Research Report

Hempel's raven paradox, Hume's problem of induction, Goodman's grue paradox, Peirce's abduction, Flagpole problem are clarified in quantum language

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Abstract

Recently we proposed "quantum language" (or, "the linguistic Copenhagen interpretation of quantum mechanics", "measurement theory") as the language of science. This theory asserts the probabilistic interpretation of science (= the linguistic quantum mechanical worldview), which is a kind of mathematical generalization of Born's probabilistic interpretation of quantum mechanics. In this preprint, we consider the most fundamental problems in philosophy of science such as Hempel's raven paradox, Hume's problem of induction, Goodman's grue paradox, Peirce's abduction, flagpole problem, which are closely related to measurement. We believe that these problems can never be solved without the basic theory of science with axioms. Since our worldview has the axiom concerning measurement, these problems can be solved easily. Hence there is a reason to assert that quantum language gives the mathematical foundations to science[1].

Key phrases: Philosophy of Science, Linguistic Copenhagen Interpretation, Probabilistic Interpretation of Science, Raven Paradox, Problem of Induction (≈ the law of large numbers), Grue Paradox, Abduction (≈ Fisher's maximaum likelihood method), Flagpole problem

1 Review: Quantum language (= Measurement theory)

1.1 Quantum language is the language to describe science

Quantum language (or, "the linguistic Copenhagen interpretation of quantum mechanics", "measurement theory (=MT)") is proposed as the language of science, which is not only characterized as the metaphysical and linguistic turn of quantum mechanics but also the quantitative turn of Descartes=Kant epistemology and the dualistic turn of statistics. Thus, the location of this theory in the history of scientific worldviews is as follows (*cf.* refs.[10, 16, 20]):
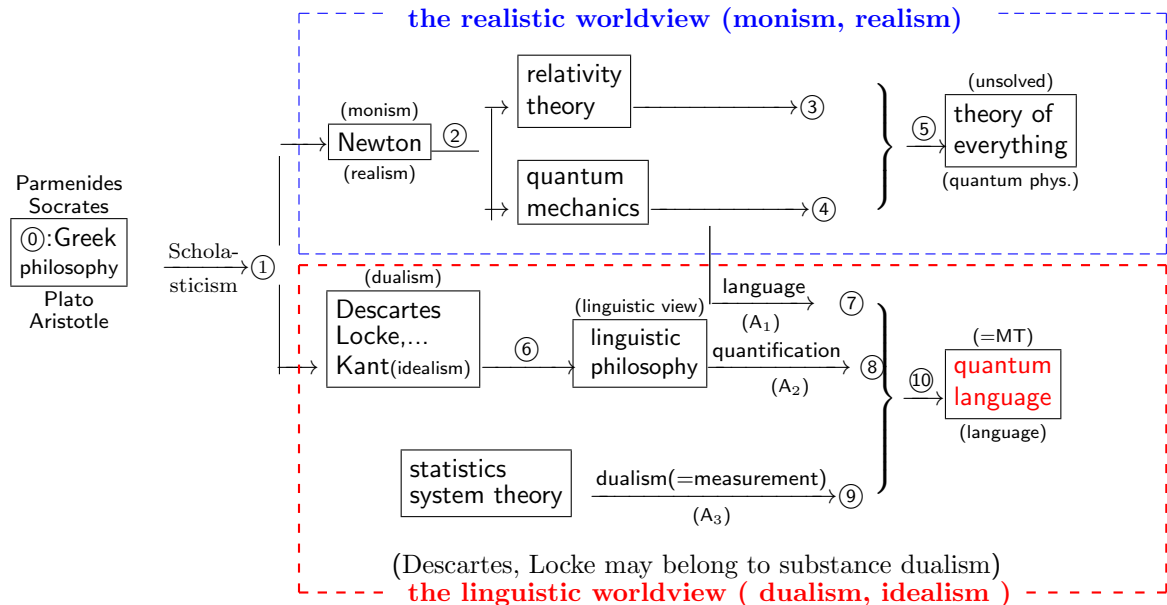

Figure 1: [**The location of quantum language in the history of scientific worldviews**]

Fig. 1 says that quantum language (=the linguistic Copenhagen interpretation) has the following three aspects

(A$_1$) the linguistic Copenhagen interpretation is the true figure of so-called Copenhagen interpretation (⑦ in Fig. 1), *cf.* refs. [5, 8, 9, 15, 18]

(A$_2$) the scientific final goal of dualistic idealism (i.e., Descartes=Kant philosophy) (⑧ in Fig. 1), *cf.* refs. [10, 16, 17, 19]

(A$_3$) statistics (= dynamical system theory) with the concept "measurement" (⑨ in Fig. 1), *cf.* refs. [6, 7, 8, 12, 13]

Hence, it is natural to assume that

(A$_4$) quantum language proposes the probabilistic interpretation of science (= the linguistic quantum mechanical worldview), and thus, it is just the language to describe science. Thus, we assert that science is built on dualism ($\approx$ ["observer"+"matter"]$\approx$ measurement) and idealism ($\approx$ metaphysics $\approx$ language).

which is the most important assertion of quantum language. Also, we assume that to make the language to describe science is one of main purposes of philosophy of science.

As criticism of philosophy of science, there is criticism that scientific philosophy is not very useful for scientists. We agree to this criticism. However, as mentioned in the above (A$_1$) and (A$_3$), we say that quantum language is one of the most useful theories in science.

Remark 1. Since space and time are independent in quantum language, the theory of relativity (and further, the theory of everything: ⑤ in Fig. 1) cannot be described by quantum language. We think that the theory of relativity is too special, an exception. It is too optimistic to expect that all scientific propositions can be written in quantum language. However, we want to assert the (A$_4$), that is, quantum language is the most fundamental language for almost all familiar science. We believe that arguments without a worldview do not bring about the success of philosophy of science.

1.2 No scientific argument without scientific worldview

It is well known that the following problems are the most fundamental in philosophy of science:

(A$_5$) Hempel's raven paradox, Goodman's grue paradox, Hume's problem of induction, Peirce's abduction, the flagpole problem etc.

In Sections 2∼4, we clarify these problems under the linguistic quantum mechanical worldview (A$_4$), since we believe that there is no scientific argument without scientific worldview (or, without scientific language). And we conclude that the reason that these problems are not yet clarified depends on lack of the concept of measurement in philosophy of science.

1.3 Mathematical Preparations

Following refs. [8, 9, 10, 20] (all our results until present are written in ref. [20]), we shall review quantum language, which has the following form:

$$\underset{\text{(= measurement theory)}}{\boxed{\boxed{\text{Quantum language}}}} = \underset{\text{(Axiom 1)}}{\boxed{\text{measurement}}} + \underset{\text{(Axiom 2)}}{\boxed{\text{causality}}} + \underset{\text{(how to use Axioms 1 and 2)}}{\boxed{\text{linguistic (Copenhagen) interpretation}}} \qquad (1)$$

which asserts that "measurement" and "causality" are the most important concepts in science.

Consider an operator algebra $B(H)$ (i.e., an operator algebra composed of all bounded linear operators on a Hilbert space H with the norm $\|F\|_{B(H)} = \sup_{\|u\|_H=1} \|Fu\|_H$), and consider the triplet $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$ (or, the pair $[\mathcal{A}, \mathcal{N}]_{B(H)}$), called a *basic structure*. Here, $\mathcal{A}(\subseteq B(H))$ is a C^*-algebra, and $\mathcal{N}$ ($\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)$) is a particular C^*-algebra (called a W^*-algebra) such that $\mathcal{N}$ is the weak closure of $\mathcal{A}$ in $B(H)$.

The measurement theory (= "quantum language" = "the linguistic Copenhagen interpretation") is classified as follows.

$$(B) \quad \underset{(= \text{ quantum language})}{\text{measurement theory}} = \begin{cases} (B'_1): \text{ quantum system theory} \quad (\text{when } \mathcal{A} = \mathcal{C}(H)) \\ \\ (B'_2): \text{ classical system theory} \quad (\text{when } \mathcal{A} = C_0(\Omega)) \end{cases}$$

The Hilbert space method for the mathematical foundations of quantum mechanics is essentially due to von Neumann (*cf.* ref. [26]). He devoted himself to quantum (B'_1). On the other hand, in most cases, we devote ourselves to classical (B'_2), and not (B'_1). However, the quantum (B'_1) is convenient for us, in the sense that the idea in (B'_1) is often introduced into classical (B'_2).

When $\mathcal{A} = \mathcal{C}(H)$, the C^*-algebra composed of all compact operators on a Hilbert space H, the (B'_1) is called quantum measurement theory (or, quantum system theory), which can be regarded as the linguistic aspect of quantum mechanics. Also, when $\mathcal{A}$ is commutative (that is, when $\mathcal{A}$ is characterized by $C_0(\Omega)$, the C^*-algebra composed of all continuous complex-valued functions vanishing at infinity on a locally compact Hausdorff space Ω (*cf.* refs. [22, 25, 27])), the (B'_2) is called classical measurement theory.

Also, note that, when $\mathcal{A} = \mathcal{C}(H)$, i.e., quantum cases,

(B_1) $\mathcal{A}^* = Tr(H)$ (=trace class), $\mathcal{N} = B(H)$, $\mathcal{N}_* = Tr(H)$ (i.e., pre-dual space), thus, ${}_{Tr(H)}(\rho, T)_{B(H)} = \text{Tr}_H(\rho T)$ $(\rho \in Tr(H), T \in B(H))$.

Also, when $\mathcal{A} = C_0(\Omega)$, i.e., classical cases,

(B_2) $\mathcal{A}^* =$ "the space of all signed measures on Ω", $\mathcal{N} = L^\infty(\Omega, \nu) (\subseteq B(L^2(\Omega, \nu)))$, $\mathcal{N}_* = L^1(\Omega, \nu)$, where ν is some measure on Ω (with the Borel field $\mathcal{B}$, thus, ${}_{L^1(\Omega, \nu)}(\rho, T)_{L^\infty(\Omega, \nu)} = \int_\Omega \rho(\omega) T(\omega) \nu(d\omega)$ $(\rho \in L^1(\Omega, \nu), T \in L^\infty(\Omega, \nu))$ (*cf.* ref. [25]).

In Sections 2∼4 later, we devote ourselves to a compact space Ω with a probability measure ν (i.e., $\nu(\Omega) = 1$) and thus, $C_0(\Omega)$ is simply denoted by $C(\Omega)$.

Let $\mathcal{A}(\subseteq B(H))$ be a C^*-algebra, and let $\mathcal{A}^*$ be the dual Banach space of $\mathcal{A}$. That is, $\mathcal{A}^* = \{\rho \mid \rho \text{ is a}$ continuous linear functional on $\mathcal{A}\}$, and the norm $\|\rho\|_{\mathcal{A}^*}$ is defined by $\sup\{|\rho(F)| \mid F \in \mathcal{A} \text{ such that } \|F\|_{\mathcal{A}}(= \|F\|_{B(H)}) \leq 1\}$. Define the *mixed state* $\rho (\in \mathcal{A}^*)$ such that $\|\rho\|_{\mathcal{A}^*} = 1$ and $\rho(F) \geq 0$ for all $F \in \mathcal{A}$ such that $F \geq 0$. And define the mixed state space $\mathfrak{S}^m(\mathcal{A}^*)$ such that

$$\mathfrak{S}^m(\mathcal{A}^*) = \{\rho \in \mathcal{A}^* \mid \rho \text{ is a mixed state}\}.$$

A mixed state $\rho(\in \mathfrak{S}^m(\mathcal{A}^*))$ is called a *pure state* if it satisfies that "$\rho = \theta \rho_1 + (1 - \theta)\rho_2$ for some $\rho_1, \rho_2 \in \mathfrak{S}^m(\mathcal{A}^*)$ and $0 < \theta < 1$" implies "$\rho = \rho_1 = \rho_2$". Put

$$\mathfrak{S}^p(\mathcal{A}^*) = \{\rho \in \mathfrak{S}^m(\mathcal{A}^*) \mid \rho \text{ is a pure state}\},$$

which is called a *state space*. It is well known (*cf.* ref. [25]) that $\mathfrak{S}^p(\mathcal{C}(H)^*) = \{|u\rangle\langle u|$ (i.e., the Dirac notation) $\mid \|u\|_H = 1\}$, and $\mathfrak{S}^p(C_0(\Omega)^*) = \{\delta_{\omega_0} \mid \delta_{\omega_0}$ is a point measure at $\omega_0 \in \Omega\}$, where $\int_\Omega f(\omega) \delta_{\omega_0}(d\omega) = f(\omega_0)$ $(\forall f \in C_0(\Omega))$. The latter implies that $\mathfrak{S}^p(C_0(\Omega)^*)$ can be also identified with Ω (called a *spectrum space* or simply *spectrum*) such as

$$\underset{\text{(state space)}}{\mathfrak{S}^p(C_0(\Omega)^*)} \ni \delta_\omega \leftrightarrow \omega \in \underset{\text{(spectrum)}}{\Omega}$$

In this paper, Ω and $\omega(\in \Omega)$ is respectively called a state space and a state.

