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Modelling Swimmers' Speeds over the Course of a Race

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Outline

1. Introduction
2. Data
3. Modelling considerations
4. Model fitting
5. Result
6. Conclusion

Introduction

Swimming science has been attracting many researchers in various areas including; Biomechanics, Race analysis, Training methods etc..

Their interests are

- drag and propulsion on swimmer (Biomechanics);
- relation between stroke ratio [cycle/m] and speed (Race analysis);
- evaluation of swimmers (Training methods).

Swimming speed is a key to integrate these areas

Data

Race: 2004 Japan Swimming Championships

- Male 200M freestyle race
- 34 swimmers

Data collection:

- International common methods (Matsui *et al*., 1997)
- Video recording by JSF
- Elapsed times of each swimmer at fixed points (21 points in a race)

- Elapsed times measured when swimmer’s head reached a check point
- The official time stamp recorded on each video frame

Comments:

- Hard to define when swimmer’s head reached a check point
- Need a lot of experts

Modelling considerations

Our approach:

- Model swimming speed $v\left(x\right)$ at the location x [m], considering
  - dynamics of swimming (drag and propulsion);
  - individual effect.
- Model elapsed times by

$$\text{Elapsed time} = \int \frac{1}{v\left(x\right)}\, dx + \text{noise}.$$

- Fit the elapsed model to the data

  Note: Only elapsed times at fixed points are available

Swimming speed model

Consider applied forces on swimmers

$$\frac{dv(t)}{dt} = -\alpha v(t)^2 + \beta, \ (\alpha > 0, \beta \geq 0)$$

α: drag parameter, β: propulsion parameter

Key assumptions:

- drag proportional to squared swimming speed
- constant propulsion

This model

- builds on Newtonian physics: $F = m\frac{dv(t)}{dt}$;
- extends well known model proposed by Amar (1920) which ignores propulsion in swimming.

The solution of the deferential equation represented as a function of location x

$$v(x) = \begin{cases} v_0 e^{-\alpha(x-x_0)} & (\beta = 0), \\ \sqrt{ce^{-2\alpha(x-x_0)} + \kappa} & (\beta > 0), \end{cases}$$

x_0: initial distance; v_0: initial velocity; $\kappa = \beta/\alpha$; $c = {v_0}^2 - \kappa$.

Comments:

- NOT realistic to assume that the parameters stay constant over the race even in a lap
- Need to introduce several phases within which parameters might be constant

Parsimonious parameterisation

- Keep model parsimonious to have converge estimates
- NOT free from the limited number of observations
- Should parameters relate directly on physical phenomena

  α: drag parameter, β: propulsion parameter

Modelling process involves

- building a candidate model;
- fitting the model;
- summarising the model (parameters);
- using diagnostics (residuals).

$$v(x;\boldsymbol{\theta}) = \begin{cases} \sqrt{c_1 e^{-2\alpha_1 (x-x_0)} + \kappa_1} & (x_2 \le x < 50), \quad \text{First phase} \\ \sqrt{c_2 e^{-2\alpha_2 (x-x_1)} + \kappa_2} & (x_2 \le x < 50), \quad \text{Middle phase} \\ \sqrt{c_3 e^{-2\alpha_3 (x-x_2)} + \kappa_3} & (x_2 \le x < 50), \quad \text{Last phase} \end{cases}$$

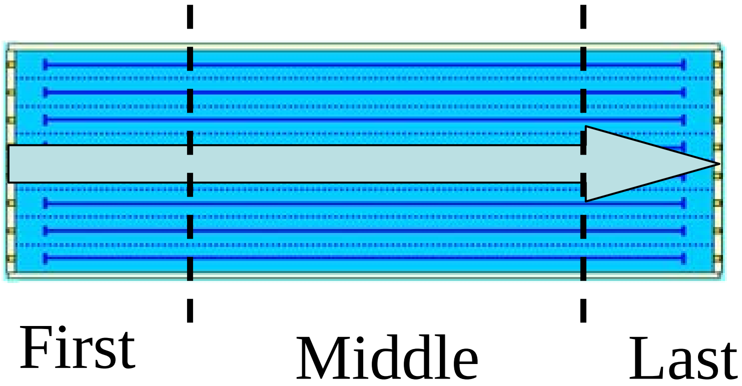
$$\boldsymbol{\theta} = (\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, x_1, x_2),\ c_l = v(x_{l-1})^2 + \kappa_l,\ \kappa_l = \beta_l/\alpha_l,\ l = 1, 2, 3.$$

Number of parameters = 8

3 phases introduced in a lap

Key assumptions:

- First phase
  - drag and propulsion
- Middle phase
  - drag and propulsion
- Last phase
  - drag and propulsion

