Cherry Bud Workshop 2008 *Discovery through Data Science*
Keio University, Yokohama, JAPAN

Discovery of a structural model for neuronal activation

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Introduction

Joint work with neuroscientists in Keio University.

To construct suitable data driven models of neuronal activation which

- incorporate knowledge in neuroscience;
- develop the joint research further.

Outline

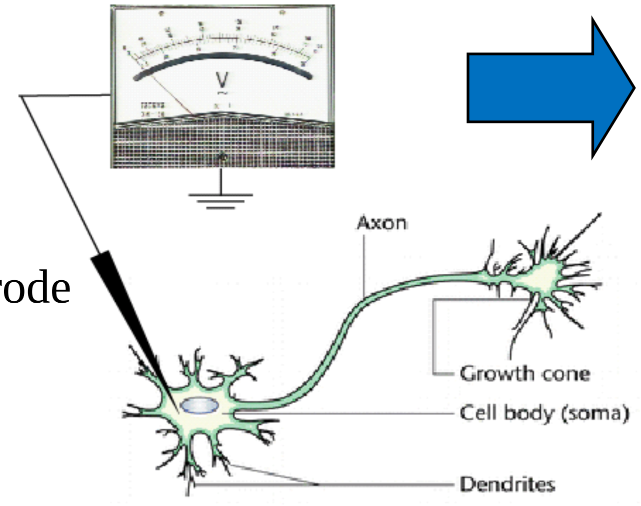
- Introduction
- Data
- Biological backgrounds
- Model
- Results
- Conclusion

Data

Earthworm’s membrane potential is measured

- by intra-cellar recording;
- for 40 sec with 0.05 msec time resolution (879000 observations).

Electrode



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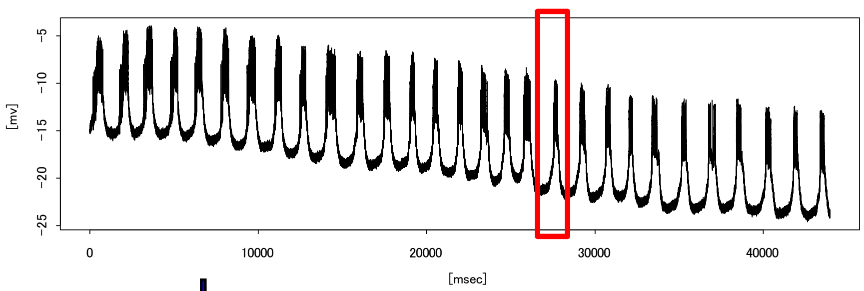
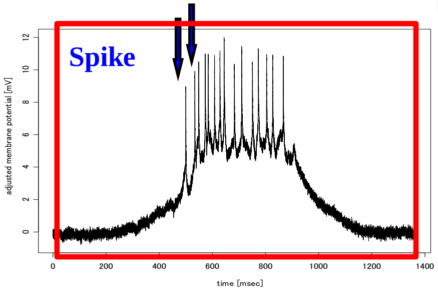
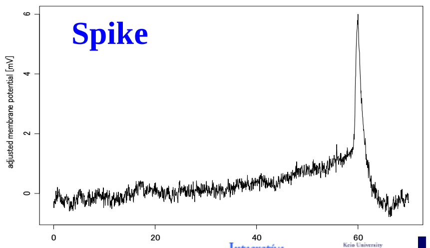
27 clusters

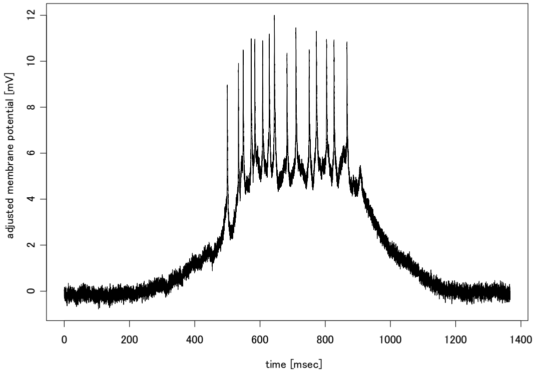




Cluster



Build a model for a cluster.



Cluster defined:

- from 500 msec ahead the first spike;
- to 500 msec behind the last spike.

Potential adjusted as both ends take zero.

Comments:

Modelling challenges include:

- the sudden stop of spikes;

There have been many attempts that model:

Membrane potential

- Conductance based models:

  Hodgkin & Huxley (1952), Rose & Hindmarsh (1989), Wilson (1999) etc.
- Integrated fire models:

  Izhikevich (2003, 2004) etc.