Definition 2. [Essentially continuous] An element $F(\in \mathcal{N})$ is said to be *essentially continuous* at $\rho_0(\in \mathfrak{S}^p(\mathcal{A}^*))$, if there uniquely exists a complex number α such that

- if $\rho (\in \mathcal{N}_*, \|\rho\|_{\mathcal{N}_*} = 1, \rho \geq 0)$ converges to $\rho_0(\in \mathfrak{S}^p(\mathcal{A}^*))$ in the sense of weak* topology of $\mathcal{A}^*$, that is,

$$\rho(G) \longrightarrow \rho_0(G) \quad (\forall G \in \mathcal{A}(\subseteq \mathcal{N})), \tag{2}$$

  then $\rho(F)$ converges to α.

Remark 3. In quantum cases, the (2) implies that

$$\rho(G)(= {}_{Tr(H)}(\rho, G)_{B(H)}) \longrightarrow \rho_0(G)(= {}_{Tr(H)}(\rho_0, G)_{B(H)}) \qquad (\text{if } \rho \xrightarrow[\text{weak* topology}]{} \rho_0)$$

Thus, the (2) always hols in quantum cases. On the other hand, in classical cases, the (2) implies that

$$\rho(G)(=\ _{\mathcal{M}(\Omega)}\big(\rho,G\big)_{L^{\infty}(\Omega,\nu)}) \longrightarrow \rho_0(G)(=\ _{\mathcal{M}(\Omega)}\big(\rho_0,G\big)_{L^{\infty}(\Omega,\nu)}) \qquad (\text{if } \rho \xrightarrow[\text{weak}^* \text{ topology}]{} \rho_0 \ (\text{ in }\ \mathcal{M}(\Omega))$$

Thus, it does not always hold in classical cases. In this sense, the quantum system theory (B'$_1$) is easier than the classical system theory (B'$_2$).

The following definition is due to E.B. Davies (*cf.* ref. [1]).

Definition 4. [Observable]   An *observable* $\mathsf{O} = (X, \mathcal{F}, F)$ in $\mathcal{N}$ is defined as follows:

(i) [σ-field] X is a set, $\mathcal{F}(\subseteq 2^X \equiv \mathcal{P}(X)$, the power set of X) is a σ-field of X, that is, "$\Xi_1, \Xi_2, ... \in \mathcal{F} \Rightarrow \cup_{n=1}^{\infty}\Xi_n \in \mathcal{F}$", "$\Xi \in \mathcal{F} \Rightarrow X \setminus \Xi(\equiv \{x \mid x \in X, x \notin \Xi\} \equiv \Xi^c,$ i.e., the complement of $\Xi) \in \mathcal{F}$".

(ii) [Countable additivity] F is a mapping from $\mathcal{F}$ to $\mathcal{N}$ satisfying: (a): for every $\Xi \in \mathcal{F}$, $F(\Xi)$ is a non-negative element in $\mathcal{N}$ such that $0 \leq F(\Xi) \leq I$, (b): $F(\emptyset) = 0$ and $F(X) = I$, where 0 and I is the 0-element and the identity in $\mathcal{N}$ respectively. (c): for any countable decomposition $\{\Xi_1, \Xi_2, \ldots, \Xi_n, \ldots\}$ of Ξ $\big($i.e., $\Xi, \Xi_n \in \mathcal{F}$ $(n = 1, 2, 3, ...)$, $\cup_{n=1}^{\infty}\Xi_n = \Xi$, $\Xi_i \cap \Xi_j = \emptyset$ $(i \neq j)\big)$, it holds that $F(\Xi) = \sum_{n=1}^{\infty} F(\Xi_n)$ in the sense of weak* topology in $\mathcal{N}$.

1.4   Axiom 1 [Measurement] and Axiom 2 [Causality]

Quantum language (1) is composed of two axioms (i.e., Axioms 1 and 2) as follows. With any *system S*, a basic structure $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$ can be associated in which the measurement theory (A) of that system can be formulated. A *state* of the system S is represented by an element $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$ and an *observable* is represented by an observable $\mathsf{O} = (X, \mathcal{F}, F)$ in $\mathcal{N}$. Also, the *measurement of the observable* O *for the system* S *with the state* ρ is denoted by $\mathsf{M}_{\mathcal{N}}(\mathsf{O}, S_{[\rho]})$ $\big($ or more precisely, $\mathsf{M}_{\mathcal{N}}(\mathsf{O} := (X, \mathcal{F}, F), S_{[\rho]})$ $\big)$. An observer can obtain a measured value x $(\in X)$ by the measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O}, S_{[\rho]})$.

The Axiom 1 presented below is a kind of mathematical generalization of Born's probabilistic interpretation of quantum mechanics. And thus, it is a statement without reality.

Now we can present Axiom 1 in the W^*-algebraic formulation as follows.

> **Axiom 1** [Measurement, the probabilistic interpretation of science].   *The probability that a measured value* x $(\in X)$ *obtained by the measurement* $\mathsf{M}_{\mathcal{N}}(\mathsf{O} := (X, \mathcal{F}, F), S_{[\rho]})$ *belongs to a set* $\Xi(\in \mathcal{F})$ *is given by* $\rho(F(\Xi))$ *if* $F(\Xi)$ *is essentially continuous at* $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$.

Remark 5. Recall Remark 3. In quantum cases (i.e, the cases that $\rho \in \mathfrak{S}^p(Tr(H))) \subseteq Tr(H)$, $F(\Xi) \in B(H)$), the probability $\rho(F(\Xi))(=\ _{Tr(H)}\big(\rho, T\big)_{B(H)} = \text{Tr}_H(\rho T))$ is always defined (*cf.* (B$_1$)). That is, the $F(\Xi)$ is always essentially continuous at any $\rho \in \mathfrak{S}^p(Tr(H))$). On the other hand, in the classical cases (i.e., the cases that $\omega \in \Omega$, $F(\Xi) \in L^{\infty}(\Omega, \nu)$), it is not guaranteed that $F(\Xi)$ is essentially continuous at ω $(\in \Omega)$). Thus, put $\mathcal{F}_{\omega_0} = \{\Xi \in \mathcal{F} : F(\Xi)$ is essentially continuous at ω_0 }. If $\mathcal{F}_{\omega_0} = \mathcal{F}$, the measurement $\mathsf{M}_{L^{\infty}(\Omega,\nu)}(\mathsf{O} := (X, \mathcal{F}, F), S_{[\omega_0]})$ makes the sample probability space $(X, \mathcal{F}, [F(\cdot)](\omega_0))$, which is usual in statistics. Thus, roughly speaking, statistics starts from "sample probability space", on the other hand, quantum language starts from "measurement".
(ii): Axiom 1 is the quantitative realization of the spirit: "there is no science without measurements". And, we think that Axiom 1 means the probabilistic interpretation of science since it is a kind of mathematical generalization of Born's probability interpretation of quantum mechanics.

Example 6. [Exact measurement] Consider a basic structure $[C(\Omega_1) \subseteq L^{\infty}(\Omega_1, \nu_1) \subseteq B(L^2(\Omega_1, \nu_1))]$, where $\Omega_1 = [0, 2]$ is the state space (where the closed interval in the real line $\mathbb{R}$). and the ν_1 is the Lebesgue measure on the Borel σ-field $\mathcal{B}_{\Omega_1}$, that is, the smallest σ-field that contains all opensets in Ω_1. Define the exact observable $\mathsf{O}_e = (X(=\Omega_1), \mathcal{B}_{\Omega_1}, F_e)$ in $L^{\infty}(\Omega_1, \nu_1)$ such that

$$[F_e(\Xi)](\omega) = 1 \ (\omega \in \Xi \in \mathcal{B}_{\Omega_1}, \quad = 0 \ (\omega \notin \Xi \in \mathcal{B}_{\Omega_1}.$$

Thus, we have the measurement $\mathsf{M}_{L^{\infty}(\Omega_1, \nu_1)}(\mathsf{O}_e = (X, \mathcal{B}_{\Omega_1}, F_e), S_{[\omega_0^0]})$. Then we have the following statement

- Let $D(\subseteq X = \Omega_1)$ be any open set such that $\omega_0^0 \in D$. Then Axiom 1 says that the probability that the measured value $x(\in X)$ obtained by the measurement $\mathsf{M}_{L^\infty(\Omega_0, \nu_0)}(\mathsf{O}_e = (X, \mathcal{F}, F_e), S_{[\omega_0^0]})$ belongs to D is given by 1.

This implies that $x = \omega_0^0$, since D is arbitrary open set such that $\omega_0^0 \in D$. Also, it should be noted that $F_e(\Xi)$ is not essentially continuous at ω_0^0 if $\Xi = [0, \omega_0^0]$. Thus, $(X, \mathcal{F}, [F_e(\cdot)](\omega_0^0))$ is not always a probability space.

Next, we explain Axiom 2. Let $[\mathcal{A}_1, \mathcal{N}_1]_{B(H_1)}$ and $[\mathcal{A}_2, \mathcal{N}_2]_{B(H_2)}$ be basic structures. A continuous linear operator $\Phi_{1,2} : \mathcal{N}_2$ (with weak* topology) $\to \mathcal{N}_1$(with weak* topology) is called a *Markov operator,* if it satisfies that (i): $\Phi_{1,2}(F_2) \geq 0$ for any non-negative element F_2 in $\mathcal{N}_2$, (ii): $\Phi_{1,2}(I_2) = I_1$, where I_k is the identity in $\mathcal{N}_k$, $(k = 1, 2)$. In addition to the above (i) and (ii), we assume that $\Phi_{1,2}(\mathcal{A}_2) \subseteq \mathcal{A}_1$ and $\sup\{\|\Phi_{1,2}(F_2)\|_{\mathcal{A}_1} \mid F_2 \in \mathcal{A}_2 \text{ such that } \|F_2\|_{\mathcal{A}_2} \leq 1\} = 1$.

It is clear that the dual operator $\Phi_{1,2}^* : \mathcal{A}_1^* \to \mathcal{A}_2^*$ satisfies that $\Phi_{1,2}^*(\mathfrak{S}^m(\mathcal{A}_1^*)) \subseteq \mathfrak{S}^m(\mathcal{A}_2^*)$. If it holds that $\Phi_{1,2}^*(\mathfrak{S}^p(\mathcal{A}_1^*)) \subseteq \mathfrak{S}^p(\mathcal{A}_2^*)$, the $\Phi_{1,2}$ is said to be deterministic. If it is not deterministic, it is said to be non-deterministic. Also note that, for any observable $\mathsf{O}_2 := (X, \mathcal{F}, F_2)$ in $\mathcal{N}_2$, the $(X, \mathcal{F}, \Phi_{1,2}F_2)$ is an observable in $\mathcal{N}_1$.

Definition 7. [Sequential causal operator; Heisenberg picture of causality]   Let $(T, \leq)$ be a tree like semi-ordered set such that "$t_1 \leq t_3$ and $t_2 \leq t_3$" implies "$t_1 \leq t_2$ or $t_2 \leq t_1$". The family $\{\Phi_{t_1, t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}\}_{(t_1, t_2) \in T_\leqq^2}$ is called a **sequential causal operator**, if it satisfies that

(i) For each $t (\in T)$, a basic structure $[\mathcal{A}_t \subseteq \mathcal{N}_t \subseteq B(H_t)]$ is determined.