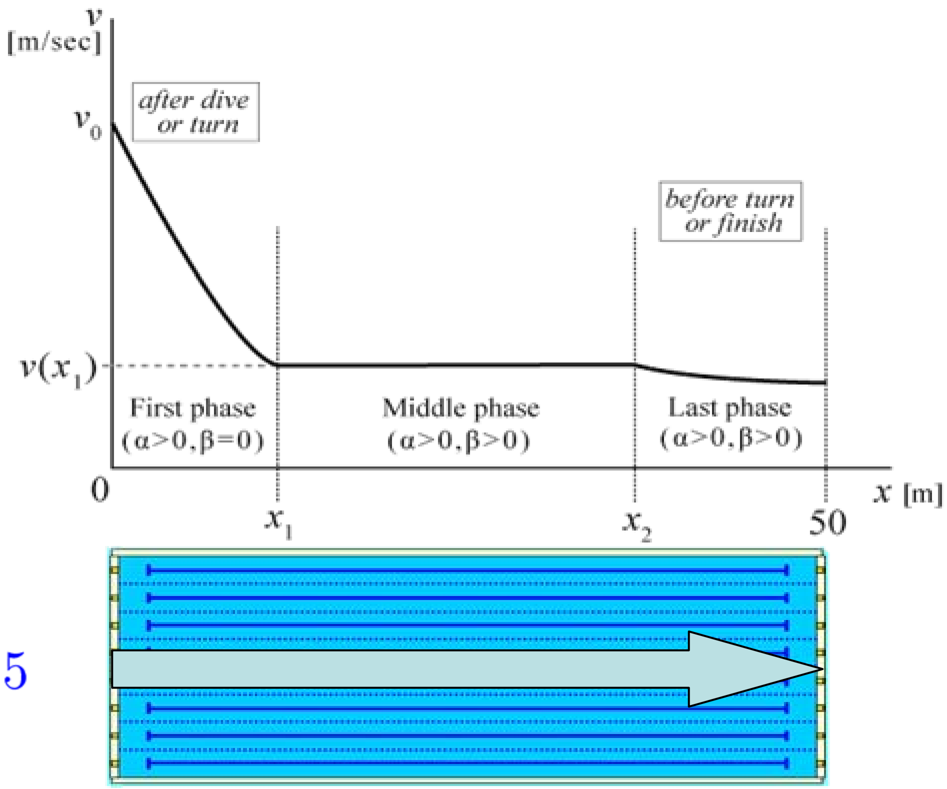


Model for one lap

3 phases introduced in a lap

Key assumptions:

- First phase
  - drag but NO propulsion
- Middle phase
  - drag and propulsion
  - constant speed (equilibrium)
- Last phase
  - drag and propulsion
  - adopt the value of drag $a = 0.425$ (Toussaint, 1988)



Finally, swimming speed model for one lap is

$$v(x;\boldsymbol{\theta}) = \begin{cases} v_0 e^{-\alpha x} & (0 \le x < x_1), & \text{First phase} \\ v(x_1) & (x_1 \le x < x_2), & \text{Middle phase} \\ \sqrt{c_2' e^{-2a(x-x_2)} + \kappa'} & (x_2 \le x < 50), & \text{Last phase} \end{cases}$$

$$\boldsymbol{\theta} = (v_0, \alpha, x_1, x_2, \beta),\ c_2' = v(x_1)^2 + \kappa' \text{ and } \kappa' = \beta/a.$$

Number of parameters = 5

Model over laps; common parameters

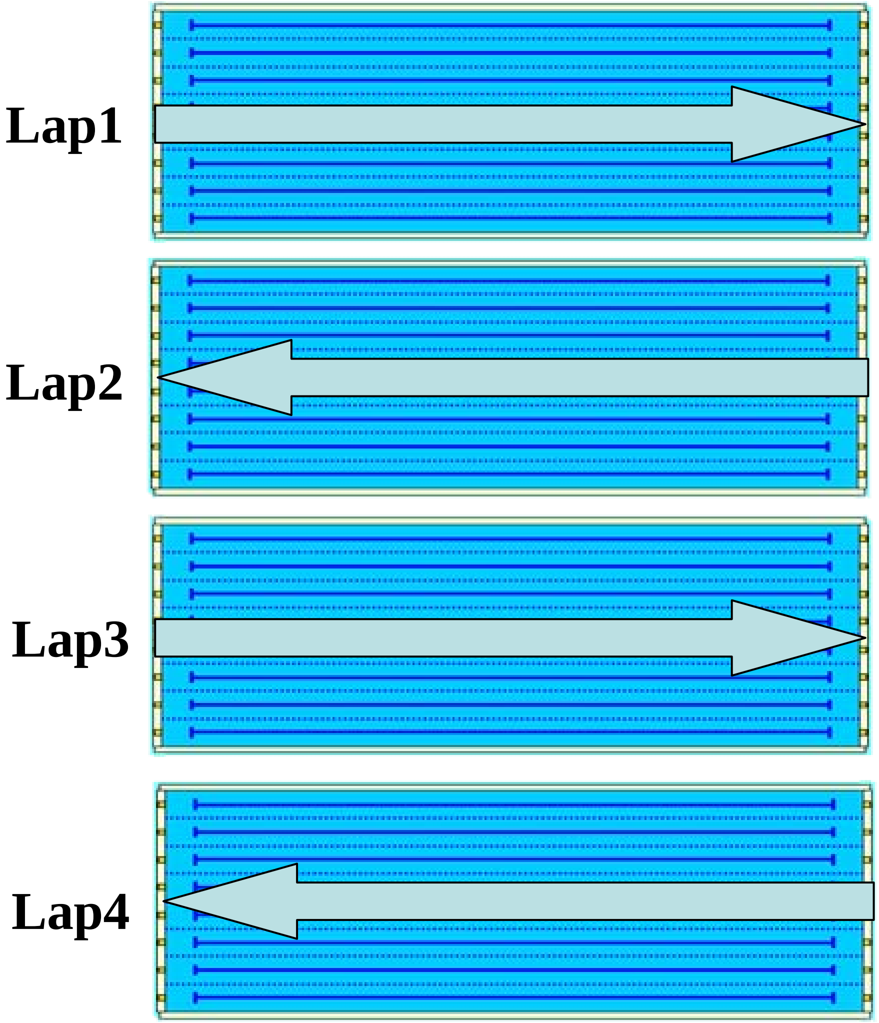
$$v_j\left(x;\boldsymbol{\theta}\right) = v\left(x - x_{0j};\boldsymbol{\theta}_j\right),$$

$$\boldsymbol{\theta}_j = (v_{0j}, \alpha_j, x_{1j}, x_{2j}, \beta_j),$$
$$\boldsymbol{x_0} = (0, 50, 100, 150)$$