Spike occurrence time

- Point process models:

  Cox & Isham (1980), Kass & Ventura (2001), Ventura et al. (2002), Kass et al. (2005) etc.

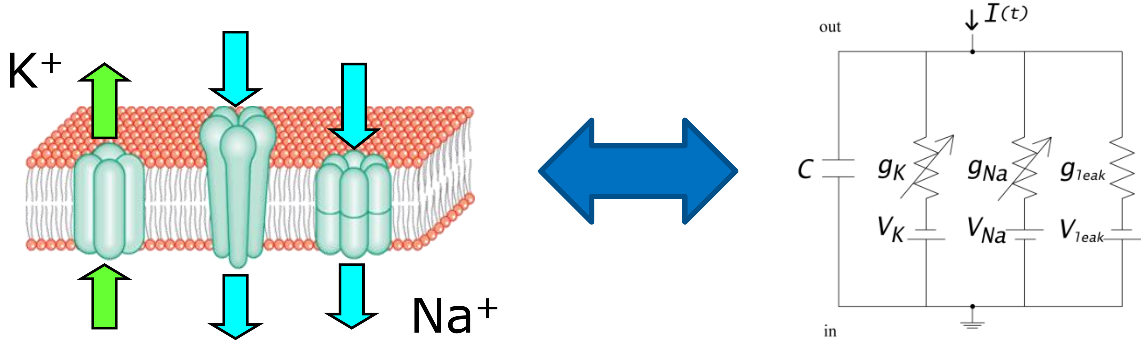
Comments:

No bridges between these two approaches.

Biological backgrounds (1)

Mechanisms:

Ion exchange



Hodgkin-Huxley model (Hodgkin & Huxley, 1952) assumes that

- the membrane behaves like electric circuits;
- channels switch depending on membrane potential.

$$C \frac{dV(t)}{dt} = I(t) - \sum_i g_i(t, V(t))(V(t) - V_i)$$

$V(t)$: Membrane potential [V];

$I(t)$: Synaptic current [A];

C : Capacitance [F];

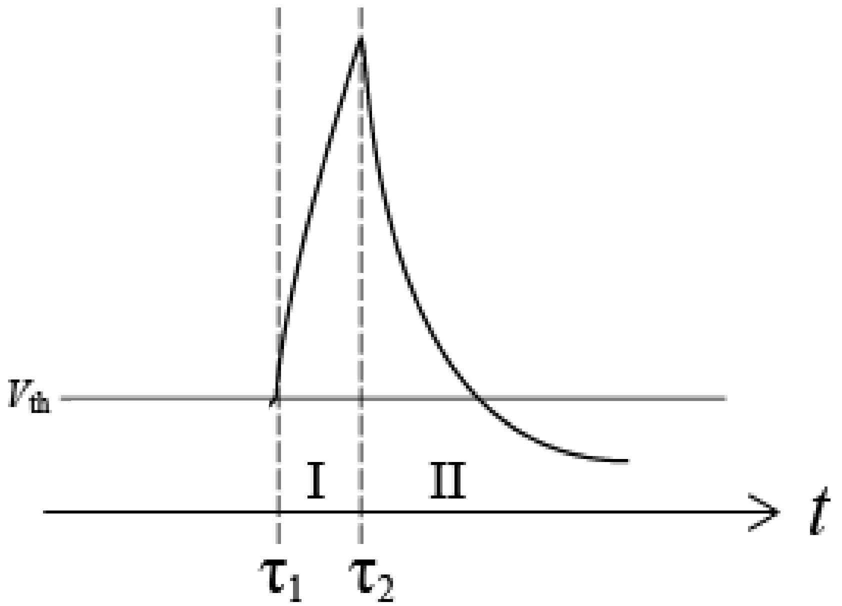
$g_i(t, V(t))$: Conductance [S];

V_i : Battery [V].

Biological backgrounds (2)

H-H model shows **two phase** in a spike:

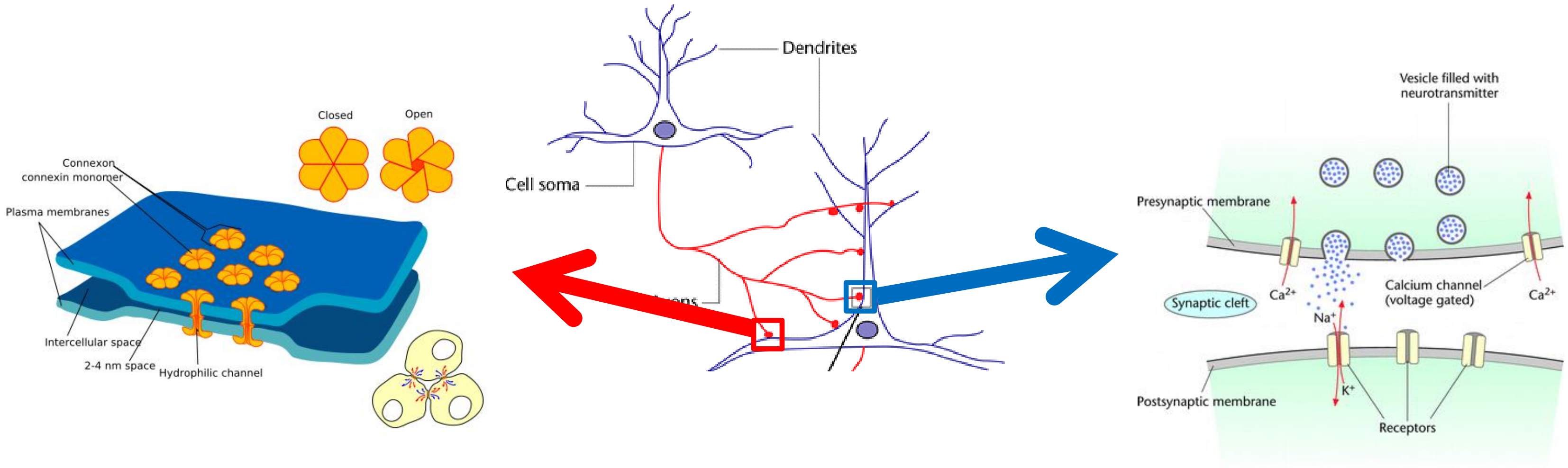
- I: Firing phase;
- II: Refractory phase.



Comments:

Two phases are NOT enough for the description.

Biological backgrounds (3)



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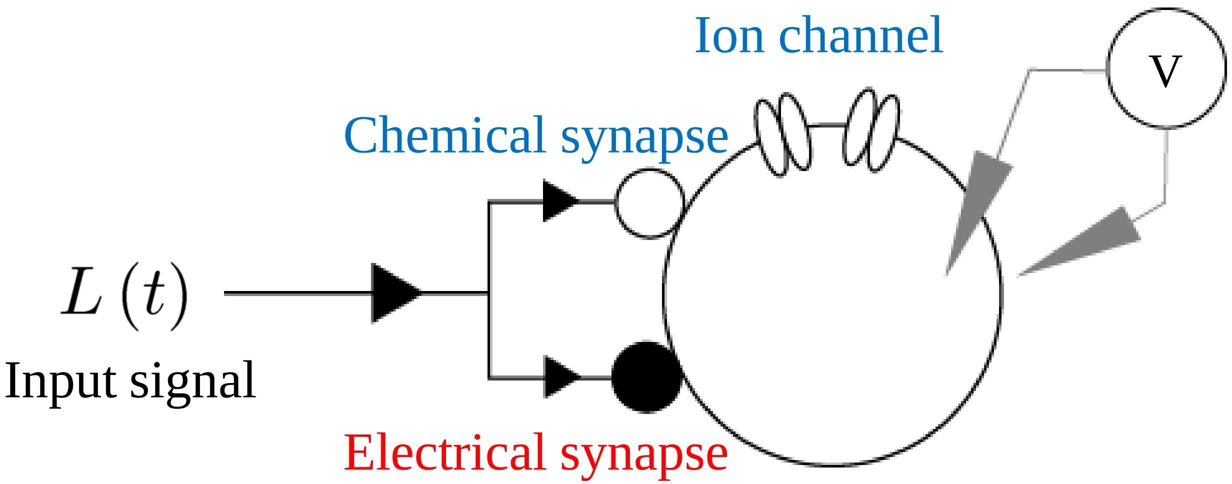
Two types of inputs:

- Electrical synapse (No delay);
- Chemical synapse (delay, degitise).

Membrane potential accumulated as

$$V(t) \sim L(t) + S(t)$$

A simple system



Input signal: $L(t)$

Cumulative potential changes caused through chemical synapse is given by

$$S(t) = \sum_{j=1}^{N} s(t - T_j)$$

Spike occurrence times

$$\{T_j; j = 1, 2, \ldots, N\}$$

Intensity function is given by

$$\lambda(t) = \begin{cases} \kappa_1 \left(\frac{dL(t)}{dt} \right)_+ & t \le \tau_1 \\ \kappa_2 \left(\frac{dL(t)}{dt} \right)_+ & \tau_1 < t \end{cases}$$