(ii) For each $(t_1, t_2) \in T_\leqq^2$, a causal operator $\Phi_{t_1, t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}$ is defined such as $\Phi_{t_1, t_2}\Phi_{t_2, t_3} = \Phi_{t_1, t_3}$ $(\forall(t_1, t_2), \forall(t_2, t_3) \in T_\leqq^2)$. Here, $\Phi_{t,t} : \mathcal{N}_t \to \mathcal{N}_t$ is the identity operator.

Now we can propose Axiom 2 (i.e., causality). (For details, see ref. [20].)

Axiom 2[Causality]; For each $t(\in T=$"tree like semi-ordered set")), consider the basic structure:

$$[\mathcal{A}_t \subseteq \mathcal{N}_t \subseteq B(H_t)]$$

Then, the **chain of causalities** is represented by a **sequential causal operator** $\{\Phi_{t_1, t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}\}_{(t_1, t_2) \in T_\leqq^2}$.

1.5   The linguistic Copenhagen interpretation (= the manual to use Axioms 1 and 2)

It is well known (*cf.* ref. [4]) that the Copenhagen interpretation of quantum mechanics has not been established yet. For example, about the right or wrong of the wave function collapse, opinions are divided in the Copenhagen interpretation. Thus, the Copenhagen interpretation is often called "so-call Copenhagen interpretation". However, we believe that the linguistic Copenhagen interpretation of quantum language (B) (i.e., both quantum (B_1') and classical (B_2')) is uniquely determined. For example, for the quantum linguistic opinion about the wave function collapse, see ref. [15] or §11.2 in ref. [20]. As mentioned in (A_1), we believe that the linguistic Copenhagen interpretation is the true figure of so called Copenhagen interpretation.
Now we explain the linguistic Copenhagen interpretation in what follows. In the above, Axioms 1 and 2 are kinds of spells, (i.e., incantation, magic words, metaphysical statements), and thus, it is nonsense to verify them experimentally. Therefore, what we should do is not "to understand" but "to use". After learning Axioms 1 and 2 by rote, we have to improve how to use them through trial and error. We may do well even if we do not know the linguistic Copenhagen interpretation (= the manual to use Axioms 1 and 2). However, it is better to know the linguistic Copenhagen interpretation, if we would like to make progress quantum language early. We believe that the linguistic Copenhagen interpretation is the true Copenhagen interpretation, which does not belong to physics.

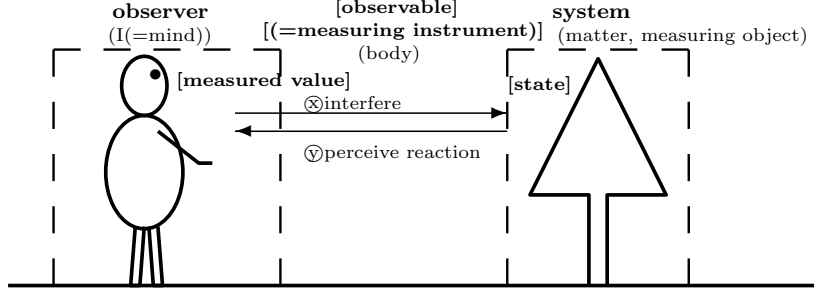The essence of the manual is as follows: In Fig. 2, we remark:

Figure 2: [Descartes Figure]: Image of "measurement(=ⓧ+ⓨ)" in mind-matter dualism

(C$_1$) ⓧ: it suffices to understand that "interfere" is, for example, "apply light".
ⓨ: perceive the reaction.

That is, "measurement" is characterized as the interaction between "observer" and "measuring object (= matter)". However,

(C$_2$) in measurement theory (=quantum language), "interaction" must not be emphasized.

Therefore, in order to avoid confusion, it might better to omit the interaction "ⓧ and ⓨ" in Fig. 2. After all, we think that:

(C$_3$) it is clear that there is no measured value without observer (i.e., "I", "mind"). Thus, we consider that measurement theory is composed of three key-words: "measured value","observable","state" (*cf.* § 3.1(p.63) in [20]).

$$\boxed{\text{measured value}}, \qquad \boxed{\text{observable (= measuring instrument)}} \quad , \quad \boxed{\text{state}},$$
(I, observer, mind)   (body(= sensory organ), thermometer, eye, ear, compass)   (matter)

Hence, quantum language is based on dualism, i.e., a kind of mind-matter dualism.
The linguistic Copenhagen interpretation says that

(D$_1$) **Only one measurement is permitted**. And therefore, the state after a measurement is meaningless since it cannot be measured any longer. Thus, the collapse of the wavefunction is prohibited (*cf.* ref. [15]; projection postulate). We are not concerned with anything after measurement. Strictly speaking, the phrase "*after the measurement*" should not be used. Also, the causality should be assumed only in the side of system, however, a state never moves. Thus, the Heisenberg picture should be adopted, and thus, the Schrödinger picture should be prohibited.

(D$_2$) "Observer"(="I") and "system" are completely separated in order not to make self-reference propositions appear. Hence, the measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O}:=(X,\mathcal{F},F), S_{[\rho]})$ does not depend on the choice of observers. That is, any proposition (except Axiom 1) in quantum language is not related to "observer"(="I"), therefore, there is no "observer's space and time" in quantum language. And thus, it does not have tense (i.e., past, present, future).

(D$_3$) there is no probability without measurements (*cf.* Bertrand's paradox (in § 9.12 of ref.[20]))

(D$_4$) Leibniz's relationalism concerning space-time (e.g, time should be regarded as a parameter), (*cf.* ref. [19]).

(D$_5$) A family of measurements $\{ \mathsf{M}_{\mathcal{N}_i}(\mathsf{O}_i:=(X_1,\mathcal{F}_i,F_i), S_{[\rho_i]}) : \quad i=1,2,3,...\}$ is realized as the paralell measurement $\mathsf{M}_{\otimes_{i=1}^{\infty}\mathcal{N}_i}(\otimes_{i=1}^{\infty}\mathsf{O}_i:=(\times_{i=1}^{\infty}X_1, \boxtimes_{i=1}^{\infty}\mathcal{F}_i, \otimes_{i=1}^{\infty}F_i), S_{[\otimes_{i=1}^{\infty}\rho_i]})$ (*cf.* Definition 9 later). For details about the tensor product " $\otimes_{i=1}^{\infty}$", see ref. [20].

and so on.

Remark 8. (i): In ref. [5] (1991), we proposed the mathematical formulation of Heisenberg's uncertainty relation. However, so-called Copenhagen interpretation is not firm (*cf.* ref. [4]). Thus, in order to understand our work (i.e., Heisenberg's uncertainty relation) deeply, we proposed the linguistic Copenhagen interpretation (= quantum language) as the true Copenhagen interpretation. For details of Heisenberg's uncertain relation, see §4.3 in ref. [20].

(ii): We consider that the above (D_1) is closely related to Kolmogorov's extension theorem (*cf.* ref. [21]), which says that only one probability space is permitted. For details, see §4.1 in ref. [20].

(iii): The formula (1) says that scientific explanation is to explain phenomena in terms of "measurement" (Axiom 1) and "causality" (Axiom 2). If we are allowed to use the famous metaphor of Kant's Copernican revolution, to do familir sciences is to see this world through colored glasses of measurement and causality (*cf.* [19]), or to use the metaphor of Wittgenstein's saying, the limits of quantum language are the limits of familir science. Therefore, the explanation problem of scientific philosophy is automatically clarified in quantum language.

(iv): Violating the linguistic Copenhagen interpretation (D_2), we have many paradoxes of self-reference type such as "brain in a vat", "five-minute hypothesis", "I think, therefore I am", "McTaggart's paradox". Cf. ref. [19] or §10.8 in ref. [20].

(v): We want to understand that Zeno's paradox is not a problem concerning geometric series or spatial division, but the problem concerning the worldview. That is, "Propose a certain scientific worldview, in which Zeno's paradox should be studied !" That is because we think that there is no scientific argument without scientific language ($\approx$ scientific worldview). And our answer (*cf.* § 14.4 in ref. [20]) is "If Zeno's paradox is a problem in science, it should be studied in quantum language". That is because our assertion is "Quantum language is the language of science". Also, Monty Hall problem, two envelope problem, three prisoners problem etc. are not only mathematical puzzles but also profound problems in quantum language (*cf.* refs. [13, 20]).

As the further explanation of parallel measurement in the linguistic Copenhagen interpretation (D_5), we have to add the following definition.

Definition 9. [Parallel measurement (*cf.* [20])] Though the parallel measurement can be defined in both classical and quantum systems, we, for simplicity, devote ourselves to classical systems as follows. Let $[C(\Omega) \subseteq L^\infty(\Omega.\nu) \subseteq [L^\infty(\Omega, \nu)]$ be a classical basic structure, where we assume, for simplicity, that Ω is compact space and ν is a measure such that $\nu(\Omega) = 1$ and $\nu(D) > 0$ ($\forall$open set $D \subseteq \Omega$). Consider a family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X_i, \mathcal{F}_i, F_i), S_{[\omega_i]}) \mid i = 1, 2, ..., N\}$. However, the linguistic Copenhagen interpretation (D_1) says "Only one measurement is permitted". Therefore, instead of the family of measurements, we consider the parallel measurement $\bigotimes_{i=1}^N \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X_i, \mathcal{F}_i, F_i), S_{[\omega_i]})$, which is defined by

$$\bigotimes_{i=1}^N \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X_i, \mathcal{F}_i, F_i), S_{[\omega_i]})$$

$$= \mathsf{M}_{L^\infty(\bigtimes_{i=1}^N \Omega, \bigotimes_{i=1}^N \nu)}(\otimes_{i=1}^N \mathsf{O}_i := (\bigtimes_{i=1}^N X_i, \boxtimes_{i=1}^N \mathcal{F}_i, \otimes_{i=1}^N F_i), S_{[(\omega_1, \omega_2, ..., \omega_N)]})$$

where $\bigtimes_{i=1}^N \Omega$ is the infinite product compact space of Ωs, $\bigotimes_{i=1}^N \nu$ is the infinite product probability of νs. Also, $\boxtimes_{i=1}^N \mathcal{F}_i (\subseteq \mathcal{P}(\bigtimes_{i=1}^N X_i))$ is the infinite product σ-field, i.e., the smallest σ-field that includes

$$\left\{ \bigtimes_{i=1}^N \Xi_i | \quad \Xi_i \in \mathcal{F}_i \right\}$$

And further, define the observable $\bigotimes_{i=1}^N F_i$ in $L^\infty(\bigtimes_{i=1}^N \Omega, \bigotimes_{i=1}^N \nu)$ which satisfies that

$$[(\otimes_{i=1}^N F_i)((\bigtimes_{i=1}^N \Xi_i)](\omega_1, \omega_2, ..., \omega_N) = \bigtimes_{i=1}^N [F_i(\Xi_i)](\omega_i) \quad \forall \Xi_i \in \mathcal{F}_i, \omega_i \in \Omega$$

Then, Axiom 1 [measurement] says that

(D_6) the probability that a measured value obtained by the parallel measurement $\bigotimes_{i=1}^N \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X_i, \mathcal{F}_i, F_i), S_{[\omega_i]})$ belongs to $\bigtimes_{i=1}^N \Xi_i$ is given by $\bigtimes_{i=1}^N [F_i(\Xi_i)](\omega_i)$, if $F_i(\Xi_i)$ is essentially continuous at ω_i ($\forall i = 1, 2, ..., N$).

