^T\int_{\mathbb{R}_0}\frac{D_{t,0}\theta_{s,x}}{1-\theta_{s,x}}\tilde{N}^{\mathbb{Q}}(ds,dx)\Bigg]. \end{aligned}$$

We next calculate $D_{t,z}\log Z_T$.
By conditions (1) and (2) in Assumption 3.1, Proposition 2.8, Proposition 2.9 and Proposition 2.10 show that

$$\begin{aligned} D_{t,z}\log Z_T &= -D_{t,z}\int_0^T u_s dW_s - \frac{1}{2}D_{t,z}\int_0^T u_s^2 ds \\ &\quad + D_{t,z}\int_0^T\int_{\mathbb{R}_0}x^{-1}\log(1-\theta_{s,x})x\tilde{N}(ds,dx) \\ &\quad + D_{t,z}\int_0^T\int_{\mathbb{R}_0}(\log(1-\theta_{s,x})+\theta_{s,x})\nu(dx)ds \\ &= -\int_0^T D_{t,z}u_s dW_s - \frac{1}{2}\int_0^T D_{t,z}(u_s)^2 ds \\ &\quad + \int_0^T\int_{\mathbb{R}_0}D_{t,z}\log(1-\theta_{s,x})\tilde{N}(ds,dx) \\ &\quad + \int_0^T\int_{\mathbb{R}_0}(D_{t,z}\log(1-\theta_{s,x}) + D_{t,z}\theta_{s,x})\,\nu(dx)ds + \frac{\log(1-\theta_{t,z})}{z}. \end{aligned} \tag{3.14}$$

Now we calculate $D_{t,z}(u_s)^2$. Corollary 2.6 implies

$$D_{t,z}(u_s)^2 = 2u_s D_{t,z}u_s + z(D_{t,z}u_s)^2, \tag{3.15}$$

because, $u \in \mathbb{D}^{1,2}$ and condition (1) in Assumption 3.1 hold. From equations (3.14) and (3.15), we have

$$\begin{aligned} D_{t,z} \log Z_T = & -\int_0^T D_{t,z} u_s dW_s^{\mathbb{Q}} - \frac{1}{2}\int_0^T z(D_{t,z}u_s)^2 ds \\ & + \int_0^T \int_{\mathbb{R}_0} ((1-\theta_{s,x}) D_{t,z} \log(1-\theta_{s,x}) + D_{t,z}\theta_{s,x})\, \nu(dx) ds \\ & + \int_0^T \int_{\mathbb{R}_0} D_{t,z} \log(1-\theta_{s,x}) \tilde{N}^{\mathbb{Q}}(ds,dx) + z^{-1}\log(1-\theta_{t,z}). \end{aligned}$$

□

We next introduce a Clark-Ocone type formula under change of measure for canonical Lévy processes.

Theorem 3.4

$$F = \mathbb{E}_{\mathbb{Q}}[F] + \sigma \int_0^T \mathbb{E}_{\mathbb{Q}}\left[D_{t,0}F - FK_t \middle| \mathcal{F}_{t-}\right] dW_t^{\mathbb{Q}} + \int_0^T \int_{\mathbb{R}_0} \mathbb{E}_{\mathbb{Q}}[F(\tilde{H}_{t,z}-1) + z\tilde{H}_{t,z} D_{t,z}F | \mathcal{F}_{t-}] \tilde{N}^{\mathbb{Q}}(dt,dz), a.s.$$

holds, where

$$K_t = \int_0^T D_{t,0}u_s dW_s^{\mathbb{Q}} + \int_0^T \int_{\mathbb{R}_0} \frac{D_{t,0}\theta_{s,x}}{1-\theta_{s,x}} \tilde{N}^{\mathbb{Q}}(ds,dx).$$