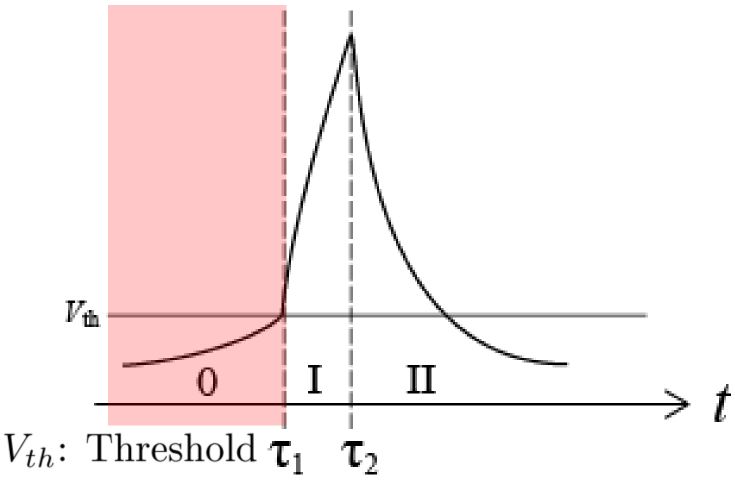
Three phase model

$$C\frac{dV(t)}{dt} = I(t) - g(t,V)(V(t) - E)$$

$$\begin{cases} g(t,V) = \text{constant} \\ I(t) = 0 \end{cases}$$

$$V(t) = \alpha e^{\beta t} + \gamma,$$

$$(\alpha = V(0) - \gamma, \beta = -g/C, \gamma = E)$$



$$V(t;\boldsymbol{\theta}) = \begin{cases} \alpha_0 e^{\beta_0 t} + \gamma_0, & t < \tau_1 & (0) \\ \alpha_1 e^{\beta_1 (t-\tau_1)} + \gamma_1, & \tau_1 \leq t < \tau_2 & (\mathrm{I}) \\ \alpha_2 e^{\beta_2 (t-\tau_2)} + \gamma_2, & \tau_2 \leq t & (\mathrm{II}) \end{cases}$$

(0) **Pre-firing phase**

(I) Firing phase

(II) Refractory phase

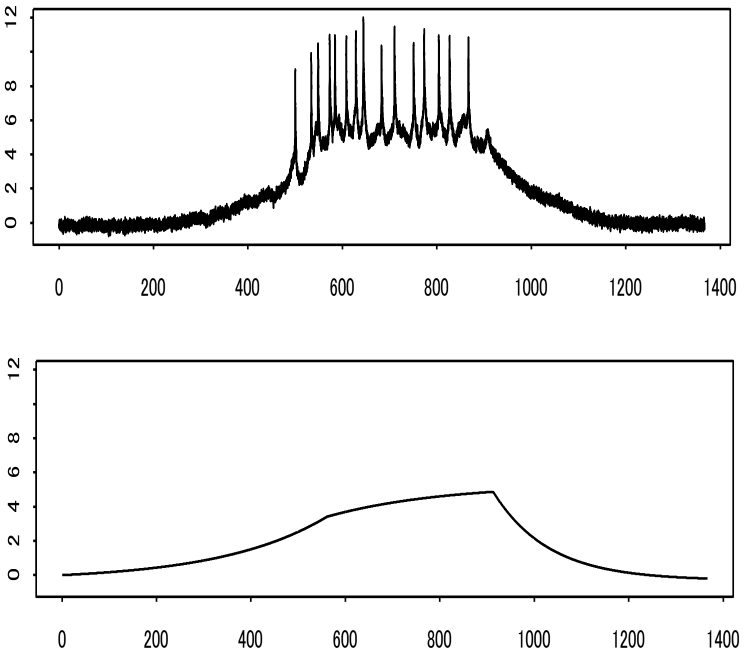
where $\boldsymbol{\theta} = (\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1, \gamma_2, \tau_1, \tau_2)$

Model of the input

$$V\left(t\right)=L\left(t\right)+S\left(t\right)+U\left(t\right)$$

Assume the three phase model for the input as

$$L(t)=\begin{cases} a_0 e^{b_0 t}+w_0, & -\infty < t < t^* \\ a_1 e^{b_1 (t-t^*)}+w_1, & t^* \le t < t^{**} \\ a_2 e^{b_2 (t-t^{**})}+w_2, & t^{**} \le t < \infty \end{cases}$$