Remark 10. The above finite parallel measurement can be generalized to the case that the index set Λ is infinite. That is,

$$\bigotimes_{\lambda \in \Lambda} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_\lambda := (X_\lambda, \mathcal{F}_\lambda, F_\lambda), S_{[\omega_\lambda]})$$

$$= \mathsf{M}_{L^\infty(\times_{\lambda \in \Lambda} \Omega, \otimes_{\lambda \in \Lambda} \nu)}(\bigotimes_{\lambda \in \Lambda} \mathsf{O}_\lambda := (\underset{\lambda \in \Lambda}{\times} X_\lambda, \boxtimes_{\lambda \in \Lambda} \mathcal{F}_\lambda, \otimes_{\lambda \in \Lambda} F_\lambda), S_{[(\omega_\lambda)_{\lambda \in \Lambda}]})$$

The existence of the parallel measurement is guaranteed in both classical and quantum systems. Cf. § 4.2 in ref. [20]. It is not so difficult to extend the above finite parallel measurements to infinite parallel measurements for mathematicians. However, in this paper, we are not concerned with the infinite parallel measurement. That is because our concern is not mathematics but foundations of philosophy of science.

Here we add the following definition, which will be used in [Step (III)] in Section 2.2.

Definition 11. [Implication: *cf.* refs. [6, 20]] Consider a basic structure:

$$[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$$

Let $\mathsf{O}_1 = (X_1, \mathcal{F}_1, F_1)$ and $\mathsf{O}_2 = (X_2, \mathcal{F}_2, F_2)$ be observables in $\mathcal{N}$. Let $\mathsf{O}_{12} = (X_1 \times X_2, \mathcal{F}_1 \boxtimes \mathcal{F}_2, F_{12})$ be an observable such that $F_1(\Xi_1) = F_{12}(\Xi_1 \times X_2)$ and $F_2(\Xi_2) = F_{12}(X_1 \times \Xi_2)$ $(\forall \Xi_1 \in \mathcal{F}_1, \Xi_2 \in \mathcal{F}_2)$. Let $\rho \in \mathfrak{S}^p(\mathcal{A}^*)$, $\Gamma_1 \in \mathcal{F}_1$, $\Gamma_2 \in \mathcal{F}_2$. Then, if it holds that

$$\rho(F_{12}(\Gamma_1 \times (X_2 \setminus \Gamma_2))) = 0$$

this is denoted by

$$[\mathsf{O}_1; \Gamma_1] \underset{\mathsf{M}_{\mathcal{N}}(\mathsf{O}_{12}, S_{[\rho]})}{\Longrightarrow} [\mathsf{O}_2; \Gamma_2] \quad \text{or, equivalently} \quad [\mathsf{O}_1; X_1 \setminus \Gamma_1] \underset{\mathsf{M}_{\mathcal{N}}(\mathsf{O}_{12}, S_{[\rho]})}{\Longleftarrow} [\mathsf{O}_2; X_2 \setminus \Gamma_2]$$

That is because the probability that a measured value $(x_1.x_2)$ obtained by the measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O}_{12} := (X_1 \times X_2, \mathcal{F}_1 \boxtimes \mathcal{F}_2, F_{12}), S_{[\rho]})$ belongs to $\Gamma_1 \times (X_2 \setminus \Gamma_2)$ is equal to 0.

Remark 12. [Syllogism] Using Definition 11, we showed that

- Syllogism always holds in classical systems (*cf.* ref. [6])

- Syllogism does not always hold in quantum systems (*cf.* ref. [11], § 8.7 in ref. [20])

1.6 Inference; Fisher's maximum likelihood method

We begin with the following notation:

Notation 13. $[\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}, S_{[*]})]$: Consider a measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, \mathcal{F}, F), S_{[\omega_0]})$ formulated in the basic structure $[C(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu))]$. Here, note that

(E_1) in most cases that the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, \mathcal{F}, F), S_{[\omega_0]})$ is taken, it is usual to think that the state $\omega_0 (\in \Omega)$ is unknown.

That is because

(E_2) the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}, S_{[\omega_0]})$ may be taken in order to know the state ω_0.

Therefore, when we want to stress that we do not know the state ω_0, the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, \mathcal{F}, F), S_{[\omega_0]})$ is often denoted by $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, \mathcal{F}, F), S_{[*]})$

Theorem 14. [Inference; Fisher's maximum likelihood method (*cf.* ref. [7] or §5.2 in ref. [20]] For simplicity, assume that X is finite set. Assume that the measured value $x (\in X)$ is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, 2^X, F), S_{[*]})$. Then, the unknown state $[*]$ can be inferred to be $\omega_0 (\in \Omega)$ such that

$$[F(\{x\})](\omega_0) = \max_{\omega \in \Omega}[F(\{x\})](\omega)$$

Proof. It is an easy consequence of Axiom 1 (*cf.* §5.2 in ref. [20]). $\qquad\square$

Remark 15. [Inference and Control *cf.* §5.2 in ref. [20]]   The inference problem is characterized as the reverse problem of measurements. That is, we consider that

(F$_1$) (state ω_0, observable O) $\xrightarrow[\text{measurement (Axiom 1)}]{\mathsf{M}_{L^\infty(\Omega,\nu)}\ (\mathsf{O}:=(X,2^X,F),\ S_{[\omega_0]})}$ measured value x_0

On the other hand

(F$_2$) (measured value x_0, observable O) $\xrightarrow[\text{inference (reverse Axiom 1)}]{\mathsf{M}_{L^\infty(\Omega,\nu)}\ (\mathsf{O}:=(X,2^X,F),\ S_{[*]})}$ state ω_0

Thus, (F$_1$) and (F$_2$) are in reverse problem.

Also, we note, from the mathematical point of view, that inference problem (F$_3$) and control problem (F$_4$) are essentially the same as follows.

(F$_3$) [Inference problem; statistics]: when measured value x_0 is obtained, infer the unknown state ω_0!

and

(F$_4$) [Control problem; dynamical system theory]: Settle the state ω_0 such that measured value x_0 will be obtained!

Thus, we think, from the theoretical point of view, that statistics and dynamical system theory are essentially the same. Thus, we consider that statistics (= dynamical system theory) is the mathematical representation of classical mechanical worldview. On the other hand, quantum language is regarded as the mathematical representation of quantum mechanical worldview.

2   Hempel's raven paradox in the linguistic quantum mechanical worldview

2.1   What is Hempel's raven paradox?

Although all results mentioned in this paper hold in both classical and quantum systems, we, for simplicity, devote ourselves to classical systems.

In this section we discuss Hempel's raven paradox (*cf.* ref. [3]) in the linguistic quantum mechanical worldview. There may no consensus among philosophers on the problem "What is Hempel's raven paradox?". Some people may even think there is no paradox in Hempel's raven problem. Thus, we mention our opinion about "What is Hempel's raven paradox?" in what follows. Let U be the unversal set of all birds. Let $B(\subseteq U)$ be a set of all black birds. Let $R(\subseteq U)$ be a set of all ravens. Thus, the statement: "any raven is black" is logically denoted by

(G$_1$) "Any raven is black" : $(\forall x\ )[x \in R \longrightarrow x \in B]$ *i.e.*, $R \subseteq B \subseteq U$,

Its contraposition is denoted by

(G$_2$) "Every non-black bird is a nonraven" : $(\forall x\ )[x \in U \setminus B \longrightarrow x \in U \setminus R]$ *i.e.*, $U \setminus B \subseteq U \setminus R$

Of course, the two (G$_1$) and (G$_2$) are equivalent. However, If (G$_1$) and (G$_2$) are equivalent, then we have the following questions (i.e., raven problem):

(G$_3$) Why is the actual verification of (G$_2$) much more difficult than the actual verification of (G$_1$)?

(G$_4$) Why can the truth of "(G$_1$): any raven is black" be known by (G$_2$), i.e., without seeing a raven also at once?

Some may think that these are nonsense questions. However, in this section, we clarify the true meaning of (G$_3$) and (G$_4$) in the linguistic quantum mechanical worldview. And further, we conclude that Hempel's raven paradox may suggest the importance of measurement in science, that is, the language of science is not logic but quantum language.

Remark 16. Throughout this section, we assume that U (the universal set of all birds) is finite, i.e., $\sharp[U] < \infty$ (where $\sharp[S]$ is denoted by the number of elements of a set S). Some may misunderstand that the questions (G$_3$) and (G$_4$) are due to the fact such that $\sharp[R] \ll \sharp[U \setminus B] \approx \infty$. This is wrong. That is, as shown in the next section, the questions arise even in the case that $\sharp[U \setminus B] \ll \sharp[R]$. In order to avoid misunderstanding, we assume that U is a finite set.

2.2 The measurement theoretical answer to the raven paradox

Although most results mentioned in Sections 2∼4 hold in both classical and quantum systems, we, for simplicity, devote ourselves to classical systems:
Let U be the universal set of all birds. Let $B(\subseteq U)$ be a set of all black birds. Let R be the set of all ravens. Assume that U is finite. Thus, *"Any raven is black"* is logically denoted by

(H$_1$) $R \subseteq B \subseteq U$,   i.e., $(\forall x\;)[x \in R \longrightarrow x \in B]$;   "any raven is black"

This is logically equivalent to the following (H$_2$) and (H$_3$): (i.e., (H$_1$) $\Leftrightarrow$ (H$_2$) $\Leftrightarrow$ (H$_3$))

(H$_2$) $[U\backslash B] \subseteq [U\backslash R] \subseteq U$,   i.e., $(\forall x\;)[x \in U \setminus B \longrightarrow x \in U \setminus R]$;   "every non-black bird is a nonraven"

(H$_3$) $R \bigcap [U \setminus B] = \emptyset$,   i.e., $(\neg \exists x\;)[[x \in R] \wedge [x \in U \setminus B]]$;   "a non-black raven does not exist"

In what follows we try to explain the measurement theoretical (i.e., dualistic) representations of the logical (or set theoretical) statements (H$_1$), (H$_2$) and (H$_3$):
Let Ω be the state space. Without loss of generality (and, for simplicity), the state space Ω is assumed to be a compact space. Let ν be a measure on Ω such that $\nu(\Omega) = 1$ and $\nu(V) > 0$ ($\forall$ open set $V(\subseteq \Omega)$). Thus we have the following classical basic structure:

$$[C(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu))]$$

Consider the state subspaces $\Omega_R(\subseteq \Omega)$ and $\Omega_B(\subseteq \Omega)$ such that $\Omega_R \subseteq \Omega_B$. And define the state map $\widetilde\omega : U \to \Omega$, that is, the state of a bird $t(\in U)$ is denoted by $\widetilde\omega(t)(\in \Omega)$. And further assume that

$$\widetilde\omega(R) \subseteq (\Omega_R)^\circ, \quad \widetilde\omega(B \setminus R) \subseteq (\Omega_B \setminus \Omega_R)^\circ, \quad \widetilde\omega(U \setminus B) \subseteq (\Omega \setminus \Omega_B)^\circ, \tag{3}$$

where D° is the interior of $D(\subseteq \Omega)$, i.e., $D^\circ \equiv \bigcup\{O \mid O \subseteq D(\subseteq \Omega), O \text{ is open}\}$. This condition is necessary for "essentially continuity" in Axiom 1 (also, see Remark 5).

Remark 17. Without loss of generality, we can assume that $\Omega = \{\omega_1, \omega_2, \omega_3\}$ (with discrete topology), $\Omega_R = \{\omega_1\}$, $\Omega_B = \{\omega_1, \omega_2\}$ and

$$\widetilde\omega(t) = \omega_1 \;(t \in R), \quad = \omega_2 \;(t \in B \setminus R), \quad = \omega_3 \;(t \in U \setminus B)$$

However, we devote ourselves to the above general situation.

[Step (I)] The measurement theoretical representation of (H$_1$); "Any raven is black"
Now let us study the measurement theoretical representation of (H$_1$): "Any raven is black". Define the observable $\mathsf{O}_B = (X, 2^X, F)$ in $L^\infty(\Omega, \nu)$ such that

$$X = \{b, \overline{b}\}, \quad [F(\{b\})](\omega) = 1 \;\;(\text{if } \omega \in \Omega_B), \qquad [F(\{b\})](\omega) = 0 \;\;(\text{if } \omega \in \Omega \setminus \Omega_B),$$
$$[F(\{\overline{b}\})](\omega) = 1 - [F(\{b\})](\omega) \;\;(\omega \in \Omega) \tag{4}$$

where "b" and "$\overline{b}$" means "black" and "non-black" respectively.