Proof. First we denote $\Lambda_t := Z_t^{-1} = e^{-\log Z_t}, t \in [0,T]$. Then by the Itô formula (see, e.g., Theorem 9.4 of [4]), we have

$$\begin{aligned} d\Lambda_t = & \Lambda_{t-}\left\{ \frac{1}{2}u_t^2 - \int_{\mathbb{R}_0} (\log(1-\theta_{t,z}) + \theta_{t,z})\nu(dz) \right\} dt \\ & + \Lambda_{t-}u_t dW_t + \frac{1}{2}\Lambda_{t-}u_t^2 dt + \int_{\mathbb{R}_0} \Lambda_{t-}\left(\frac{1}{1-\theta_{t,z}} - 1\right) \tilde{N}(dt,dz) \\ & + \int_{\mathbb{R}_0} \left[\Lambda_{t-} \cdot \frac{1}{1-\theta_{t,z}} - \Lambda_{t-} + \Lambda_{t-}\log(1-\theta_{t,z})\right] \nu(dz)dt \\ = & \Lambda_{t-}\left[u_t^2 dt + u_t dW_t + \int_{\mathbb{R}_0} \frac{\theta_{t,z}^2}{1-\theta_{t,z}} \nu(dz)dt + \int_{\mathbb{R}_0} \frac{\theta_{t,z}}{1-\theta_{t,z}} \tilde{N}(dt,dz) \right] \\ = & \Lambda_{t-}\left[u_t dW_t^{\mathbb{Q}} + \int_{\mathbb{R}_0} \frac{\theta_{t,z}}{1-\theta_{t,z}} \tilde{N}^{\mathbb{Q}}(dt,dz) \right]. \end{aligned}$$

Denoting $Y_t := \mathbb{E}_{\mathbb{Q}}[F|\mathcal{F}_t], t \in [0,T]$, we have $Y_t = \Lambda_t \mathbb{E}[Z_T F|\mathcal{F}_t]$ by condition (5) in Assumption 3.1 and the Bayes rule (see, e.g., Lemma 4.7 of [4]). From (5) in Assumption 3.1, Corollary 2.6 implies that $Z_T F \in \mathbb{D}^{1,2}$. Hence, Lemma 2.4 implies that $\mathbb{E}[Z_T F|\mathcal{F}_t] \in \mathbb{D}^{1,2}$ holds. We apply Proposition 2.11 to $\mathbb{E}[Z_T F|\mathcal{F}_t]$, then, by Lemma 2.4, we have

$$\mathbb{E}[Z_T F|\mathcal{F}_t] = \mathbb{E}[Z_T F] + \int_0^t \int_{\mathbb{R}} \mathbb{E}[D_{s,z}(Z_T F)|\mathcal{F}_{s-}] Q(ds,dz).$$

Denoting $V_t := \mathbb{E}[Z_T F|\mathcal{F}_t]$, we have $Y_t = \Lambda_t V_t$. Itô's product rule implies that