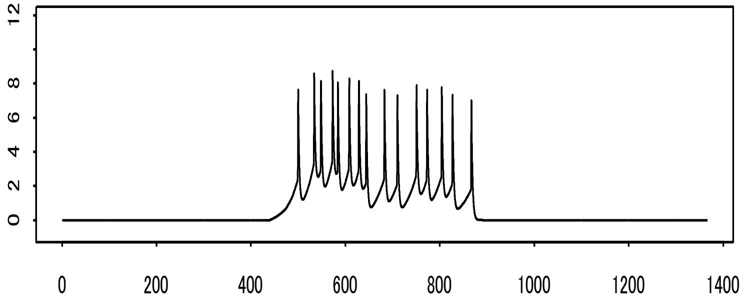


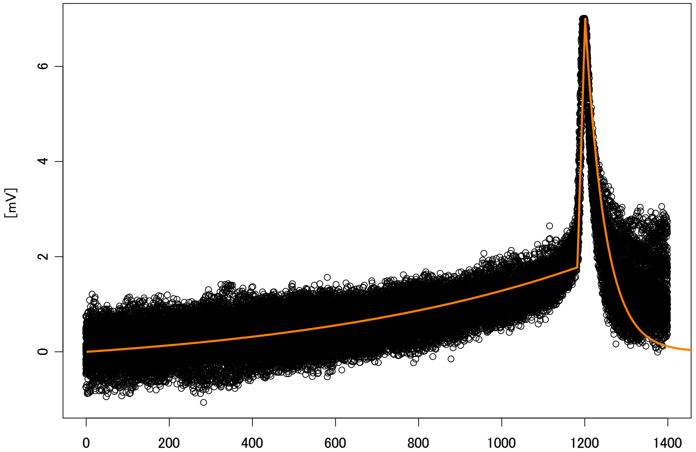
Model of a spike

$$V(t) = L(t) + S(t) + U(t)$$

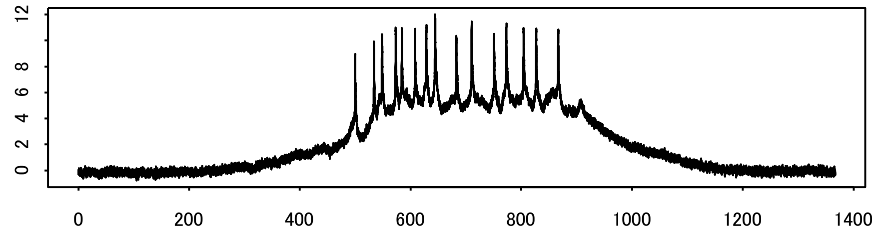
$$S(t) = \sum_{j=1}^{N} s(t - T_j)$$

$$s(t) = \begin{cases} \alpha_1 e^{\beta_1 t} + \gamma_1, & T_j < t < \tau_1 \\ \alpha_2 e^{\beta_2 (t-\tau_1)} + \gamma_2, & \tau_1 \leq t < \tau_2 \\ \alpha_3 e^{\beta_3 (t-\tau_2)} + \gamma_3, & \tau_2 \leq t < \infty \end{cases}$$





$V(t)$



0
2
4
6
8
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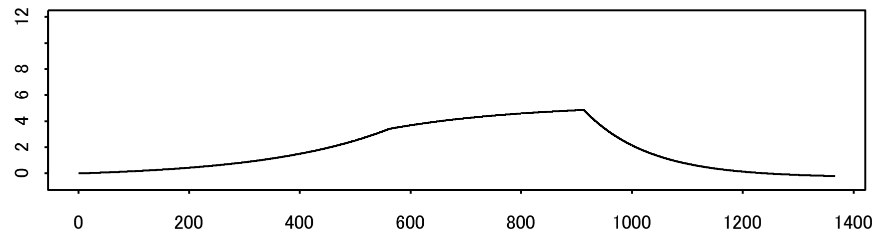
$L(t)$



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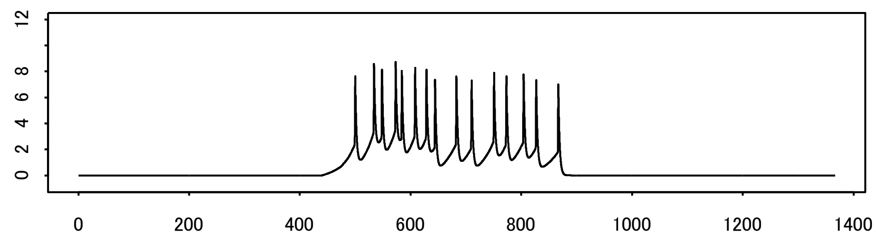
$S(t)$



