Robustness Aspects in Risk Management

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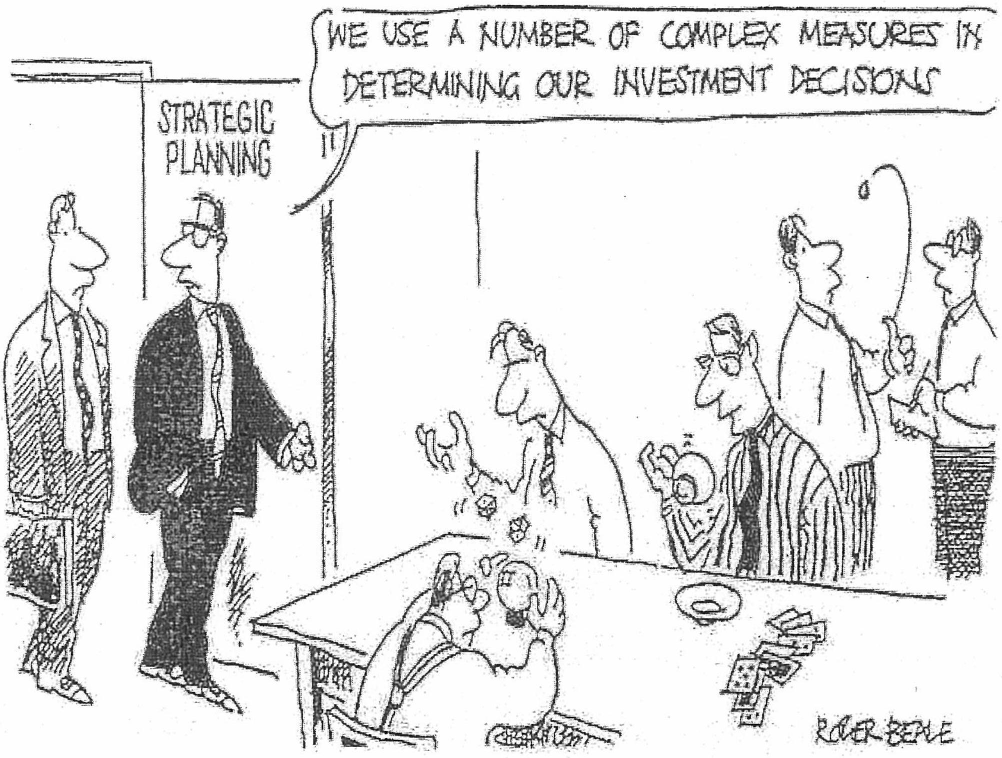

WE USE A NUMBER OF COMPLEX MEASURES IN DETERMINING OUR INVESTMENT DECISIONS
STRATEGIC PLANNING
ROGER BEALE

Outline

- Introduction
  - Stylized Facts
- Dynamic Location-Scale Models
  - ARCH, GARCH, ...
- Estimation and Inference for Single Factor Models
  - Robust Parametric Inference
  - Indirect Inference
  - Nonparametric Inference
- Robust Inference for Dynamic Location-Scale Models
- Other Topics and Outlook

♦ Introduction

Stylized Facts

- **Volatility clustering**
  Variance of daily price changes vary over time (heteroscedasticity)
  $\Rightarrow$ Model conditional distribution at time t given the information $\mathcal{F}_{t-1}$ up to time $t-1$

- **Low signal-to-noise ratio**

- Financial markets are complex; therefore **financial models** are at best only **approximate descriptions** of the underlying structure
  $\Rightarrow$ Robustness issue

♦ Dynamic Location-Scale Models

$\{y_t\}_{t\in\mathbb{Z}}$ a real valued strictly stationary random sequence
y_t has a conditionally Gaussian distribution
$y_t|\mathcal{F}_{t-1} \sim \mathcal{N}(\mu_t(\theta), \sigma_t^2(\theta))$ i.e.

$$\begin{aligned} y_t &= \mu_t(\theta) + \varepsilon_t(\theta), \\ \varepsilon_t^2(\theta) &= \sigma_t^2(\theta) + \nu_t(\theta), \end{aligned}$$

where
$\mu_t(\theta) = E[y_t|\mathcal{F}_{t-1}]$ and $\sigma_t^2(\theta) = var[y_t|\mathcal{F}_{t-1}]$ parameterize the conditional mean and the conditional variance of y_t given the information $\mathcal{F}_{t-1}$ up to time $t-1$.

- ARMA Box & Jenkins(1975)

$$\begin{aligned} \mu_t(\theta) &= \rho_0 + \rho_1 y_{t-1} \\ \sigma_t^2(\theta) &= \sigma^2 \end{aligned}$$

$\rho_0 \in \mathbb{R},\ |\rho_1| < 1.$

- ARCH Engle(1982)

$$\begin{aligned} \mu_t(\theta) &= \rho_0 + \rho_1 y_{t-1} \\ \sigma_t^2(\theta) &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2(\theta) \\ &= \alpha_0 + \alpha_1 (y_{t-1} - \rho_0 - \rho_1 y_{t-2})^2 \end{aligned}$$

$\rho_0 \in \mathbb{R},\ |\rho_1| < 1,\ \alpha_0 > 0,\ 0 \le \alpha_1 < 1.$

- GARCH Bollerslev(1986)

$$\begin{aligned} \mu_t(\theta) &= 0 \quad (y_t = \varepsilon_t) \\ \sigma_t^2(\theta) &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \delta_1 \sigma_{t-1}^2(\theta) \\ &= \alpha_0/(1-\delta_1) + \alpha_1 \sum_{j=0}^{\infty} \delta_1^j y_{t-1-j}^2 \end{aligned}$$

$\alpha_0, \alpha_1, \delta_1 > 0,\ \ \alpha_1 + \delta_1 < 1.$
GARCH model is an ARCH model with an infinite number of lagged y variables.

- Double threshold ARCH
  Glosten, Jagannathan, Runkle(1993)
  Li & Li(1996)

$$
\begin{aligned}
\mu_t(\theta) &= \rho_0 + (\rho_1 + \rho_2\, d_{1,t-1})\, y_{t-1} \\
\sigma_t^2(\theta) &= \alpha_0 + (\alpha_1 + \alpha_2\, d_{2,t-1}) \\
&\quad (y_{t-1} - \rho_0 - (\rho_1 + \rho_2\, d_{1,t-2})\, y_{t-2})^2 \\
&\quad + \alpha_3\, d_{1,t-1}
\end{aligned}
$$

with the dummy variable
$d_{1,t-1} = 1$ if $\rho_0 + \rho_1\, y_{t-1} > 0$ and 0 othw.
$d_{2,t-1} = 1$ if $\varepsilon_{t-1}(\theta) < 0$ and 0 othw.

Estimation

Conditional moment condition:
$E_\theta[\psi(y_1, ..., y_m; a(\mathbb{P}_\theta^m)|\mathcal{F}_{m-1}] = 0.$

For example, $\psi = s$, the conditionally Gaussian score function

$$\begin{aligned} s(y_1, ..., y_m; \theta) &= -k_{1,m} + k_{2,m}\, u_m(\theta) \\ &\quad + k_{1,m}\, u_m(\theta)^2, \end{aligned}$$

$u_m(\theta) = \varepsilon_m(\theta)\sigma_m(\theta)^{-1}$,
$k_{1,m} := \frac{1}{2\sigma_m^2(\theta)} \frac{\partial \sigma_m^2(\theta)}{\partial \theta}$,
$k_{2,m} := \frac{1}{\sigma_m(\theta)} \frac{\partial \mu_m(\theta)}{\partial \theta}$
defines a conditionally unbiased estimator of θ.

