# Comparison of Multivariate Data Representations: Three Eyes are Better than One

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# Multivariate Data Representation

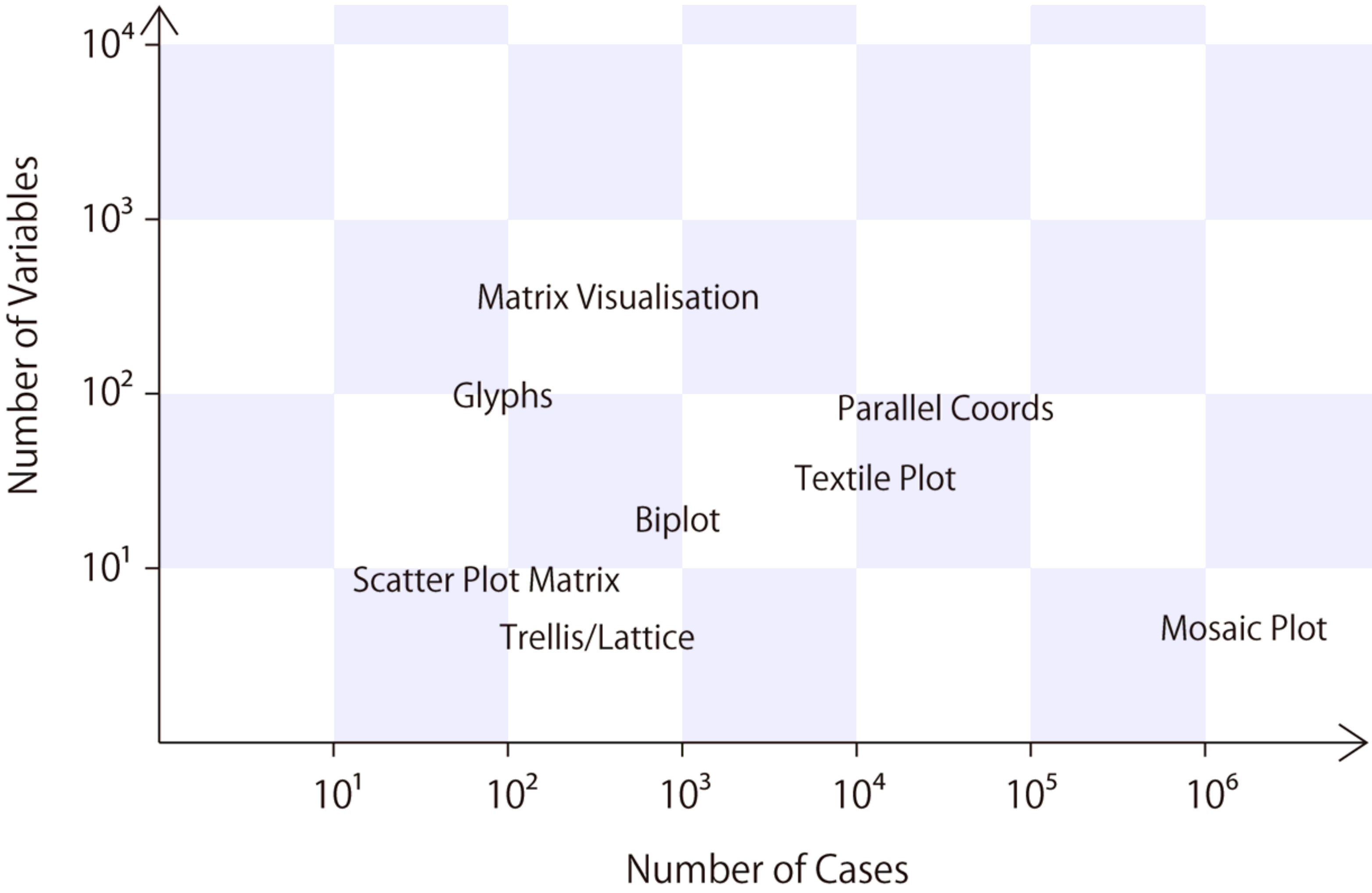
- **Numerical data**
  - Biplot (Gabriel 1971)
  - Scatter plot matrix (Cleveland 1984)
  - GGobi (Cook, D. et.al 2007)
  - Glyphs (Anderson 1957, Chernoff 1973, Fienberg 1979)
  - Parallel coordinate plot (Inselberg 1985, Wegman 1990)
  - Matrix visualization (Chen 2002)
- **Categorical data**
  - Mosaic plot (Hartigan 1981)
  - MANET (Unwin et.al. 1996)
- **Numerical/Categorical data**
  - Trellis/Lattice (Chambers 1992, Cleveland 1993)
  - Textile plot (Kumasaka and Shibata 2008)

- **General**
  - Graphics of Large Datasets (Unwin et.al. 2006)
  - Handbook of Data Visualization (Chen, et.al. 2008)

# CONTENT

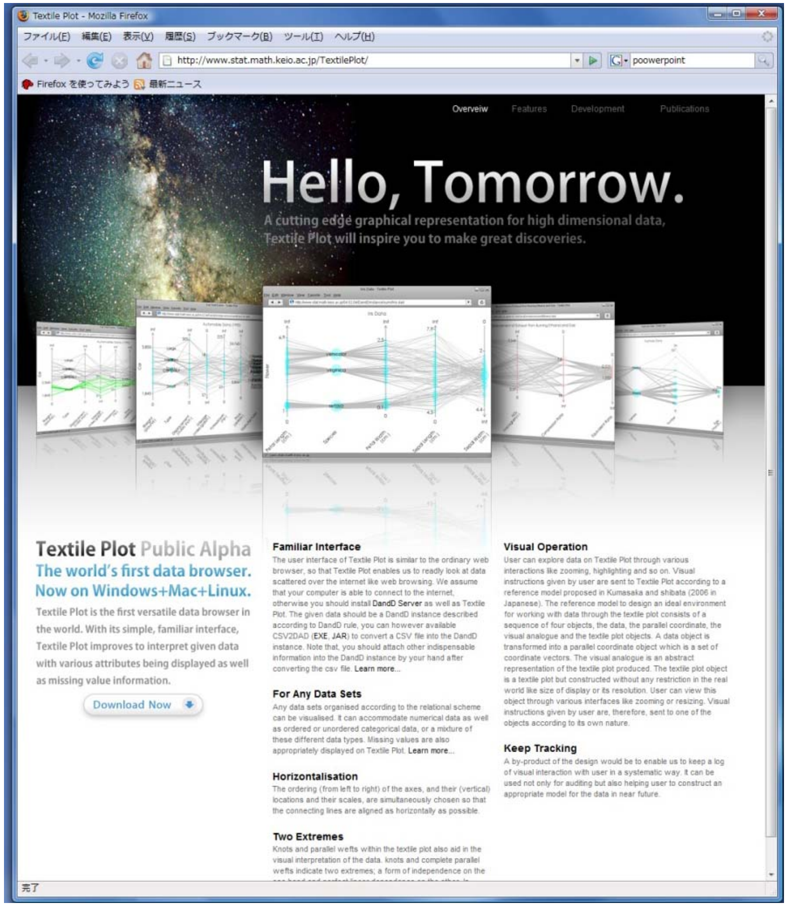
- 3 multivariate data representations
  - Parallel coordinate plot
  - Mosaic plot
  - Textile plot
- Visual data analysis
  - Decathlon data
  - Wine data
  - Animal data
  - Titanic data

# Capability of the Number of Cases and Variables on a Display



# TEXTILE PLOT (Kumaska and Shibata 2006, 2007, 2008)

- Parallel coordinate system
- Horizontalisation criterion
- Any type of data
- Order of Axes




http://stat.math.keio.ac.jp/TextilePlot/

# Transform Datavector into Coordinate Vector

$$\underset{n \times p}{\begin{pmatrix} \\ x_{ij} \\ \\ \end{pmatrix}} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_p) \quad \Rightarrow \quad \underset{n \times p}{\begin{pmatrix} \\ y_{ij} \\ \\ \end{pmatrix}} = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_p)$$

Data Vector $\boldsymbol{x}_j$ ($n$- dimensional)

Numerical

$$\boldsymbol{y}_j = \alpha_j \mathbf{1} + \beta_j \boldsymbol{x}_j$$

Non-numerical

$\mathbf{X}_j : n \times (q_j - 1)$ matrix

$q_j$: number of levels

contrasts

$$\mathbf{C} = \begin{pmatrix} 0 & \cdots & 0 \\ 1 & \ddots & \vdots \\ \vdots & \ddots & 0 \\ 1 & \cdots & 1 \end{pmatrix}$$

