Pathways Lecture Series in Mathematics, KEIO

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Optimization and Chaos

We start by introducing the simplest of all optimal growth models, namely the aggregative model. Under the standard assumption of a neoclassical production function, all optimal paths converge monotonically to a unique non-trivial steady state. If one allows for increasing returns to scale at small capital-labor ratios, then there can be two steady state solutions and it depends on the initial capital-labor ratio to which steady state the economy converges. Then we deal with two-sector models of optimal growth. After introducing the general framework of two-sector models, we discuss the possibility of optimal cycles and optimal chaos. The former is shown to occur in an economy in which both sectors have Cobb-Douglas production functions whereas the latter is demonstrated in an economy with technologies that are (approximately) described by Leontief production functions.

Lastly we introduce external effects and show how the resulting equilibrium problem can have a continuum of different solutions, then we survey a few results on optimal growth models that are formulated in continuous time, too. Since continuous-time dynamical systems can exhibit complicated dynamics only for a sufficiently high dimension of the state space, the results on cycles and chaos are somewhat harder to derive than in the discrete-time case. We state results on the occurrence of Hopf bifurcations and closed orbits in multi-sector optimal growth models without external effects, and on indeterminacy of equilibria in models with externalities. For both cases we use a framework with Cobb-Douglas technologies.