But non-robust
$\Rightarrow$ More general ψ functions (conditional M-estimators);

Mancini, Ronchetti, Trojani (2005)
J. Am. Stat. Ass. (MATLAB; very fast)

♦ Estimation and Inference for Single Factor Models

Data set of Chan et al. (1992):

One-month yields based on the average of bid and ask prices for Treasury bills normalized to reflect a standard month of 30.4 days. They are monthly observations covering the period from June 1964 to December 1989, for a total of 307 observations.

Ahn and Gao (1999): McCulloch and Kwon (1993) dataset over the period from December 1946 to February for comparability with the Ahn and Gao (1999) study.

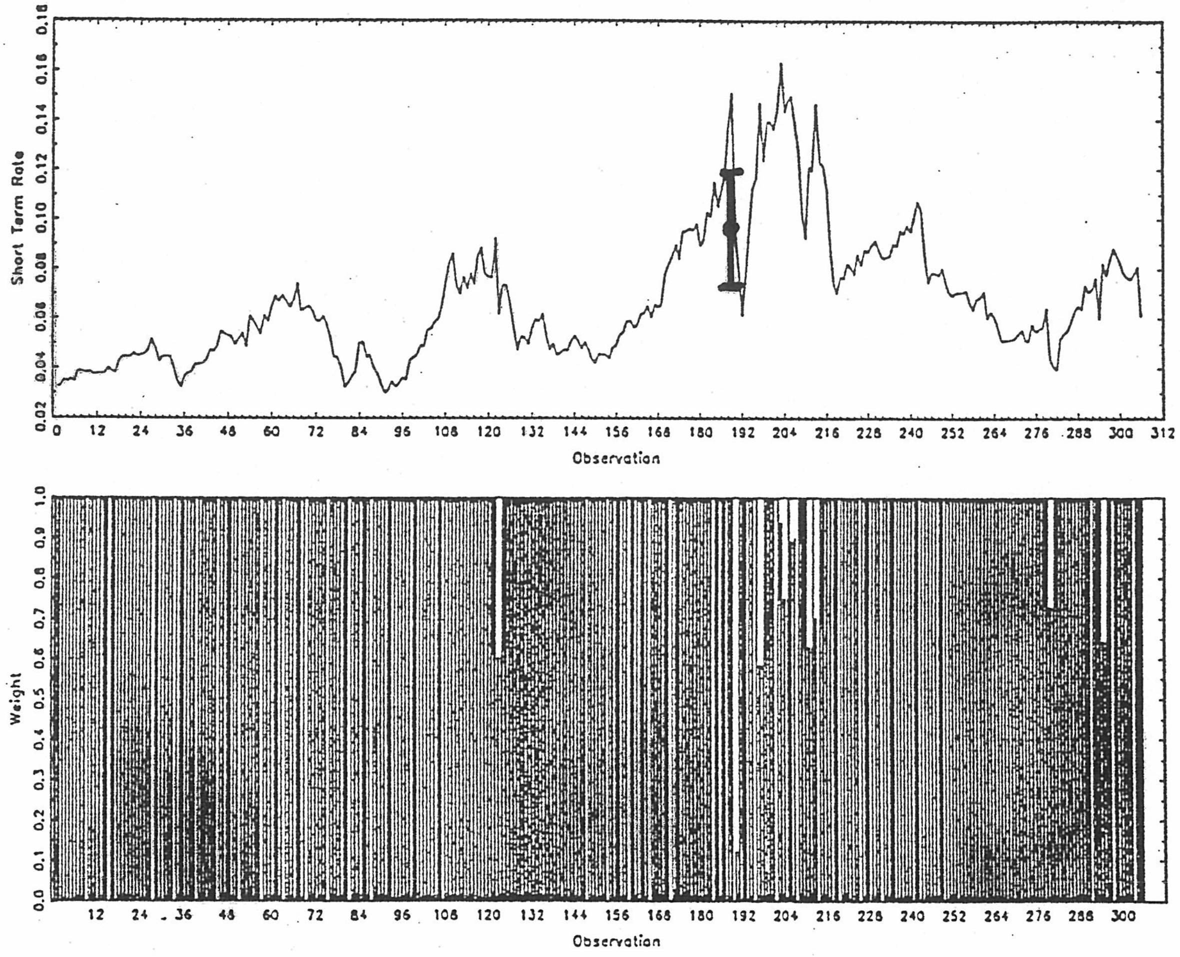
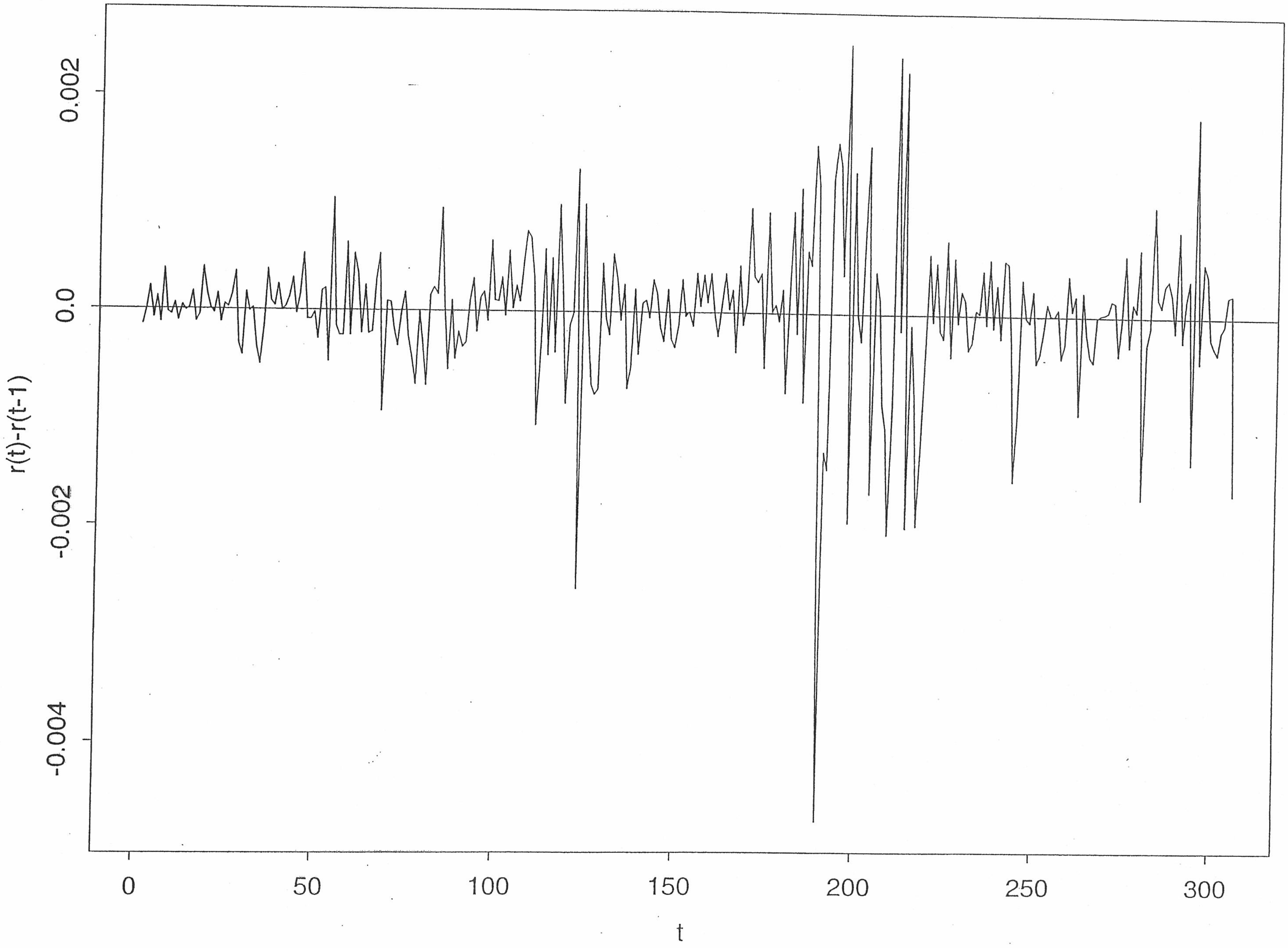