Unordered

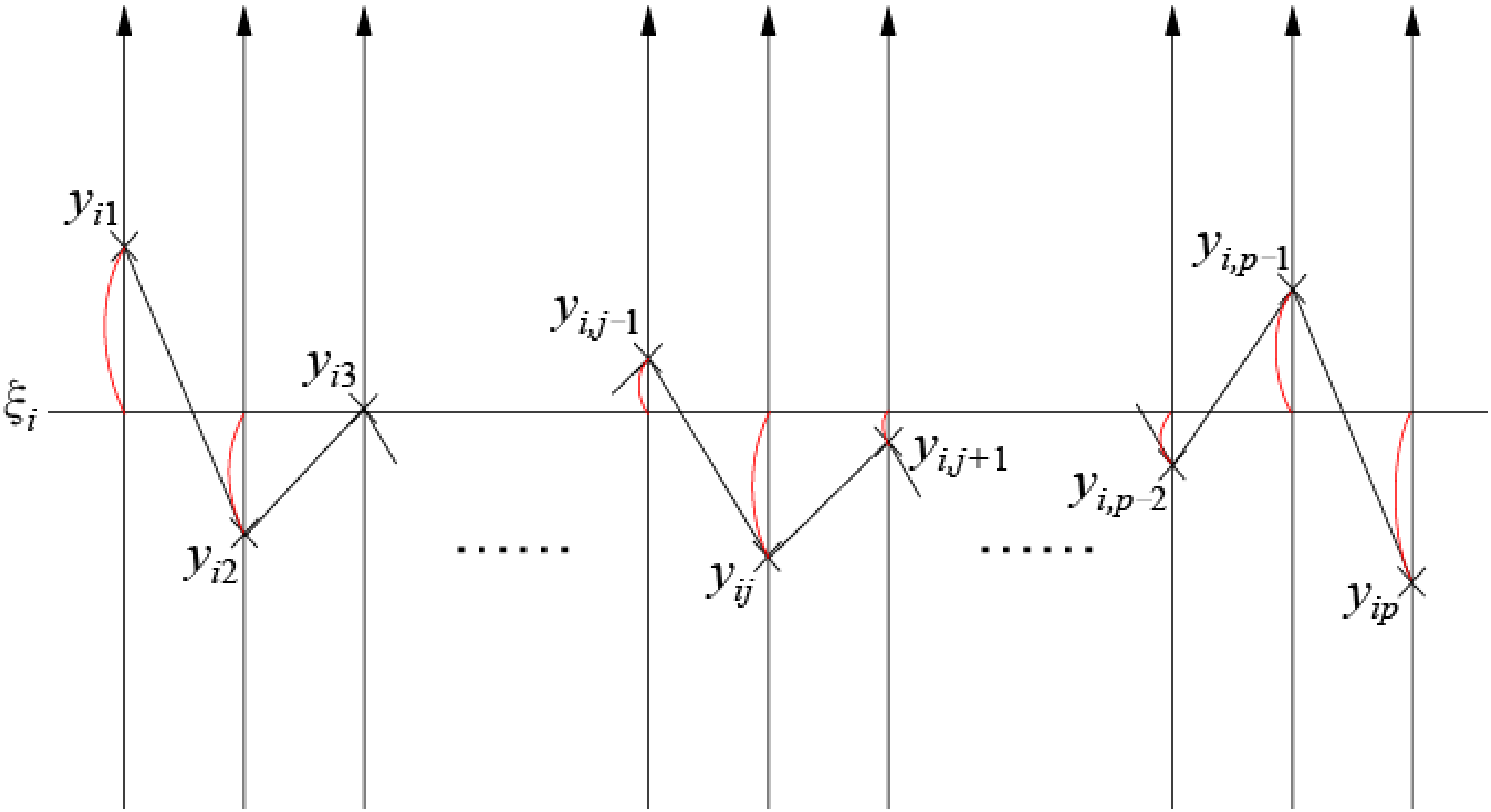
$$\boldsymbol{y}_j = \alpha_j \mathbf{1} + \mathbf{X}_j \boldsymbol{\beta}_j$$

Ordered

$$\boldsymbol{y}_j = \alpha_j \mathbf{1} + \mathbf{X}_j \boldsymbol{\beta}_j$$

$$\boldsymbol{\beta}_j \leq \mathbf{0} \text{ or } \geq \mathbf{0}$$

# HORIZONTALISATION CRITERION

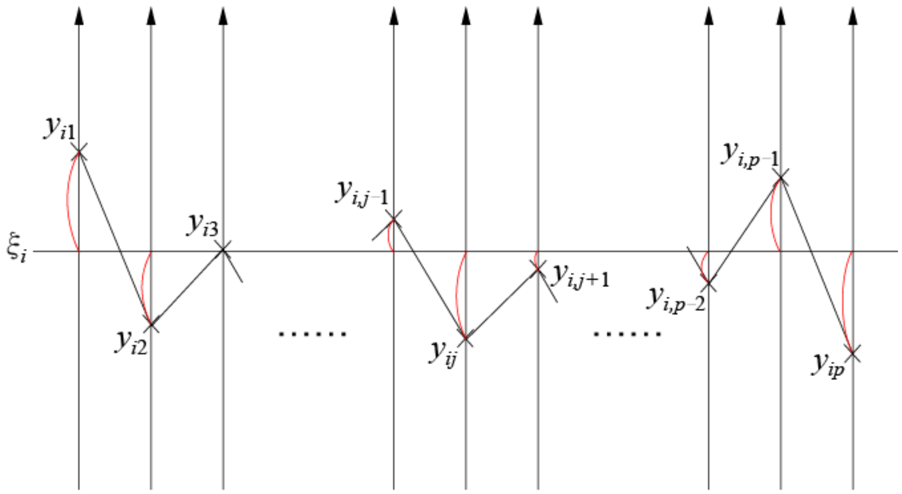



$$\sum_{j=1}^{p}(y_{ij}-\xi_i)^2$$

# Optimisation Problem

Minimise $$\sum_{i=1}^{n}\sum_{j=1}^{p}(y_{ij}-\xi_i)^2 = \sum_{j=1}^{p}\left\|\boldsymbol{y}_j-\boldsymbol{\xi}\right\|^2 \underset{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\xi}}{\rightarrow} \min$$

Subject to $$\sum_{j=1}^{p}\|\boldsymbol{y}_j-\bar{y}_{\cdot j}\mathbf{1}\|^2 = np$$




Location Parameter Vector

$$\boldsymbol{\alpha} = (\alpha_1,\ldots,\alpha_p)^T$$

Scale Parameter Vector

$$\boldsymbol{\beta} = (\beta_1,\ldots,\beta_p)^T$$

Ideal Coordinate Vector

$$\boldsymbol{\xi} = (\xi_1,\ldots,\xi_n)^T$$

# SOLUTION $(\mathbf{1}^T \boldsymbol{x}_j = 0 \text{ and } \|\boldsymbol{x}_j\| = 1;\ j = 1, \ldots, p)$

$$\sum_{j=1}^{p} \|\boldsymbol{y}_j - \boldsymbol{\xi}\|^2 = \sum_{j=1}^{p} \|\boldsymbol{y}_j - \boldsymbol{m}\|^2 + p\|\boldsymbol{m} - \boldsymbol{\xi}\|^2$$

$$\Rightarrow \quad \hat{\boldsymbol{\xi}} = \boldsymbol{m} = \frac{1}{p}\sum_{j=1}^{p} \boldsymbol{y}_j \qquad (\boldsymbol{m}:\ \text{mean vector})$$

$$\sum_{j=1}^{p} \|\boldsymbol{y}_j - \boldsymbol{m}\|^2 = \sum_{j=1}^{p} \|\boldsymbol{y}_j - \bar{y}_{\bullet j}\mathbf{1}\|^2 - p\|\boldsymbol{m} - \bar{y}_{\bullet\bullet}\mathbf{1}\|^2 + \sum_{j=1}^{p} \|\bar{y}_{\bullet j}\mathbf{1} - \bar{y}_{\bullet\bullet}\mathbf{1}\|^2$$