Now, for any $t \in R(\equiv \{r_1, r_2, ..., r_{\sharp[R]}\})$, we have the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_B := (X, 2^X, F), S_{[\widetilde\omega(t)]})$, i.e., the measurement of whether the raven $t(\in R)$ is black. Axiom 1 [measurement] says that, for any (= arbitrary) raven $t(\in R)$,

(I_1) the probability that a measured value obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_B := (X, 2^X, F), S_{[\widetilde{\omega}(t)]})$ is equal to $b(\in X)$ is given by 1 $(\equiv [F(\{b\})](\widetilde{\omega}(t)) = 1)$.

This is one of the measurement theoretical representations of the statement: *"Any raven is black"*. However, the above term "for any raven" is too logical and not concrete. Thus, we may, by using the parallel measurement (*cf.* (D_6) in Definition 9), rewrite the (I_1) to the following:

(I_2) the probability that a measured value $(x_t)_{t \in R} (\equiv (x_i)_{i=1}^{\sharp[R]} \in X^R)$ obtained by the parallel measurement $\bigotimes_{t \in R} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_B := (X, 2^X, F), S_{[\widetilde{\omega}(t)]})$ is equal to $(b)_{t \in R} (\equiv (b)_{i=1}^{\sharp[R]} \in X^R)$ is given by 1 $(\equiv \times_{t \in R}[F(\{b\})](\widetilde{\omega}(t)) = 1)$

which is a formal (i.e., measurement theoretical) expression of the (H_1). Also, this may mean *"All ravens are black"* rather than *"Any raven is black"*. For completeness, note that "any (=arbitrary)" and "all" are distinguished in quantum language.

[Step (II)]; The measurement theoretical representation of (H_2); "Every non-black bird is a nonraven"

Here let us study the measurement theoretical representation of (H_2); "Every non-black bird is a nonraven". Define the observable $\mathsf{O}_R = (Y, 2^Y, G)$ in $L^\infty(\Omega, \nu)$ such that

$$
\begin{aligned}
Y = \{r, \overline{r}\}, \quad [G(\{r\})](\omega) = 1 \quad (\text{if } \omega \in \Omega_R), \qquad [G(\{r\})](\omega) = 0 \quad (\text{if } \omega \in \Omega \setminus \Omega_R), \\
[G(\{\overline{r}\})](\omega) = 1 - [G(\{r\})](\omega) \quad (\forall \omega \in \Omega)
\end{aligned} \tag{5}
$$

where "r" and "$\overline{r}$" means "raven" and "nonraven" respectively. Now, for any $t \in U \setminus B$, we have the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_R := (Y, 2^Y, G), S_{[\widetilde{\omega}(t)]})$, i.e., the measurement of whether the non-black bird $t \in U \setminus B$ is a nonraven. Axiom 1 [measurement] says that, for any non-black bird $t \in U \setminus B$,

(J_1) the probability that a measured value obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_R := (Y, 2^Y, G), S_{[\widetilde{\omega}(t)]})$ is equal to $\overline{r}(\in Y)$ is given by 1 $(\equiv [G(\{\overline{r}\})](\widetilde{\omega}(t)) = 1)$.

This is one of the measurement theoretical representations of the statement: *"Every non-black bird is a nonraven"*. However, the above term "for any non-black bird" is too logical and not concrete. Thus, we may, by using the parallel measurement (*cf.* (D_6) in Definition 9), rewrite the (J_1) to the following:

(J_2) the probability that a measured value $(y_t)_{t \in U \setminus B} (\in Y^{U \setminus B})$ obtained by the parallel measurement $\bigotimes_{t \in U \setminus B} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_R := (Y, 2^Y, G), S_{[\widetilde{\omega}(t)]})$ is equal to $(\overline{r})_{t \in U \setminus B}$ is given by 1,

which is a formal (i.e., measurement theoretical) expression of the (H_2). Also, this may mean *"All the birds non-black are nonravens"* rather than *"Any non-black bird is a nonraven"*.

Note that the argument of [Step(I)] and that of [Step(II)] are essentially the same (i.e., the role of R and the role of $U \setminus B$ are symmetrical). However, the following [Step(III)] is different from them.

[Step (III)]; The measurement theoretical representation of (H_3); "A non-black raven does not exist"

Let $\mathsf{O}_B := (X, 2^X, F)$ and $\mathsf{O}_R := (Y, 2^Y, G)$ be as in [Step (I)] and [Step (II)] respectively. Define the observable $\mathsf{O}_{BR} := (X \times Y = \{(b, r), (b, \overline{r}), (\overline{b}, r), (\overline{b}, \overline{r})\}, 2^{X \times Y}, H)$ in $L^\infty(\Omega, \nu)$ such that, for any $\omega \in \Omega$,

$$
\begin{cases}
[H(\{(b, r)\})](\omega) = [F(\{b\})](\omega) \cdot [G(\{r\})](\omega) = [G(\{r\})](\omega) \\
[H(\{(b, \overline{r})\})](\omega) = [F(\{b\})](\omega) \cdot [G(\{\overline{r}\})](\omega) \\
[H(\{(\overline{b}, r)\})](\omega) = [F(\{\overline{b}\})](\omega) \cdot [G(\{r\})](\omega) = 0 \\
[H(\{(\overline{b}, \overline{r})\})](\omega) = [F(\{\overline{b}\})](\omega) \cdot [G(\{\overline{r}\})](\omega) = [F(\{\overline{b}\})](\omega)
\end{cases}
$$

Then we see:

(K_1) Let t be any bird in U. Assume that a measured value $(x, y) \in X \times Y$ is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_{BR} := (X \times Y, 2^{X \times Y}, H), S_{[\widetilde{\omega}(t)]})$. Axiom 1 [measurement] says that the probability that $(x, y) = (\overline{b}, r)$ is equal to 0 $(\equiv [H(\{(\overline{b}, r)\})](\widetilde{\omega}(t)) = 0)$. When we paraphrase, Axiom 1 [measurement] says that the probability that $(x, y) \in \{(b, r), (b, \overline{r}), (\overline{b}, \overline{r})\}$ is equal to 1.

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This, of course, means "*A non-black raven does not exist*". Or, using Definition 11 [implication], we can describe

$$[\mathsf{O}_B; \{\bar{b}\}] \underset{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_{BR}, S_{[\widetilde{\omega}(t)]})}{\Longrightarrow} [\mathsf{O}_R; \{\bar{r}\}] \text{ or, equivalently } [\mathsf{O}_B; \{b\}] \underset{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_{BR}, S_{[\widetilde{\omega}(t)]})}{\Longleftarrow} [\mathsf{O}_R; \{r\}] \quad (\forall t \in U)$$

However, the above term "Let t be any bird" is too logical and not concrete. Thus, we may, by using the parallel measurement (*cf.* (D_6) in Definition 9), rewrite the (K_1) to the following:

(K_2) Assume that a measured value $(x_t, y_t)_{t \in U} \in (X \times Y)^U$ is obtained by the parallel measurement $\bigotimes_{t \in U} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_{BR} := (X \times Y, 2^{X \times Y}, H), S_{[\widetilde{\omega}(t)]})$. Axiom 1 [measurement] says that the probability that $(x_t, y_t) \in \{(b, r), (b, \bar{r}), (\bar{b}, \bar{r})\}$ ($\forall t \in U$) is equal to 1.

which is a formal (i.e., measurement theoretical) expression of the (H_3). Also, as shown in the following [Step(IV)], this (K_2) is the best compared to (I_2) and (J_2).

[Step (IV)]; Answers to Hempel's problems (G_3) and (G_4)

Although (H_1), (H_2) and (H_3) are equivalent, the measurements (I_2), (J_2) and (K_2) are not equivalent. In (I_2) of [Step (I)], we consider the parallel measurement $\bigotimes_{t \in R} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_B := (X, 2^X, F), S_{[\widetilde{\omega}(t)]})$, which includes $\sharp[R]$ measurements. However, in (K_2) of [Step (III)], we consider the parallel measurement $\bigotimes_{t \in U} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_{BR} := (X \times Y, 2^{X \times Y}, H), S_{[\widetilde{\omega}(t)]})$, which includes more measurements than the parallel measurement $\bigotimes_{t \in R} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_B := (X, 2^X, F), S_{[\widetilde{\omega}(t)]})$. Thus, some people think that the actual verification of (H_1) (or, (H_2)) may be more easier than that of (H_3). However, this is not true. That is because if the (I_2) asserts "all ravens are black" ($R \subseteq B$), we have to prove that R is the set of all ravens. That is,

(L_1) Before the measurement (I_2), we have to prove that R is the set of all ravens, namely, we have to obtain the measured value $(y_t)_{t \in U}$ by the parallel measurement $\bigotimes_{t \in U} \mathsf{M}_{L^\infty(\Omega,\nu)} (\mathsf{O}_R := (Y, 2^Y, G), S_{[\widetilde{\omega}(t)]})$, and we define R (the set of all ravens) by $\{t \in U \mid y_t = r\}$. And further, we take the parallel measurement $\bigotimes_{t \in R} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_B := (X, 2^X, F), S_{[\widetilde{\omega}(t)]})$ in (J_2).

This procedure (L_1) is realized by the parallel measurement $\bigotimes_{t \in U} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_{BR} := (X \times Y, 2^{X \times Y}, H), S_{[\widetilde{\omega}(t)]})$ in(K_2) of [Step (III)] (if it is proved that U is the set of all birds). That is, in any of the three cases $((I_2)$, (J_2) and (K_2)), it is discussed under the premise that "U is the set of all ravens" is known. Summing up, we conclude that

(L_2) If "(G_1); $R \subseteq B(\subseteq U)$" means "All ravens are black", we have to prove "R is the set of all ravens", not just define it. To do so, we have to examine all the birds ($\forall t \in U$). That is, we have to prove that $U(\equiv \{b_1, b_2, ..., b_{\sharp[U]}\})$ is the set of all birds. However, it is usually difficult to prove that U is the set of all birds as there may be some birds in unexplored land. Therefore, in most case, it is impossible to be convinced of "All ravens are black". Similarly, to assert " every non-black bird is a nonraven" or "a non-black raven does not exist", we have to examine all the birds ($\forall t \in U$). This is imposible in most cases.

Therefore, we can completely understand the questions (G_3) and (G_4). As the answer to the (G_3) and (G_4), some may directly find the (L_2) without quantum language (and thus, without the arguments [Step(I)∼Step(IV)]). If so, they may be somewhat excellent. However, it is not worth so much. That is because we think that the reason why Hempel's problem is famous is that many researchers know the (L_2) (i.e., the difference between definition and proof) unconsciously. Thus, we think that to solve Hempel's raven problems (G_3) and (G_4) is to answer the following:

(L_3) Propose a worldview! And further derive the assertion (L_2) from the axioms of its worldview!

As shown above [Step(I)∼Step(IV)], we derived the (L_2) from Axiom 1 [measurement] in the linguistic quantum mechanical worldview.

Remark 18. (i): As seen in the above, the logical implication "$\longrightarrow$" has various interpretation in quantum language. In this paper we are not concerned with Axiom 2 [Causality], which is also related to "implication". That is because ["state at time t_1" $\xrightarrow[\text{Axiom 2 [causality]}]{t_1 < t_2}$ "state at time t_2"] can be regarded as a kind of implication. But it's a little unreasonable to regard causality as an implication. For example, consider the following famous puzzle:

(L$_4$) Describe the contraposition of "If he is not scolded, he does not study"!

This is not so difficult as puzzle. Also, some may associate temporal logic. But, we think, from the quantum linguistic point of view, that this puzzle is unnatural. That is because we consider that the language of science is not logic but quantum language.