Figure 1: Short Term Rate and Weights of the Robust GMM Estimator ($c = 6$).


0.002
0.0
-0.002
-0.004
r(t)-r(t-1)
0
50
100
150
200
250
300
t

♦ Single Factor Short Rate Models

Basic Setting

$$dr_t = \mu(r_t)dt + \sigma(r_t)dW_t \quad ,$$

where r_t is the short rate at time t and $(W_t)_{t\geq 0}$ is a standard Brownian motion in $\mathbb{R}$

- Chan, Karolyi, Longstaff, Sanders (1992) (CKLS), linear drift term

$$\begin{aligned} \mu(r_t) &= \alpha + \beta r_t \\ \sigma(r_t) &= \sigma r_t^{\gamma} \end{aligned}$$

- Ahn and Gao (1999), quadratic drift term

$$\begin{aligned} \mu(r_t) &= \alpha + \alpha_1 r_t + \alpha_2 r_t^2 \\ \sigma(r_t) &= \sigma r_t^{3/2} \end{aligned}$$

- Ait Sahalia (1996)

$$\begin{aligned} \mu(r_t) &= \alpha + \alpha_1 r_t + \alpha_2 r_t^2 + \alpha_3 r_t^{-1} \\ \sigma(r_t) &= \beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta_3} \end{aligned}$$

Alternative Models of the Short Rate[22]

Model	α	β	σ	γ	restrictions
Merton		0		0	(0 attainable)
Vasicek				0	$\beta < 0$ (0 attainable)
Cox Ingersoll Ross				$\frac{1}{2}$	$\beta < 0$ and $2\alpha \geq \sigma^2$
Dothan	0	0		1	—
Geometric Brownian Motion	0			1	$\beta < 0$, (0 attainable)
Brennan Schwartz				1	$\beta < 0$ and $\alpha > 0$
Variable Rate	0	0		$\frac{3}{2}$	(0 attainable)
Constant Elasticity of Variance	0				$\beta < 0$, (0 attainable)

[22] Natural restrictions have to be imposed on the parameter values to ensure that that the drift is mean-reverting at high interest rate values (infinity not attainable) and zero is unattainable; c.f. Aït-Sahalia (1996).

♦ GMM Estimation of CKLS Models

Crude discretization

$$r_t - r_{t-1} = \alpha + \beta r_{t-1} + \epsilon_t$$

where $E(\epsilon_t) = 0$ and $E(\epsilon_t^2) = \sigma^2 r_{t-1}^{2\gamma}$

Orthogonality conditions used in CKLS:

$$\begin{aligned} E(\epsilon_t) &= 0 \\ E(\epsilon_t r_{t-1}) &= 0 \\ E(\eta_t) &= 0 \\ E(\eta_t r_{t-1}) &= 0 \end{aligned}$$

where $\eta_t = \epsilon_t^2 - \sigma^2 r_{t-1}^{2\gamma}$

♦ GMM Estimators

$\mathcal{X} := (X_n)_{n \in N}$ stationary ergodic sequence defined on an underlying probability space $(\Omega, \mathcal{F}, \mathrm{IP})$.

Parametric model
$\mathcal{P} := \{P_\theta, \theta \in \Theta\}$, $\theta \in \Theta \subset R^p$.
True parameter vector: θ_0.

Method of moments:

$$\frac{1}{n}\sum_{i=1}^{n} X_i = E_\theta X_1 = g_1(\theta)$$
$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 = E_\theta X_1^2 = g_2(\theta)$$

....

Equivalently:

$$\sum_{i=1}^{n}[X_i - g_1(\theta)] = 0$$
$$\sum_{i=1}^{n}[X_i^2 - g_2(\theta)] = 0$$

....

i.e.

$$E_\theta h(X_1;\theta) = 0$$

where $h(X_1;\theta) = (h_1(X_1;\theta), h_2(X_1;\theta), \ldots)'$
Orthogonality conditions

GMM:

Estimate indirectly some function

$$a : \mathcal{P} \to \mathcal{A} := a(\mathcal{P}) \subset R^k$$

of *parameters of interest* by introducing a function

$$h : R^N \times \mathcal{A} \to R^H$$

enforcing a set of orthogonality conditions

$$E_{\theta_0} h(X_1; a(P_{\theta_0})) = 0 \quad , \qquad (1)$$

on the structure of the underlying model.

GMM Estimators and Tests

- GMME: they are defined as the solution that minimizes the squared norm of the empirical moment conditions in an appropriate metric

$$\widehat{\vartheta}_T = \arg\inf_{\vartheta} \left\| \frac{1}{T} \sum_{t=1}^{T} h\left(X_t; \vartheta\right) \right\|^2 \underset{as}{\backsim} \mathcal{N}\left(\vartheta_0, \frac{\Sigma}{T}\right)$$

- GMM Specification Test ($H \geq k$):
  - Null $\mathcal{H}_0$: $E\left[h\left(X; \vartheta\right)\right] = 0$
  - Hansen's (1982) statistic:

$$T \cdot \left\| \frac{1}{T} \sum_{t=1}^{T} h\left(X_t; \widehat{\vartheta}_T\right) \right\|^2 \underset{\substack{as \\ \mathcal{H}_0}}{\backsim} \mathcal{X}^2\left(H - k\right)$$

$\mathcal{W} := (W_n)_{n \in N}$ sequence of weighting symmetric positive definite matrices converging a.s to W_0, the inverse of the covariance matrix of $h(X_1, a(P_{\theta_0}))$,

Generalized method of moments estimator (GMME) associated with $\mathcal{W}$: $(\tilde{a}(P_{\theta_n}))_{n \in N}$ solution to (Hansen, 1982)

$$\min_{a \in \mathcal{A}} E_{\theta_n} h'(X_1; a) W_n E_{\theta_n} h(X_1; a) \quad n \in N,$$

where $P_{\theta_n} := \frac{1}{n} \sum \delta_{X_i}$ is the empirical distribution of $X_1, .., X_n$ and δ_x denotes the point mass distribution at $x \in R^N$.