200M freestyle race consists of 4 laps

Key assumptions:

- $\alpha_2 = \alpha_3 = \alpha_4$
- $x_{12} = x_{13} = x_{14}$
- $x_{21} = x_{22} = x_{23}$



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Number of parameters = 14

Individuals effects

A simple description including individual effects is to assume that the swimming speed of swimmer i in lap j can be written as

$$v_{ij}\left(x;\boldsymbol{\theta}\right) = \mu_i v_j\left(x;\boldsymbol{\theta}\right),$$

where μ_i is individual parameter, $v_j\left(x;\boldsymbol{\theta}\right)$ is common swimming speed over swimmers in lap j.

Individual parameters measure how faster or slower than the common swimming speed.

Elapsed time model

If $T_{ij}(k)$ denotes the elapsed time of swimmer i at the distance $x_j(k)$ where is check point k in lap j, the elapsed time model is given by

$$T_{ij}(k) = \int_0^{x_j(k)} \frac{1}{\mu_i v_j(x)}\, dx + \sigma B_i(x_j(k)),$$

The model is now

$$\Delta T_{ij}(k) = \frac{1}{\mu_i} \int_{x_j(k-1)}^{x_j(k)} \frac{1}{v_j(x)}\, dx + \sigma \sqrt{\Delta x_j(k)}\, \varepsilon_{ijk},$$

where $\Delta T_{ij}(k) = T_{ij}(k) - T_{ij}(k-1)$, $\Delta x_j(k) = x_j(k) - x_j(k-1)$,

$$\varepsilon_{ijk} \sim \mathcal{N}\left(0, \sigma^2 \Delta x_j(k)\right)$$

Model fitting

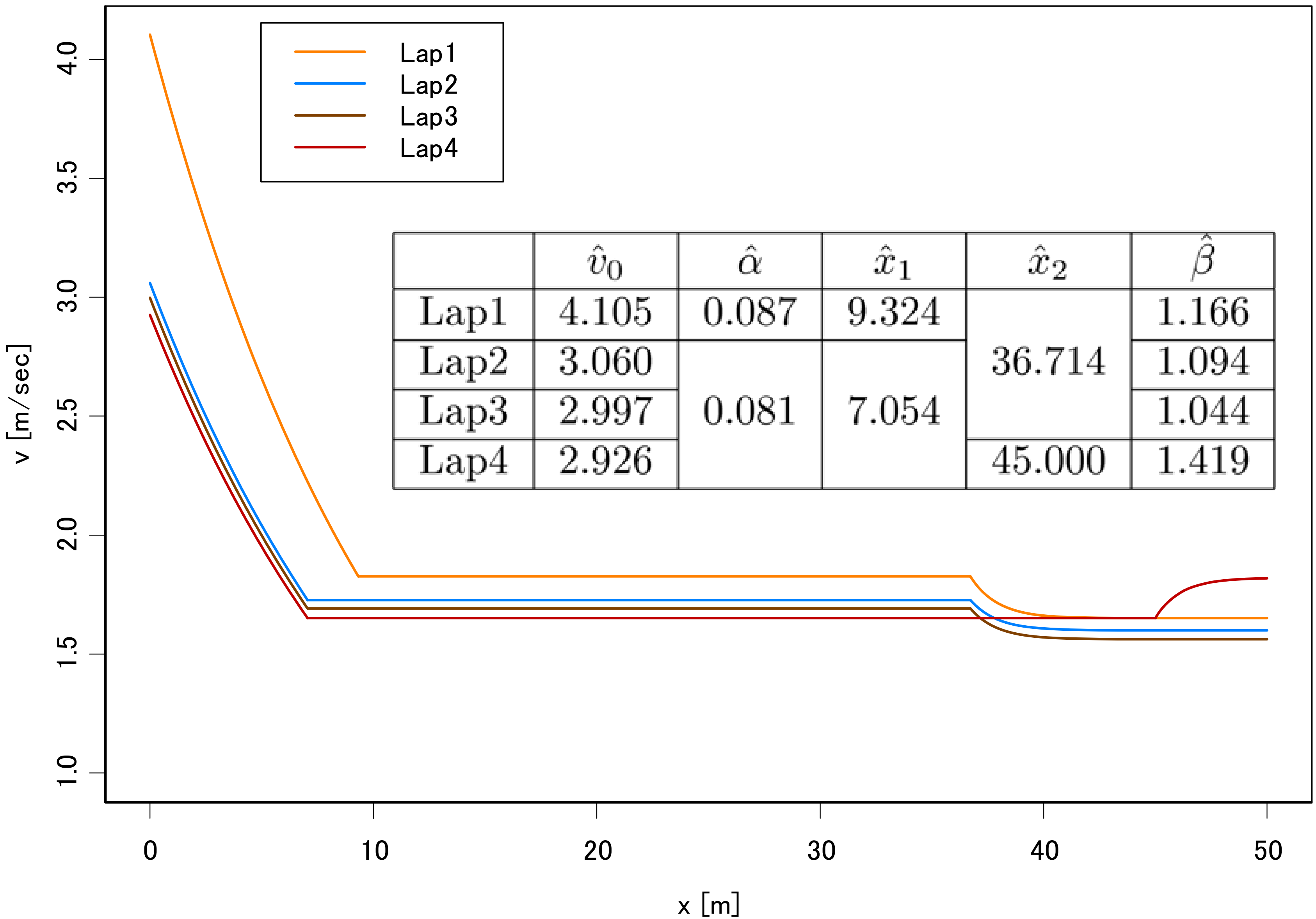
Use nonlinear weighted least squared method (nlminb, on S-PLUS).