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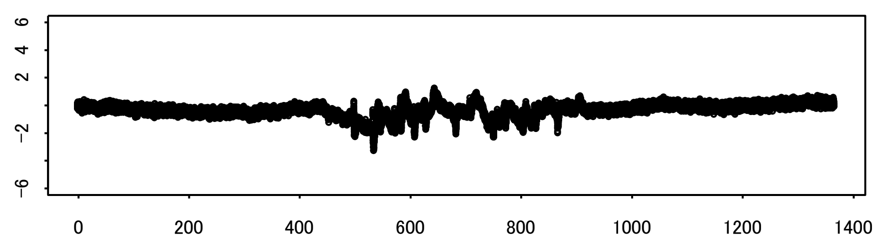
$U(t)$



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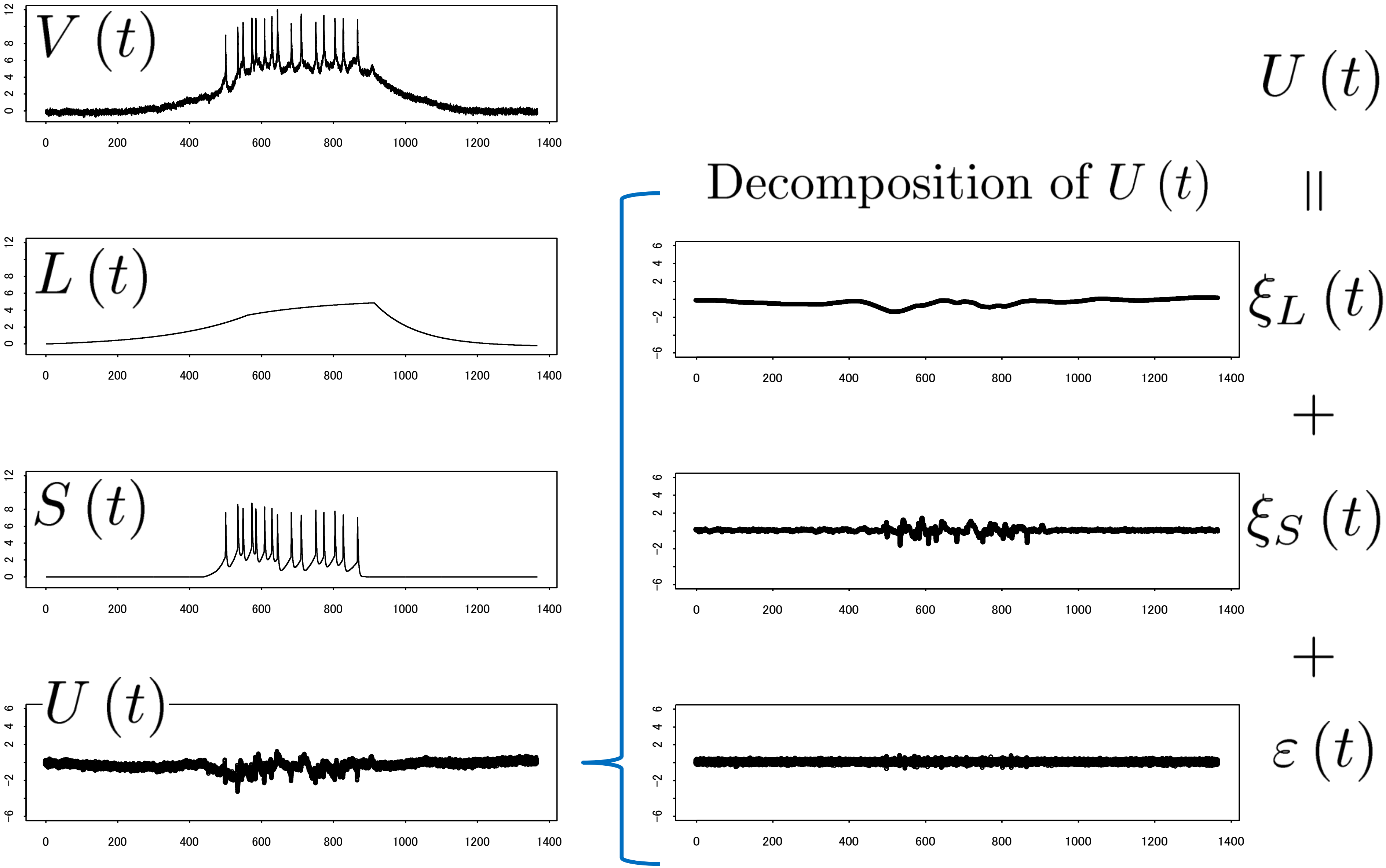
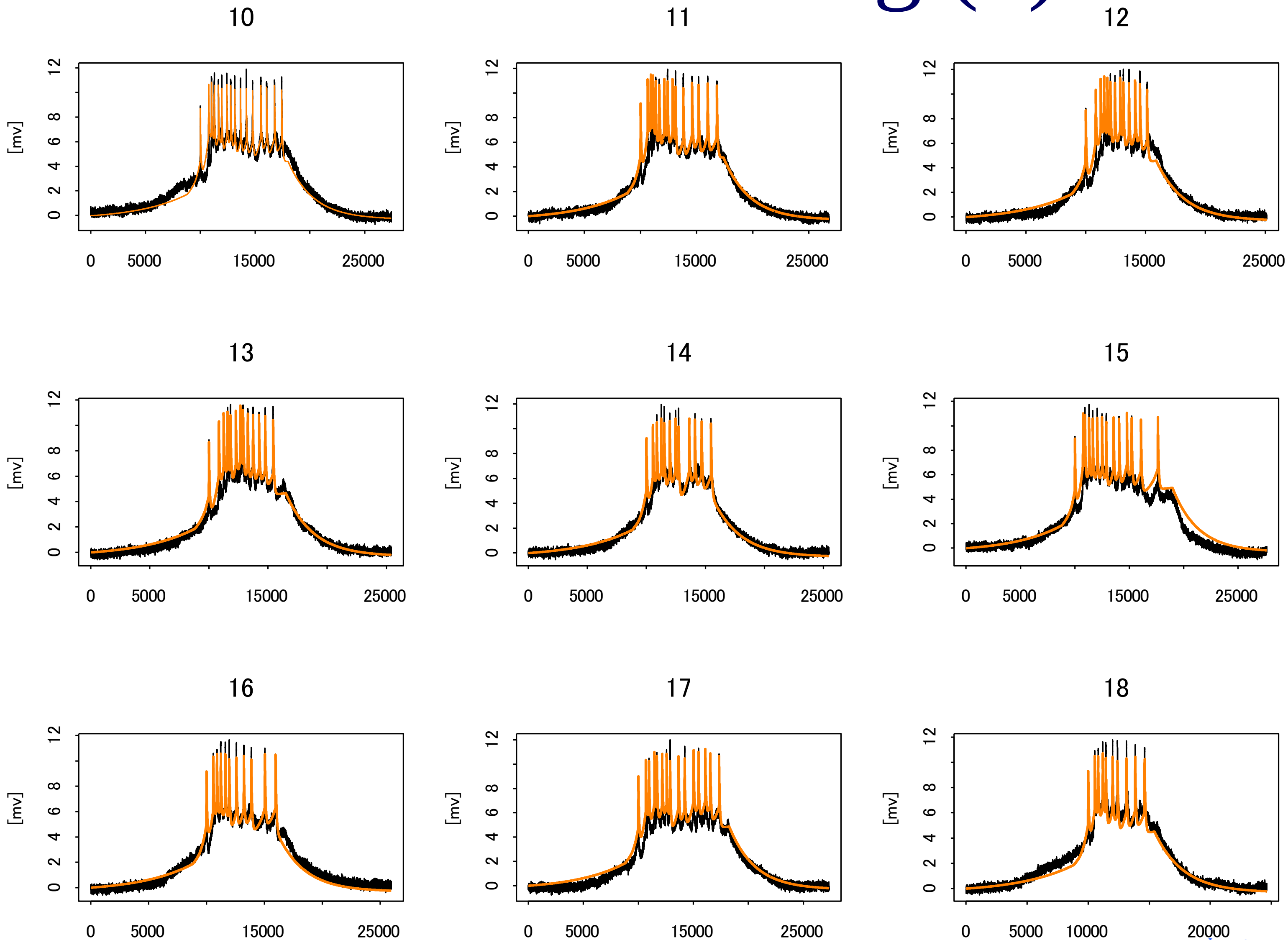
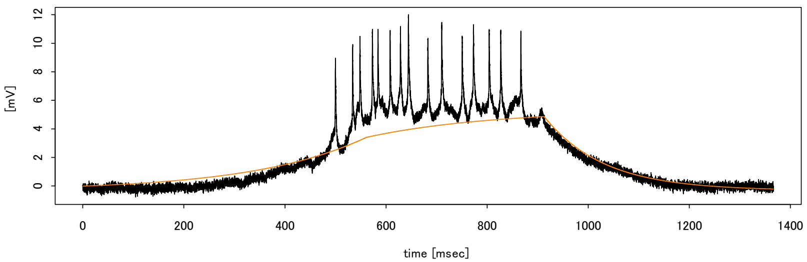


