KEIO-YONSEI NUMBER THEORY WORKSHOP

3rd floor of Building 16, Faculty of Science and Technology, Keio University

December 3 (Mon)

10:00–11:00 Soogil Seo (Yonsei Univ)

On the towers of torsion Bertandias and Payan modules

11:20–12:20 Ken-ichi Bannai (Keio Univ)

On the de Rham realization of the polylogarithm for certain algebraic Tori

13:30–14:30 Dong Hyeok Lim (Yonsei Univ)

On the Gross-Kuzmin Conjecture

15:00–16:00 Yoshinosuke Hirakawa

A heuristic recipe for insoluble equations of the form $x^3 + py^3 = Cz^p$

16:30–17:30 Hideki Matsumura (Keio Univ)

A unique pair of triangles

18:15– Reception party at Faculty Lounge

December 4 (Tues)

10:00–11:00 Kazuto Ota (Keio Univ)

Anticyclotomic Iwasawa main conjecture for modular forms

11:20–12:20 Chan-Ho Kim (KIAS)

On the Fitting ideals of Selmer groups of elliptic curves with supersingular reduction

13:30–14:30 Mahiro Atsuta (Keio Univ)

Fitting ideals of the 2-components of class groups of number fields

15:00–15:40 Yusuke Tanuma (Keio Univ)

Arithmetic properties of Hecke-Mahler series and its derivatives

16:00–16:40 Takaaki Tanaka (Keio Univ)

Algebraic independence properties of a certain map defined on the set of orbits of the action of Klein four-group

Soogil Seo

On the towers of torsion Bertandias and Payan modules

Abstract: For an odd prime p, let K/k be a Galois p-extension and S be a set of primes of k containing the primes lying over p. For the p^r-th roots of unity in K, we describe the so called Sha group in terms of the Galois groups of certain subfields of K corresponding to S. As an application, we investigate a tower of extension fields $\{k_{T^i}\}_{i>0}$ where $k_{T^{i+1}}$ is defined as the fixed field of a free part of the Galois group of the Bertrandias and Payan extension of k_{T^i} over k_{T^i}. This is called a tower of torsion parts of the Bertrandias and Payan extensions over k. We find a relation between the degrees $[k_{T^{i+1}} : k_{T^i}]$ over the towers. Using this formula we investigate whether the towers are stationary or not.

Ken-ichi Bannai

On the de Rham realization of the polylogarithm for certain algebraic Tori

Abstract: The classical polylogarithm function may be interpreted as a period function of a certain variation of mixed $\mathbb{R}$-Hodge structures on the projective line minus three points and may be interpreted as a Deligne-Beilinson class, called the polylogarithm, whose construction is motivic. The de Rham realization of the polylogarithm is given by a certain rational function which is a generating function for critical values of Riemann-Zeta function and more generally Lerch Zeta functions. In this talk, we give a construction of the de Rham realization of the polylogarithm in the equivariant de Rham cohomology of a certain algebraic torus associated to a totally real field, equivariance being respect to the action of the totally positive units of the totally real field. We then relate the rational function defining this class with the generating function of critical values of Lerch-type Shintani Zeta functions of the associated totally real field. This is joint work with Kei Hagihara, Kazuki Yamada, and Shuji Yamamoto.

Dong Hyeok Lim

On the Gross-Kuzmin Conjecture

Abstract: In 1972, L.V.Kuzmin conjectured that the characteristic ideal of $\varprojlim A_n$ is divisible by T at most once. To study this conjecture, Kuzmin introduced the notions of local and global universal norm groups. His work implicitly used p-adic class field theory, which was effective. But this made his work hard to grasp. Recently, Seo Soogil introduced an equivalent form of the conjecture that comprehends the notions of universal norms, but not using p-adic class field theory. In this talk, I'll introduce Kuzmin, and Seo Soogil's works and some small results using the new equivalent form.