$$\text{minimise} \sum_{i=1}^{34} \sum_{j=1}^{4} \sum_{k=1}^{5} \frac{r_{ijk}^2}{\Delta x_j(k)},$$

where $r_{ijk} = \Delta T_{ij}(k) - \frac{1}{\mu_i} \int_{x_j(k-1)}^{x_j(k)} \frac{1}{v_j(x)} \, dx,$

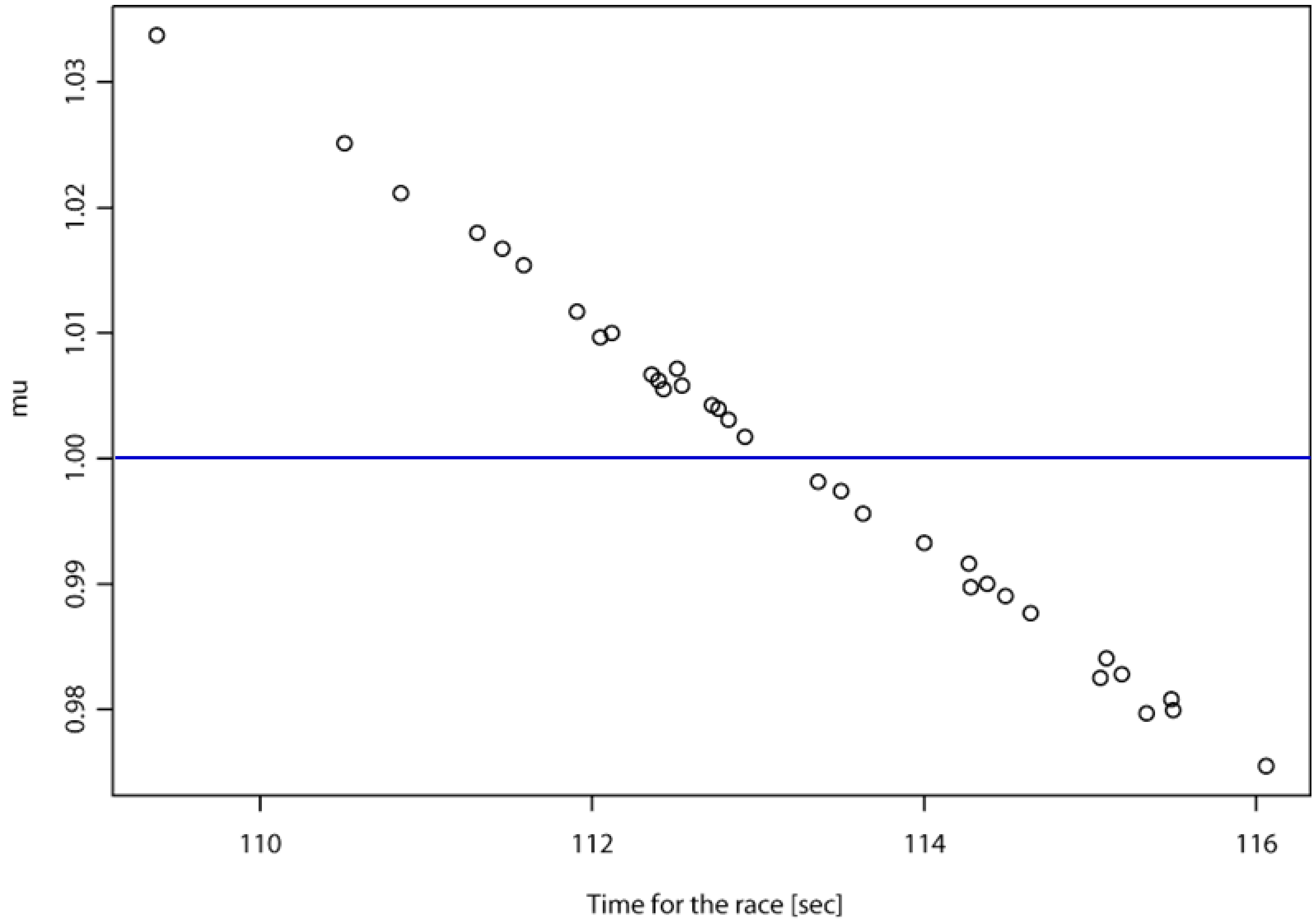
Note: r_{ijk} expected to be iid $\mathcal{N}\left(0, \sigma^2 \Delta x_j(k)\right)$.