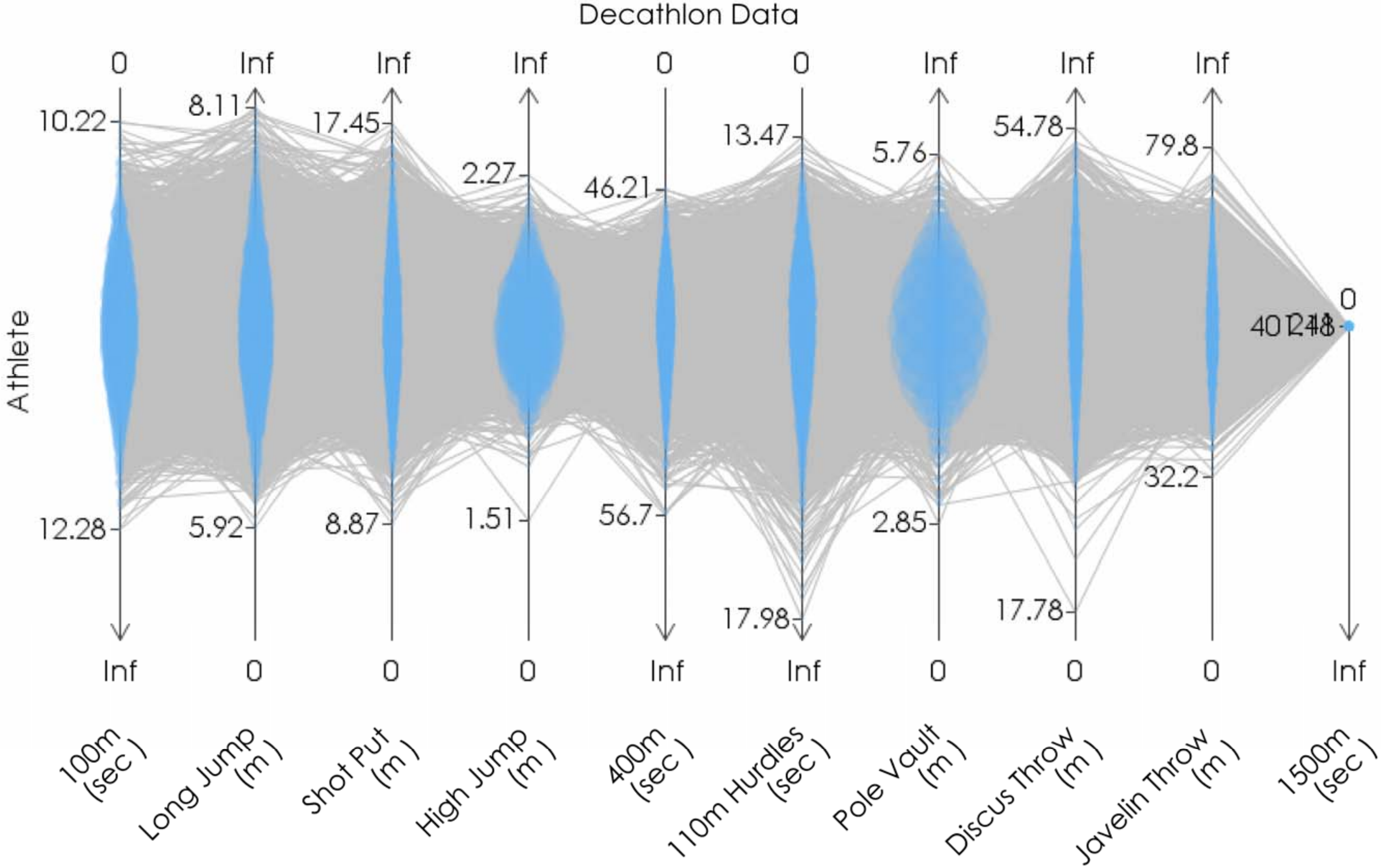
$$\Rightarrow \quad \hat{\alpha}_j = \alpha_0; \quad j = 1, \ldots, p \qquad (\bar{y}_{\cdot\cdot} = \textstyle\sum_{j=1}^{p} \bar{y}_{\cdot j}/p)$$

$$\sum_{j=1}^{p} \|\boldsymbol{y}_j - \bar{y}_{\bullet j}\mathbf{1}\|^2 - p\|\boldsymbol{m} - \bar{y}_{\bullet\bullet}\mathbf{1}\|^2 = \|\boldsymbol{\beta}\|^2 - \frac{1}{p}\boldsymbol{\beta}^T\mathbf{R}\boldsymbol{\beta}$$

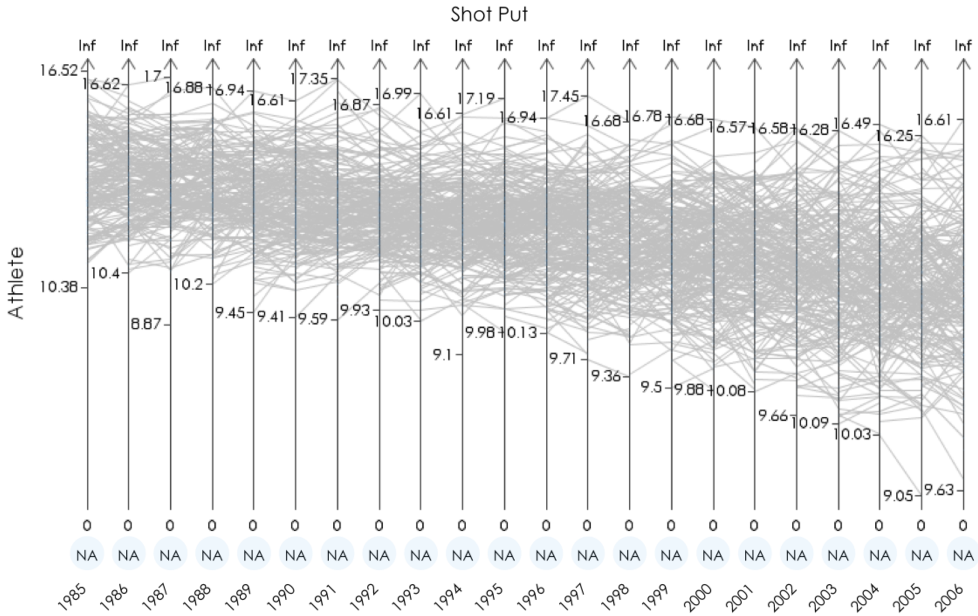
$\Rightarrow$ $\hat{\boldsymbol{\beta}}$: Eigenvector of sample correlation matrix $\mathbf{R}$ with the largest eigenvalue, such that $\|\hat{\boldsymbol{\beta}}\|^2 = np$.

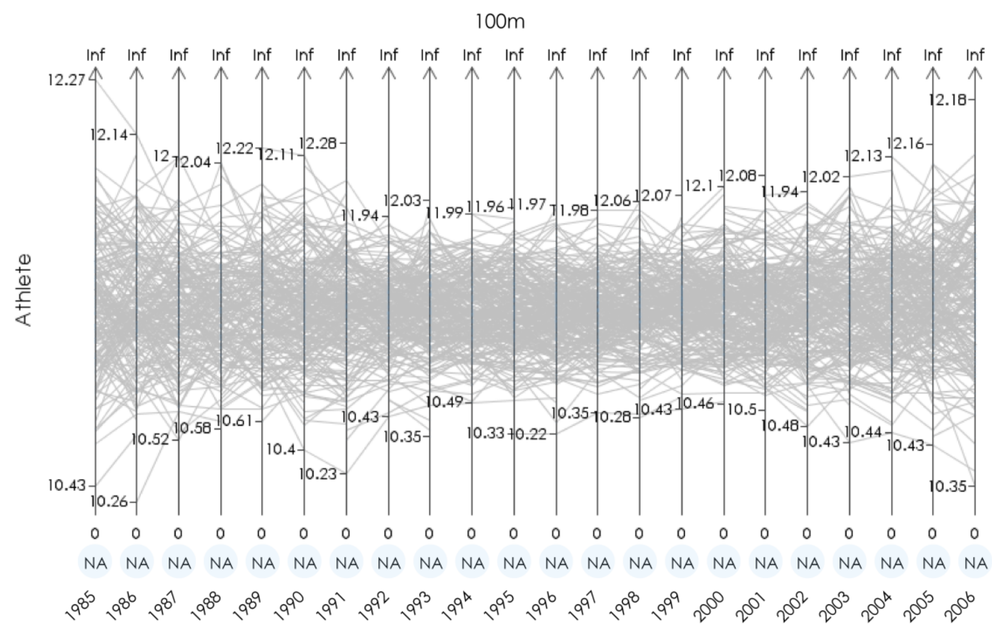
$(\Sigma_{j=1}^{p} \|\boldsymbol{y}_j - \bar{y}_{\bullet j}\mathbf{1}\|^2 = \|\boldsymbol{\beta}\|^2 = np)$