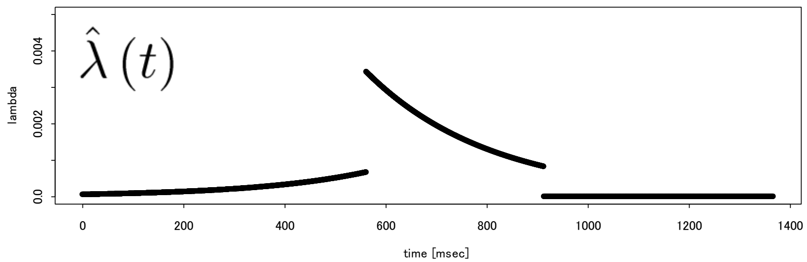
$V(t)$
$L(t)$
$S(t)$
$U(t)$
Decomposition of $U(t)$
$U(t)$
$||$
$\xi_L(t)$
$+$
$\xi_S(t)$
$+$
$\varepsilon(t)$

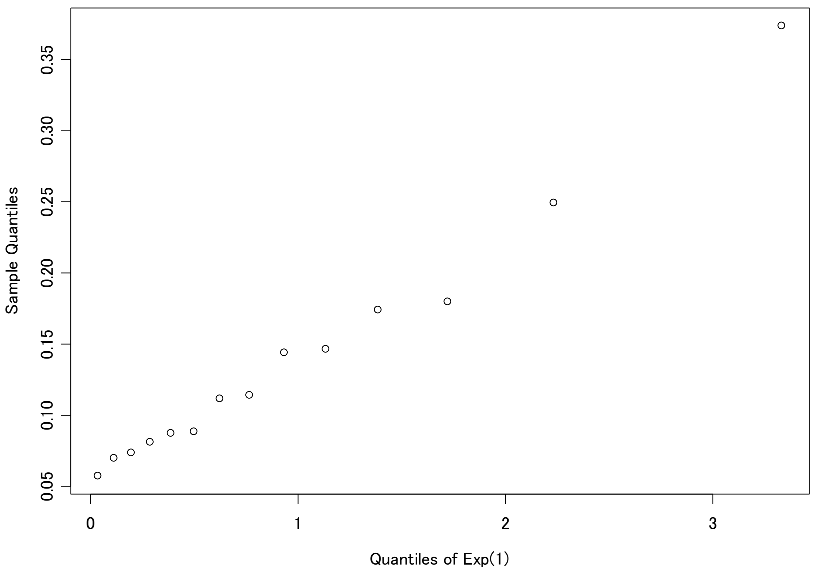
Model checking (1)



Model checking (2)







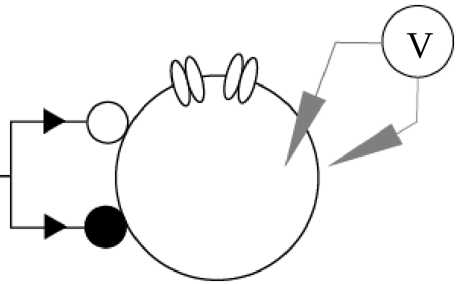
$$\lambda(t) = \begin{cases} \kappa_1 \left(\dfrac{dL(t)}{dt} \right)_+ & t \leq \tau_1 \\ \\ \kappa_2 \left(\dfrac{dL(t)}{dt} \right)_+ & \tau_1 < t \end{cases}$$

$$\Lambda_j = \int_0^{T_j} \hat{\lambda}(u)\, du$$

$$Z_j = \Lambda_{j+1} - \Lambda_j$$

Summary of the proposed model

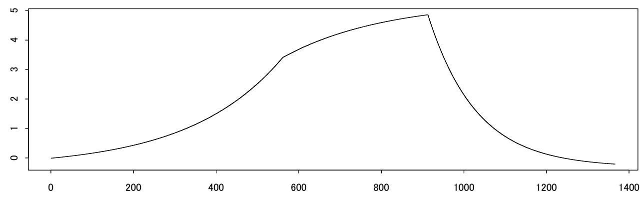
Input signal
$L(t)$



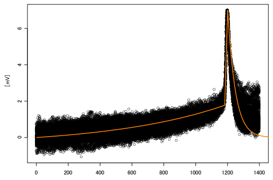
$$\lambda(t) = \frac{dL(t)}{dt}$$ **Intensity**

$$V(t) = L(t) + S(t) + U(t)$$

Electrical synapse



$$S(t) = \sum_{j=1}^{N} s(t - T_j)$$



Chemical synapse + Ion channel

Conclusion

Contributions from

Neuroscience

- H-H model
- Two types of inputs

Data Science

- Pre-firing phase
- Relation between the input and intensity function

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Thank you for kind attention.
Comments and suggestions welcomed!

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