Common swimming speeds

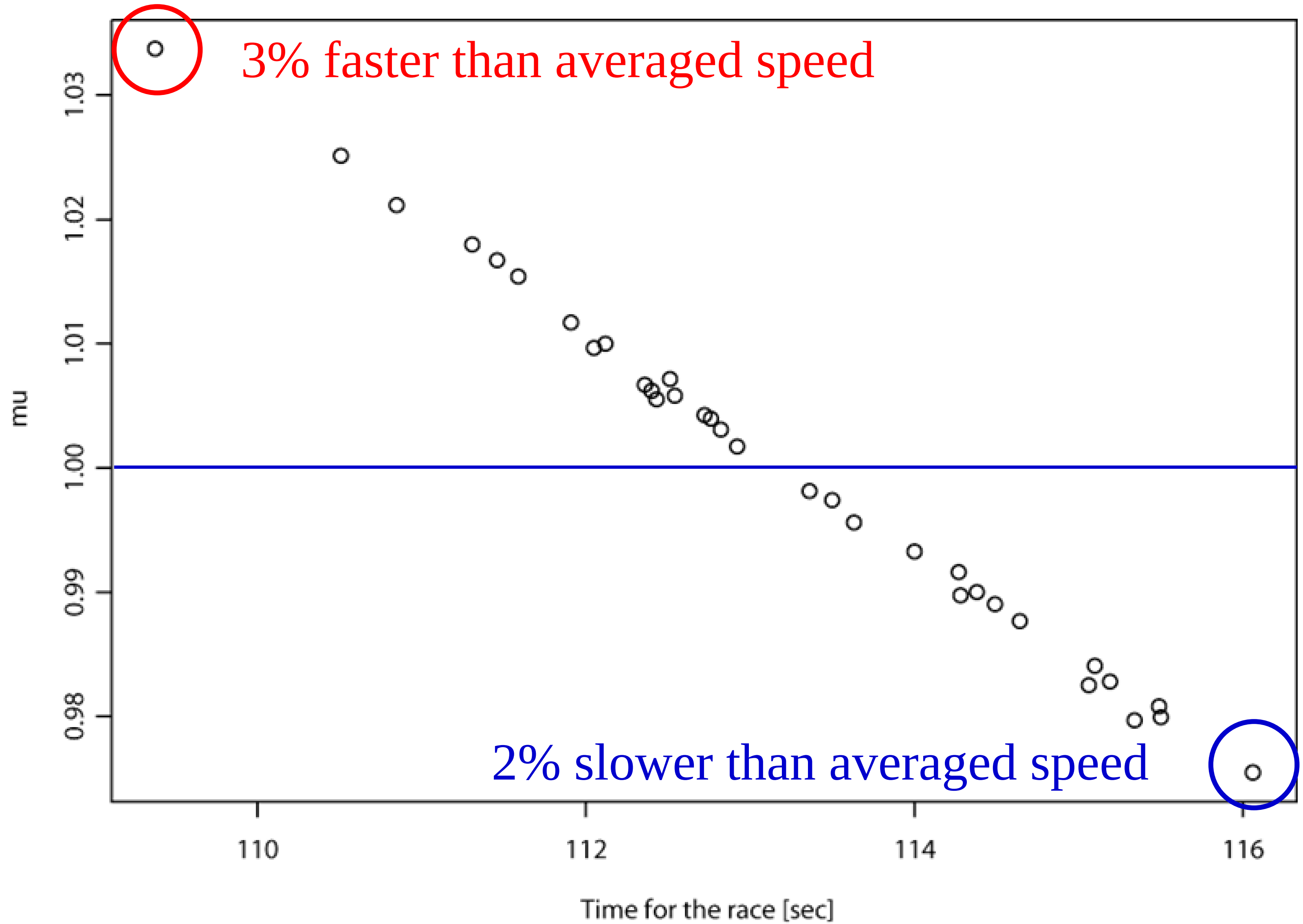


<table>
<tr><th></th><th>$\hat{v}_0$</th><th>$\hat{\alpha}$</th><th>$\hat{x}_1$</th><th>$\hat{x}_2$</th><th>$\hat{\beta}$</th></tr>
<tr><td>Lap1</td><td>4.105</td><td>0.087</td><td>9.324</td><td rowspan="3">36.714</td><td>1.166</td></tr>
<tr><td>Lap2</td><td>3.060</td><td rowspan="3">0.081</td><td rowspan="3">7.054</td><td>1.094</td></tr>
<tr><td>Lap3</td><td>2.997</td><td>1.044</td></tr>
<tr><td>Lap4</td><td>2.926</td><td>45.000</td><td>1.419</td></tr>
</table>