# TEXTILE PLOT OF DECATHLON DATASET (PERFORMANCES NOT POINTS)



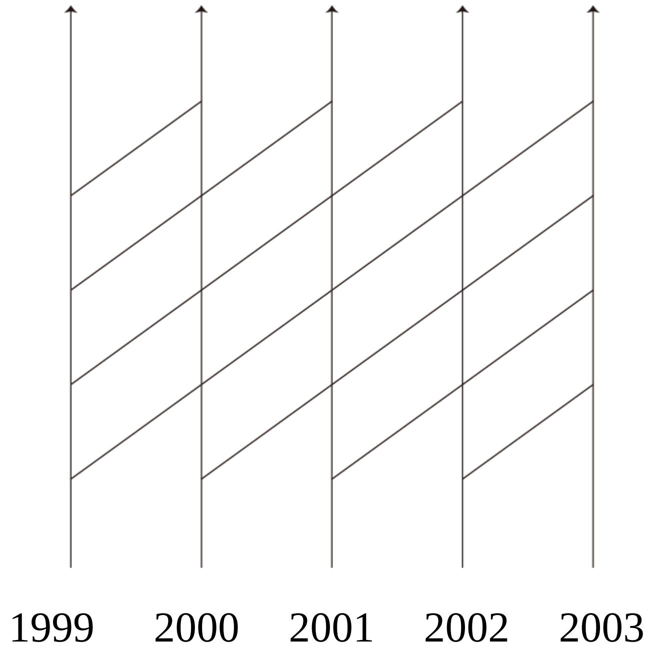
# Re-organised Decathlon Dataset



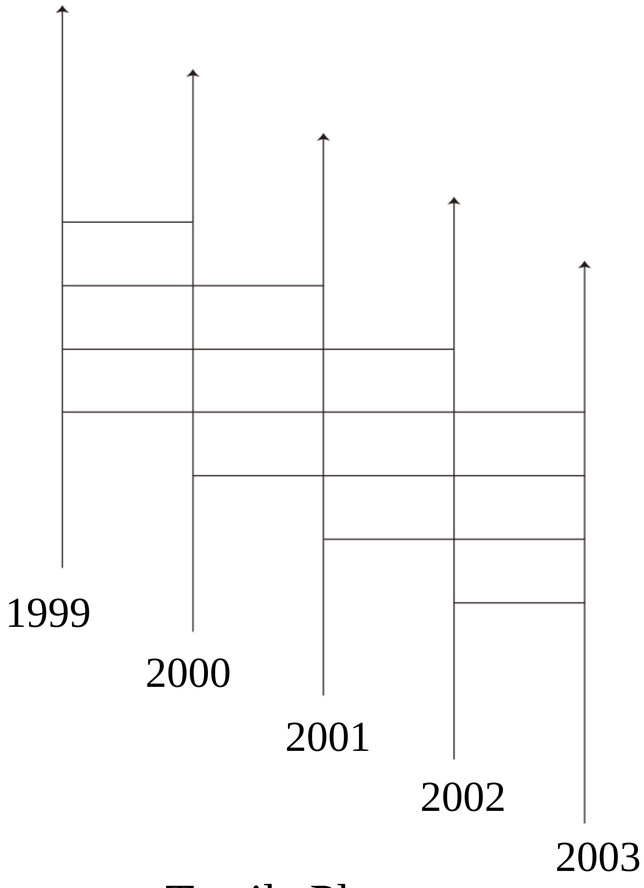
# Trend on Textile Plot

Assumption: All athletes improve their performances year by year, and stop their careers at their peak.




Parallel Coordinate Plot
with Common Scaling

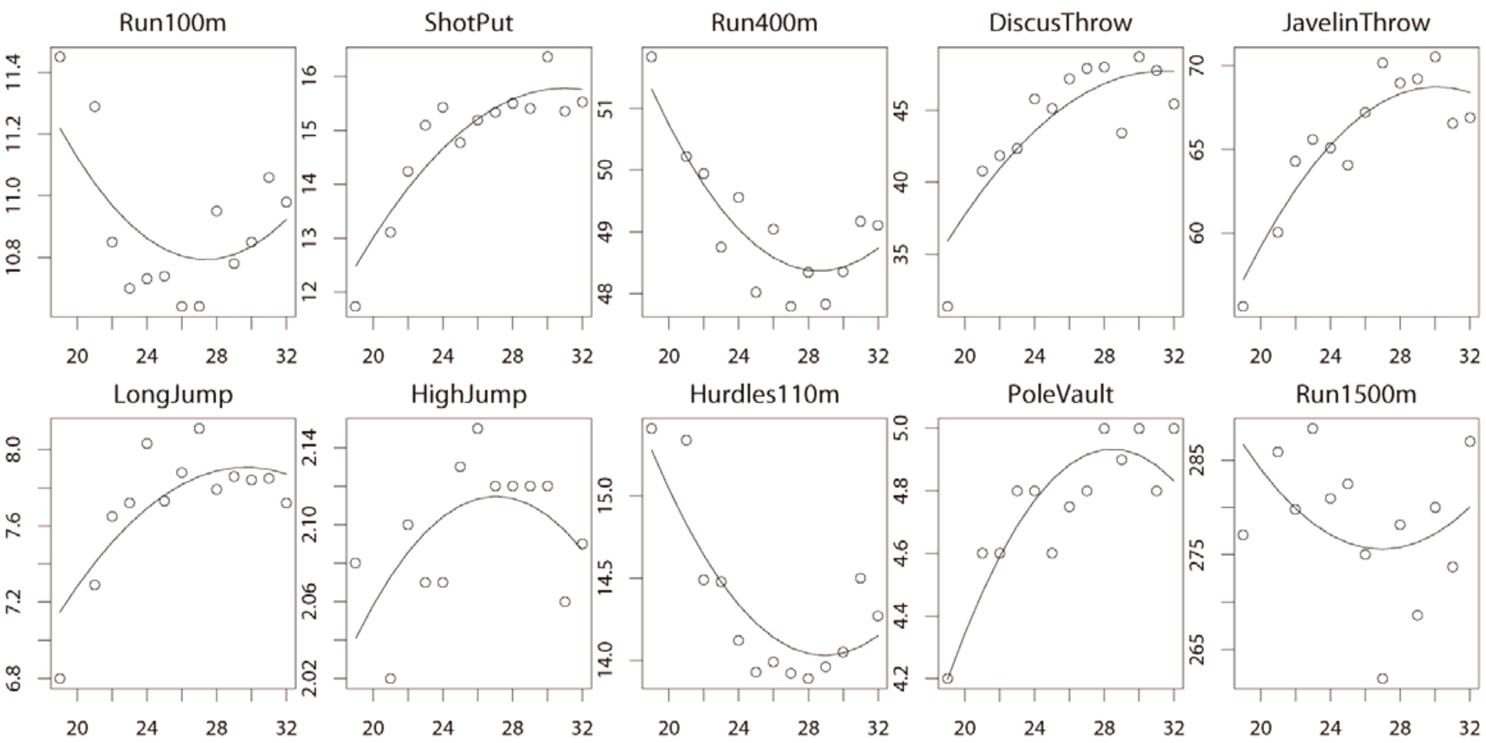
Horizontalisation




Textile Plot

# FURTHER CONFIRMATORY DATA ANALYSIS


- Performances of Mr. Roman Sebrle (best record holder)




$$Performance_{ik} = \alpha_{ik} + \beta_k (Age_i - \gamma_{ik})^2 + \varepsilon_{ik},$$

