慶應義塾大学 21世紀COEプログラム
「統合数理科学：現象解明を通した数学の発展」

Integrative Mathematical Sciences 統合数理科学

Keio University

# Pathways Lecture Series in Mathematics, KEIO

## Speaker : Prof. Raphael Douady

(Research Director of Riskdata,
Organiser of the Courant Institute Mathematical Finance Seminar)

## Place : Room 14-203, 2nd Floor, Bldg.14
Faculty of Science and Technology
KEIO University

## Mathematical Challenges in Finance

**Lecture 1** 14:45-16:15 Tueday, June 12, 2007
Market dynamics, pseudo-differential operators, "topology" of financial assets

**Lecture 2** 16:30-18:00 Wednesday, June 13, 2007
Control problems, derivative products, parabolic PDE's and numerical techniques

**Lecture 3** 16:30-18:00 Thursday, June 14, 2007
Optimal investment problems, risk budgeting approach, cross-asset class and alternative investments

Unlike economy, which is a recognised science, finance denotes a set of techniques to implement and optimise business activity. If economy is the most quantitative of social sciences, "doing finance" requires mastering wide portions of the mathematical and statistical fields. In the past 20 years, in parallel with the tremendous growth of computation capacity, both mathematical finance and statistics have deeply evolved. The number one tool of risk mitigation is securitization and the creation of derivative instruments, leading to two "dual" problems: their pricing and their dynamic management. Since seminal works of Black, Merton and Scholes in 1973, the base paradigm of option pricing is "arbitrage theory, leading to solving a parabolic PDE of Kolmogorov type, with boundary conditions given by constraints or asymptotic properties derived from contractual clauses, including possible free boundaries.
A range questions rises with interest rate models in a possibly infinite dimensional phase space. New numerical techniques, using Monte-Carlo simulations and a Galerkin representation of the phase space have been specifically developed to address free boundary problems without dimension limitations. For certain derivative securities, volatility itself – or equivalently the "trading time" – must be made stochastic. It is also a well established statistical fact (Mandelbrot, Pareto) that market randomness has "fat tails" and cannot be faithfully represented by a diffusion process. Both departures from "vanilla" models require appropriate modelling, statistics and numerical treatment.
In this series of 3 lectures, we shall present some important mathematical challenges in finance from the angle of risk: risk measurement, risk management, risk/return optimisation. In the first lecture, we shall explore the statistics of financial market: the implied topological structure of financial markets and, when applicable, their differential structure, as well as Mandelbrot works on extreme (not so) rare events. Then we shall introduce the most commonly used risk measures. The second lecture will be devoted to derivative securities pricing and hedging, related control problems and some innovative numerical techniques.
The third lecture will address the question of optimal investment: the simple Markowitz framework and the "risk budgeting" practice, the problem of fat tails and nonlinearities and, finally, questions related to investment across several asset classes, including credit-sensitive assets and so-called "alternative investments" such as hedge funds.

問合わせ先: 慶應義塾大学21世紀COEプログラム 統合数理科学 渉外担当
横浜市港北区日吉3-14-1 Tel：045-566-1442 Fax：045-566-1768
e-mail：igarashi@math.keio.ac.jp URL：http://coe.math.keio.ac.jp

21 世紀 COEプログラム