A SUPPLEMENT TO THE THEOREM B

We shall intend to give a more elegant proof as for last half part of Theorem B. That is as follows.

(ii) In the case a point $0 \in E$.

We have defined $\sigma(0)$ as follows

$$\sigma(0) = \frac{\overline{\sigma}(u) + \underline{\sigma}(u)}{2}.$$

But $\overline{\sigma}(u) = \sigma(0+)$ and $\underline{\sigma}(u) = \sigma(0-)$ by Lemma B_1, we have also as follows

$$\sigma(0) = \frac{\sigma(0+) + \sigma(0-)}{2}.$$

Let us put $\sigma(0+) - \sigma(0-) = d > 0$ and define

$$\sigma^*(u) = \sigma(u) - dh(u)$$

where $h(u)$ is the Heaviside operator, that is as follows

$$h(u) = \begin{cases} 1 & (u > 0) \\ \frac{1}{2} & (u = 0) \\ 0 & (u < 0). \end{cases}$$

Let us define also

$$D^* = D \cup \{0\}.$$

We shall intend to treat the case $\sigma^*(u)$ on the set D^* and we shall prove that it satisfies the same properties as the case $\sigma(u)$ on the set D. This enable us to reduce the case (ii) of point $0 \in E$ to the case (i) of point $0 \in D$.

In the first, let us remark that $\sigma^*(u)$ is a bounded and monotone increasing function. Let us define

$$\sigma^*_\varepsilon(u) = \sigma_\varepsilon(u) - dh(u).$$

Then at any point $u \in D$ the limit

$$\lim_{\varepsilon \to 0} \sigma^*_\varepsilon(u) = \sigma^*(u)$$

exists and equals to

$$\sigma^*(u) = \sigma(u) - dh(u).$$

Since we have supposed to $D = D_0$, $\sigma^*(u)$ is continuous at any point $u \in D$.

Next let us remark $\sigma^*(u)$ at the point 0, then we have

$$\sigma^*(0+) = \sigma(0+) - dh(0+) = \sigma(0-)$$
$$\sigma^*(0-) = \sigma(0-) - dh(0-) = \sigma(0-)$$
$$\sigma^*(0) = \sigma(0) - dh(0) = \sigma(0-)$$

respectively. Thus $\sigma^*(u)$ is continuous at the point $u = 0$.

Now we have for any pair (u', u'') such as $u' < 0 < \varepsilon < u''$ and $u', u'' \in D$

$$\sigma_\varepsilon^*(u') \le \sigma_\varepsilon^*(0) \le \sigma_\varepsilon^*(u'').$$

We shall take it as $\{\varepsilon\} \downarrow 0$, then we have

$$\sigma^*(u') \le \underline{\lim}_{\varepsilon \to 0} \sigma_\varepsilon^*(0) \le \overline{\lim}_{\varepsilon \to 0} \sigma_\varepsilon^*(0) \le \sigma^*(u'').$$

Let us define as follows

$$\sup_{\substack{u'<o \\ u'\in D}} \sigma^*(u') = \underline{\sigma}^*(0) \quad \text{and} \quad \inf_{\substack{u''>0 \\ u''\in D}} \sigma^*(u'') = \overline{\sigma}^*(0)$$

then we have

$$.\ \underline{\sigma}^*(0) \le \underline{\lim}_{\varepsilon \to 0} \sigma_\varepsilon^*(0) \le \overline{\lim}_{\varepsilon \to 0} \sigma_\varepsilon^*(0) \le \overline{\sigma}^*(0).$$

Here we have $\underline{\sigma}^*(0) = \sigma^*(0-)$ and $\overline{\sigma}^*(0) = \sigma^*(0+)$ by Lemma B_1 and $\sigma^*(0-) =$ $\sigma^*(0) = \sigma^*(0+)$ by the continuity of $\sigma^*(u)$ at the point $u = 0$, therefore we have

$$\lim_{\varepsilon \to 0} \sigma_\varepsilon^*(0) = \sigma^*(0).$$

Now we could reduce the case (ii) $\sigma^*(u)$ on $D^* = D \cup \{0\}$ at the point 0 to the case (i) $\sigma(u)$ on D at the point 0.

Thus we have proved

$$\lim_{\varepsilon \to 0} \sigma_\varepsilon^*(+\varepsilon) = \sigma^*(0+)$$

where $\sigma_\varepsilon^*(\varepsilon) = \sigma_\varepsilon(\varepsilon) - dh(\varepsilon) = \sigma_\varepsilon(\varepsilon) - d$ and $\sigma^*(0+) = \sigma(0-)$.

Therefor we have

$$\lim_{\varepsilon \to 0} \sigma_\varepsilon(+\varepsilon) = \sigma(0+).$$

Similarly we have

$$\lim_{\varepsilon \to 0} \sigma_\varepsilon^*(-\varepsilon) = \sigma^*(0-)$$

where $\sigma_\varepsilon^*(-\varepsilon) = \sigma_\varepsilon(-\varepsilon) - dh(-\varepsilon) = \sigma_\varepsilon(-\varepsilon)$ and $\sigma^*(0-) = \sigma(0-)$.

Therefore we have

$$\lim_{\varepsilon \to 0} \sigma_\varepsilon(-\varepsilon) = \sigma(0-).$$

Thus we shall prove by the same arguments as above

$$\lim_{\varepsilon \to 0} \sigma_\varepsilon(u \pm \varepsilon) = \sigma(u \pm 0)$$

at any point u respectively.