An analysis of tax revenue forecast errors

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Outline

1. Background
2. Treasury's tax forecasting process
3. Towards a model framework
4. Forecast error decompositions
5. Data analysis
6. Conclusions

1. Background

Treasury tax revenue forecasts have persistently underestimated actual tax flows over the past 6 years.

Objectives

- To better determine the major sources of tax revenue forecast errors; and
- to identify the potential for methodological improvements.

Comments

- Study builds on the review Schoefisch (2005).
- An IMF study showed that Treasury's tax forecasting performance (1995-2003) compared well with agencies in other countries.
- New Zealand is not unique in terms of the persistent underestimation of tax. Other countries are also reviewing their procedures.
- The literature on tax revenue forecasting is sparse and mainly the preserve of official government agencies, IMF etc. Very little in the academic literature.

2. Treasury's tax forecasting process

- Similar to that used in other countries.
- Based on rating up past tax revenues by growth rates in related macroeconomic variables such as GDP which also need to be forecast.
- Spreadsheet-based modelling rather than statistical modelling.

Example

For general income tax, nonlinear models such as

$$G_q = G_{q-4}\left(1 + \frac{E_q - E_{q-4}}{E_{q-4}}\right)\left(1 + 1.2\frac{W_q - W_{q-4}}{W_{q-4}}\right)$$

have been adopted where E_q denotes total employment, W_q total salaries, and q indexes quarters.

Other tax types are modelled similarly.

Notes

- Within each tax type, models have been modified over years (no one tax type model).
- Forecasts further modified by judgemental factors.

3. Towards a model framework

Treasury's forecasting procedures suggest multiplicative models. A simple example is

$$Y_t = \alpha X_t^{\beta} e_t$$

where

$$Y_t = \text{tax revenue}, \qquad X_t = \text{macro predictor such as GDP}$$

and e_t is multiplicative error with $E(e_t) = 1$.

X_t^{β} can be thought of as a proxy for the relevant tax base. Then above is a tax model with α interpreted as a mean tax rate.

Many other variants possible.

In terms of growth rates

$$\Delta \log Y_t = \beta \Delta \log X_t + \epsilon_t$$

where now β is an elasticity and the ϵ_t are additive errors (possibly stationary) with $E(\epsilon_t) = 0$.

Given X_t and independent ϵ_t, the best predictor of $\Delta \log Y_t$ is

$$\Delta \log \hat{Y}_t = \beta \Delta \log X_t$$

yielding

$$\hat{Y}_t = \hat{Y}_{t-1}(1 + \beta \frac{X_t - X_{t-1}}{X_{t-1}})$$

since

$$\Delta \log Y_t = \log Y_t - \log Y_{t-1} \approx \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

to a good approximation. Treasury's tax forecasting procedures can now be seen as optimal for such multiplicative models.

The linkage between a model and its forecast function is far from unique. However, the simplicity of

- these models, and
- the corresponding growth rate models

make this a suitable model framework within which the Treasury methods can be embedded.

This is the strategy that has beeen adopted here.

4. Forecast error decompositions

To better understand the source and nature of Treasury's tax revenue forecast errors, the following forecast error decompositions were developed.

- The disaggregation of total tax revenue forecast errors into component tax types.

- The decomposition of individual tax revenue forecast errors into a component due to forecasting the macro predictor (proxy tax-base) and a component due to forecasting the ratio of tax revenue to proxy tax-base (tax ratio).

- The decomposition of the (tax ratio) forecast errors into a trend measuring the underlying mean tax rate and a random error component.

Need a benchmark model to help with the last two.

Benchmark model

The taxation process suggests the simple structural model

$$Y_t = R_t X_t \qquad R_t = \alpha_t e_t$$

with multiplicative errors e_t, $E(e_t) = 1$. The observed tax ratio

$$R_t = \frac{Y_t}{X_t}$$

has a mean tax rate α_t, called the tax ratio trend, which is assumed to evolve smoothly over time.

Taking logarithms yields the additive model

$$\log Y_t = \log R_t + \log X_t \qquad \log R_t = \log \alpha_t + \epsilon_t$$

where $\epsilon_t = \log e_t$ will be assumed to be white noise and independent of α_t.

Decomposition of total tax revenue by tax type

The total tax revenue is

$$Y(t) = \sum_{j=1}^{p} Y_j(t)$$

where the $Y_j(t)$ denote the component tax revenues. The proportionate forecast errors

$$e(Y_j(t)) = \log \widehat{Y}_j(t) - \log Y_j(t) \approx \frac{\widehat{Y}_j(t) - Y_j(t)}{Y_j(t)}$$

satisfy the simple decomposition

$$e(Y(t)) = \sum_{j=1}^{p} P_j(t) e(Y_j(t))$$

where

$$P_j(t) = \frac{Y_j(t)}{Y(t)} = \text{tax share}$$

Individual tax revenue decomposition

For an individual tax revenue following the benchmark model

$$Y_t = R_t X_t$$

assume that the forecasts satisfy

$$\widehat{Y}_t = \widehat{R}_t \widehat{X}_t$$

so that

$$\log \widehat{Y}_t - \log Y_t = \log \widehat{R}_t - \log R_t + \log \widehat{X}_t - \log X_t$$