$$\begin{aligned}
dY_t &= \Lambda_{t-} dV_t + V_{t-} d\Lambda_t + d[\Lambda, V]_t \\
&= \Lambda_{t-} \left\{ \sigma \mathbb{E}[D_{t,0}(Z_T F)|\mathcal{F}_{t-}] dW_t + \int_{\mathbb{R}_0} \mathbb{E}[D_{t,z}(Z_T F)|\mathcal{F}_{t-}] z \tilde{N}(dt, dz) \right\} \\
&\quad + V_{t-}\Lambda_{t-} \left\{ u_t dW_t^{\mathbb{Q}} + \int_{\mathbb{R}_0} \frac{\theta_{t,z}}{1-\theta_{t,z}} \tilde{N}^{\mathbb{Q}}(dt, dz) \right\} \\
&\quad + \Lambda_{t-} \left\{ \sigma u_t \mathbb{E}[D_{t,0}(Z_T F)|\mathcal{F}_{t-}] + \int_{\mathbb{R}_0} \frac{\theta_{t,z}}{1-\theta_{t,z}} \mathbb{E}[D_{t,z}(Z_T F)|\mathcal{F}_{t-}] z \nu(dz) \right\} dt \\
&\quad + \Lambda_{t-} \int_{\mathbb{R}_0} \frac{\theta_{t,z}}{1-\theta_{t,z}} \mathbb{E}[D_{t,z}(Z_T F)|\mathcal{F}_{t-}] z \tilde{N}(ds, dz) \\
&= \Lambda_{t-} \mathbb{E}[\sigma D_{t,0}(Z_T F)|\mathcal{F}_{t-}] dW_t^{\mathbb{Q}} + \Lambda_{t-} \mathbb{E}[Z_T F u_t|\mathcal{F}_{t-}] dW_t^{\mathbb{Q}} \\
&\quad + \Lambda_{t-} \int_{\mathbb{R}_0} \frac{\mathbb{E}[D_{t,z}(Z_T F)|\mathcal{F}_{t-}]}{1-\theta_{t,z}} z \tilde{N}^{\mathbb{Q}}(dt, dz) + \Lambda_{t-} \int_{\mathbb{R}_0} \mathbb{E}\left[Z_T F \frac{\theta_{t,z}}{1-\theta_{t,z}} \middle| \mathcal{F}_{t-} \right] \tilde{N}^{\mathbb{Q}}(dt, dz). \qquad (3.16)
\end{aligned}$$

Now we shall calculate $D_{t,0}(Z_T F)$ and $D_{t,z}(Z_T F)$. As for $D_{t,0}(Z_T F)$, by (5) in Assumption 3.1, Corollary 2.6 yields that

$$D_{t,0}(Z_T F) = F D_{t,0} Z_T + Z_T D_{t,0} F. \qquad (3.17)$$

Therefore combining (3.17) with (3.5), we can conclude

$$\begin{aligned}
D_{t,0}(Z_T F) &= F D_{t,0} Z_T + Z_T D_{t,0} F \\
&= F Z_T \left[-\sigma^{-1} u_t - \int_0^T D_{t,0} u_s dW_s^{\mathbb{Q}} - \int_0^T \int_{\mathbb{R}_0} \frac{D_{t,0}\theta_{s,x}}{1-\theta_{s,x}} \tilde{N}^{\mathbb{Q}}(ds, dx) \right] + Z_T D_{t,0} F \\
&= Z_T \left[D_{t,0} F - F \left(\sigma^{-1} u_t + K_t \right) \right]. \qquad (3.18)
\end{aligned}$$

Next we calculate $D_{t,z}(Z_T F)$. From condition (5), Corollary 2.6 implies that

$$D_{t,z}(Z_T F) = F D_{t,z} Z_T + Z_T D_{t,z} F + z D_{t,z} Z_T \cdot D_{t,z} F. \qquad (3.19)$$

From (3.6),

$$D_{t,z} Z_T = z^{-1} Z_T [(1-\theta_{t,z}) \tilde{H}_{t,z} - 1]. \qquad (3.20)$$

Therefore, combining (3.19) and (3.20), we obtain

$$\begin{aligned}
D_{t,z}(Z_T F) &= z^{-1} Z_T [(1-\theta_{t,z}) \tilde{H}_{t,z} - 1] F + Z_T D_{t,z} F + Z_T [(1-\theta_{t,z}) \tilde{H}_{t,z} - 1] D_{t,z} F \\
&= Z_T \left[z^{-1} \{ (1-\theta_{t,z}) \tilde{H}_{t,z} - 1 \} F + (1-\theta_{t,z}) \tilde{H}_{t,z} D_{t,z} F \right]. \qquad (3.21)
\end{aligned}$$