Under appropriate regularity conditions the GMME exists, is strongly consistent and asymptotically normally distributed with asymptotic covariance matrix

$$\begin{aligned}\Sigma_{\theta_0}(W_0) &= [E_{\theta_0}\frac{\partial h'(X_1;a(P_{\theta_0}))}{\partial a}W_0 \times \\ &\quad E_{\theta_0}\frac{\partial h(X_1;a(P_{\theta_0}))}{\partial a'}]^{-1}.\end{aligned}$$

In our case:

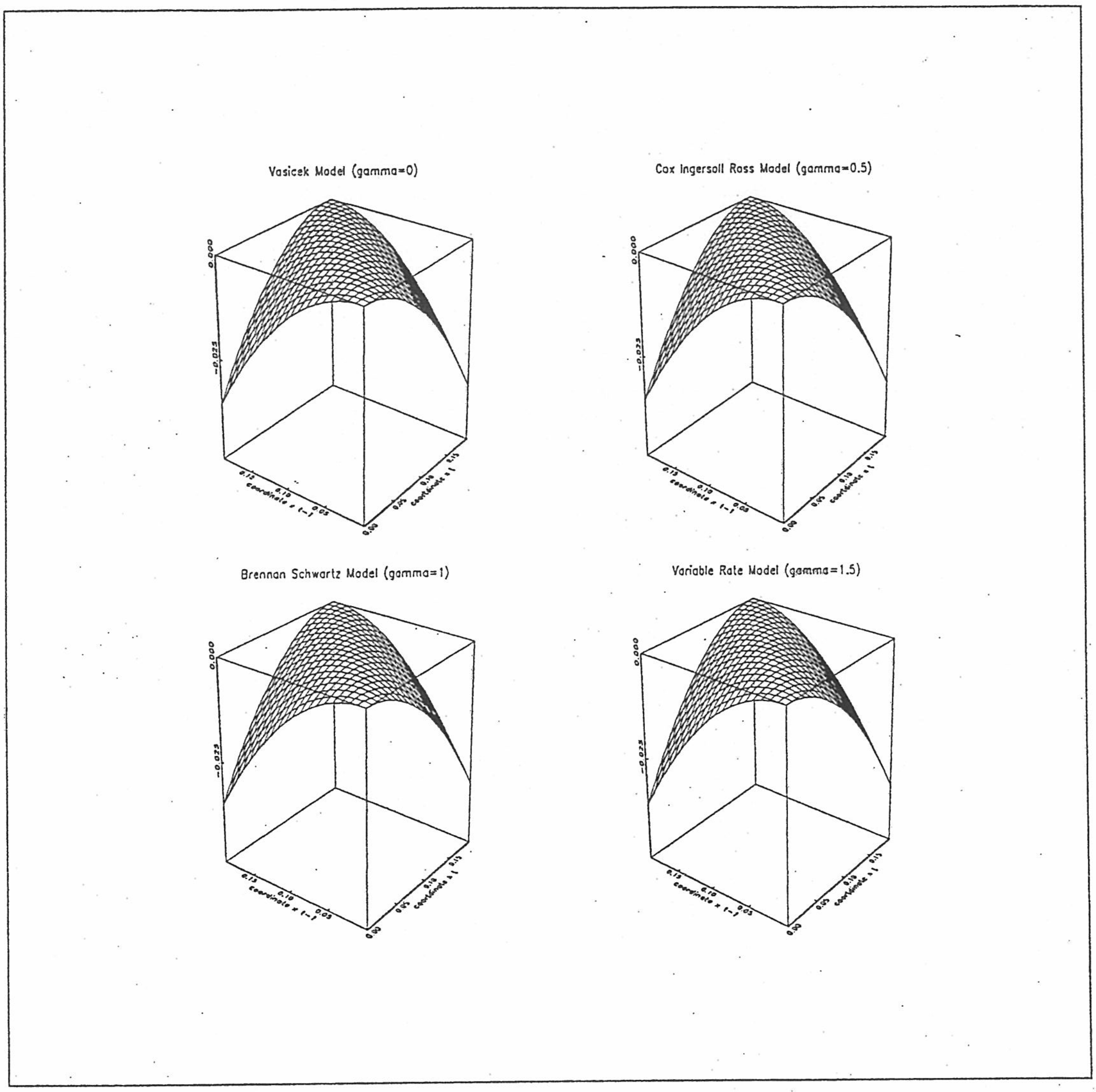
$\mathcal{X} = (r_{t-1}, r_t)'_{t \geq 0}$
Orthogonality function h:

$$h(x, y; \alpha, \beta, \sigma, \gamma) =$$

$$\begin{array}{l} y - x - \alpha - \beta x \\ (y - x - \alpha - \beta x)x \\ (y - x - \alpha - \beta x)^2 - \sigma^2 x^{2\gamma} \\ ((y - x - \alpha - \beta x)^2 - \sigma^2 x^{2\gamma})x. \end{array}$$

h is unbounded in x and y;

The classical GMM estimator and tests corresponding to this orthogonality function are therefore not robust.



Function $-h$ in the Vasicek, the Cox Ingersoll and Ross, the Brennan Schwartz and the Variable Rate model of CKLS ($\gamma = 0, 0.5, 1, 1.5$ respectively).