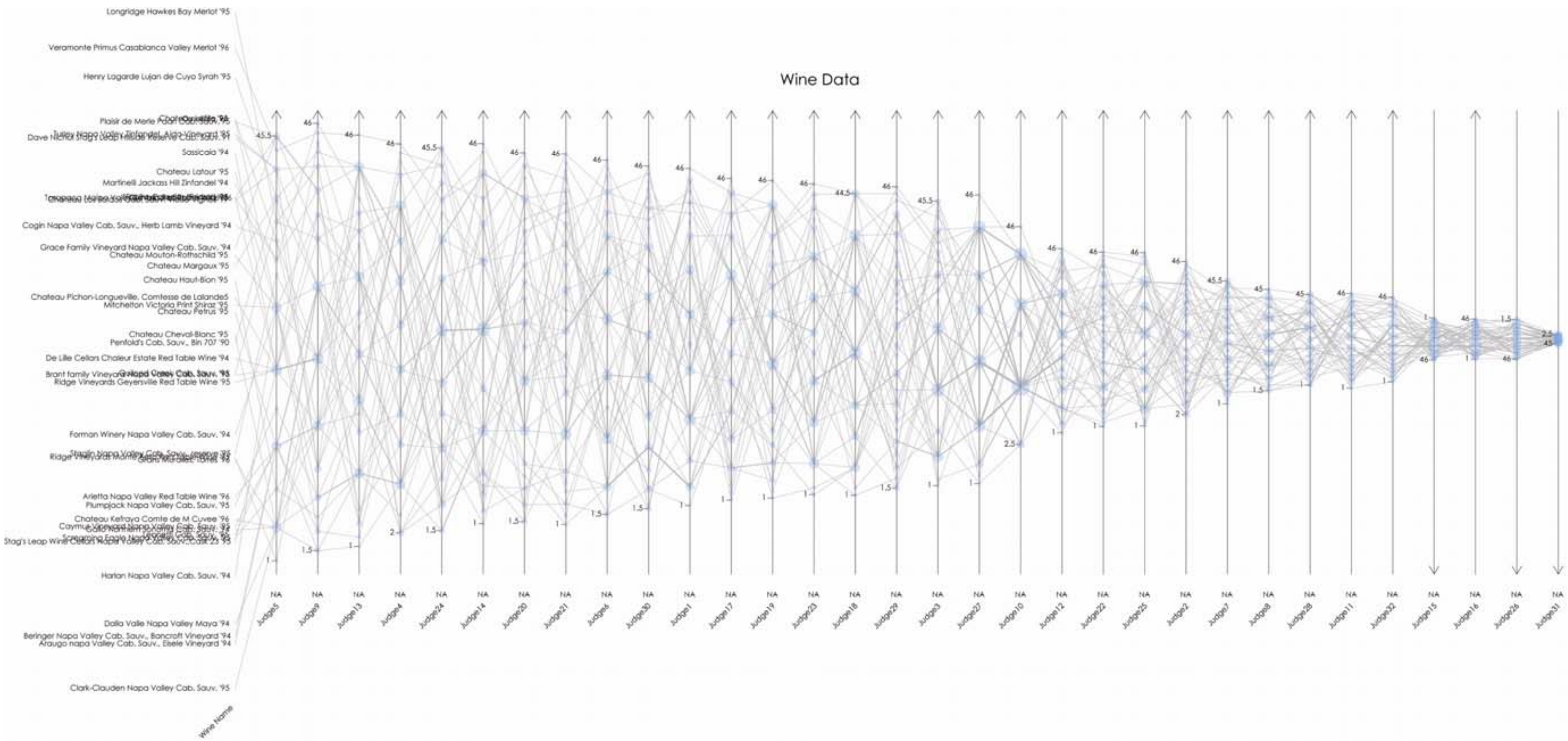
$i$: athlete, $k$: event

# Wine Dataset (Liquid Assets: www.liquidasset.com)

- Cabernet challenge 1999
- Case
  - 47 (only 46 rated) Cabernet Sauvignons: 34 US, 9 French, 2 Italian, 2 others
  - Vintages from 1994 to 1996
- Variable
  - 33 judges (Californian) ranked the wines

- Analysis goals:
  - Which wines were rated best?
  - Is the ranking of wines clear-cut?
  - Do the judges have similar opinions?
  - Are there clusters of judges?

# TEXTILE PLOT OF WINE DATASET



## Graphics for Multivariate Numerical Data

- Textile plot or parallel coordinate plot gives an overview of the data
- Re-organisation of data is always useful to know another aspects of the data
- Textile plots suggest potential avenues for subsequent further confirmatory data analysis

# TRANSFORM DATAVECTOR INTO COORDINATE VECTOR

$$\underset{n \times p}{\begin{pmatrix} \\ x_{ij} \\ \\ \end{pmatrix}} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_p) \quad \Rightarrow \quad \underset{n \times p}{\begin{pmatrix} \\ y_{ij} \\ \\ \end{pmatrix}} = (\boldsymbol{y}_1, \ldots, \boldsymbol{y}_p)$$

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$$\mathbf{C} = \begin{pmatrix} 0 & \cdots & 0 \\ 1 & \ddots & \vdots \\ \vdots & \ddots & 0 \\ 1 & \cdots & 1 \end{pmatrix}$$

Unordered

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Ordered

$$\boldsymbol{y}_j = \alpha_j \mathbf{1} + \mathbf{X}_j \boldsymbol{\beta}_j$$

$\boldsymbol{\beta}_j \leq \mathbf{0}$ or $\geq \mathbf{0}$

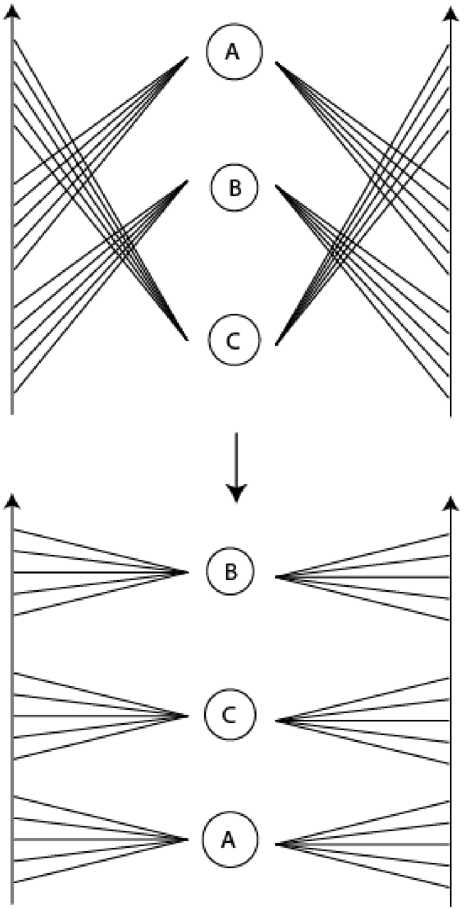
# Categorical Datavector

- Coordinate vector transformation
  - Determine optimised default position for each level
  - Introduction of a set of contrasts

Ex: Using a *treatment* contrast
$$\begin{array}{c} A \\ B \\ C \end{array} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\boldsymbol{x} = \begin{pmatrix} A \\ A \\ B \\ C \\ C \end{pmatrix} \Rightarrow \mathbf{X} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

Coordinate vector:
$$\boldsymbol{y} = \alpha\mathbf{1} + \mathbf{X}\boldsymbol{\beta} = \begin{pmatrix} \alpha \\ \alpha \\ \alpha + \beta_1 \\ \alpha + \beta_2 \\ \alpha + \beta_2 \end{pmatrix}$$



# ORDERED CATEGORICAL DATAVECTOR

- Order of levels are retained on the axis
  - Introduction of the specific contrasts
  - Additional constraints

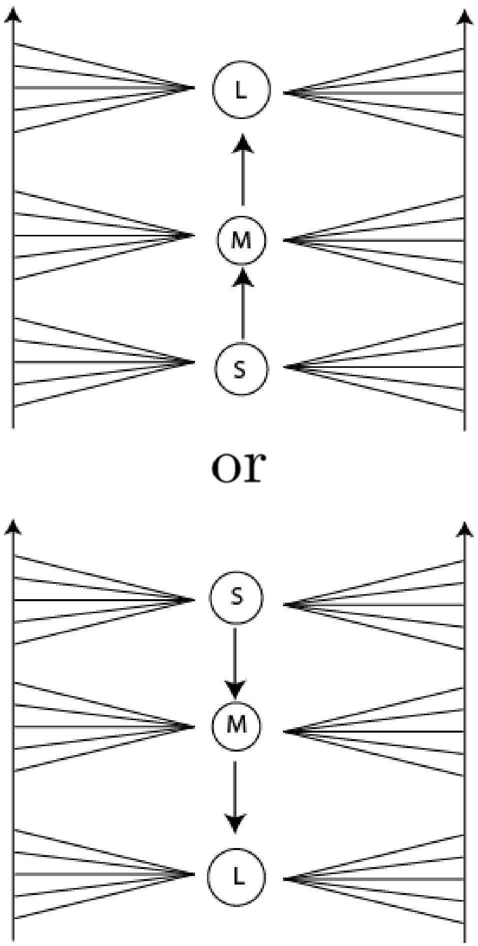
Ex:
$$\begin{array}{l} Small \\ Medium \\ Large \end{array} \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\boldsymbol{x} = \begin{pmatrix} Small \\ Small \\ Medium \\ Large \\ Large \end{pmatrix} \Rightarrow \mathbf{X} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Coordinate vector:

$$\boldsymbol{y} = \alpha\mathbf{1} + \mathbf{X}\boldsymbol{\beta} = \begin{pmatrix} \alpha \\ \alpha \\ \alpha + \beta_1 \\ \alpha + \beta_1 + \beta_2 \\ \alpha + \beta_1 + \beta_2 \end{pmatrix}$$