This gives the primary forecast error decomposition

$$e(Y_t) = e(R_t) + e(X_t)$$

Individual tax ratio decomposition

Here

$$R_t = \alpha_t e_t$$

and it is assumed that

$$\widehat{R}_t = \widehat{\alpha}_t$$

This yields a further forecast error decomposition

$$e(R_t) = e(\alpha_t) + n_t$$

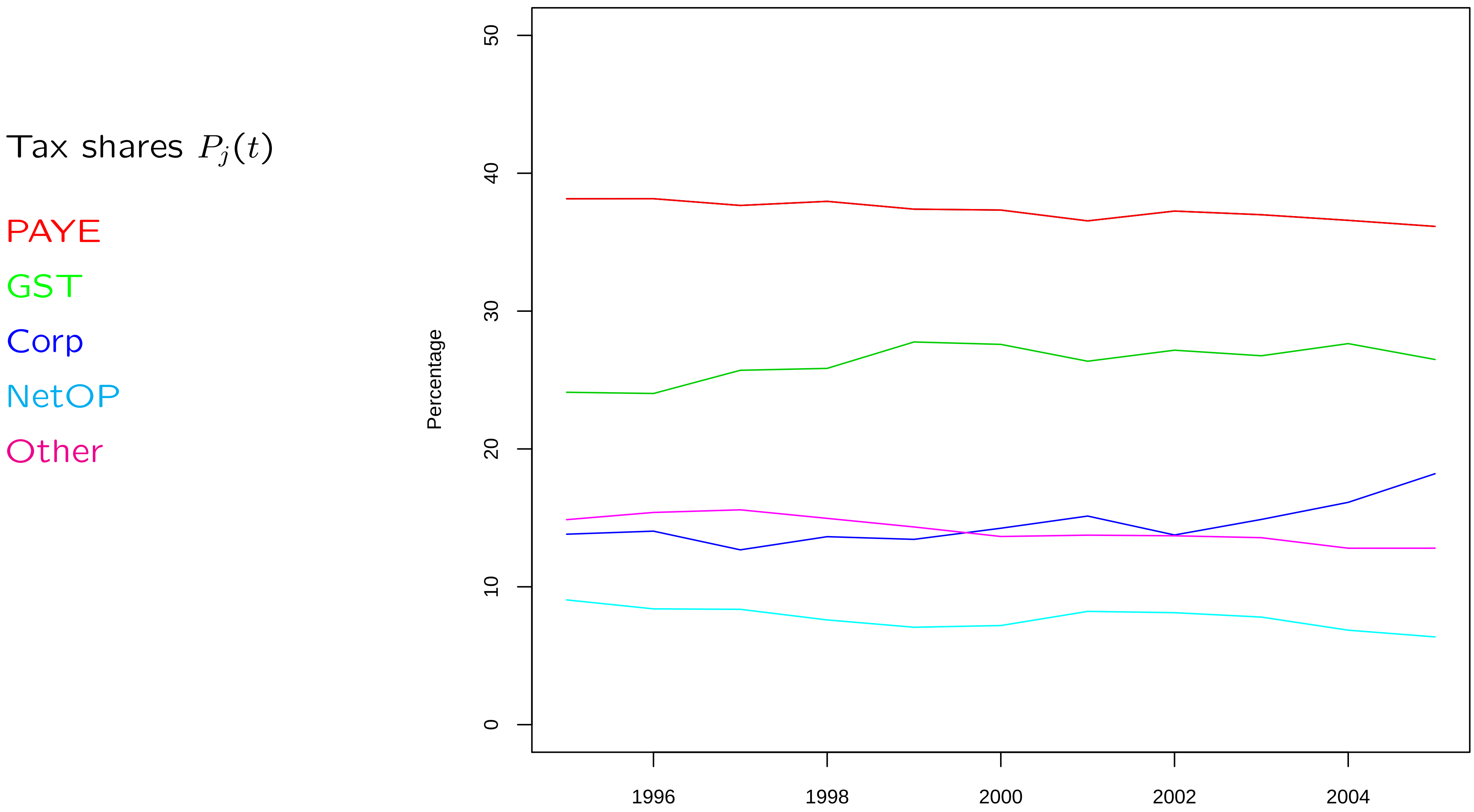
where n_t represents non-systematic white noise error.

The size of n_t provides a measure of best accuracy that can be achieved with the benchmark model.