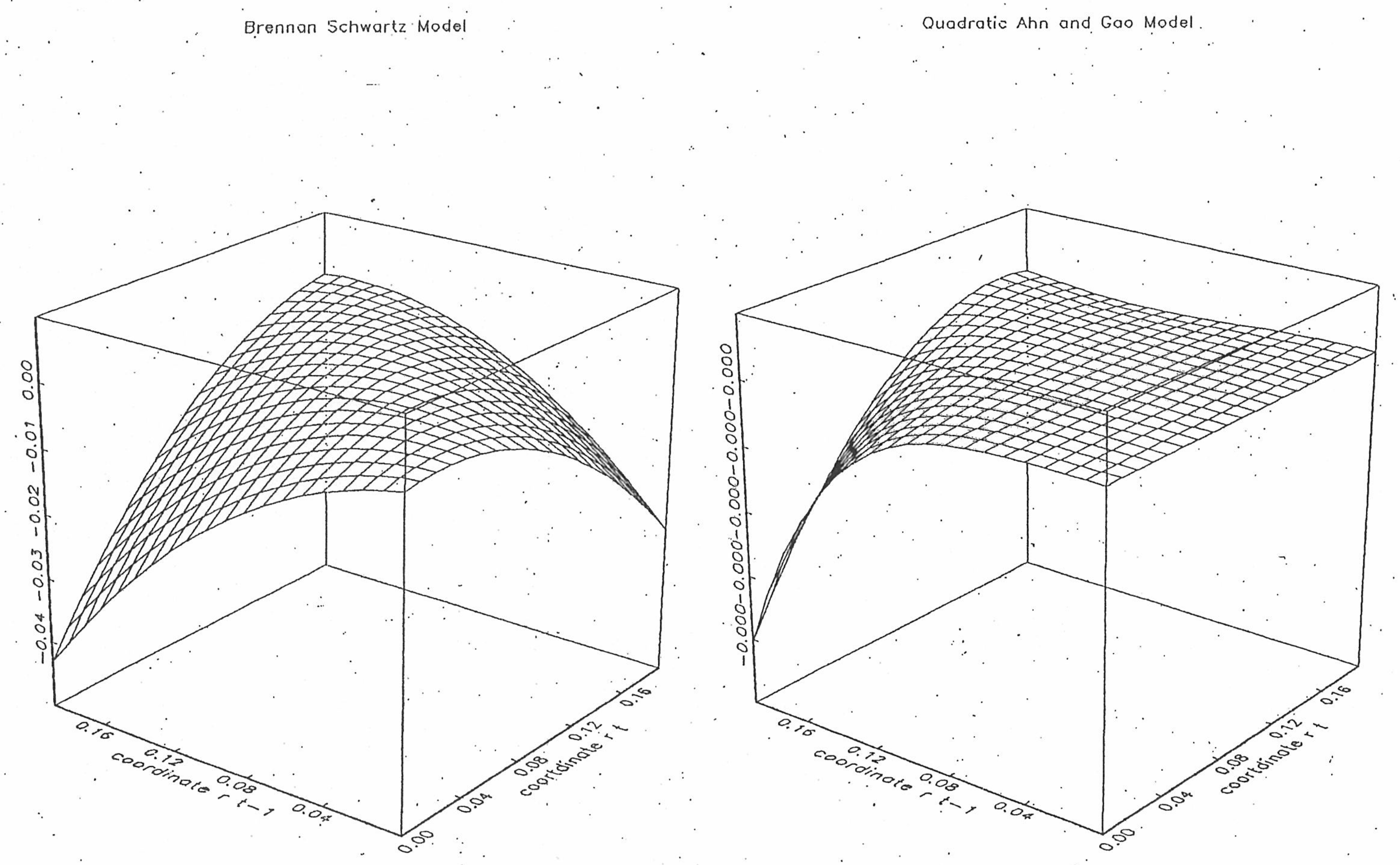


Figure 1: Function $-h$ (cf. (16)) in the Brennan Schwartz model and when only the paremeter σ is estimated with one single orthogonality condition given by (14) and when a quadratic drift term is added as in the case of Ahn and Gao.

Table II: Classical GMM Estimates of Alternative Models for the Short-Term Interest Rate

[T]he parameters are estimated by the classical GMM induced by the original orthogonality function h; [t]-statistics are in parentheses. The value of Hansen's statistics (ξ^G for brevity), are reported with p-values [i]n parentheses and associated degrees of freedom ($d.f.$).

Model	$\tilde{\alpha}$	$\tilde{\beta}$	$\tilde{\sigma}$	$\tilde{\gamma}$	ξ^G	$d.f$
Unrestricted	0.003402 (1.85)	−0.049345 (−1.55)	0.139192 (0.77)	1.499898 (5.95)	−	−
Merton	0.000458 (1.44)	0.0	0.000035 (7.27)	0.0	6.75910 (0.034)	2
Vasicek	0.000529 (0.33)	−0.001262 (−0.04)	0.000035 (7.17)	0.0	6.79973 (0.009)	1
Cox et al.	0.001065 (0.67)	−0.010222 (−0.36)	0.000620 (7.64)	0.5	4.89890 (0.027)	1
Dothan	0.0	0.0	0.009765 (7.97)	1.0	5.60148 (0.133)	3
Geometric Brownian Motion	0.0	0.008427 (1.50)	0.009873 (8.04)	1.0	3.15564 (0.206)	2
Brennan & Schwartz	0.002018 (1.24)	−0.026152 (−0.92)	0.009880 (8.09)	1.0	2.21381 (0.137)	1
Variable Rate	0.0	0.0	0.002505 (7.83)	1.5	6.30606 (0.098)	3
Constant Elasticity of Variance	0.0	0.008585 (1.53)	0.024516 (0.58)	1.171155 (3.59)	2.97738 (0.084)	1

Classical Analysis

- Models that allow for values of $\gamma \geq 1$ are not rejected – using Hansen's statistic – while models where $\gamma \in [0, 1)$ are.

- The estimates $\tilde{\gamma}$ in the corresponding models are strongly significant.

How stable is this analysis?

How reliable are these conclusions?

Sensitivity analysis of the $p-$value of Hansen's statistic

Change one observation: ϵ=1/306=0.3%.

Vary the value corresponding to April 1980 (observed value: 0.0942) from 0.075 to 0.113 by steps of size 0.001.

Variability of the short-term rate changes around April 1980 is very high: a change from a 15% to a 9.5% interest rate level just before April 1980
$\Rightarrow$ the magnitude of the sensitivity analysis seems to be realistic with respect to the structure of the short-rate observations over this particular period.