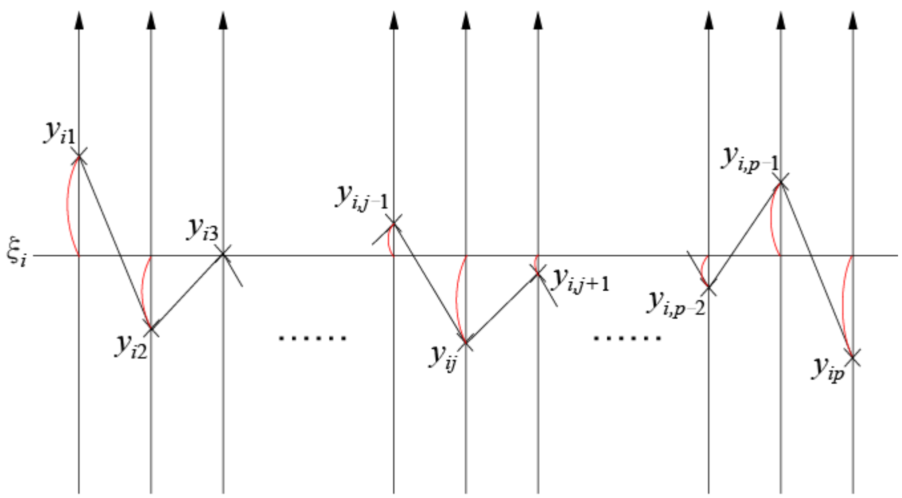
$$\beta_1, \beta_2 \geq 0 \quad \text{or} \quad \beta_1, \beta_2 \leq 0$$



# Optimisation Problem

Minimise $$\sum_{i=1}^{n}\sum_{j=1}^{p}(y_{ij}-\xi_i)^2 = \sum_{j=1}^{p}\left\|\boldsymbol{y}_j-\boldsymbol{\xi}\right\|^2 \underset{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\xi}}{\longrightarrow} \min$$

Subject to $$\sum_{j=1}^{p}\|\boldsymbol{y}_j-\bar{y}_{\cdot j}\mathbf{1}\|^2 = np \quad (\boldsymbol{\beta}_j \geq \mathbf{0} \text{ or } \boldsymbol{\beta}_j \leq \mathbf{0})$$




Location Parameter Vector
$$\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_p)^T$$

Scale Parameter Vector
$$\boldsymbol{\beta} = (\boldsymbol{\beta}_1^T, \ldots, \boldsymbol{\beta}_p^T)^T$$

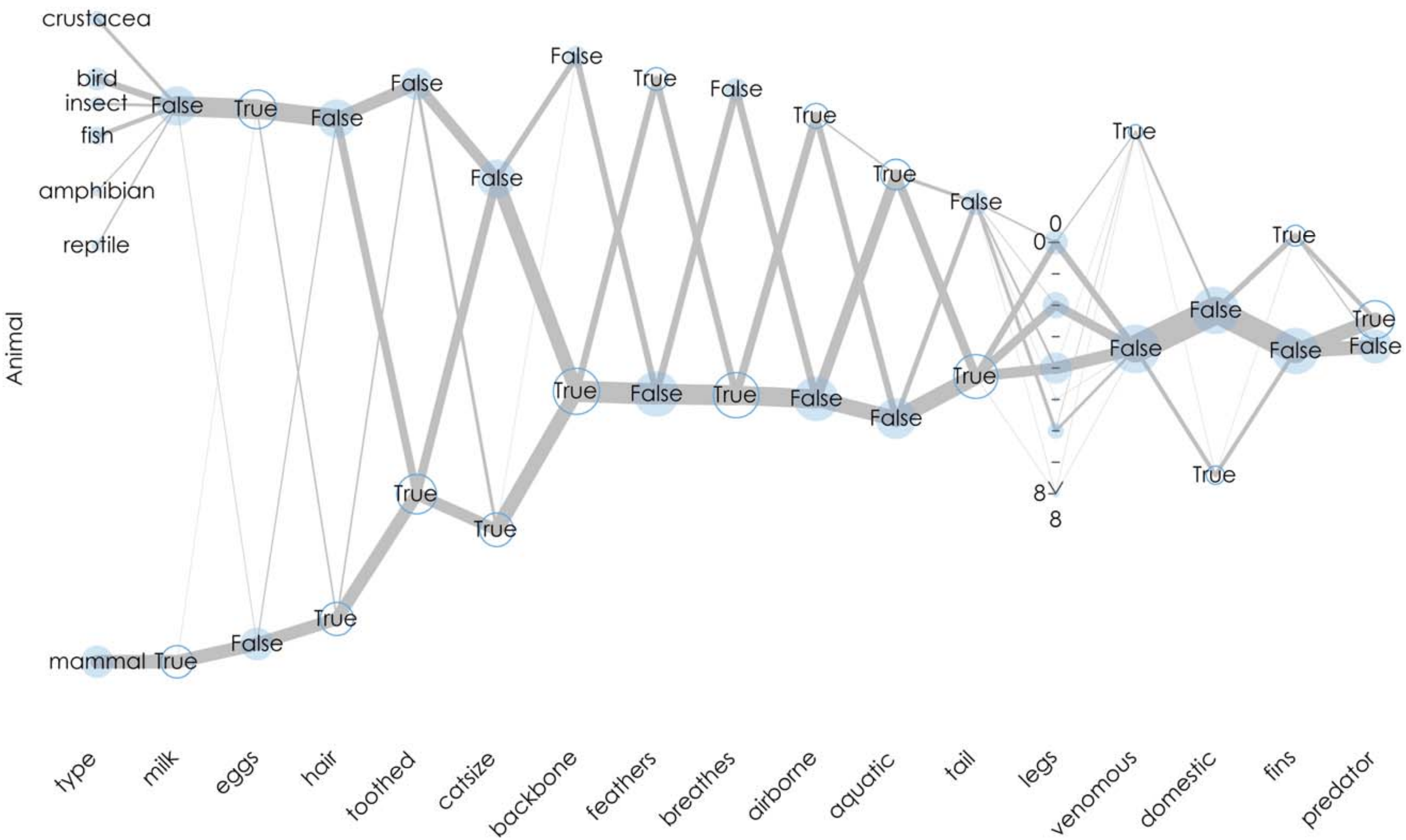
Ideal Coordinate Vector
$$\boldsymbol{\xi} = (\xi_1, \ldots, \xi_n)^T$$

# Animal Data (UCI Machine Learning Group 2008)

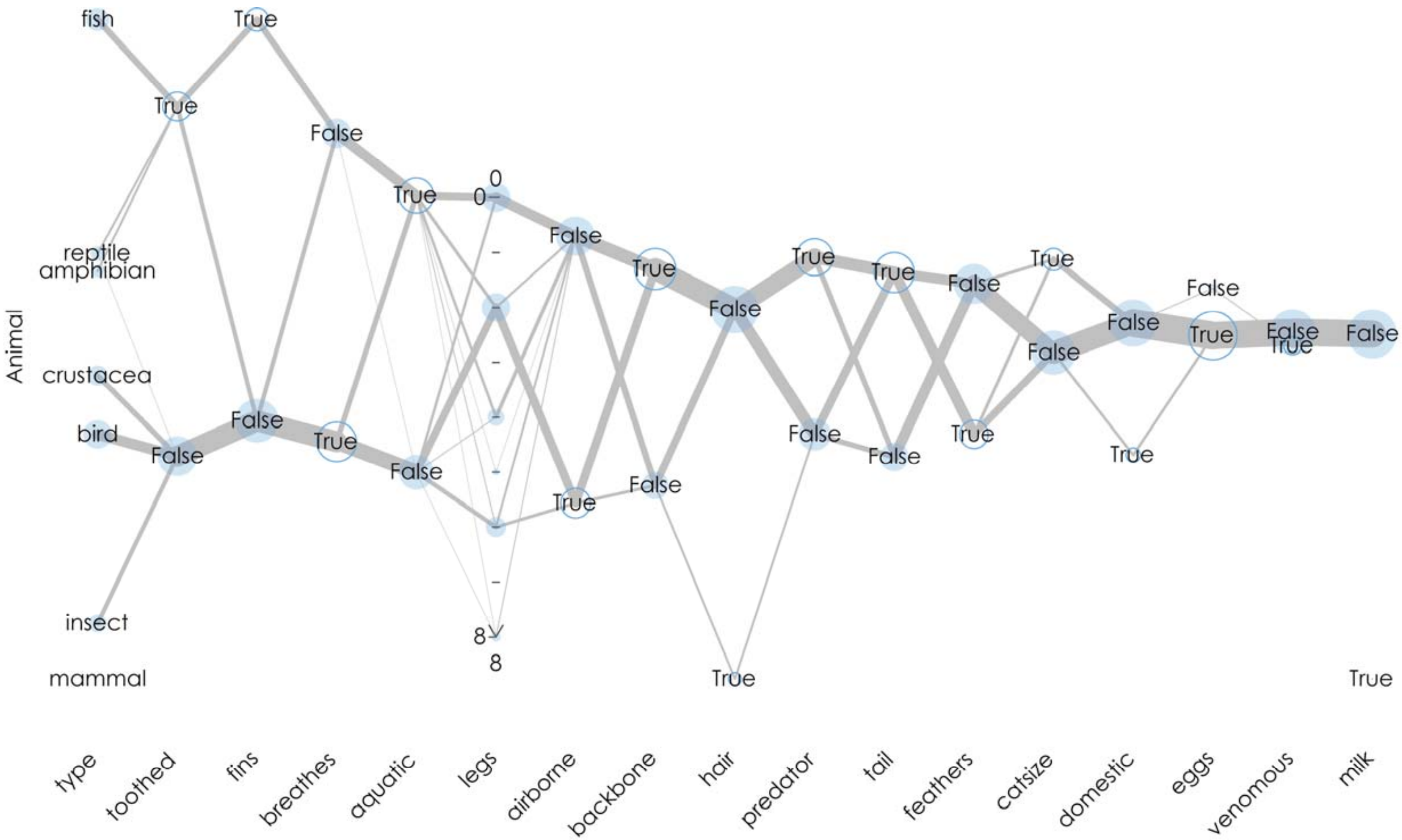
- Case
  - 101 animals
  - Invalid cases
    - Two frogs
    - Girl?
- Response
  - Animal type
    - Mammal
    - Reptile (爬虫類)
    - Amphibian (両生類)
    - Fish
    - Insect
    - Bird
    - Invertebrate (無脊椎動物)