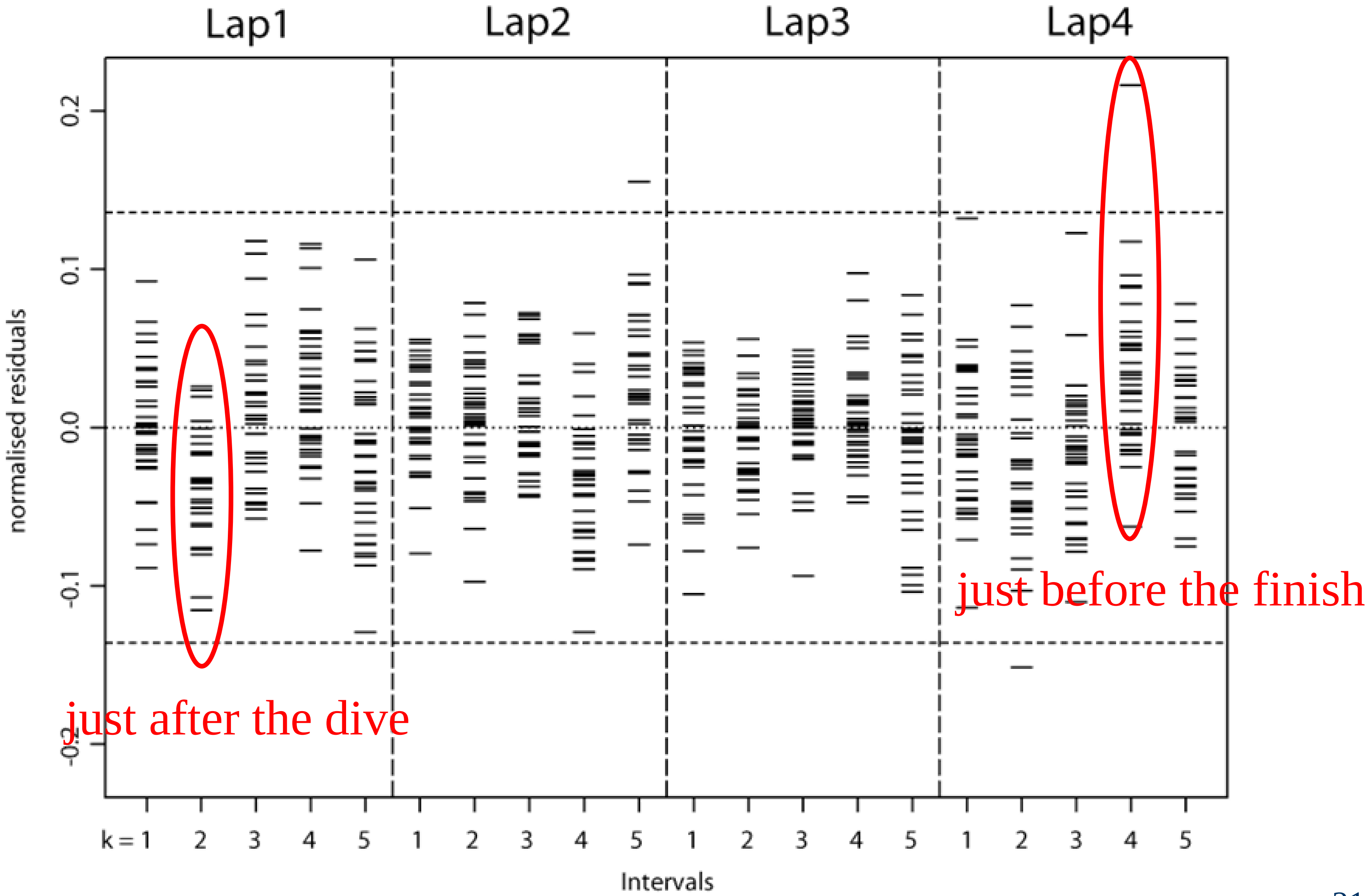
Individual parameters



Individual parameters



Normalised residual plot $\left\{\hat{\varepsilon}_{ijk} = \frac{\hat{r}_{ijk}}{\sqrt{\Delta x_j(k)}}; i = 1, 2, \ldots, 34\right\}$