(ii): For the sake of completeness, we sum up and add the following correspondence:

$$
\begin{array}{ccc}
\begin{array}{c}(\text{H}_1):\ R \subseteq B \subseteq U \\ \text{any raven is black}\end{array} & \xleftarrow{\ \text{simplified form}\ } & \begin{array}{c}(\text{I}_2):\bigotimes_{t \in R}\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_B := (X, 2^X, F), S_{[\widetilde{\omega}(t)]}) \\ \text{measured value: } (b)_{t\in R}\end{array} \\[2mm]
\parallel & & \\[2mm]
\begin{array}{c}(\text{H}_2):\ [U \setminus B] \subseteq [U \setminus R] \subseteq U \\ \text{every non-black bird is a nonraven}\end{array} & \xleftarrow{\ \text{simplified form}\ } & \begin{array}{c}(\text{J}_2):\bigotimes_{t \in U\setminus B}\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_R := (Y, 2^Y, G), S_{[\widetilde{\omega}(t)]}) \\ \text{measured value: } (\overline{r})_{t\in U\setminus B}\end{array} \\[2mm]
\parallel & & \\[2mm]
\begin{array}{c}(\text{H}_3):\ R \bigcap [U \setminus B] = \emptyset \\ \text{a non-black raven does not exis}\end{array} & \xleftarrow{\ \text{simplified form}\ } & \begin{array}{c}(\text{K}_2):\ \bigotimes_{t \in U}\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_{BR} := (X \times Y, 2^{X\times Y}, H), S_{[\widetilde{\omega}(t)]}) \\ \text{measured value: } (x_t, y_t) \in \{(b,r), (b,\overline{r}), (\overline{b},\overline{r})\}\ (\forall t \in U)\end{array}
\end{array}
$$

$$(6)$$

2.3 Falsification test

As seen in the above (L$_2$), in most case, it is impossible to be convinced of "All ravens are black". Thus, our next problem is to answer the problem "How do we believe it ?". For example, assume that

(M$_1$) It was found that one hundred ravens were black continuously.

What we can do is to reject the null hypothesis of the (M$_1$). For instance, it is usual to assume the following null hypothesis:

(M$_2$) Non-black ravens can be observed at 7 time of a rate to 100 times.

A simple calculation shows that this null hypothesis (M$_2$) is represented in quantum language as the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}'_B := (X, 2^X, F'), S_{[\widetilde{\omega}(r)]})$, (where $\widetilde{\omega}: U \to \Omega$ is the state map (*cf.* the formula (3)) such that $\widetilde{\omega}(r) \in \Omega^\circ_R\ (\forall r \in R)$) where

$$X = \{b, \overline{b}\}, \quad [F'(\{b\})](\omega) = \frac{93}{100} \quad (\text{if } \omega \in \Omega_R), \qquad 0 \leq [F'(\{b\})](\omega) \leq 1 \quad (\text{if } \omega \in \Omega \setminus \Omega_R),$$

$$[F'(\{\overline{b}\})](\omega) = 1 - [F'(\{b\})](\omega) \quad (\forall \omega \in \Omega)$$

which is a slight modification of the formula (4). And thus, under the null hypothesis (M$_2$), we calculate, by Axiom 1 [measurement], that

(M$_3$) the probability that a measured value $(x_i)_{i=1}^{100}(\in X^{100})$ obtained by the parallel measurement $\bigotimes_{r \in \{r_1, r_2, ..., r_{100}\} \subseteq R}\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}'_B := (X, 2^X, F'), S_{[\widetilde{\omega}(r)]})$ satisfies that $x_i = b\ (i = 1, 2, ..., 100)$ is given by $(93/100)^{100}(< 0.048)$. That is, the probability that (M$_1$) is realized (i.e., we meet one hundred black ravens continuously) is less than $0.048\ (> (93/100)^{100}))$.

Thus, we may reject the null hypothesis (M$_2$) since probability 0.048 is quite rare.

Remark 19. Note that the above argument is popular as statistical hypothesis testingin in statistics, though statistics does not have the concept of "measurement". In our worldview (i.e., linguistic quantum mechanical worldview), we consider that Popper's falsificationism (*cf,* ref. [24]) and statistical hypothesis testing are almost the same. However his theory was supported by philosophers rather than scientists since his proposal was not proposed under a certain scientific worldview.

3 Hume's problem of induction and falsification tests

3.1 Problem of Induction in the linguistic quantum mechanical world view

Although David Hume (1711-1776), British experimentalist, suspected the justification of induction, in this section we show that the justification is easily solved in our worldview. If we expect a scientific answer to Hume's problem, we must start with the scientific definition of "the uniformity principle of nature", i.e., the following Definition 20 [The uniformity principle of nature]. Some may feel that the uniformity principle of nature (i.e., the condition in Definition 20) is too strong. However, we think that it is impossible to propose the different quantitative definition of the uniformity principle of nature that leads to a result like Theorem 21 [Inductive reasoning] (i.e., *If similar measurements are performed, the similar measured values are obtained*).

Definition 20. [The uniformity principle of nature] Let $[C(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu))]$ be a classical basic structure such that Ω is compact and $\nu(\Omega) = 1$. A family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ is said to satisfy the *uniformity principle of nature* (*concerning* μ), if there exists a probability space $(X, \mathcal{F}, \mu)$ such that

$$[F_i(\Xi)](\omega_i) = \mu(\Xi) \qquad \forall \Xi \in \mathcal{F}, \forall i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$$

Under this definition, we assert the following theorem, which should be regarded as the fundamental theorem in philosophy of science.

Theorem 21. [Inductive reasoning, the quantum linguistic solution of Hume's problem of induction]. Let $[C(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu))]$ be a basic structure such that Ω is compact and $\nu(\Omega) = 1$. Assume that a family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ satisfies the uniformity principle of nature (concerning μ). Let $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., x_N) \in \times_{i=-n}^N X$ be a measured value by the parallel measurement $\bigotimes_{i=-n}^N \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F_i), S_{[\omega_i]})$. Then, we see that

$$\frac{\sharp\{k \mid x_k \in \Xi, k = -n, -n+1, ..., -1, 0\}}{n} \approx \mu(\Xi)(= [F_i(\Xi)](\omega_i)) \quad (\Xi \in \mathcal{F}, i = -n, -n+1, ..., -1, 0, 1, 2, ..., N) \tag{7}$$

where n is sufficiently large. Here $\sharp[\Theta]$ is the number of elements in a set Θ.

Proof. Let $\Xi_i \in \mathcal{F}$ $(i = -n, -n+1, ..., -1, 0, 1, ..., N)$. Axiom 1 [measurement] says that the probability that a measured value $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., x_N)$ obtained by the parallel measurement $\bigotimes_{i=-n}^N \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F_i), S_{[\omega_i]})$ belongs to $\times_{i=-n}^N \Xi_i$ is given by $\times_{i=-n}^N[F_i(\Xi_i)](\omega_i) = \times_{i=-n}^N \mu(\Xi_i)$. Thus, the sequence $\{x_i\}_{i=-n}^N$ can be regarded as independent random variables with the identical distribution μ. Hence, using the law of large numbers, we can immediately get the formula (7). Also, this theorem is a direct consequence of the law of large numbers for parallel measurements (*cf.* refs. [7], or § 4.2 in ref.[20]). □

Remark 22. (i): Recall that the law of large numbers (which is almost equivalent to Theorem 21) says that

$$\text{"frequency probability"} = \text{"the probability in Axiom 1"} \quad (cf. \text{ ref. [7]})$$

though the probability in Axiom 1 has the several aspects. Also, note that the law of large numbers in statistics (*cf.* ref. [21]) has already been accepted as the fundamental theorem in science. Therefore, even if Theorem 21 ([Inductive reasoning]+(7)) is called the fundamental theorem in philosophy of science, we don't think it's exaggerated. We believe that our proposal (i.e., Theorem 21) is completely true in our worldview. Thus, we think that the solution of Hume's problem of induction was practically already found as the law of large numbers. In the framework of our worldview, we are convinced that the above is the definitive solution to Hume's problem. However, there may be another idea if some start from another worldview. Hence, as described at the end of this paper, we hope that many philosophers propose various mathematical foundations of scientific philosophy, in which Hume's problem of induction are discussed from the various viewpoints.

(ii): In Definition 20 [The uniformity principle of nature] and Theorem 21 [Inductive reasoning], we consider the family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$. This may be too general. Usually, it suffices to consider that $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$, i.e., $F = F_i$ $(-n \le \forall i \le N)$.

(iii): It may be understandable to consider two measurements: $\bigotimes_{i=-n}^0 \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, \mathcal{F}, F_i), S_{[\omega_i]})$ and $\bigotimes_{i=1}^N \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, \mathcal{F}, F_i), S_{[\omega_i]})$. The reason that we do not consider two measurements is due to the linguistic Copenhagen interpretation (D_1), i.e., *only one measurement is permitted*.

Example 23. [Coin tossing]. Let us discuss the unfair coin tossing as the most understandable example of Theorem 21 [Inductive reasoning]. Consider a basic structure $[C(\Omega) \subseteq L^\infty(\Omega,\nu) \subseteq B(L^2(\Omega,\nu))]$. Let $\{\omega_i\}_{i=-n}^N$ be a sequence in Ω, where ω_i is the state of i-th coin tossing $(i = -n, -n+1, ..., 0, 1, 2, 3, ..., N)$. Let $\mathsf{O} = (X, 2^X, F)$ be an observable in $L^\infty(\Omega,\nu)$ such that

$$X = \{H, T\}, (\text{ where } H\text{: head, } U\text{: tail}) ,$$
$$[F(\{H\})](\omega_i) = \mu(\{H\}) = 2/3, \quad [F(\{T\})](\omega_i) = \mu(\{T\}) = 1/3 \quad (\forall i = -n, -n+1, ..., -1, 0, 1, 2, ..., N) \tag{8}$$

That is, a family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O} := (X, 2^X, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ satisfies the uniformity principle of nature (concerning μ). Let $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., N) \in \times_{i=-n}^N X$ be a measured value obtained by the parallel measurement $\bigotimes_{i=-n}^N \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, 2^X, F), S_{[\omega_i]})$, i.e., infinite coin throws. Here, Theorem 21 [Inductive reasoning] say that it is natural to assume that, for sufficiently large n,

$$(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0) = (\underbrace{\text{T H H T H H H T T ..... T H H}}_{n+1}) \tag{9}$$

(where the number of Hs $\approx 2n/3$, Ts $\approx n/3$)

Then we can believe that we see that $x_i = H$ with probability 2/3 [resp. $x_i = T$ with probability 1/3] for each $i = 1, 2, ..., N$. It should be noted that even without knowing (8), we can conclude that if we know (9).

Example 24. [Induction concerning raven problem]. Let $R \subseteq B \subseteq U$ be as in Section 2.2. Consider a basic structure $[C(\Omega) \subseteq L^\infty(\Omega,\nu) \subseteq B(L^2(\Omega,\nu))]$. Let $\mathsf{O}_B = (X, 2^X, F)$ be an observable in $L^\infty(\Omega,\nu)$ such as defined in the formula (4), that is,

$$X = \{b, \bar{b}\}, \quad [F(\{b\})](\omega) = 1 \quad (\text{if } \omega \in \Omega_B), \qquad [F(\{b\})](\omega) = 0 \quad (\text{if } \omega \in \Omega \setminus \Omega_B),$$
$$[F(\{\bar{b}\})](\omega) = 1 - [F(\{b\})](\omega) \quad (\forall \omega \in \Omega)$$

Let $\{\omega_i\}_{i=-n}^N$ be a sequence in $\Omega_R^\circ (\subseteq \Omega)$. Clearly, a family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_B := (X, 2^X, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ satisfies the uniformity principle of nature (concerning μ) such that

$$[F(\{b\})](\omega_i) = \mu(\{b\}) = 1, \quad [F(\{\bar{b}\})](\omega_i) = \mu(\{\bar{b}\}) = 0, \qquad (\; i = -n, -n+1, ..., 0, 1, 2, 3, ..., N)$$

Let $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., x_N) \in \times_{i=-n}^N X$ be a measured value obtained by the parallel measurement $\bigotimes_{i=-n}^N \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}_B := (X, 2^X, F), S_{[\omega_i]})$, We see, of course, that $x_i = b$ $(i = -n, -n+1, .., -1, 0)$. And thus, we can believe, by Theorem 21 [Inductive reasoning], that $x_1 = x_2 = ... = x_N = b$.

3.2 Grue paradox cannot be represented in quantum language

If our understanding of inductive reasoning (mentioned in the above) is true, we can solve the grue paradox (*cf.* ref. [2]). Let us mention it as follows.