5. Data analysis

Decomposition of total tax revenue by tax type

$$e(Y(t)) = \sum_{j=1}^{p} P_j(t) e(Y_j(t))$$

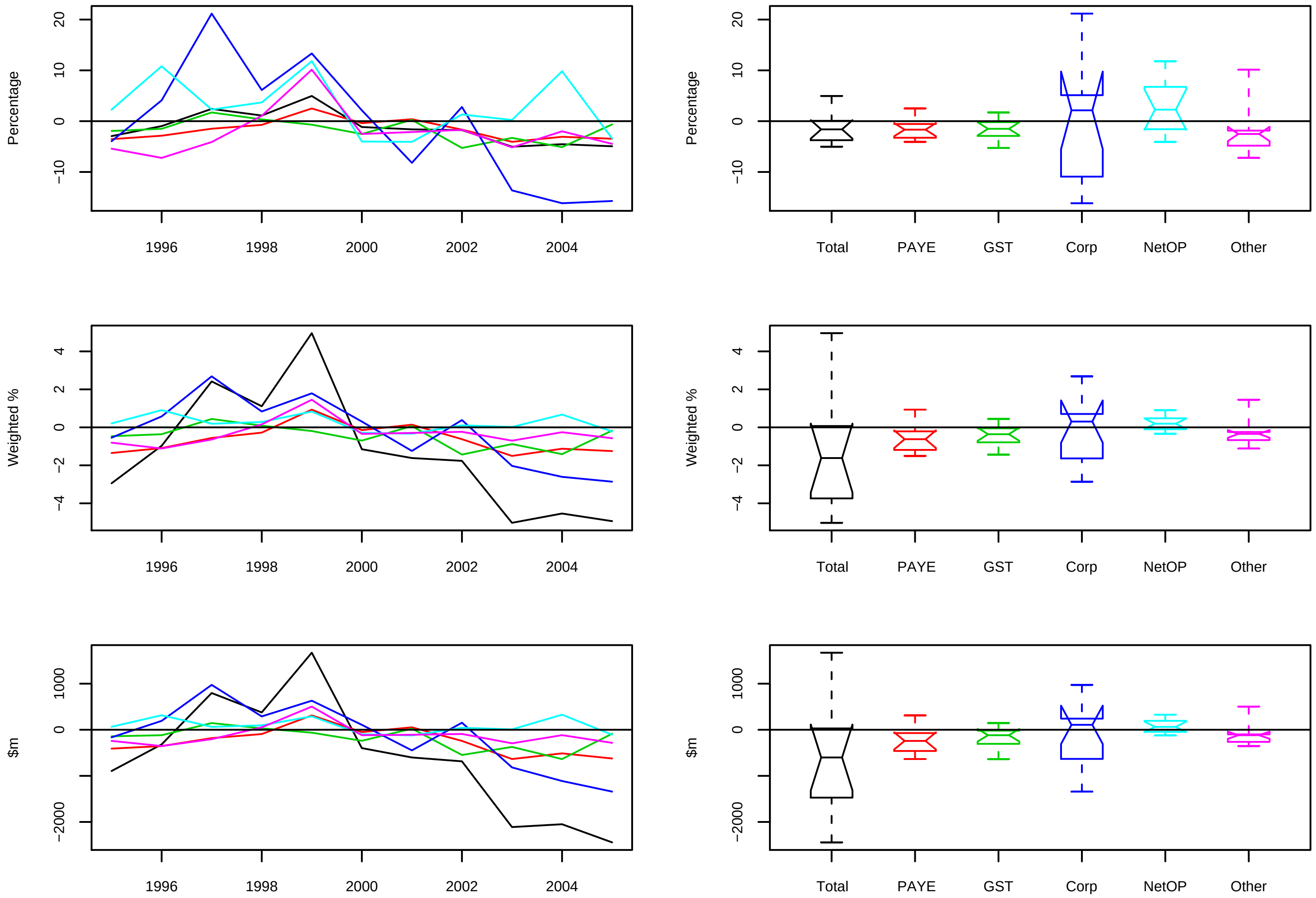


Tax shares $P_j(t)$
PAYE
GST
Corp
NetOP
Other
Percentage
0
10
20
30
40
50
1996
1998
2000
2002
2004

Percentage forecast errors

Weighted percentage forecast errors

Actual forecast errors



Individual tax decompositions

Tax revenue decomposition:

$$e(Y_t) = e(R_t) + e(X_t)$$

Tax ratio decomposition

$$e(R_t) = e(\alpha_t) + n_t$$

The tax ratio trends α_t were estimated by the Hodrick-Prescott filter, although other trend estimates could have been used.

