慶應義塾大学 21世紀COEプログラム
「統合数理科学：現象解明を通した数学の発展」

# Pathways Lecture Series in Mathematics, KEIO

**Speaker : Prof. Maciej Zworski (UC, Berkeley )**

**Date : March 4-6, 2008 Time : 16:30 - 18:00**

**Place : Room 12-209, 2nd Floor, Bldg.12**
**Faculty of Science and Technology**
**Yagami Campus, KEIO University**

## Soliton Dynamics in External Fields

**Abstract**: A bright soliton in dimension one is obtained by solving

$$-\frac{1}{2}\eta'' - \eta^3 + \frac{\mu^2}{2}\eta = 0\,,$$

which gives $\eta(x) = \mu \operatorname{sech}(\mu x)$. Consequently, $u(x,t) = e^{i\gamma(t)}e^{iv(x-a(t))}\eta(x-a(t))$, with $a(t) = a_0 + vt$ and a suitably chosen $\gamma(t)$, solves the nonlinear Schrödinger equation,

$$iu_t = -\frac{1}{2}u_{xx} - |u|^2 u\,,$$

providing a perfect profile moving with the velocity $v$ and exhibiting a lot of stability.

I will describe recent theoretical and numerical results on the motion of solitons when an external potential is added:

$$iu_t = -\frac{1}{2}u_{xx} - |u|^2 u + V(x)u\,, \qquad (1)$$

for a variety of different potentials $V(x)$. The motivation comes, for instance, from the study of Bose-Einstein condensates. The results on which the talks are based were obtained in collaboration with Justin Holmer, with additional contributions by Kiril Datchev and Jeremy Marzuola.

**Lecture 1.** General properties of nonlinear Schrödinger equations: Hamiltonian structure, Lagrangian dynamics, stability of solitons, complete integrability of cubic NLS on the line.

**Lecture 2.** Fast soliton scattering by the delta function potentials. In that case the soliton splits into two waves which decompose into solitons and radiation. The scattering matrix of the delta potential determines the precise amplitudes and phases of the scattered solitons. This is the "quantum case" compared to the case presented in the next lecture.

**Lecture 3.** Semiclassical interaction of solitons with external potentials of two kinds: small irregular potentials (such as $V(x) = h^2\delta_0(x)$) or slowly varying potentials ($V(x) = W(hx)$). In that case the solution to (1) with initial condition given by a soliton stays within $\mathcal{O}(h^2)$ to a soliton evolving according to natural classical equations in the parameters $\gamma, a, v$, and $\mu$ (a four dimensional symplectic manifold). What is particularly appealing is the striking numerical improvement corresponding to the theoretical improvement in semiclassical errors, $h \to h^2$, compared to earlier works in the subject.

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