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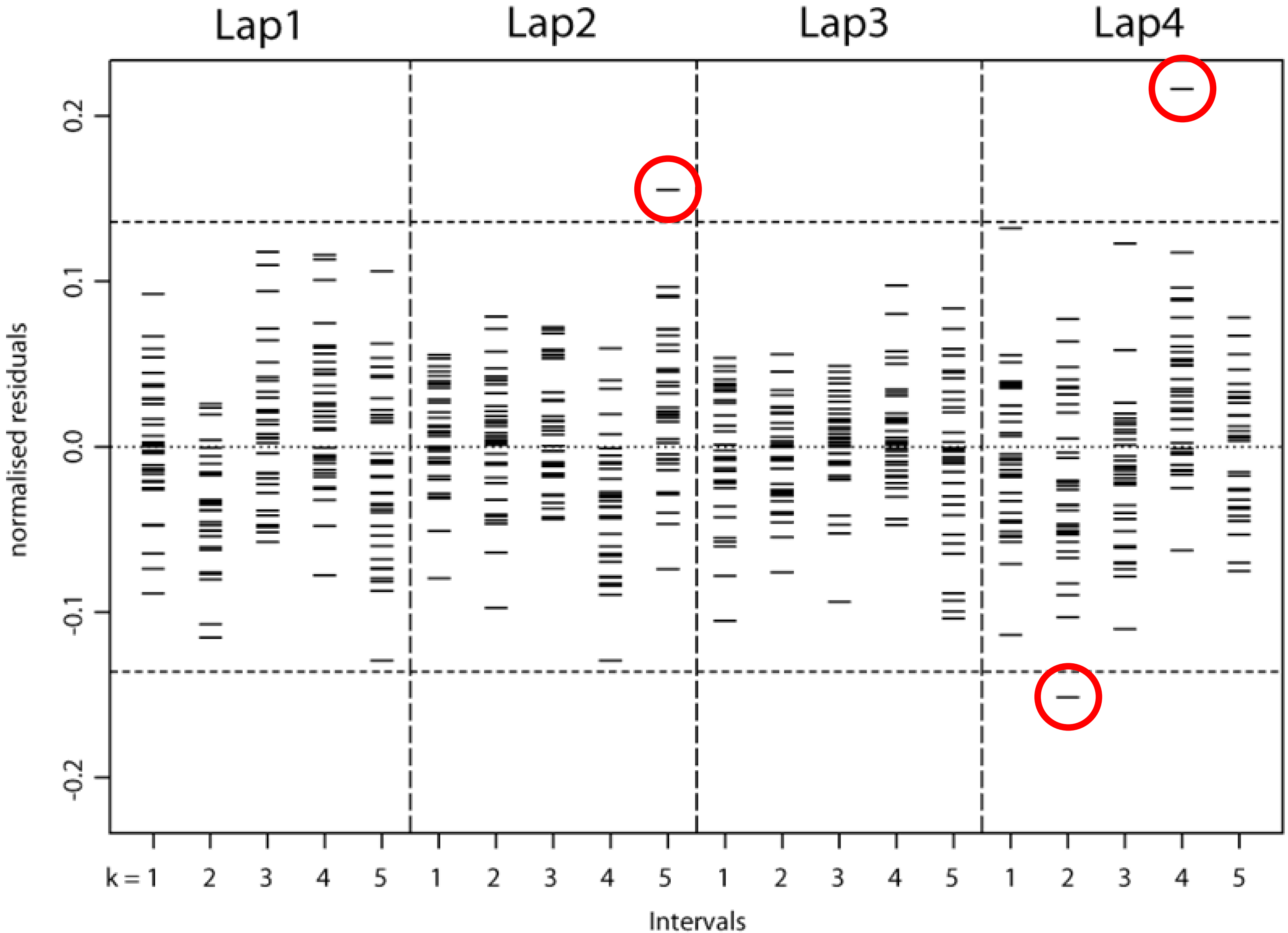
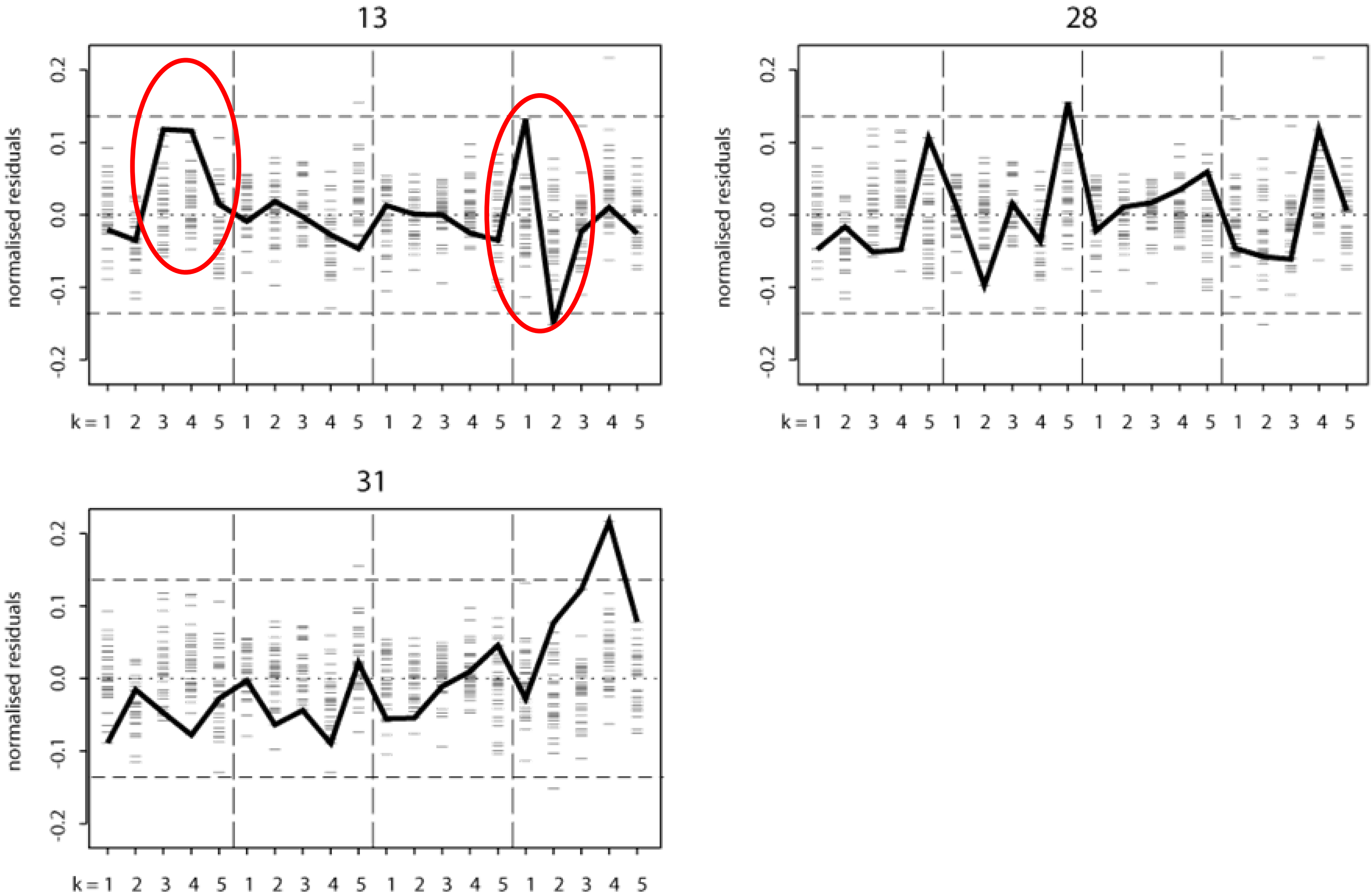
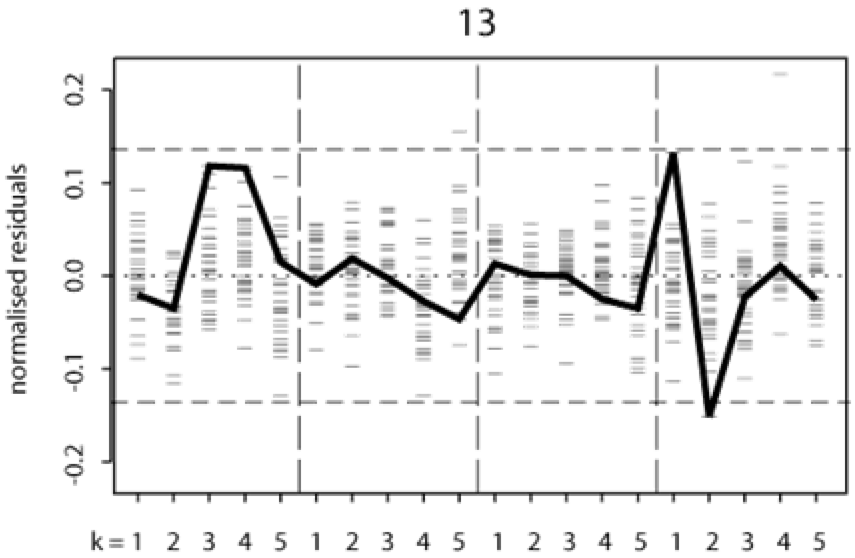
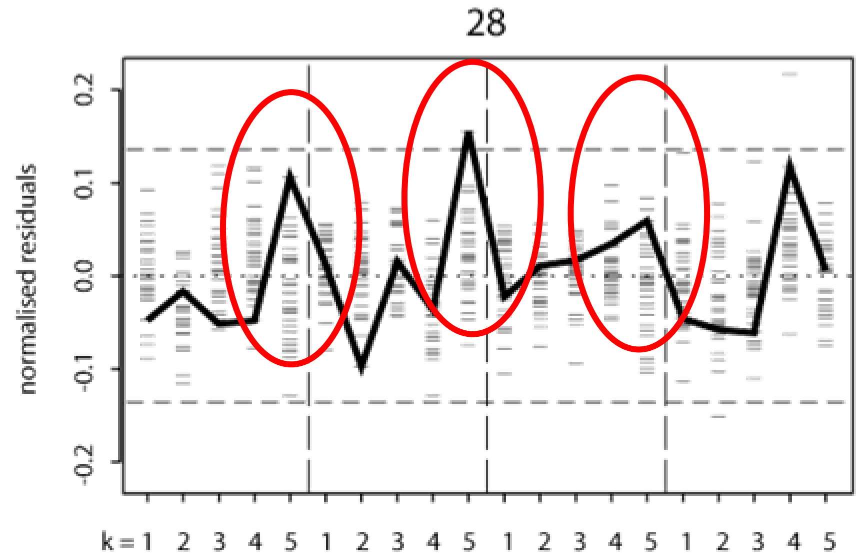