Total tax revenue

Y_t X_t R_t α_t

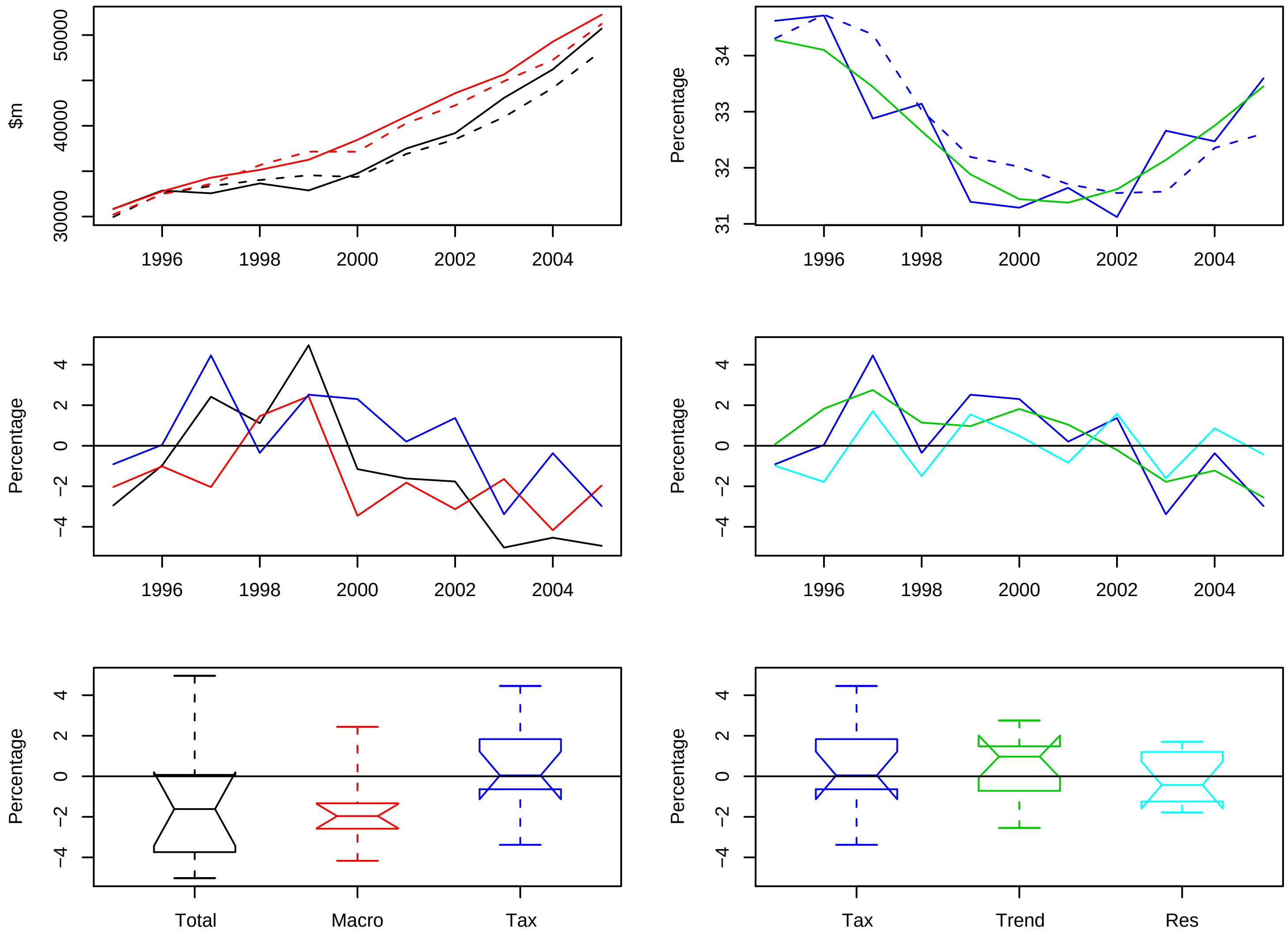
Forecasts - - - -

$e(Y_t)$ $e(X_t)$

$e(R_t)$

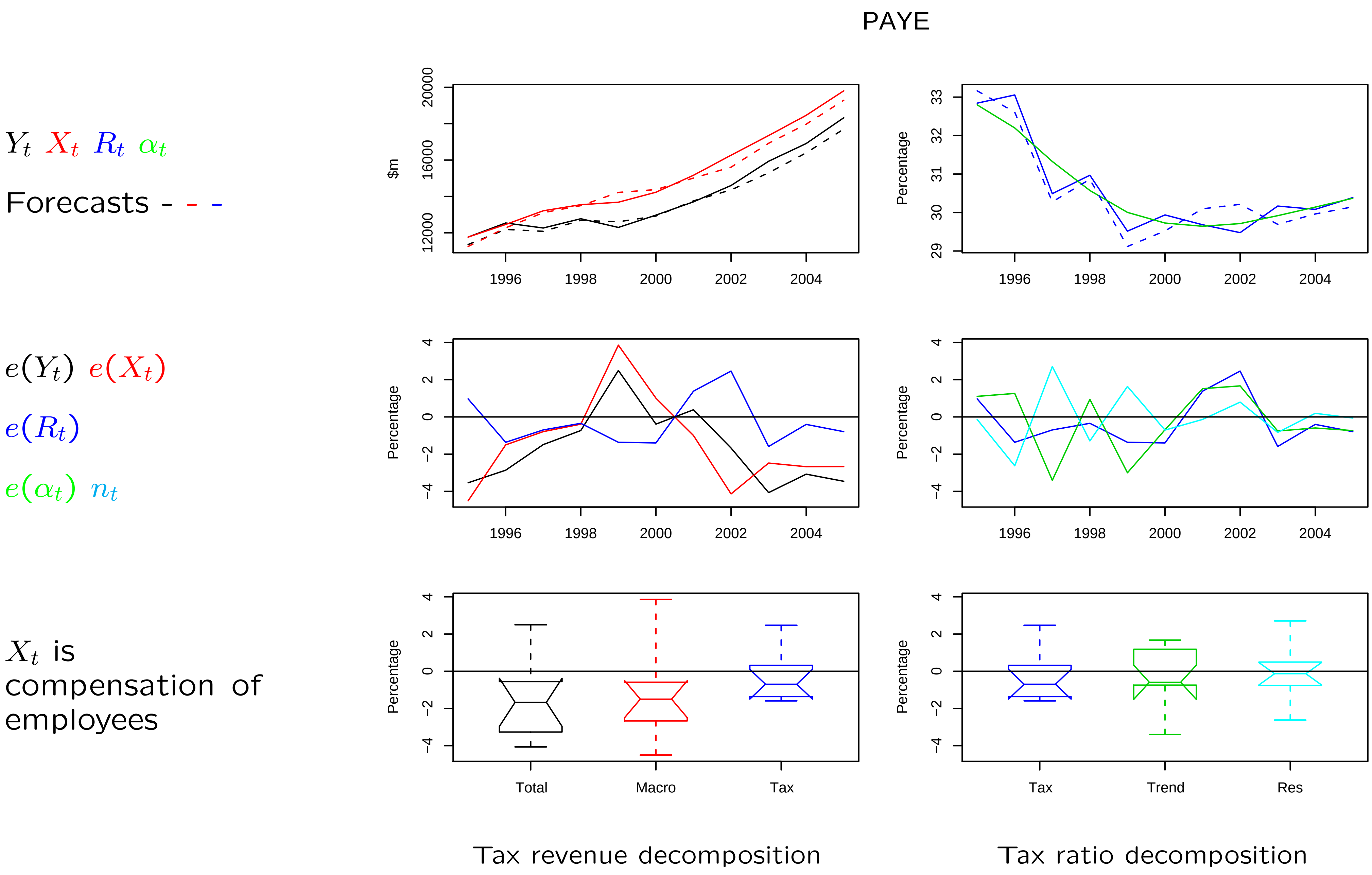
$e(\alpha_t)$ n_t

X_t is GDP



Tax revenue decomposition

Tax ratio decomposition


PAYE
Y_t X_t R_t α_t
Forecasts - - -
$e(Y_t)$ $e(X_t)$
$e(R_t)$
$e(\alpha_t)$ n_t
X_t is compensation of employees
$m
Percentage
1996
1998
2000
2002
2004
Total
Macro
Tax
Trend
Res
Tax revenue decomposition
Tax ratio decomposition

