My memories of Iwasawa

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Iwasawa 2017 was very meaningful for me. It was a celebration of the 100th anniversary of Iwasawa's birth. It was also a personal anniversary. I first met Iwasawa in the Fall of 1967, just 50 years before. I feel so fortunate that he accepted me as his student. I began to study the theory of Γ-extensions at that time, which we now would call the theory of $\mathbf{Z}_p$-extensions. Actually, Iwasawa adopted that terminology only a few years later. So much has changed in the intervening 50 years. When I was a graduate student, the subject was quite accessible. One could learn virtually everything that was known by studying a relatively small number of papers, mostly Iwasawa's own papers. This is far from true now. Just listening to the diverse lectures at Iwasawa 2017 was a dramatic reminder of how fruitful Iwasawa's ideas have turned out to be and how the subject has really blossomed over the past 50 years. That may be the most important meaning of Iwasawa 2017 for me. It has been a profound privilege for me to witness this blossoming and to be part of it to some extent.

When I was a graduate student, I often met Iwasawa at the afternoon tea in Fine Hall. We would chat briefly and then he would ask if I made any progress on the various questions that he had suggested for me to work on. At some point, I had made some progress on one of the questions. I had managed to find an example where I thought that the λ and μ invariants in his famous formula vanish, but the ν-invariant is nonzero. We went to his office to discuss the example and, with his help, it turned out to work. Iwasawa had actually asked me to look for an example of a totally real number field F and a prime p where the λ-invariant was positive (for what we now call the cyclotomic $\mathbf{Z}_p$-extension of F). I did manage to find quite a variety of examples where both λ and μ vanish and ν is positive, mostly examples where F is a real quadratic field. However, I never succeeded in finding an example where F is totally real and λ (or μ) is positive. There is probably a very good reason why I failed. It became a well-known conjecture that such examples don't exist. This topic became one chapter in my Ph.D.

thesis. I'm not certain what Iwasawa believed at the time or what he eventually came to believe about that question.

I often felt that Iwasawa was purposely not telling me everything that he knew about various questions that we discussed. I think that he wanted me to accomplish whatever I could on my own. One of the questions he suggested to me involved explicit reciprocity laws. I remember saying that I needed to read the existing literature on that topic, but Iwasawa advised me against that. His philosophy was that one might have a much better chance of discovering a fresh approach to something if one resisted the natural tendency to read everything about a topic. I tried to follow his advice to some extent, but I must admit that I did read Iwasawa's own papers quite thoroughly.

Iwasawa's lectures were beautiful. It was obvious that he prepared them carefully. He lectured without notes. I recall just one time when he glanced at a small piece of paper that he had in his pocket. He then put it back and continued his lecture. One of his courses was a rather complete introduction to the theory of Γ-extensions. I remember spending many evenings carefully rewriting my notes from his lectures. I learned a lot from doing that. Xeroxed copies of my handwritten notes were later circulated, but strangely I don't have my own copy. I have no idea how they disappeared.

I visited Iwasawa at his home in Princeton several times while I was a graduate student and rather regularly even afterwards. Mrs. Iwasawa would bring some tea and some sweets. Sometimes their dog zeta would come in. Our conversations were very relaxed. There were many moments of silence. Iwasawa would say something and then, after a pause, I might say something. I had a chance to visit Iwasawa at his home in Tokyo a few times. It was just like before. Their new house even reminded me of the one in Princeton. My last visit was just six months before Iwasawa passed away.

There was an outing during Iwasawa 2017 to the village where Iwasawa grew up and to the gravesite for him and his wife. It was a very moving experience. It is a Buddhist cemetery and all of us performed a traditional ritual to honor him.

I want to mention one more memory. I was spending a year at the Institute for Advanced Study just two years after finishing graduate school. Iwasawa's home in Princeton was very close to the Institute. My parents were visiting me one weekend. I was showing them around and,

purely by chance, we met Iwasawa and his wife. They were just out for a walk. It gives me a very good feeling that my parents and my teacher were able to meet each other.

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