Consider a basic structure $[C(\Omega) \subseteq L^\infty(\Omega,\nu) \subseteq B(L^2(\Omega,\nu))]$. Let $\Omega_g, \Omega_b, \Omega_o$ be the subsets of the state space Ω such that $\Omega_g \cap \Omega_b = \emptyset$ and $\Omega_o = \Omega \setminus (\Omega_g \cup \Omega_b)$. Let $\mathsf{O} = (X \equiv \{g, b, o\}, 2^X, F)$ be the observable in $L^\infty(\Omega,\nu)$ such that

$$[F(\{g\})](\omega) = 1 \; (\omega \in \Omega_g), \quad = 0 \; (\omega \in \Omega \setminus \Omega_g) \qquad [F(\{b\})](\omega) = 1 \; (\omega \in \Omega_b), \quad = 0 \; (\omega \in \Omega \setminus \Omega_b)$$
$$[F(\{o\})](\omega) = 1 - [F(\{g\})](\omega) - [F(\{b\})](\omega) \; (\omega \in \Omega) \tag{10}$$

where "g", "b", "o" respectively means "green", "blue", "others".

Let $\{e_{-n}, e_{-n+1}, ..., e_{-1}, e_0, e_1, e_2, ..., e_N\}$ be the set of (green) emeralds. And assume that $\omega_i (\in \Omega_g^\circ)$ is the state of emerald e_i ($i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$).

A family of measurements $\{\mathsf{M}_{L^\infty(\Omega, \nu)}(\mathsf{O}_i := (X, 2^X, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ clearly satisfies the uniformity principle of nature, that is, there exists an probability space $(X, 2^X, \mu)$ such that

$$[F(\Xi)](\omega_i) = \mu(\Xi) \quad \forall \Xi \in 2^X, \forall i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$$

where $\mu(\{g\}) = 1$, $\mu(\{b, o\}) = 0$.

Let $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., x_N) \in \times_{i=-n}^N X$ be a measured value obtained by the parallel measurement $\bigotimes_{i=-n}^N \mathsf{M}_{L^\infty(\Omega, \nu)} (\mathsf{O} := (X, 2^X, F), S_{[\omega_i]})$. We see, of course, that $x_i = g$ ($i = -n, -n+1, .., -1, 0$). And thus, we can believe, by Theorem 21 [Inductive reasoning], that $x_1 = x_2 = ... = x_N = g$. For the sake of completeness, note that we can predict $x_1 = x_2 = ... = x_N = g$ only by the data $x_{-n} = x_{-n+1} = ... = x_0 = g$. This is usual arguments concerning Theorem 21 [Inductive reasoning].

On the other hand, Goodman's grue paradox is as follows (*cf.* ref. [2]).

(N$_1$) Define that Y has a grue property iff Y is green at time i such that $i \leq 0$ and Y is blue at time i such that $0 < i$. Suppose that we have examined the emeralds at $-n, -n+1, ... -1, 0$, and found them to all be green (and hence also grue). Then, "so-called inductive reasoning" says that emeralds at $1, 2, ..., N$ have the grue property (and hence blue) as well as green. Thus, a contradiction is gotten.

However, we think that this (N$_1$) cannot be described in quantum language. If we try to describe the (N$_1$), we may consider as follows.

(N$_2$) Let $\{e_{-n}, e_{-n+1}, ..., e_{-1}, e_0, e_1, e_2, ..., e_N\}$ be the set of emeralds. Let $\omega_i (\in \Omega_g^\circ)$ be the state of emerald e_i ($i = -n, -n+1, ..., -1, 0$), and let $\omega_i (\in \Omega_b^\circ)$ be the state of emerald e_i ($i = 1, 2, ..., N$). However, it should be noted that a family of measurements $\{\mathsf{M}_{L^\infty(\Omega, \nu)}(\mathsf{O}_i := (X, 2^X, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ does not satisfy the uniformity principle of nature. That is because

$$[F(\{g\})](\omega_i) = 1 \ (i = -n, -n+1, ..., 0), \qquad [F(\{g\})](\omega_i) = 0 \ (i = 1, 2, ..., N)$$

Hence Theorem 21 [Inductive reasoning] cannot be applied.

Or,

(N$_3$) Let $\{e_{-n}, e_{-n+1}, ..., e_{-1}, e_0, e_1, e_2, ..., e_N\}$ be the set of emeralds. And let $\omega_i (\in \Omega_g^\circ)$ is the state of emerald e_i such that $\omega = \omega_i$ ($i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$). Let $\mathsf{O}_i = (X, 2^X, F_i)$ be the observable ($i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$) such that O_i is the same as $\mathsf{O}(= (X \equiv \{g, b, o\}, 2^X, F))$ in (10) (if $i = -n, -n+1, ..., -1, 0$), and $\mathsf{O}_i = (X, 2^X, F_i)$ (if $0, 1, 2, ..., N$) is defined by $F_i(\{g\}) = F(\{b\})$, $F_i(\{b\}) = F(\{g\})$, $F_i(\{o\}) = F(\{o\})$. However, in this case, it should be noted that a family of measurements $\{\mathsf{M}_{L^\infty(\Omega, \nu)}(\mathsf{O}_i := (X, 2^X, F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ does not satisfy the uniformity principle of nature. That is because

$$[F_i(\{g\})](\omega_i) = [F(\{g\})](\omega_i) = 1 \qquad (i = -n, -n+1, ..., 0),$$
$$[F_i(\{g\})](\omega_i) = [F(\{b\})](\omega_i) = 0 \qquad (i = 1, 2, ..., N)$$

Hence Theorem 21 [Inductive reasoning] cannot be applied.

Therefore Goodman's grue paradox (N$_1$) cannot be described in quantum language.

Remark 25. We believe that there is no scientific argument without scientific worldview. Thus, we can immediately conclude that Goodman's discussion (N$_1$) is doubtful since his argument is not based on any scientific worldview. In this sense, the above arguments (N$_2$) and (N$_3$) may not be needed. That is, the confusion of grue paradox is due to lack of the understanding of Hume's problem of induction in the linguistic quantum mechanical worldview, and not lack of the term "grue" is non-projectible (*cf.* ref. [2]). Thus, we think that to solve Goodman's grue paradox is to answer the following:

(N$_4$) Propose a worldview! And further formulate Hume's induction as the fundamental theorem in the worldview! In this formulation, confirm that Goodman's paradox is eliminated naturally.

What we did is this.

4 The measurement theoretical representation of abduction

4.1 Deduction and abduction in "logic"

A typical example of deduction is as follows: $\Big($ In the following, (O_1') and (P_1') are often omitted. $\Big)$

(O_1) All the beans in this bag B_1 are white: [bag $B_1 \longrightarrow$ "w" ($\approx$ white)]

(O_1') All the beans in that bag B_2 are white or black fifty-fifty (or generally, the ratio of white beans to black beans is $p/(1-p)$ where $0 < p < 1$): [bag $B_2 \longrightarrow$ "w" ($\approx$ white) or "b" ($\approx$ black)]

(O_2) This bean is from this bag B_1: [bag B_1]

(O_3) Therefore, this bean is white: ["w" ($\approx$ white)]

It is, of course, obvious and ordinary.
On the other hand, C.S, Peirce (*cf.* ref. [23]) proposed abduction. The example of abduction is as follows:

(P_1) All the beans in this bag B_1 are white: [bag $B_1 \longrightarrow$ "w" ($\approx$ white)]

(P_1') All the beans in that bag B_2 are white and black fifty-fifty (or generally, the ratio of white beans to black beans is $p/(1-p)$): [bag $B_2 \longrightarrow$ "w" ($\approx$ white) or "b" ($\approx$ black)]

(P_2) This bean (from B_1 or B_2) is white: ["w" ($\approx$ white)]

(P_3) Therefore, this bean is from this bag B_1 : [bag B_1]

This is wrong from the logical point of view. However, the abduction (abductive reasoning) is known as one of useful tools to find a best solution. Also, note that $[(O_2) \longrightarrow (O_3)]$ and $[(P_2) \longrightarrow (P_3)]$ are in reverse relation.

4.2 The measurement theoretical representation of deduction and abduction

In this section, we show that the abduction $[(P_1)\text{-}(P_3)]$ can be justified in quantum language. Consider the state space $\Theta = \{\theta_1, \theta_2\}$ with the discrete topology, and the classical basic structure $[C(\Theta) \subseteq L^\infty(\Theta, \nu) \subseteq B(L^2(\Theta, \nu))]$, where $\nu(\{\theta_1\}) = \nu(\{\theta_2\}) = 1/2$. Assume that

$$\theta_1 \approx \text{the state of the bag } B_1, \quad \theta_2 \approx \text{the state of the bag } B_2,$$

Assume that 1000 white beans belong to bag B_1, and further, 500 white beans and 500 black beans belong to the bag B_2. Thus we have the observable $\mathsf{O} = (\{w, b\}, 2^{\{w,b\}}, F)$ in $L^\infty(\Theta, \nu)$ such that

$$[F(\{w\})](\theta_1) = 1 \qquad [F(\{b\})](\theta_1) = 0$$
$$[F(\{w\})](\theta_2) = p \qquad [F(\{b\})](\theta_2) = 1 - p \qquad (0 < p < 1)$$

where "w" and "b" means "white" and "black" respectively.
Thus, we have the measurement $\mathsf{M}_{L^\infty(\Theta,\nu)}(\mathsf{O} := (\{w, b\}, 2^{\{w,b\}}, F), S_{[\theta_i]})$, $i = 1, 2$. For example, Axiom 1 [measurement] says that

(Q_1) [measurement]: The probability that the measured value w is obtained by $\mathsf{M}_{L^\infty(\Theta,\nu)}(\mathsf{O} := (\{w, b\}, 2^{\{w,b\}}, F), S_{[\theta_1]})$ is equal to 1

This is the same as the deduction (i.e., (O_1)–(O_3)).
Next, under the circumstance that bags B_1 and B_2 cannot be distinguished, we consider the following inference problem:

(Q_2) [inference problem]: When the measured value w is obtained by the measurement $\mathsf{M}_{L^\infty(\Theta,\nu)}(\mathsf{O} := (\{w, b\}, 2^{\{w,b\}}, F), S_{[*]})$, which do you infer, $[*] = \theta_1$ or $[*] = \theta_2$?
Theorem 14 [Fisher's maximum likelihood method] says that $[*] = \theta_1$, since

$$\max\{F(\{w\})](\theta_1), F(\{w\})](\theta_2)\} = \max\{1, p\} = 1 = [F(\{w\})](\theta_1)$$

This implies (P_3).

Therefore, the above (Q_2) is the measurement theoretical representation of abduction (i.e., (P_1)–(P_3)). For the sake of completeness, note that (Q_1) and (Q_2) are in reverse problem (*cf.* Remark 15). That is, we have the following correspondence:

$$
\begin{array}{ccc}
\begin{array}{c} [(O_1),(O_1'),(O_2) \longrightarrow (O_3)] \\ \text{deduction} \end{array} & \overline{\quad\text{simplified form}\quad} & \begin{array}{c} (Q_1)\text{: measurement} \\ (\text{Axiom 1}) \end{array} \\[2mm]
\Big\downarrow \text{reverse} & & \Big\downarrow \text{reverse} \\[2mm]
\begin{array}{c} [(P_1),(P_1'),(P_2) \longrightarrow (P_3)] \\ \text{abduction} \end{array} & \overline{\quad\text{simplified form}\quad} & \begin{array}{c} (Q_2)\text{: inference} \\ (\text{Fisher's maximum likelihood method}) \end{array}
\end{array}
\tag{11}
$$

Thus, the scientific meaning of abduction can be completely clarified in the translation from logic to quantum language.

5 Flagpole problem

Let us explain the flagpole problem as follows. Suppose that the sun is at an elevation angle α° in the sky. Assume that $\tan \alpha^\circ = 1/2$. There is a flagpole which is ω_0^0 meters tall. The flagpole casts a shadow ω_1^0 meters long. Suppose that we want to explain the length of the flagpole's shadow. On Hempel's model, the following explanation is sufficient.