Goods and services tax

Y_t X_t R_t α_t

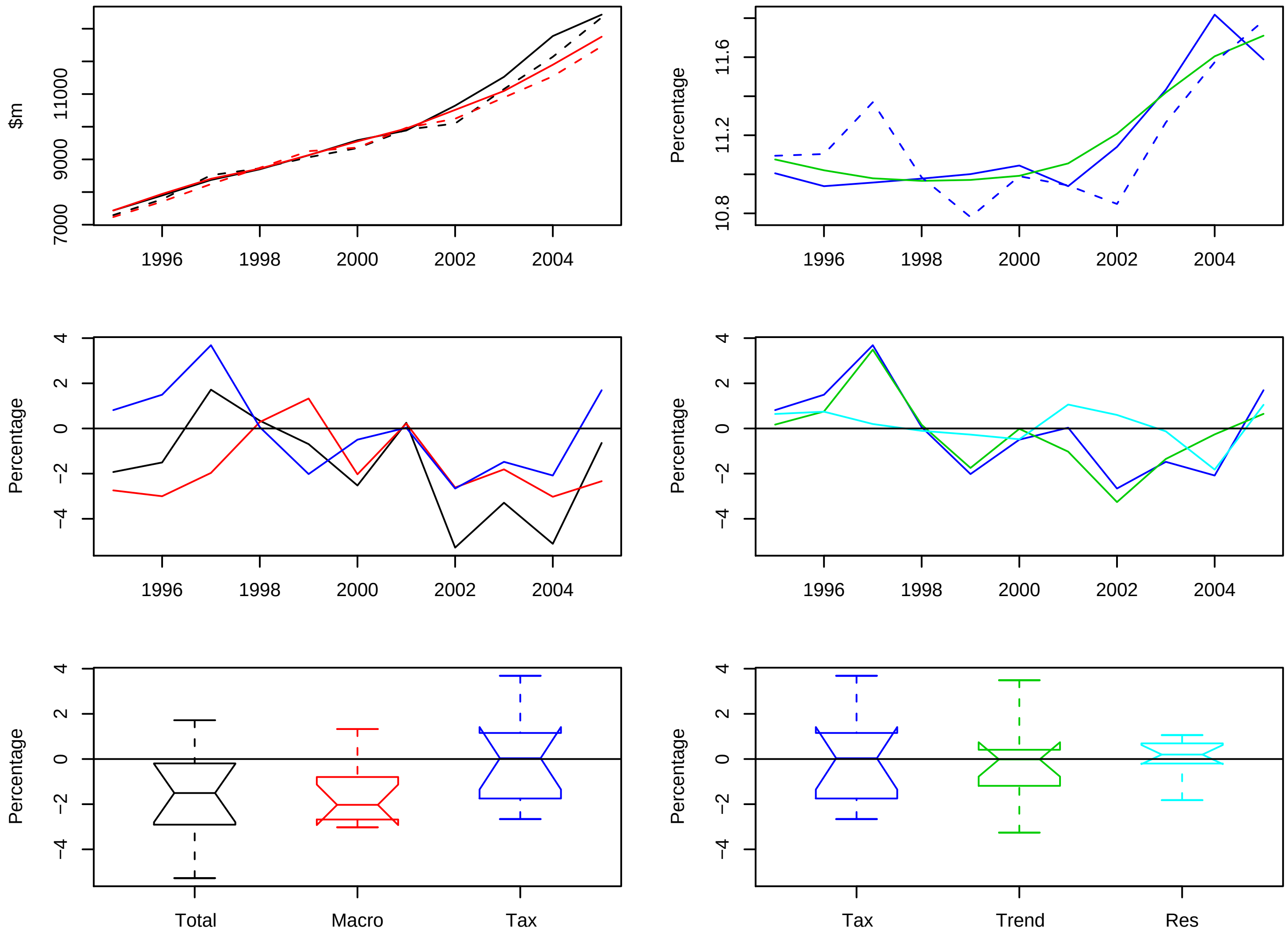
Forecasts - - -

$e(Y_t)$ $e(X_t)$

$e(R_t)$

$e(\alpha_t)$ n_t

X_t is nominal consumption



Tax revenue decomposition

Tax ratio decomposition

Y_t X_t R_t α_t

Forecasts - - - - -

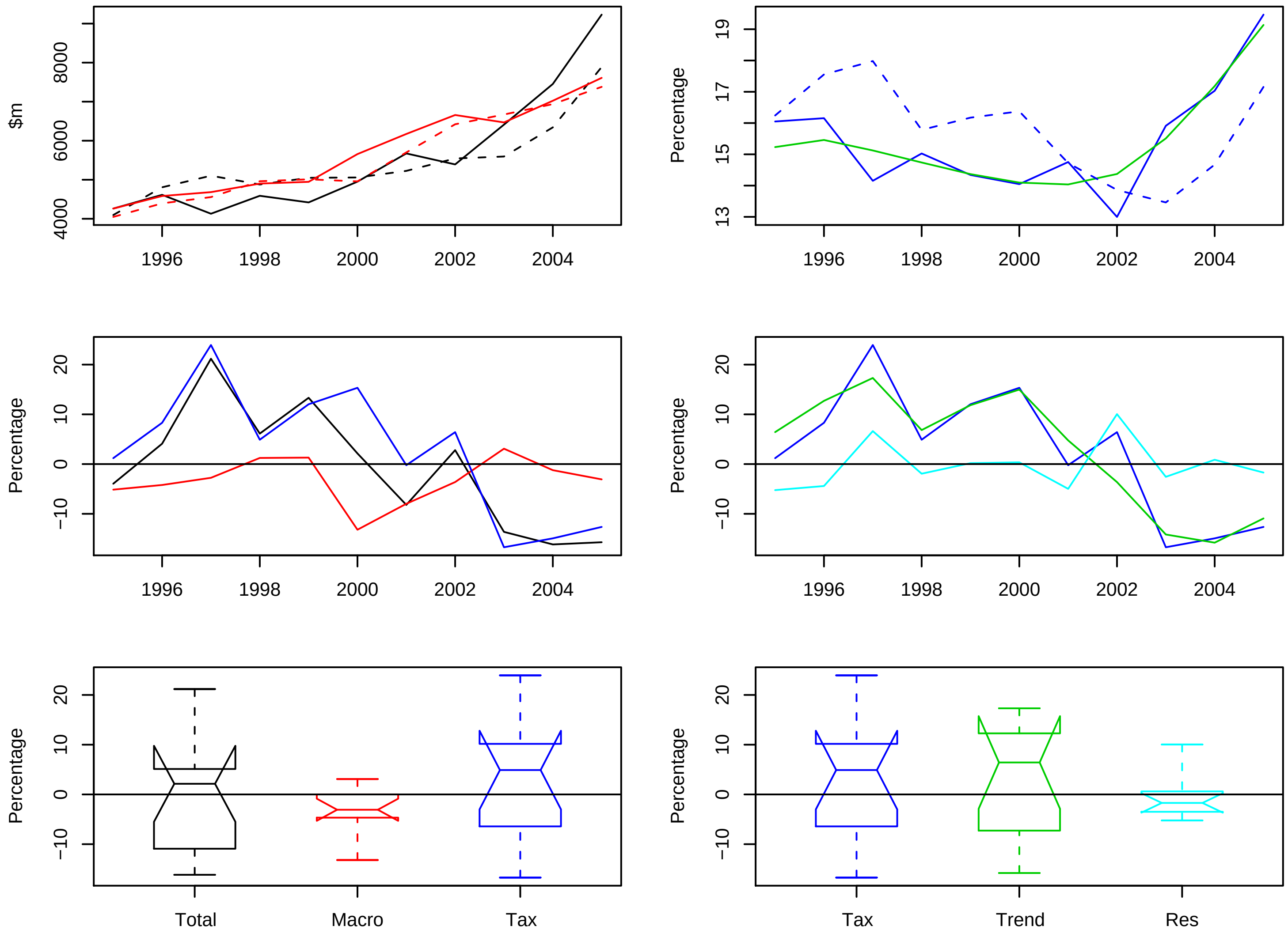
$e(Y_t)$ $e(X_t)$

$e(R_t)$

$e(\alpha_t)$ n_t

X_t is operating surplus

Corporate tax



Tax revenue decomposition

Tax ratio decomposition

Y_t X_t R_t α_t

Forecasts - - -

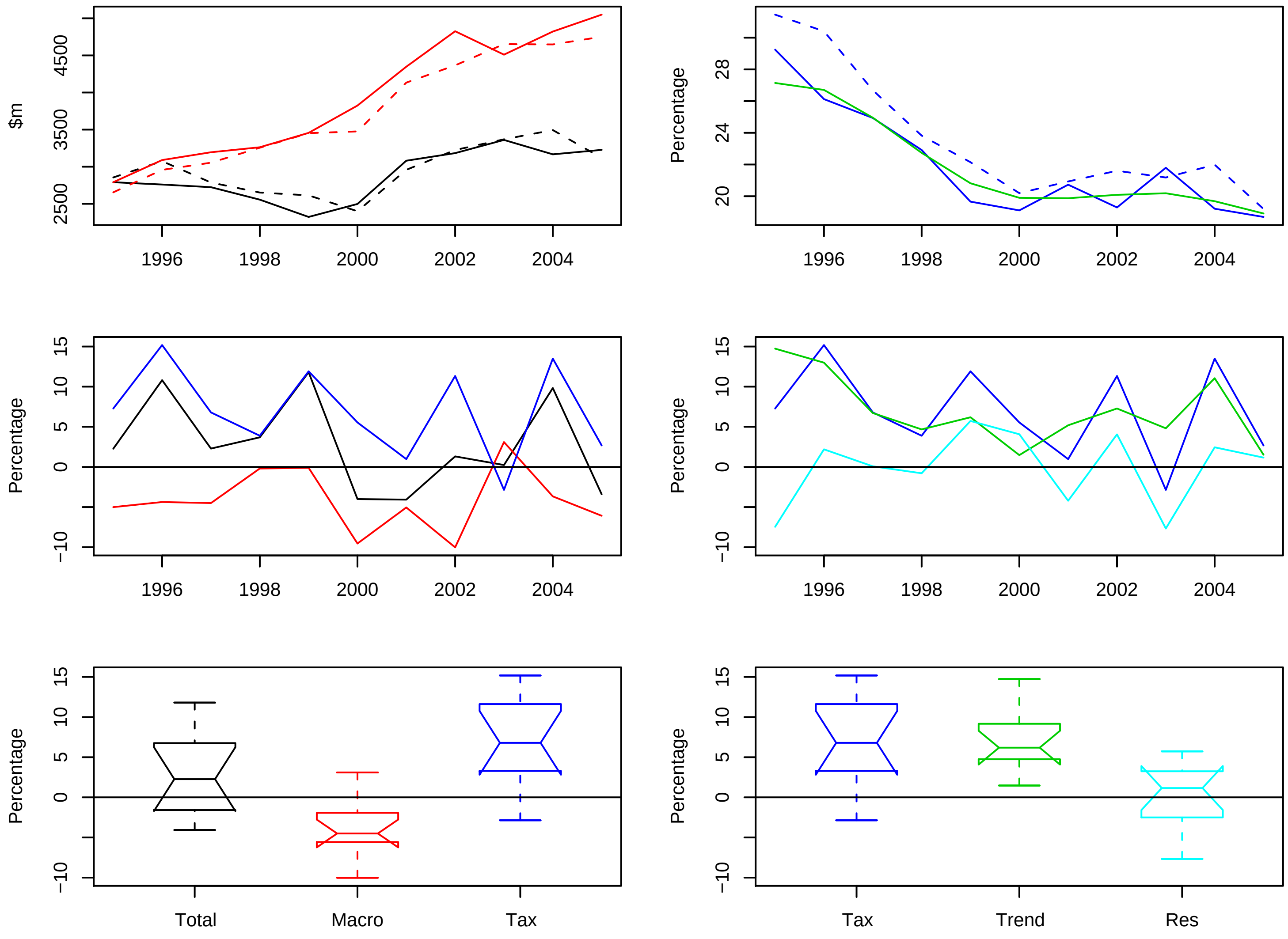
$e(Y_t)$ $e(X_t)$

