

My Recollections of Kenkichi Iwasawa

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I arrived in Princeton as a young graduate student in the year 1980/81. Iwasawa had been on the faculty there since 1967, the year in which he moved to Princeton from MIT. He was to stay in Princeton for seven more years. In 1986 he retired, and a year later he and his wife moved back to Tokyo, but I think that they were contemplating this move already in 1981.

Princeton's Fine hall, in sharp contrast to the Science Center at Harvard, for example, is built vertically. This meant that social contact between new graduate students and the professors was often limited to tea and coincidental meetings in the elevators, unless you were an avid player of Go, or were babysitting for the children of Thurston. My lack of confidence and Iwasawa's shyness did not make it easier for me to get to know him in my first year. So, although I have been attending a wonderful class of his on Function Field Arithmetic, my first acquaintance with Iwasawa Theory was through Eduardo Friedman. Eduardo was a year ahead of me, the last student of Iwasawa (if I am not mistaken). In 1981, in his second year, he had already obtained the main results of his PhD, which were published a year later in *Inventiones*, an unusual accomplishment even for a bright student at Princeton. In addition, Eduardo had an outgoing personality, was friendly with the Iwasawas, and when he saw that I was interested in number theory, volunteered to give a series of introductory talks on Iwasawa theory. He lectured to me and to Toby Orloff, a student of Shimura, and I immediately fell in love with the subject. I remember being struck by how the very simple, but ingenious idea of considering class groups in a \mathbb{Z}_p -tower, lead to an asymptotic formula for the p -class number. I did not know enough about Tate modules of Jacobians, nor was I able to appreciate the Main Conjecture, or understand the work of Mazur and Wiles which had just been announced. But the little Eduardo had taught me convinced me that this was the direction I wanted to take, so at the end of my first year I approached Prof. Iwasawa with great timidity and asked if I could become his student.

Finding a thesis problem was not an easy task. Fortunately for me, John Coates was visiting Princeton in 1981 and gave a course on the arithmetic of elliptic curves. Iwasawa was attending it too, like a regular student, and suggested that I come to his office and “explain” Coates’ lectures. In his extreme modesty, Iwasawa pretended to be a novice in the field. He made me go very carefully through Tate duality, descent and Selmer groups. He taught me some very important lessons, which I hope I have kept to this day. For example, that it is not enough to understand the definition of a cohomology group. You have to befriend it: is it finite? Compact or discrete? What is its dual? How does it behave under restriction or corestriction? I still remember how embarrassed I was when I could not answer such questions on the spot and had to go back and think about them.

I was also lucky to have met Karl Rubin, who gave me his thesis to read. I still think that reading a good Ph.D. thesis is an excellent way to get introduced to a new area. Sometimes in the middle of my second year it became known that Wiles would come to Princeton the next year, and Iwasawa suggested to me to go and work with him. This is how I missed the opportunity to become Iwasawa’s last student, but I owe him a great deal for guiding me in my first steps.

In my last year at Princeton a small question that Iwasawa asked was responsible for my first paper, a 4-page note on relative Lubin-Tate groups. Iwasawa communicated it to the editors, and also taught me the basics of mathematical writing. I remember his comment that you should never start a sentence with a mathematical symbol, even if this symbol is the letter p in the word “ p -adic”.

I believe I have taken every course that Iwasawa had offered in my four years at Princeton. In 80/81 it was a course on Function Field arithmetic, including the Riemann Hypothesis and even a chapter on Tate’s elliptic curve. In 81/82 the topic was Classical Groups – finite Chevalley groups in particular. In 82/83 he gave two courses. In the Fall, a general course on formal groups, and in the Spring, a course on local class field theory via Lubin-Tate groups. And in 83/84 he went back to what has become known as “Tate’s thesis”, the study of zeta functions via harmonic analysis on the adèles. This was developed simultaneously and independently by Iwasawa and Tate around 1950. I think that Iwasawa was even invited to talk about it at the ICM in Cambridge.

In retrospect, these four courses were a panorama of Iwasawa’s earlier work. Some followed books that he had written – the book on function fields had not been translated yet from Japanese, and the one on local class field theory was about to appear in Oxford University Press.

Notably, none of the courses dealt with Iwasawa theory. I can only speculate that with so much happening in this area in the early 80's, with the works of Coates and Wiles, and Mazur and Wiles, he consciously made a decision to go back and review his older favorite topics.

Much has been said about Iwasawa's pedagogical insight and his pedantic style. His lectures were carefully prepared in advance, and he was writing notes every year from fresh. Larry Washington once lent me his copies of Iwasawa's hand-written notes, which I zeroxed and keep with me to this day. They are a masterpiece of exposition. His handwriting and blackboard techniques were phenomenal. He always started on the upper left corner, filling all the blackboards, then erasing carefully and starting all over again. But once, while erasing, he left a certain Lemma in the middle of one of the blackboards. He gave no hint why, and resumed writing as usual at the upper left corner. Everything was so carefully planned, that just as he was about to quote the Lemma, he reached the line where he had left it.

Iwasawa was an extremely modest and generous person, but when you spent time with him, you realized that he was also very warm and had a good sense of humor. He used to invite his students to tea at his home once or twice a year, and although I had switched to Wiles, to these social events I was still invited. That's how I met Mrs. Iwasawa. I remember Iwasawa mentioning, as a matter of fact, that one of his daughters was married to a mathematician by the name of Ihara, not taking any pride in the great mathematical achievements of his son-in-law. I also remember meeting Prof. Chowla at the Iwasawas home. Chowla was a great mathematician, but outside mathematics, especially after the death of his wife, relied on the help of his friends. At that time he was staying at the Institute for Advanced Studies, and I think that Iwasawa and his wife were helping him a lot.

From time to time I find myself reflecting on my memories of Iwasawa. I was lucky to have known him – an incredible figure, both as a mathematician and as a person.