- 16 Covariates (binary)
  - Hair/Feathers/Eggs/Milk/Airborne/Aquatic/Predator/Toothed/Backbone/Breathes/Venomous/Fins/Legs/Tail/Domestic/Cat-size

- Analysis goals:
  - What features best classify animals by type?
  - How are the features related?



Animal
crustacea
bird
insect
fish
amphibian
reptile
mammal
False
True
0
8
type
milk
eggs
hair
toothed
catsize
backbone
feathers
breathes
airborne
aquatic
tail
legs
venomous
domestic
fins
predator


Animal
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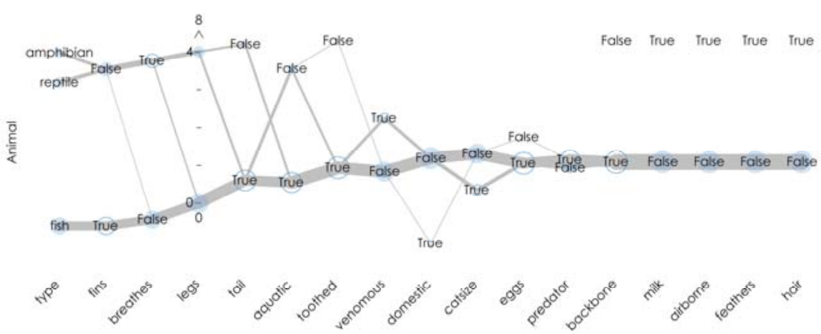
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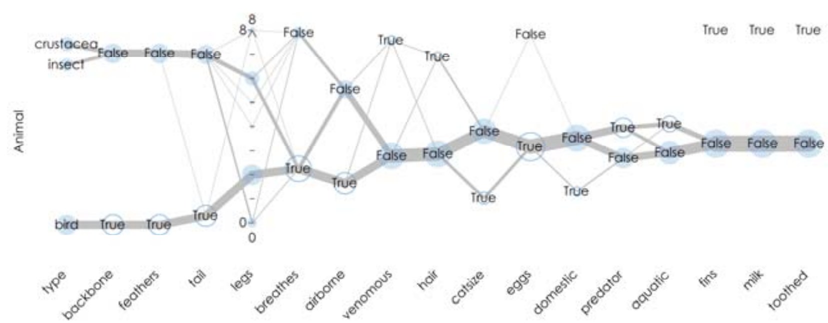
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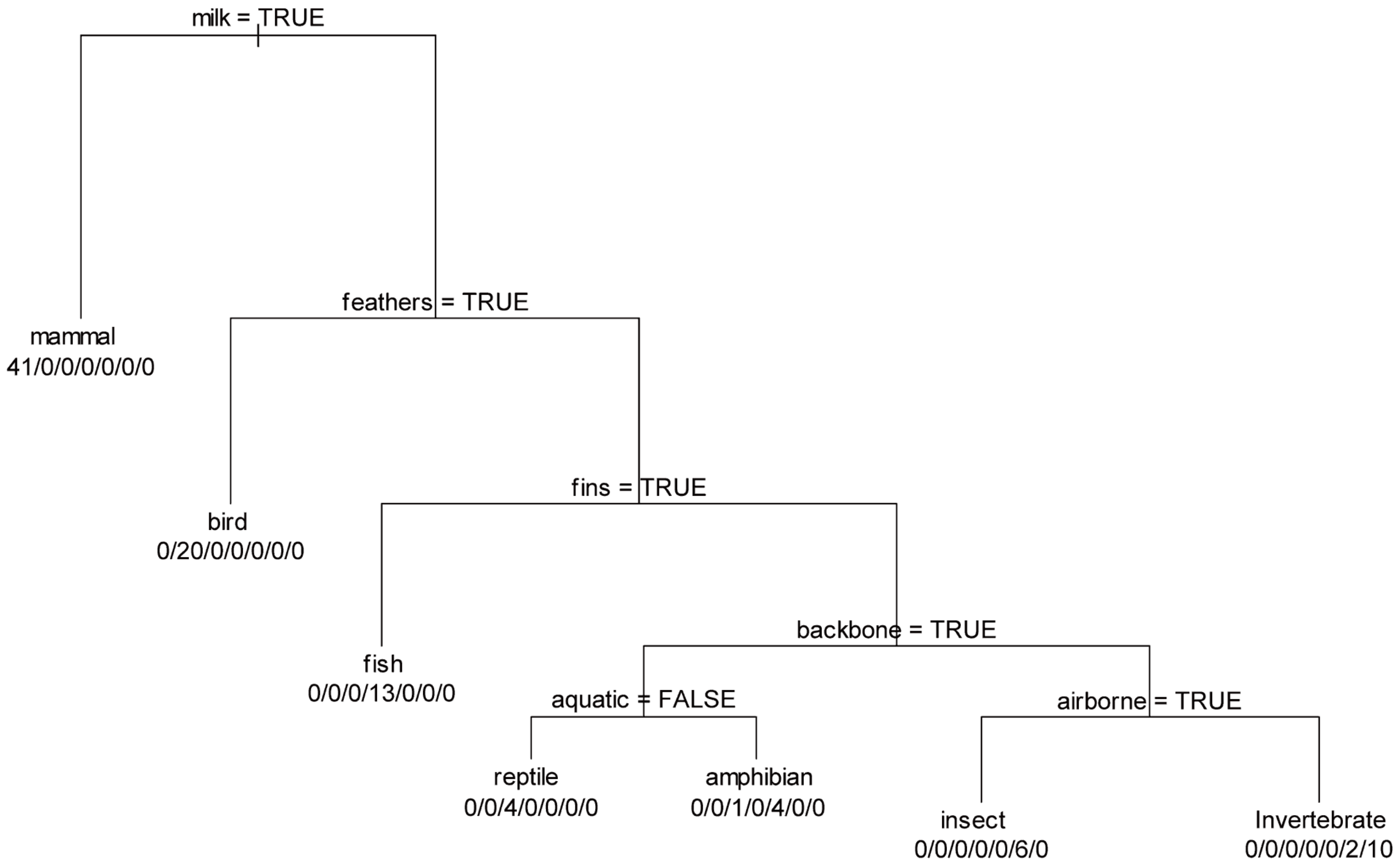
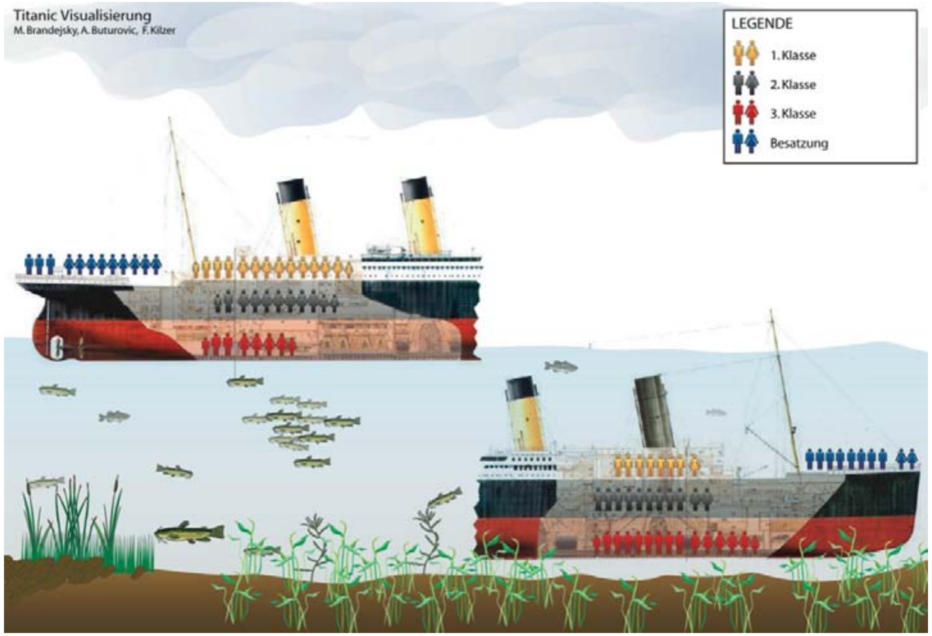
predator
aquatic
fins
milk
toothed