$e(R_t)$

$e(\alpha_t)$ n_t

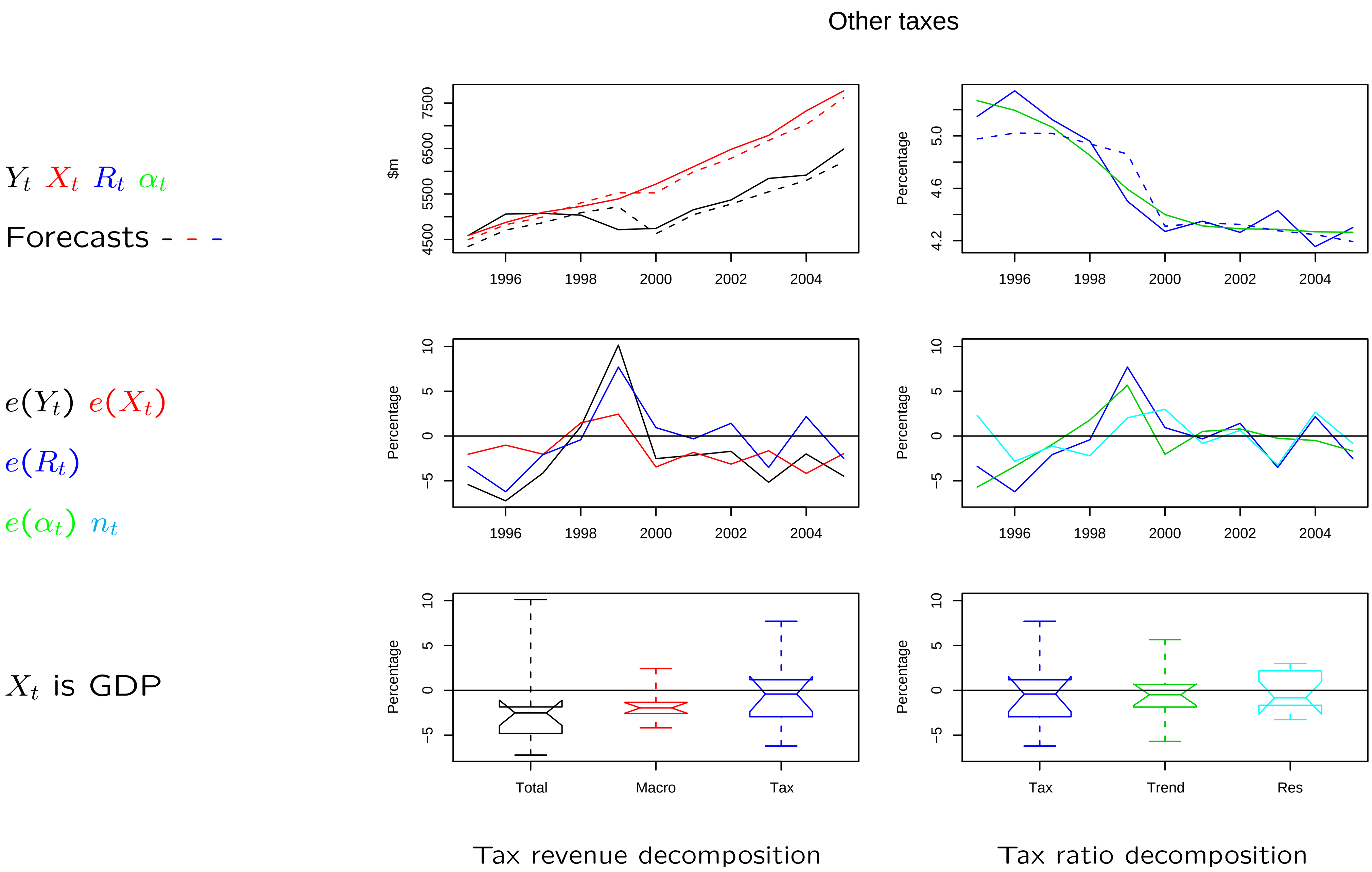
X_t is entrepreneurial income

Net other persons tax

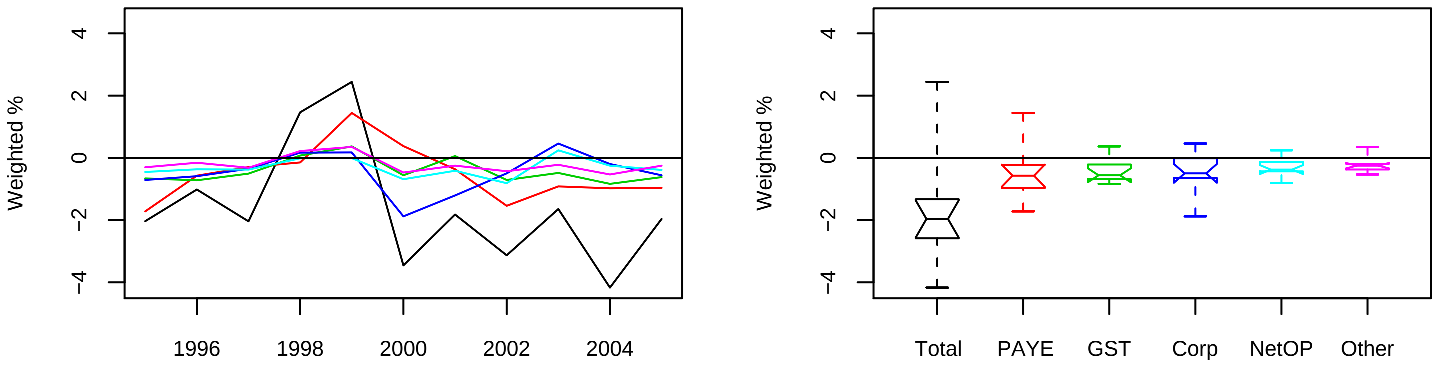


Tax revenue decomposition

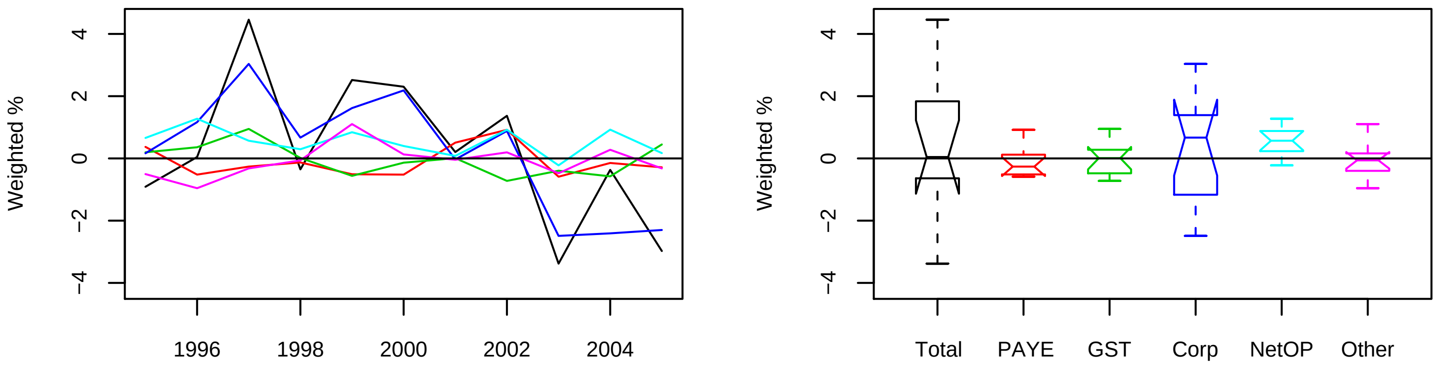
Tax ratio decomposition


Other taxes
Y_t X_t R_t α_t
Forecasts - - -
$e(Y_t)$ $e(X_t)$
$e(R_t)$
$e(\alpha_t)$ n_t
X_t is GDP
$m
Percentage
1996
1998
2000
2002
2004
Total
Macro
Tax
Trend
Res
Tax revenue decomposition
Tax ratio decomposition

Tax share weighted percentage forecast errors (X_t)



Tax share weighted percentage forecast errors (R_t)



6. Conclusions

- The main source of tax revenue underforecasting is the underforecasting of the macro variables used as tax-base proxies.

- The tax ratio forecasts were generally unbiased, but less precisely determined than the macro forecasts.

- Corporate tax is least accurately forecast and contributes the most variability to total tax.

- The size of the error due to forecasting the tax ratio trend was almost always greater than the size of the non-systematic error, indicating that better tax ratio forecasts could be achieved.

- The benchmark models have merit as competing models that could be investigated alongside other simple structural time series models in a systematic evaluation using historical data.

References

Keene, M. and Thomson, P.J. (2007) An analysis of tax revenue forecast errors. New Zealand Treasury Working Paper 07/02. To appear. http://www.treasury.govt.nz/workingpapers/

Schoefisch, U.D. (2005) Examination of the New Zealand Treasury's tax forecasting methods and processes. New Zealand Treasury.