Yoshinosuke Hirakawa

A heuristic recipe for insoluble equations of the form $x^3 + py^3 = Cz^p$

Abstract: The contents of this talk is based on joint work with Yosuke Shimizu, a graduate of Keio University last year. A considerable number of researches have been done on the Diophantine equations of the form $Ax^l + By^m = Cz^n$. Among them, Darmon and Granville (1995) proved that whenever $1/l + 1/m + 1/n < 1$, the equation $Ax^l + By^m = Cz^n$ has only finitely many coprime integer solutions (x, y, z) for each triple of non-zero integers (A, B, C). Let $p > 3$ be a prime number. In this talk, we introduce a heuristic recipe for an integer C for which $x^3 + py^3 = Cz^p$ violates the Hasse

principle for coprime integer solutions, that is, the equation has local coprime integer solutions at every prime number but no global coprime integer solutions. The key ingredients of our proof are a certain p-adic property of the fundamental unit of $\mathbb{Q}(\sqrt[3]{p})$ and the decomposition law of prime numbers in $\mathbb{Q}(\sqrt[3]{p})$. We also give an application for construction of counterexamples to the Hasse principle for rational points on non-singular plane curves of degree p.

Hideki Matsumura

A unique pair of triangles

Abstract: A triangle is called rational if the lengths of its sides are rational numbers. In this talk, we prove that there exists a unique pair (up to similitude) of a rational right triangle and a rational isosceles triangle which have the same perimeters and the same areas. The proof is reduced to determine the set of rational points on a certain hyperelliptic curve. We carry out this task using the method of Chabauty-Coleman and the 2-descent on the Jacobian variety of a hyperelliptic curve with a help of MAGMA. Here, the method of Chabauty-Coleman gives an explicit upper bound of the number of rational points on a hyperelliptic curve if its Mordell-Weil rank is sufficiently small, which is verified for the above hyperelliptic curve by the 2-descent. This is joint work with Yoshinosuke Hirakawa.

Kazuto Ota

Anticyclotomic Iwasawa main conjecture for modular forms

Abstract: We will discuss recent developments on the anticyclotomic Iwasawa main conjecture for modular forms. It predicts an equality between Selmer groups and p-adic L-functions which interpolate special values of L-functions twisted by Hecke characters of an imaginary quadratic field. In this talk, we explain our result which proves a half part of the conjecture. This is joint work with Shinichi Kobayashi.

Chan-Ho Kim

On the Fitting ideals of Selmer groups of elliptic curves with supersingular reduction

Abstract: We discuss the behavior of Fitting ideals of Selmer groups of elliptic curves with good supersingular reduction over finite subextensions of the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$. This is joint work with Masato Kurihara.

Mahiro Atsuta

Fitting ideals of the 2*-components of class groups of number fields*

Abstract: We discuss the Fitting ideals of ideal class groups of number fields. After surveying several known results and several methods, we give some new results, especially for the 2-component of the minus quotient of the class group of an imaginary field.

Yusuke Tanuma

Arithmetic properties of Hecke-Mahler series and its derivatives

Abstract: Hecke-Mahler series is the generating function $h_\omega(z) = \sum_{k=1}^{\infty}[k\omega]z^k$ of the sequence $\{[k\omega]\}_{k\geq 1}$, where ω is a real irrational number and $[\,\cdot\,]$ denotes the integral part. Hecke proved that $h_\omega(z)$ has the unit circle as its natural boundary. In what follows, we assume that ω is a real quadratic irrational number. Mahler proved that the value $h_\omega(\alpha)$

is transcendental, where α is a nonzero algebraic number inside the unit circle. Nishioka gave a complete proof of the fact that the infinite set $\{h_\omega^{(l)}(\alpha) \mid l \geq 0\}$, where α is as above, is algebraically independent, whose incomplete proof had been given by Mahler. On the other hand, Masser proved that the infinite set $\{h_\omega(\alpha) \mid \alpha \in \overline{\mathbb{Q}},\ 0 < |\alpha| < 1\}$ is algebraically independent. In this talk we discuss the algebraic independence of the values of the Hecke-Mahler series as well as its derivatives of any order at distinct nonzero algebraic numbers inside the unit circle.

Takaaki Tanaka

Algebraic independence properties of a certain map defined on the set of orbits of the action of Klein four-group

Abstract: In the previous work the speaker constructed a function of two variables taking algebraically independent values at any distinct points belonging to the cartesian product of the set of nonzero algebraic numbers and that of nonzero algebraic numbers inside the unit circle. In this talk the speaker introduces a balanced analogue of the two-variable function stated above, which is defined on a wider domain, the cartesian product of the set of complex numbers and that of complex numbers except the unit circle. The balanced function can also be regarded as a map defined on the set of orbits of the action of Klein four-group on the domain of definition of the balanced function and the latter map takes algebraically independent values at any distinct orbits represented by algebraic points.