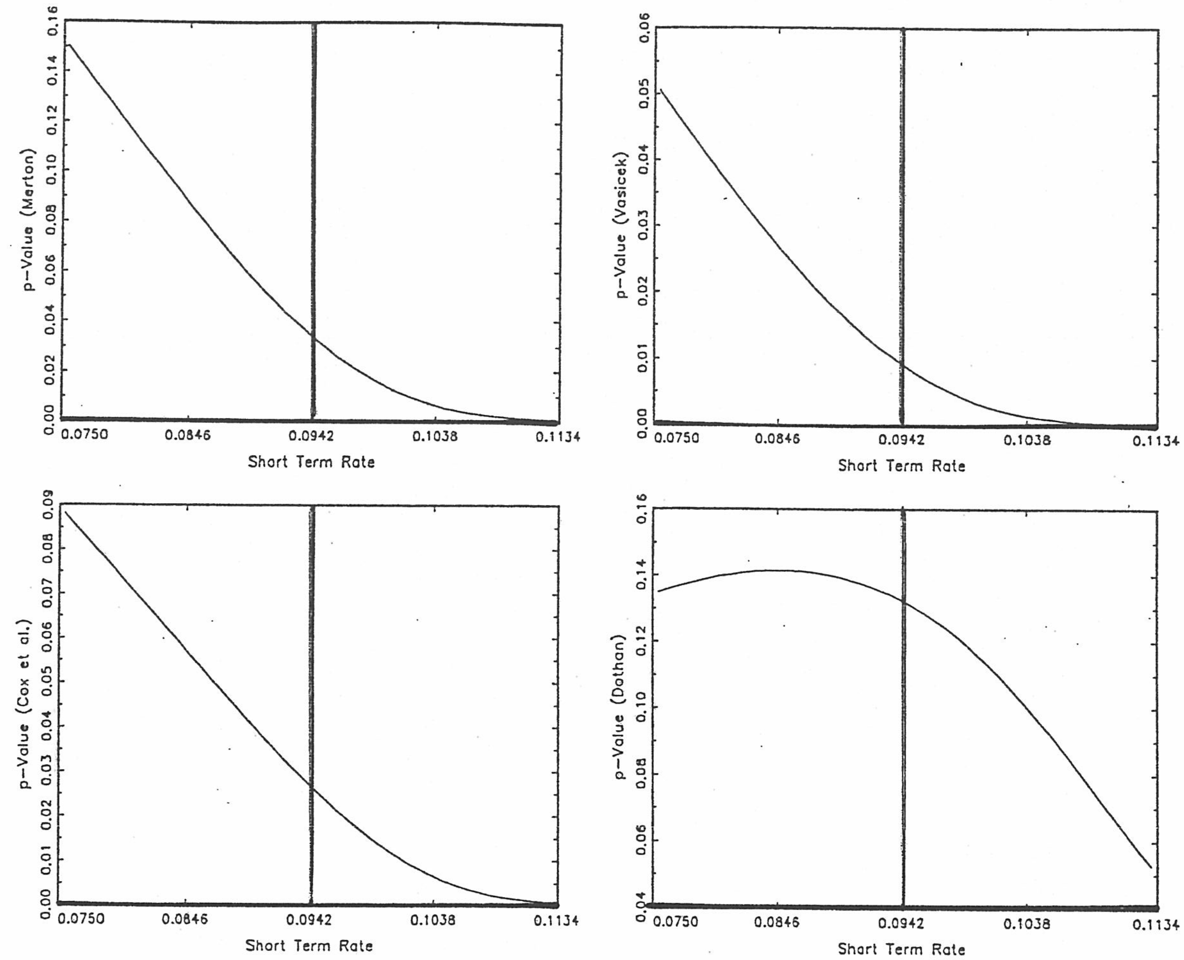


Figure 2: Sensitivity Analysis for the Classical GMM–Specification Test in Merton's, Vasicek's, Cox et al.'s and Dothan's Model.

Classical Hansen's test

- Very steep $p-$ values curves

- E.g. Merton and Cox et al. models: a change of the value of the observation of 40-80 basis points (100 basis points $= 0.01$) is sufficient for obtaining p-values *not rejecting* the model specification at a 5% significance value, as opposed to the results for the uncontaminated model.

This analysis shows that it is very difficult to distinguish – by means of the classical Hansen's test – the specification properties of the models, even when only a *single* observation is changed in the data.

Alternative Analysis: Robust Statistics

♦ Robustness in Finance

Two main research fields on robustness:

- Modelling preferences for robustness and robust decision processes of agents that take into account some forms of model misspecification in their decisions

- Developing robust statistics for the econometric analysis of financial time series using models that are possibly misspecified

Wanted

Robust procedures that take into account model misspecifications both

- when determining optimal policies in financial models
- when estimating the parameter inputs for a financial model

Robust Statistics

- deals with deviations from ideal models and their dangers for corresponding inference procedures

- primary goal is the development of procedures which are still reliable and reasonably efficient under deviations from the model used

Key tools

- Influence Function (local stability)

- Breakdown Point (global reliability)

Definition

The Influence Function (IF, Hampel (1968, 1974)) of a statistic (functional) T is defined by

$$IF(x; T, F) = \lim_{\varepsilon \downarrow 0} \frac{T((1-\varepsilon)F + \varepsilon\Delta_x) - T(F)}{\varepsilon}$$

for all x where the limit exists. Δ_x is the distribution which puts mass 1 at x.

- The IF describes the normalized influence on the statistic of an infinitesimal observation at x.

- IF is the Gâteaux derivative of T at F or the integrand in the first term of the von Mises expansion.

- Examples of "interesting" statistics: an estimator, its expectation and variance, the power and the level of a test, a portfolio allocation, etc.

Wanted

Procedures with **bounded influence function**

- IF bounded implies a bounded bias of the statistic in a contaminated neighborhood of the model

- Many models in econometrics/finance imply optimal policies/statistics with unbounded IF

- Well-known examples: OLS–, TSLS–, NLLS–methods, many ML and GMM statistics; optimal portfolios and indirect utilities in mean variance optimization problems

- Huber(1981), Wiley

- Hampel, Ronchetti, Rousseeuw, Stahel(1986), Wiley

Local stability properties of the GMME $\tilde{a}$:

$$IF(x; \tilde{a}, P_{\theta_0}) =$$
$$-\Sigma_{\theta_0}(W_0) E_{\theta_0} \frac{\partial h'(X_1; \tilde{a}(P_{\theta_0}))}{\partial a} W_0 \mathbf{h}(\mathbf{x}; \tilde{\mathbf{a}}(\mathbf{P}_{\theta_0})) \quad .$$

The IF of a GMME *is proportional to* the orthogonality function of the model

and is

bounded if and only if the function inducing the orthogonality conditions of the model is bounded.

Examples of GMME with **unbounded** orthogonality conditions:

- linear and nonlinear LS

- instrumental variables estimators

♦ Robust GMM Estimators

GMME with influence bounded by c

Huber function:

$$\mathcal{H}_c : R^H \to R^H; y \mapsto y w_c(y),$$

where $w_c(y) := \min(1, \frac{c}{\|y\|})$
New orthogonality function:

$$h_c^{A,\tau} : R^N \times \mathcal{A} \to R^H$$

$$\begin{aligned} h_c^{A,\tau}(x,a) &:= \mathcal{H}_c(A[h(x;a) - \tau]) \qquad (2) \\ &= A[h(x;a) - \tau] w_c(A[h(x;a) - \tau]) \end{aligned}$$

where the nonsingular matrix $A \in R^{H \times H}$ and the vector $\tau \in R^H$ are determined by the implicit equations:

$$E_{\theta_0} h_c^{A,\tau}(X_1, a(P_{\theta_0})) = 0 \quad , \qquad (3)$$

and

$$E_{\theta_0} h_c^{A,\tau}(X_1, a(P_{\theta_0})) h_c^{\prime A,\tau}(X_1, a(P_{\theta_0})) = I. \, (4)$$