From (3.16), (3.18), (3.21), we arrive at:

$$\begin{aligned}
dY_t &= \Lambda_{t-} \mathbb{E}\left[Z_T \left\{ \sigma D_{t,0} F - F \left(u_t + \sigma K_t \right) \right\} \middle| \mathcal{F}_{t-} \right] dW_t^{\mathbb{Q}} \\
&\quad + \Lambda_{t-} \int_{\mathbb{R}_0} \mathbb{E}\left[\left\{ Z_T F \left(\tilde{H}_{t,z} - \frac{1}{1-\theta_{t,z}} \right) + z \tilde{H}_{t,z} D_{t,z} F \right\} \middle| \mathcal{F}_{t-} \right] \tilde{N}^{\mathbb{Q}}(dt, dz) \\
&\quad + \Lambda_{t-} \mathbb{E}[Z_T F u_t|\mathcal{F}_{t-}] dW_t^{\mathbb{Q}} + \Lambda_{t-} \int_{\mathbb{R}_0} \mathbb{E}\left[Z_T F \frac{\theta_{t,z}}{1-\theta_{t,z}} \middle| \mathcal{F}_{t-} \right] \tilde{N}^{\mathbb{Q}}(dt, dz) \\
&= \sigma \Lambda_{t-} \mathbb{E}\left[Z_T \left\{ D_{t,0} F - F K_t \right\} \middle| \mathcal{F}_{t-} \right] dW_t^{\mathbb{Q}} \\
&\quad + \Lambda_{t-} \int_{\mathbb{R}_0} \mathbb{E}\left[Z_T \{ F \left(\tilde{H}_{t,z} - 1 \right) + z \tilde{H}_{t,z} D_{t,z} F \} \middle| \mathcal{F}_{t-} \right] \tilde{N}^{\mathbb{Q}}(dt, dz).
\end{aligned}$$

From (1) and (2) in Assumption 3.1, we have $K_t \in L^2(\mathbb{P})$ t-a.e. Hence, by (5) in Assumption 3.1, $\mathbb{E}_{\mathbb{Q}}[|FK_t|] = \mathbb{E}[|FK_t|Z_T] \leq \sqrt{\mathbb{E}[|K_t|^2]\mathbb{E}[|FZ_T|^2]} < \infty$. Moreover, from (5) in Assumption 3.1, we have $D_{t,0}F \in L^2(\mathbb{P})$ t-a.e. and $\mathbb{E}_{\mathbb{Q}}[|D_{t,0}F|] = \mathbb{E}[|D_{t,0}F|Z_T] \leq \sqrt{\mathbb{E}[|D_{t,0}F|^2]\mathbb{E}[Z_T^2]} < \infty$. Then, by (6) in Assumption 3.1 and $F, D_{t,0}F, FK_t \in L^1(\mathbb{Q})$ t-a.e., the Bayes rule implies

$$dY_t = \sigma \mathbb{E}_{\mathbb{Q}}\left[D_{t,0}F - FK_t \middle| \mathcal{F}_{t-}\right] dW_t^{\mathbb{Q}} + \int_{\mathbb{R}_0} \mathbb{E}_{\mathbb{Q}}[F(\tilde{H}_{t,z} - 1) + z\tilde{H}_{t,z}D_{t,z}F|\mathcal{F}_{t-}]\tilde{N}^{\mathbb{Q}}(dt, dz). \tag{3.22}$$

Since $Y_t = \mathbb{E}_{\mathbb{Q}}[F|\mathcal{F}_T] = F, Y(0) = \mathbb{E}_{\mathbb{Q}}[F|\mathcal{F}_0] = \mathbb{E}_{\mathbb{Q}}[F]$, Integrating equation (3.22) gives

$$F - \mathbb{E}_{\mathbb{Q}}[F] = \sigma \int_0^T \mathbb{E}_{\mathbb{Q}}\left[D_{t,0}F - FK_t \middle| \mathcal{F}_{t-}\right] dW_t^{\mathbb{Q}} + \int_0^T \int_{\mathbb{R}_0} \mathbb{E}_{\mathbb{Q}}[F(\tilde{H}_{t,z} - 1) + z\tilde{H}_{t,z}D_{t,z}F|\mathcal{F}_{t-}]\tilde{N}^{\mathbb{Q}}(dt, dz).$$

The proof is concluded.

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