Memories of Professor Iwasawa

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During the editing of this volume, I felt as the chief editor that it would be better if there would be an article explaining the days of Iwasawa in Japan after his retirement from Princeton. I could not find a suitable person to ask to write on this, and so I finally decided to write a short article on it myself.

My first direct memories of Professor Iwasawa are when I had the opportunity to attend his talk in a number theory conference at RIMS, Kyoto in December 1987, the year Iwasawa came back to Japan. At that time I was a Ph.D student at the University of Tokyo, and studying higher dimensional class field theory as a student of Professor K. Kato. In the conference at RIMS, I also gave a talk, which was my first experience giving a talk in a conference, on my result [5] on the description of abelian extensions of certain complete discrete valuation fields with *general* residue fields. Iwasawa talked on some problems on cyclotomic fields (see [1], [2]), and it was so impressive for me. It was done in his legendary style using no notebook nor memo, and perfectly organized. I knew nothing on Iwasawa theory at that time, but concentrated on his lecture and waited for what would happen next in the lecture. One hour had passed in a moment like a movie. The protagonist $\mathbb{Q}(\mu_p)$ who had been a simple object in my mind before his talk changed to be a mysterious and interesting target after his talk.

Next year I heard from H. Ichimura, whose speciality is Iwasawa theory, that every Saturday a seminar was held at Komaba campus, which Iwasawa attended with G. Fujisaki and S. Nakajima[1]. I became a regular member of the seminar. When I first attended it, the participants studied de Shalit's notebook on elliptic curves, but soon after that, the style of the seminar changed, and the participants could talk on any topics they liked. I had an idea to get a small partial answer to support one of Iwasawa's conjectures he talked in Kyoto on $\mathbb{Q}(\mu_p)$, using the computation of the K-group $K_4(\mathbb{Z})$ by Lee and Szczarba. I talked on

[1]In those days the department of mathematics was in Hongo campus (the main campus of Tokyo University) though it is now in Komaba campus.

this idea at the seminar, then Iwasawa was very interested in it, and suggested me to write a paper though I thought this was just a remark. Without Iwasawa's enthusiasm for cyclotomic fields, I could not have produced my paper [6]. I owe much to him.

Iwasawa himself sometimes gave talks at the seminar. One of his talks was on integral representations of finite groups [3], which was again a perfectly organized, very interesting talk, related to the class groups of cyclotomic fields.

When the notion of an Euler system was introduced by Kolyvagin, Iwasasa pointed out to us the importance of it. As a result we carefully read in the seminar Rubin's exposition, which was later revised to become [7]. In the final version [7] the base field is $\mathbb{Q}$, but an older version treats more general number fields satisfying some conditions. Iwasawa distributed his notes in which he shows that the conditions are satisfied only in the case that the base field is $\mathbb{Q}$ or an imaginary quadratic field.

Every Saturday after the seminar, we went to some restaurant to have lunch together. I still remember clearly the peaceful atmosphere of the lunch time we had with Iwasawa (I don't know why, but always with beautiful sunshine from windows in my memory). Also, he sometimes invited us to his house with a garden which was so large by the standard of Tokyo. He talked about various subjects, e.g. his experiences in the United States, what he thought recently after reading some books, etc. One can find an example of such conversation in [8].

I got a position as an Assistant at Tokyo Metropolitan University, and Iwasawa's house was 10 minutes' walk from the campus of TMU[2]. He kindly invited me several times to his house. I was very happy to talk with him on many things in mathematics, e.g. the structure of the class groups of number fields, and also on non-mathematics subjects. One day he proposed me to make a bet on how many mathematicians would get the Fields medals at the ICM held in Kyoto in 1990. I did not understand what he said because the word like "bet" I never expected from Iwasawa. But he continued to ask me how about ten thousand yen. In conclusion no bet was made, but I understood later that the bet would have been totally unfair if it had been made because Iwasawa was a member of the Fields medal committee.

Just before ICM 90, John Coates visited Tokyo Metropolitan University, and Iwasawa introduced me to John. This was also a very important encounter in my life. Iwasawa attended a big satellite conference on number theory and algebraic geometry held at TMU before the ICM.

[2]In those days the campus of TMU was in Tokyo Metropolitan Area. It moved to Hachioji city in 1991.

Mrs. Iwasawa was a lady who talked in very beautiful and noble Japanese. I had known such noble Japanese only in the literature before I met her. She told me that she went to the US with their children not by a passenger ship, but by a cargo ship, but the travel was so comfortable because the crews were so kind to them. Before I went to the US studying in Harvard for two years, she explained to me that even in Boston Area, I can buy Japanese foods at the Japanese grocery store, Yoshinoya[3], at Central Square in Cambridge, MA. I heard they had lived in Belmont, MA when Iwasawa worked at MIT.

So many things come up to my mind when I remember those old days. Iwasawa was often disappointed by there being fewer Japanese members in the Institute for Advanced Studies than before. When I applied to the position of a member of the IAS, Iwasawa and Kato kindly wrote reference letters for me, which I appreciate very much.

Kato was not a regular participant of the Saturday seminar, but he sometimes attended the seminar to talk on his new results. I remember that Iwasawa admired Kato's work [4] on an equivariant version of his Tamagawa number conjecture with Bloch.

When Wiles proved Fermat's last theorem and distributed his paper with Taylor, Iwasawa told me that he was so happy to see, and thought it interesting, that Greenberg's ideas were used there.

I think I was really lucky, I should say, more than lucky that I could get to know Iwasawa. I am very happy and honored to edit with my colleagues this volume to commemorate the 100th anniversary of his birth.

References

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[3]For Japanese readers, this store has no relation with the famous beef bowl restaurant with the same name.

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