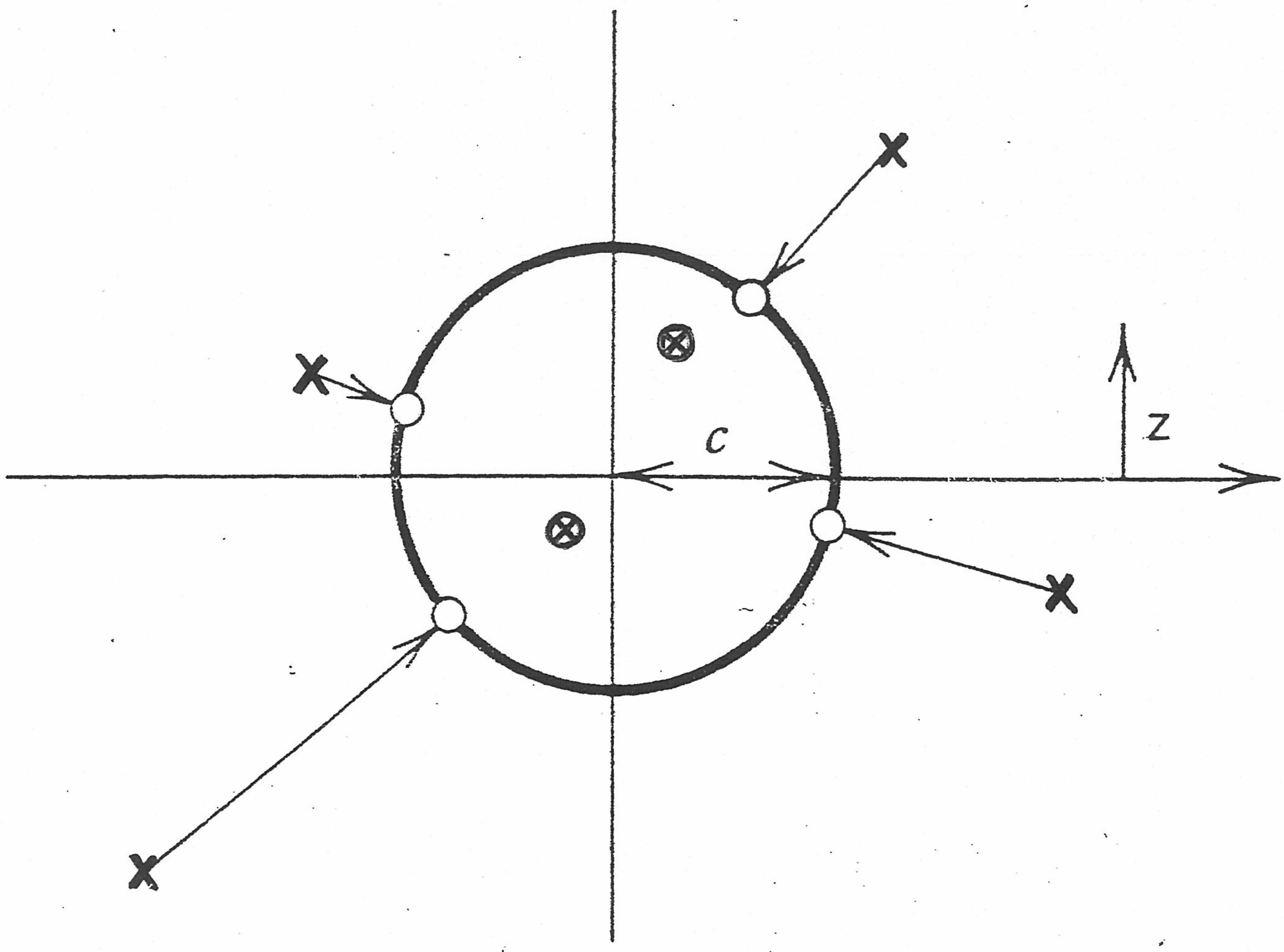


Figure 1. Sketch of the Huber function $z \mapsto h_c(z)$.

- $h_c^{A,\tau}$ can be interpreted as a *truncated version* of h. Because of the truncation, h must be shifted by τ in order to satisfy the orthogonality condition (3). Moreover, (4) ensures that c is an upper bound on the self-standardized influence of the corresponding GMME, because – by construction – the selfstandardized norm of $h_c^{A,\tau}$ is equal to its euclidean norm which itself is bounded by c.

- The GGME $\tilde{a}_c^{A,\tau}$ associated to the modified orthogonality function $h_c^{A,\tau}$ is a consistent estimator for $a(P_{\theta_0})$ that is asymptotically best and robust.

- Whereas the original moment conditions h are usually dictated by economic theory, the truncated version $h_c^{A,\tau}$ takes into account the *realistic case* that only the "majority of the data" can reasonably fit the original moment conditions. The weights $w_c(A[h(x;a) - \tau])$ assigned to each observation x can be used to detect outlying points.

- The bound imposed on the self-standardized influence of *any* GMME cannot be chosen arbitrarily small. Indeed, $c \geq \sqrt{H}$.

- No further model assumptions are needed in order to do this construction.

Robust Analysis ($c = 6$)

- Robust GMM specifications tests *reject practically all constrained models at a 5% significance level.*

- Some observations are identified as potentially influential (e.g. April 1980).

This analysis shows that it is very difficult to distinguish – by means of the classical Hansen's test – the specification properties of the models, even when only a *single* observation is changed in the data.

A robust Hansen's test should be used if one is interested in obtaining decisions that are not primarly determined by a few observations in the sample.

Models with $\gamma \geq 1$ cannot be motivated, when using *robust* model selection strategies.

♦ Extensions of the CKLS Models

Misspecification of the CKLS models
Need sophisticated multi-factor models or more complex single factor models?

- Regime-switching models (e.g. as proposed in Cai (1994), Gray (1996), Ang and Bekaert (2000b))

- Models allowing for nonlinearities in the drift and diffusion term (e.g. Ait Sahalia (1996), Stanton (1997), Jiang (1998) and Ahn and Gao (1999))

- Models adding GARCH and similar features (e.g. Brenner, Harjes and Kroner (1996), Koedijk, Nissen, Schotman and Wolff (1997) and Ball and Torous (1999))

Details can be found in :

Ronchetti, E. and Trojani, F. (2001)
"Robust Inference with GMM Estimators"
Journal of Econometrics, 101, 37-69.

Dell'Aquila, R., Ronchetti, E., and Trojani, F. (2003)
"Robust GMM Analysis of Models for the Short Rate Process"
Journal of Empirical Finance, 10, 373-397.

♦ Robust Inference for Dynamic Location-Scale Models

♦ RGMM Testing For Conditional Heteroscedasticity

ARCH(1,1)

$$\begin{aligned} y_t &= \rho_0 + \rho_1 y_{t-1} + \sigma_t u_t \quad , \\ \sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 \end{aligned}$$

where $(u_t)_{t\in\mathbb{N}}$ is a standardized i.i.d sequence of r.v.

Orthogonality conditions for a GMM estimation of the parameters $(\alpha_0, \alpha_1, \beta_0, \beta_1)$:

$$\begin{aligned} E[\epsilon_t] = 0 \quad &, \quad E[\epsilon_t y_{t-1}] = 0 \quad , \\ E[\eta_t - h_t] = 0 \quad &, \quad E[\eta_t \eta_{t-1}] = 0 \quad , \end{aligned}$$

where $\epsilon_t = y_t - \rho_0 - \rho_1 y_{t-1} \quad , \quad \eta_t = \epsilon_t^2 \quad .$

Orthogonality conditions **unbounded**!

Test $\alpha_1 = 0$ vs $\alpha_1 > 0$

Compare level and power of classical and robust GMM tests under the following distributions of $\{u_t\}$

- Standard normal $N(0, 1)$
- Contaminated normal $CN(\epsilon, K^2)$
  $\epsilon = 0.05, K = 10$
- Student t_ν , $\nu = 5, 9$
- Double exponential

- $(\rho_0, \rho_1, \alpha_0) = (0.4, 0.3, 0.25)$

- α_1 : $0 - 0.3$

- $T = 250, 500, 1000$

- 1000 simulations

- Tuning constant $c = 2.09$
  ($\epsilon = 10\%$, max bias level $= +/-0.5\%$)

Table 2: GMM and RGMM Simulation Results under $u_t \sim \mathcal{N}(0,1)$

Each entry in the Table corresponds to the empirical rejection frequency of the hypothesis $\alpha_1 = 0$ obtained using 5% critical values for the $\mathcal{X}^2$ test. The constant c for the RGMM test was set to $c = 2.09$.

