# Real and Complex Dynamics of Hénon's maps

$25^{\text{th}}$ **(Mon)**

**• Sylvain Crovisier (Université Paris-Sud)**

**Renormalization of Hénon maps with zero entropy**

De Carvalho, Lyubich and Martens have built a renormalization for the (real) Hénon maps with very small Jacobian and described the boundary of the parameters with zero entropy. With E. Pujals and C. Tresser we extend some of these results up to Jacobian 1/4: any Hénon map with zero entropy can be renormalized. As a consequence, we also obtain a characterization of the sets of periods (answering a conjecture by Gambaudo and Tresser).

**• Sebastien Biebler (Université Paris-Est)**

**Polynomial automorphisms of $C^2$ with a wandering Fatou component and high emergence**

This is a joint work with Pierre Berger. We show that there exists a locally dense set of real polynomial automorphisms of $\mathbb{C}^2$ displaying a wandering Fatou component. The proof relies on a geometric model which allows us to show the existence of a real wandering domain for a dense set of parameters for every family in an open and dense set of $C^r$-families of surface diffeomorphisms inside the Newhouse domain. We also prove that the wandering domain has a historical behaviour, which allows us to solve last Taken's problem in the $C^\infty$ case and complement the work of Kiriki-Soma. Finally, we will present the concept of emergence due to Berger and we show that our example has high emergence.

$26^{\text{th}}$ **(Tue)**

**• Yutaka Ishii (Kyushu University)**

**Crossed mapping condition**

In this lecture, I present some combinatorial properties of the complex Henon maps satisfying the crossed mapping condition.

**• Anna Florio (Université Avignon)**

**Torsion for $C^1$ diffeomorphism of the 2-dimensional annulus**

For a $C^1$ diffeomorphism isotopic to the identity on the 2-dimensional annulus $f\colon A \to A$, the torsion, denoted as Torsion$(f)$, is the limit of the average rotational velocity of tangent vectors under the action of the linearized dynamical system. We will give the definition of torsion and then discuss some examples. We will study the relation between the map Torsion$(f, \cdot)\colon A \to \mathbb{R}$ and some dynamical properties of $f$.

**• Mao Shinoda (Keio University)**

**Construction of a locally constant function without zero temperature limit**

We consider the sequence of equilibrium measures in the context of symbolic dynamical systems. Parametrizing the equilibrium measures by temperature, we pay attention to the behavior of the sequence when the temperature drops to zero. More precisely, we discuss convergence and non-convergence. In the one-dimensional case, for every locally constant function the sequence of equilibrium measures converges. However in the high-dimensional case, there exists a locally constant function whose sequence of equilibrium measures does not converge. We construct such a locally constant function in dimension two by imbedding a one-dimensional effective subshift into

a two-dimensional subshift of finite type. This is a joint work with Jean-René Chazottes in École Polytechnique.

27th **(Wed)**

• **Romain Dujardin (Université Pierre et Marie Curie)**
**New results on the $J = J^*$ problem**

It is a major open problem whether for complex Hénon maps, the Julia set coincides with the closure of the set of saddle periodic orbits. In the talk we will present several new results pointing towards this conjecture.

• **Akira Shudo (Tokyo Metropolitan University)**
**Stokes geometry for the quantum Hénon map**

Stokes geometry for the propagator of the quantum Hénon map is studied in the light of recent developments of the exact WKB analysis. As a simplest possible situation the Hénon map satisfying the horseshoe condition is closely analyzed, together with listing up local bifurcation patterns of the Stokes geometry. Our analysis reveals that the birth and death of the saddle point solutions caused by the Stokes phenomenon do not occur in a local but entirely global manner, reflecting topological nature encoded in the Stokes geometry. We derive an explicit general formula to enumerate the number of saddle point solutions in the asymptotic region and obtain its growth rate, which is shown to be less than the topological entropy of the corresponding classical dynamics.

• **Noriaki Kawaguchi (University of Tokyo)**
**Shadowing and a chain relation**

In this talk, for any continuous self-map of a compact metric space, we introduce an equivalence relation on the chain recurrent set by using chains, and discuss its implications for the maps with shadowing properties. In particular, we show that if a continuous map with the limit shadowing property has positive topological entropy, then it exhibits a strong type of chaos.

28th **(Thu)**

• **Zin Arai (Chubu University)**
**On the monodromy and period doubling bifurcations of the Hénon map**

We discuss the structure of the parameter space of the Hénon family. Our main tool is the monodromy representation that assigns an automorphism of the full shift to each loop in the hyperbolic parameter locus of the complex Hénon map. It is shown that monodromy automorphisms must satisfy a certain algebraic condition, and since the monodromy carries the information of bifurcations taking place inside the loop, this imposes some geometric restrictions on the structure of the parameter space. It follows that period doubling sequences play special roles both in real and complex families of the Hénon map.

• **Thomas Gauthier (École Polytechnique)**
**The support of the bifurcation measure has positive volume**

The moduli space $\mathcal{M}_d$ of degree $d \geq 2$ rational maps can naturally be endowed with a measure $\mu_{\text{bif}}$ detecting maximal bifurcations, called the bifurcation measure. We prove that the support of the bifurcation measure $\mu_{\text{bif}}$ has positive Lebesgue measure. To do so, we establish a general criterion for the conjugacy class of a rational map to belong to the support of $\mu_{\text{bif}}$ and we exhibit a "large" set of Collet-Eckmann rational maps which satisfy that criterion. As a consequence, we get a set of Collet-Eckmann rational maps of positive Lebesgue measure which are approximated by hyperbolic rational maps.

• **Yuushi Nakano (Tokai University)**
**Emergence via non-existence of averages**

Inspired by a recent work by Pierre Berger, we introduce the concept of pointwise emergence. This concept provides with a quantitative perspective into the study of non-existence of averages for dynamical systems. We show that high pointwise emergence on a large set appears for abundant dynamical systems: There is a dense subset of any Newhouse open set each element of which has super-polynomial pointwise emergence on a positive Lebesgue measure subset of the state space. Furthermore, the full shift has super-polynomial pointwise emergence on a residual subset of the state space. This is a joint work with Shin Kiriki and Teruhiko Soma.

$29^{\text{th}}$ **(Fri)**
• **Hiroki Takahasi (Keio University)**
**Large deviation principles for countable Markov shifts**

A number of dynamical systems lacking the uniform hyperbolicity is modeled by countable Markov shifts. We establish the large deviation principle for a countable Markov shift which satisfies strong connectivity assumptions called "finite irreducibility" or "finite primitiveness". More precisely, we assume the existence of a Gibbs state for a potential $\phi$ in the sense of Bowen, and prove the level-2 large deviation principles for the distribution of empirical means under the Gibbs state, as well as that of weighted periodic points and iterated pre-images. The rate function is written with the pressure and the free energy associated with the potential $\phi$. A paper on this result is to appear in Trans. Amer. Math. Soc.

• **Mitsuhiro Shishikura (Kyoto University)**
**Fingers in the complex parameters spaces of Arnold family and Hénon family**

In the parameter space of complex Arnold family and complex Hénon family, a structure called fingers are observed near a parabolic parameter. We analyze the parabolic bifurcation and explain how such fingers are formed.

• **Masato Tsujii (Kyushu University)**
**Exponential mixing for volume-preserving Anosov flows**

In the talk, I would like to present my recent result that an open dense set of volume-preserving Anosov flows on a three dimensional manifold exhibit exponential decay of correlations. I will also explain the reason why the generalizations to non-volume-preserving cases are very subtle (and may not be true).