# Relation to Partitioning Tree



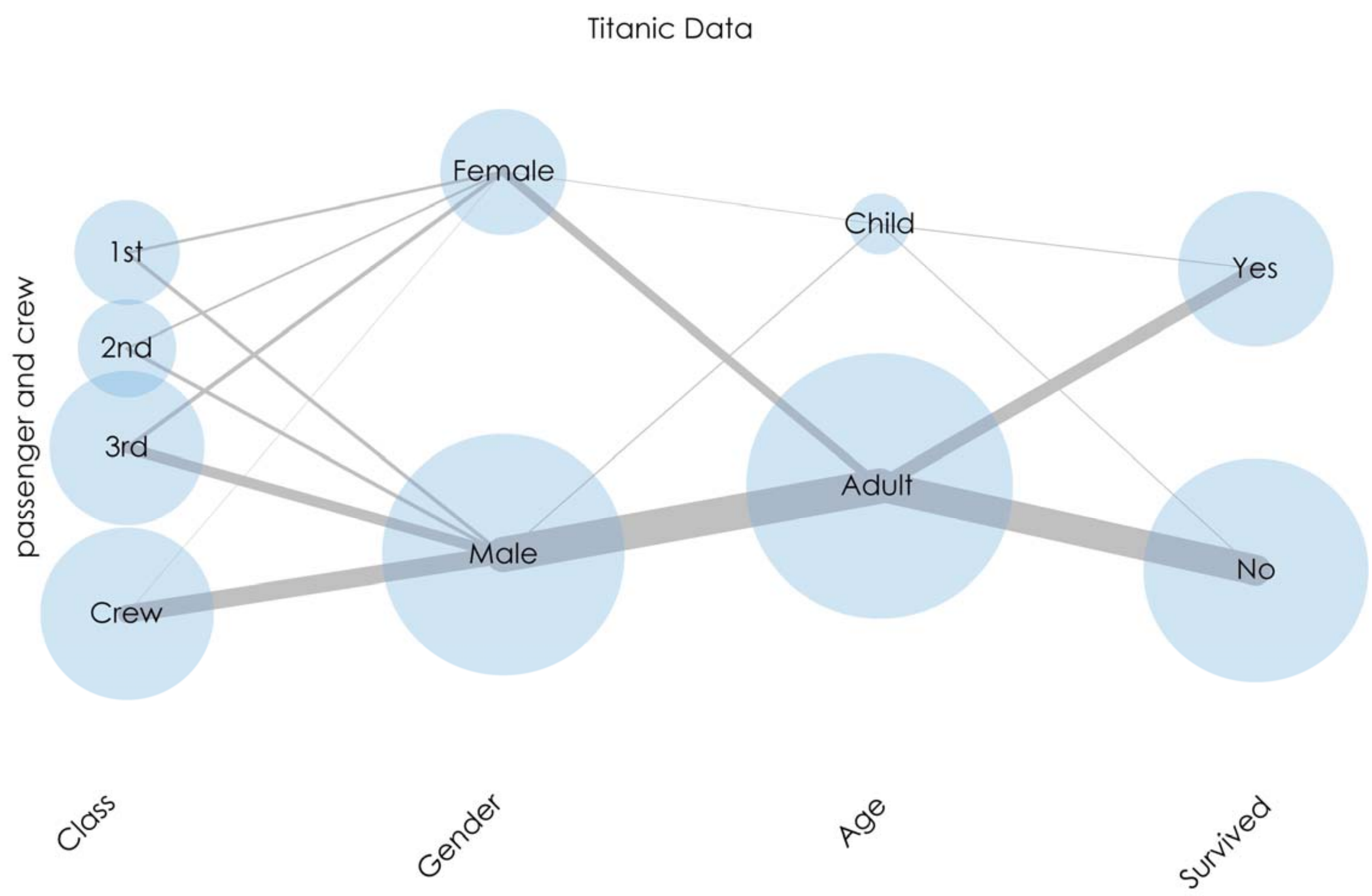
# Titanic Dataset (British Board of Trade 1990)

- Case
  - 2201 passengers and crew
- Variable
  - Class (First, Second, Third, Crew)
  - Age (Young, Old)
  - Gender (Male, Female)
  - Survived (Yes, No)




http://eagereyes.org/

# TEXTILE PLOT OF TITANIC DATASET

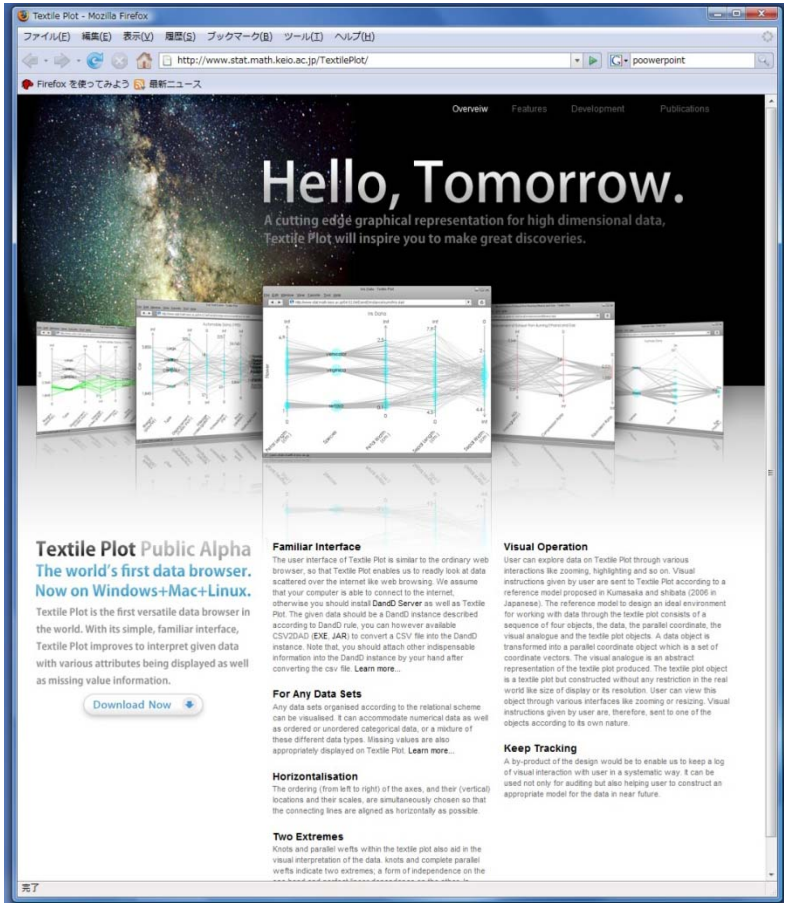


# Summary: Textile plot for Multivariate Categorical Datasets

- Textile plots provide rough idea of classifying cases
- Textile plots of multivariate categorical data emphasise absolute numbers
- Detailed conditional probability is difficult to interpret

# SOFTWARE: TEXTILE PLOT ENVIRONMENT

- Network ready
  - Based on DandD Client Server System (Yokouchi and Shibata 2004)
- Cross-platform
  - JAVA JRE 1.5
- Interactive user interfaces
  - Reference model (Kumasaka and Shibata, 2007)




http://stat.math.keio.ac.jp/TextilePlot/

# Conclusions

- Textile plot
  - Show an overview of the given data in an optimal way
  - Accommodate numerical and categorical data
  - Suggest several avenues for further exploratory or confirmatory data analysis
- Three eyes are better than one
  - Further investigations should be carried out with other graphical representations

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