	GMM			RGMM		
α_1; T	250	500	1000	250	500	1000
0.00	0.08	0.08	0.05	0.02	0.02	0.02
0.05	0.05	0.09	0.19	0.02	0.06	0.07
0.10	0.09	0.28	0.62	0.06	0.14	0.29
0.15	0.20	0.52	0.90	0.12	0.31	0.62
0.20	0.32	0.74	0.97	0.21	0.51	0.87
0.25	0.45	0.84	0.98	0.35	0.71	0.95
0.30	0.56	0.89	0.98	0.49	0.86	0.99

Table 5: GMM and RGMM Simulation Results under $u_t \sim t_5$

Each entry in the Table corresponds to the empirical rejection frequency of the hypothesis $\alpha_1 = 0$ obtained using 5% critical values for the $\mathcal{X}^2$ test. The constant c for the RGMM test was set to $c = 2.09$.

	GMM			RGMM		
α_1; T	250	500	1000	250	500	1000
0.00	0.10	0.11	0.11	0.02	0.02	0.03
0.05	0.05	0.05	0.06	0.03	0.07	0.11
0.10	0.06	0.10	0.24	0.05	0.14	0.33
0.15	0.11	0.18	0.43	0.11	0.28	0.61
0.20	0.15	0.29	0.59	0.17	0.46	0.82
0.25	0.21	0.40	0.67	0.29	0.64	0.93
0.30	0.27	0.48	0.71	0.40	0.78	0.97

Table 6: GMM and RGMM Simulation Results under $u_t \sim CN(0.05, 100)$

Each entry in the Table corresponds to the empirical rejection frequency of the hypothesis $\alpha_1 = 0$ obtained using 5% critical values for the $\mathcal{X}^2$ test. The constant c for the RGMM test was set to $c = 2.09$.

	GMM			RGMM		
α_1; T	250	500	1000	250	500	1000
0.00	0.35	0.51	0.48	0.02	0.01	0.02
0.05	0.16	0.19	0.17	0.02	0.03	0.06
0.10	0.09	0.08	0.05	0.03	0.06	0.14
0.15	0.06	0.04	0.02	0.06	0.11	0.24
0.20	0.04	0.03	0.03	0.07	0.16	0.36
0.25	0.04	0.03	0.06	0.10	0.22	0.48
0.30	0.04	0.04	0.11	0.13	0.28	0.60

- Robust test yields **stable level and power** across distributions

- Classical test shows a **drastic liberal behavior**

- The **power advantage** of the classical test under normality is **lost** for **very small deviations from normality**

Empirical Application

Apply classical and robust Wald tests for ARCH structure to weekly exchange rate returns of the Swedish krona against US dollar Period Nov. 29th, 1993 - Nov. 17th, 2003; 522 observations (from Datastream).

'Regular' time series with no clear outlier.

The first ten sample autocorrelations of squared and absolute returns are not significantly different from zero.

Moreover, the Jarque-Bera test has a p-value of 0.47 not rejecting normality.

Estimates for the parameters ρ_0, ρ_1, α_0 and α_1 of an AR(1)-ARCH(1) model and (Wald test p-values for the hypothesis that the corresponding parameter is zero):

Classical
0.02 (0.73), −0.030 (0.53), 1.86 (0), 0.06 (**0.22**)

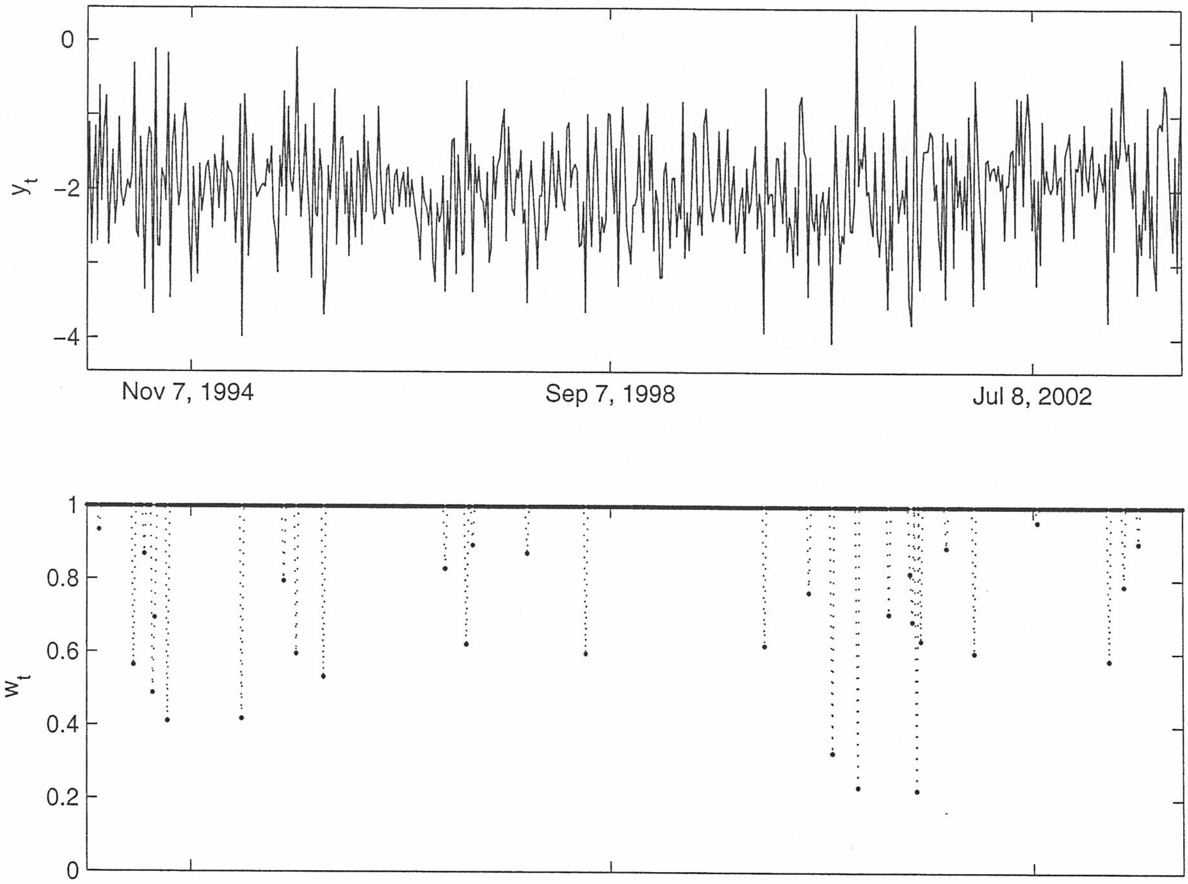
Robust ($c = 4$)
0.01 (0.88), 0.014 (0.75), 1.64 (0), 0.47 (**0**).

As in typical financial return series, the conditional mean parameters are not significantly different from zero.

Moreover, the classical estimate of the ARCH parameter α_1 is also not significant. Hence, the classical Wald test does not reject the homoscedasticity hypothesis.

By contrast, the robust estimate of this ARCH parameter is highly significant, showing that ARCH effects in the data are possibly obscured by some outlying observations detected by the robust weights.

It is interesting to notice that one would expect outliers to *enhance* the ARCH structure. Instead, because the estimation of the volatility by classical techniques is inflated, the potential ARCH structure is hidden by the presence of a few outlying observations.



Weekly exchange rate returns of the Swedish krona vs. US dollar (Nov 29, 1993 - Nov 17,2003 (top panel) and weights implied by the rob. est. of the AR(1)-ARCH(1) model with $c = 4$ (bottom panel).

ROBUST STATISTICS IN FINANCE

E. Ronchetti

2005

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