(R_1) 1. The sun is at an elevation angle α° in the sky.
     2. Light propagates linearly.
     3. The flagpole is ω_0^0 meters high.
     Then,
     4. The length of the shadow is $\omega_1^0 = \omega_0^0 / \tan \alpha^\circ = 2\omega_0^0$

This is a good explanation of "Why is that shadow $2\omega_0^0$ meters long?"
Similarly, we may consider as follows.

(R_2) 1. The sun is at an elevation angle α° in the sky.
     2. Light propagates linearly.
     3. The length of the shadow is ω_1^0
     Then,
     4. The flagpole is $\omega_0^0 (= (\tan \alpha^\circ)\omega_1^0 = \omega_1^0/2)$ meters tall.

However, the above is not sufficient as the explanation of "Why is the flagpole $\omega_0^0 (= \omega_1^0/2)$ meters tall?"

The confusion between (R_1) and (R_2) is due to the lack of measurement. In what follows, we discuss it. For each time $t = 0, 1$, consider a basic structure $[C(\Omega_t) \subseteq L^\infty(\Omega_t, \nu_t) \subseteq B(L^2(\Omega_t, \nu_t))]$, where $\Omega_0 = [0, 1]$ is the state space at time 0 (where the closed interval in the real line $\mathbb{R}$), $\Omega_1 = [0, 2]$ is the state space at time 1 and the ν_t is the Lebesgue measure. Since the sun is at an elevation angle α° in the sky, it suffices to consider to the map $\phi_{0,1} : \Omega_0 \to \Omega_1$ such that $\phi_{0,1}(\omega_0) = 2\omega_0$ ($\forall \omega_0 \in \Omega_0$). Thus, we can define the causal operator $\Phi_{0,1} : L^\infty(\Omega_1) \to L^\infty(\Omega_0)$ such that $(\Phi_{0,1}f_1)(\omega_0) = f_1(\phi(\omega_0))$ ($\forall f_1 \in L^\infty(\Omega_1), \omega_0 \in \Omega_0$).

Let $\mathsf{O}_e = (X, \mathcal{F}, F_e)$ be the exact observable in $L^\infty(\Omega_1, \nu_1)$ (*cf.* Example 6). That is, it satisfies that $X = \Omega_1, \mathcal{F} = \mathcal{B}_{\Omega_1}$ (i.e., the Borel field in Ω), $[F_e(\Xi)](\omega_1) = 1$ (if $\omega_1 \in \Xi$), $= 0$ (otherwise).

Thus, we have the measurement $\mathsf{M}_{L^\infty(\Omega_0, \nu_0)}(\Phi_{0,1}\mathsf{O}_e = (X, \mathcal{F}, \Phi_{0,1}F_e), S_{[\omega_0^0]})$. Then we have the following statement

(S_1) [Measurement]; the probability that the measured value $x (\in X)$ obtained by the measurement $\mathsf{M}_{L^\infty(\Omega_0, \nu_0)}$ $(\Phi_{0,1}\mathsf{O}_e = (X, \mathcal{F}, \Phi_{0,1}F_e), S_{[\omega_0^0]})$ is equal to $2\omega_0^0$ is given by 1.

which is the measurement theoretical representation of (R_1). That is, we consider that the (R_1) is the simplified form (or, the rough representation) of (S_1). Also,

(S_2) [Inference]; Assume that the measured value $\omega_1^0 (\in X)$ is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega_0, \nu_0)}(\Phi_{0,1}\mathsf{O}_e = (X, \mathcal{F}, \Phi_{0,1}F_e), S_{[*]})$. Then, we can infer that $[*] = \omega_1^0/2$

which is the measurement theoretical representation of (R_2). That is, we consider that the (R_2) is the simplified form (or, the rough representation) of (S_2).

Thus, we conclude that "scientific explanation" is to describe by quantum language.

6 Conclusion; To do science is to describe phenomena by quantum language

6.1 Logic (in ordinary language), statistics, quantum language

"What is science ?"is the main question of philosophy of science. For example, we have the following answers:

($\sharp_1$) Science is to describe phenomena by logic

($\sharp_2$) Science is to describe phenomena by statistics

($\sharp_3$) Science is to describe phenomena by quantum language

In what follow, let us examine the above three:

[($\sharp_1$):**Logic**]: Some may say "Science is to describe phenomena by logic", which may be due to the logical positivism (or, the tradition of Aristotle's syllogism). However, as seen in Sections 2∼4, Hempel's raven paradox, Hume's problem of induction, Goodman's grue paradox , Peirce's abduction and flagpole problem are related to the concept of measurement (= inference), and thus, these problems cannot be adequately handled by logic alone. Thus, we think that logic is the language of mathematics, and not the language of science. As seen throughout this paper, we believe that the representation using "logic" is rough in most cases.

[($\sharp_2$): **Statistics; the classical mechanical world view**]: Statistics are used everywhere in science, and thus, statistics may be the principle of science. Therefore some may say "Science is to describe phenomena in the classical mechanical worldview ($\approx$ statistics $\approx$ dynamical system theory)". This answer may be somewhat better as follows.

(T_1) economics is to describe economical phenomena by statistics (it is usual to regard economics as the application of dynamical system theory ($\approx$ statistics))

(T_2) psychology is to describe psychological phenomena by statistics

(T_3) biology is to describe biological phenomena by statistics

(T_4) medicine is to describe medical phenomena by statistics (i.e., medical statistics)

Also, since dynamical system theory is considered as a kind of mathematical generalization of Newtonian mechanics, we may be allowed to say:

(T_5) Newtonian mechanics is to describe classical mechanical phenomena by statistics (= dynamical system theory). Also, it is clear that dynamical system theory plays a central role in engineering.

though Newtonian mechanics is physics, and thus, it belongs to the realistic worldview in Fig. 1.
However, statistics ($\approx$ dynamical system theory (*cf.* Remark 15)) is too mathematical. Hence, "Science is to describe phenomena in the classical mechanical worldview ($\approx$ statistics $\approx$ dynamical system theory)" is almost the same as "Science is to describe phenomena using the mathematical theories of probability and differential equation". And thus, the framework of the classical mechanical worldview is ambiguous. Since statistics ($\approx$ dynamical system theory) does not have clear axioms, we think that it is a little unreasonable to say that statistics is the language of science.

For example, we don't know how to attack Hempel's raven problem (G_3) and (G_4) from the statistical point of view, since statistics does not have the concept of measurement. As seen below, the relationship between science and statistics is revealed by quantum language (*cf.* ⑨ in Fig. 1).

[($\sharp_3$): **Quantum language; the linguistic quantum mechanical worldview**]: We choose quantum language (i.e., the linguistic quantum mechanical worldview, or the probabilistic interpretation of science), and we assert "Science is to describe phenomena by quantum language". That is, in a similar sense of (T_1)-(T_5), we say that

(U_1) economics is to describe economical phenomena by quantum language

(U$_2$) psychology is to describe psychological phenomena by quantum language (*cf.* Chapter 18 in ref [20])

(U$_3$) biology is to describe biological phenomena by quantum language

(U$_4$) medicine is to describe medical phenomena by quantum language

(U$_5$) Newtonian mechanics is to describe classical mechanical phenomena by quantum language (in the same meaning as the (T$_5$), also recall the history: ② ⟶ ⑦ ⟶ ⑩ in Fig. 1). Also, it is clear that classical system theory (= dynamical system theory, *cf.* (B$'_2$)) plays a central role in engineering.

(U$_6$) statistical mechanics is to describe statistical mechanical phenomena by quantum language (*cf.* ref. [14])

(U$_7$) quantum mechanics (i.e., quantum information theory) is to describe quantum mechanical phenomena by quantum language (*cf.* (A$_1$)).

(U$_8$) As mentioned in the (iv) and (v) of Remark 8, a lot of paradoxes (e.g., Bertrand's paradox, McTaggart's paradox, Zeno's paradox, Monty Hall problem, etc. (*cf.* refs. [20, 13, 19])) are clarified in quantum language.

(U$_9$) As shown in Sections 2∼4, fundamental problems in philosophy of science (i.e., Hempel's raven paradox, Hume's problem of induction, Peirce's abduction, the flagpole problem, etc.) are easily solved in the linguistic quantum mechanical worldview. Therefore, quantum language guarantees that these problems are scientific. On the other hand, Goodman's grue paradox (N$_1$) cannot be described by quantum language. Thus, it is not scientific. Also, it should be noted that these results are consequences of Axiom 1 [measurement].

etc. Quantum language has the advantage of having the concept of "measurement". And thus, as seen in this paper, "logic" can be paraphrased in detail in terms of measurement, and thus, precise expression is obtained.

Remark 26. [Can logic and statistics be regarded as kinds of worldviews?] Logic and statistics has respectively various aspects. However, when we say roughly, logic is the language of mathematics, and statistics is a quite useful mathematical theory. If so, how can we regard logic and statistics as kinds of worldviews? In this paper, we see that logic and statistics respectively has aspects such as simplified forms of quantum language. That is,

$$
\left.\begin{array}{l} \text{logic} \\[2mm] \text{statistics} \end{array}\right\} \quad \frac{\text{simplified form}}{\text{elimination of dualism} (\approx \text{measurement})} \text{ quantum language}
$$

(*e.g.* see (A$_3$) in Fig. 1 and the formulas (6), (11)). If so, logic and statistics are scientific as simplified forms of quantum language.

6.2 Summing up; Quantum language is the language of science

In this paper, we clarify the following unsolved problems in the linguistic quantum mechanical worldview:

- Hempel's raven paradox in Section 2, Hume's problem of induction in Section 3.1, Goodman's grue paradox in Section 3.2, Peirce's abduction in Section 4, the flagpole problem in Section 5

That is, we sum up as follows:

(V$_1$) [Hempel's raven paradox in Section 2]: Hempel's raven problem (i.e., (G$_3$) and (G$_4$)) is related to measurement and not logic. Thus it is not solved if the concept of measurement is not clarified. Therefore, it is easily solved in the linguistic quantum mechanical worldview since it includes Axiom 1 [measurement].

(V$_2$) [Hume's problem of induction in Section 3.1]: In the linguistic quantum mechanical worldview, Hume's problem of induction (= Theorem 21 [Inductive reasoning]) is essentially the same as the law of large numbers (which is the most basic theorem in science). It may be reasonable. That is because Hume's problem of induction should be the most basic claim in science if there is the answer to this problem.

(V_3) [Goodman's grue paradox in Section 3.2]: We believe that if Hume's problem of induction is solved, Goodman's grue paradox is immediately solved as its corollary. In fact, Goodman's grue paradox can be easily solved as the corollary of Theorem 21 [Inductive reasoning] in the linguistic quantum mechanical worldview. Thus, Goodman's grue paradox is due to the lack of the understanding of Hume's problemof induction.

(V_4) [Peirce's abduction in Section 4]: As shown in the formula (11), Peirce's abduction is characterized as the simplified form of Fisher's maximum likelihood method in quantum language.

(V_5) [The flagpole problem in Section 5]: The confusion concerning the flagpole problem is due to relying only on "logic", and not using quantum language.

Hence we conclude that the reason that these problems are not yet clarified depends on lack of the worldview with the concept of measurement in philosophy of science. Again we emphasize the importance of worldview in science. That is because, if we do not have the worldview, we do not know what to rely on to proceed with the discussion. Therefore, it is no exaggeration to say that there is no science without a scientific worldview.

As mentioned in Remark 1, quantum language does not cover all sciences. However, we consider that familiar sciences are described by quantum language. And further, we believe that quantum language (i.e., the probabilistic interpretation of science) plays a central role in almost familiar sciences. That is, we believe that quantum language gives the mathematical foundations to science.

However, the answer to "What is science?" may not necessarily be determined uniquely. If so, we feel like knowing other interpretations (i.e., other scientific worldview) besides ours (i.e., the probabilistic interpretation of science). Thus, we hope that various mathematical foundations (e.g., category theoretical approach, modal logic approach, etc.) of scientific philosophy will be proposed. And we expect that fundamental problems such as raven problem, problem of induction, grue paradox, etc. will be investigated in these interpretations. And we hope that philosophy of science will progress with such competition.

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