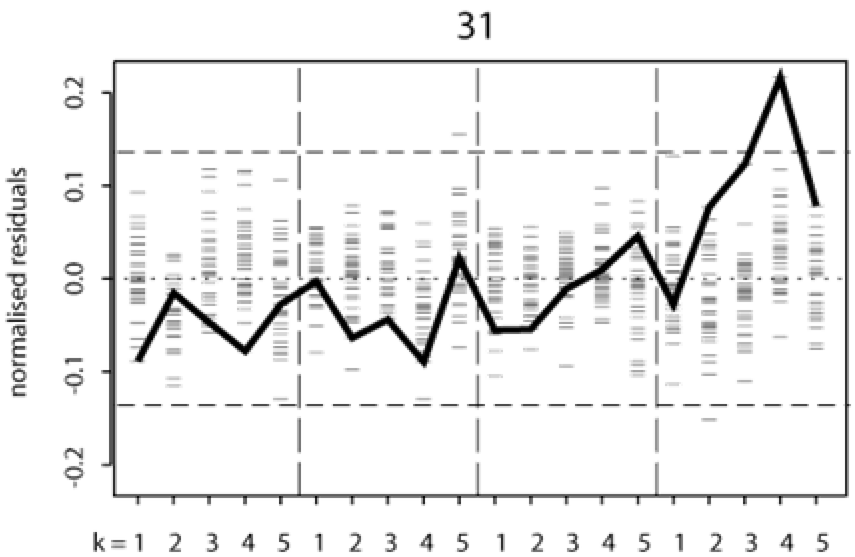
Outlying swimmers



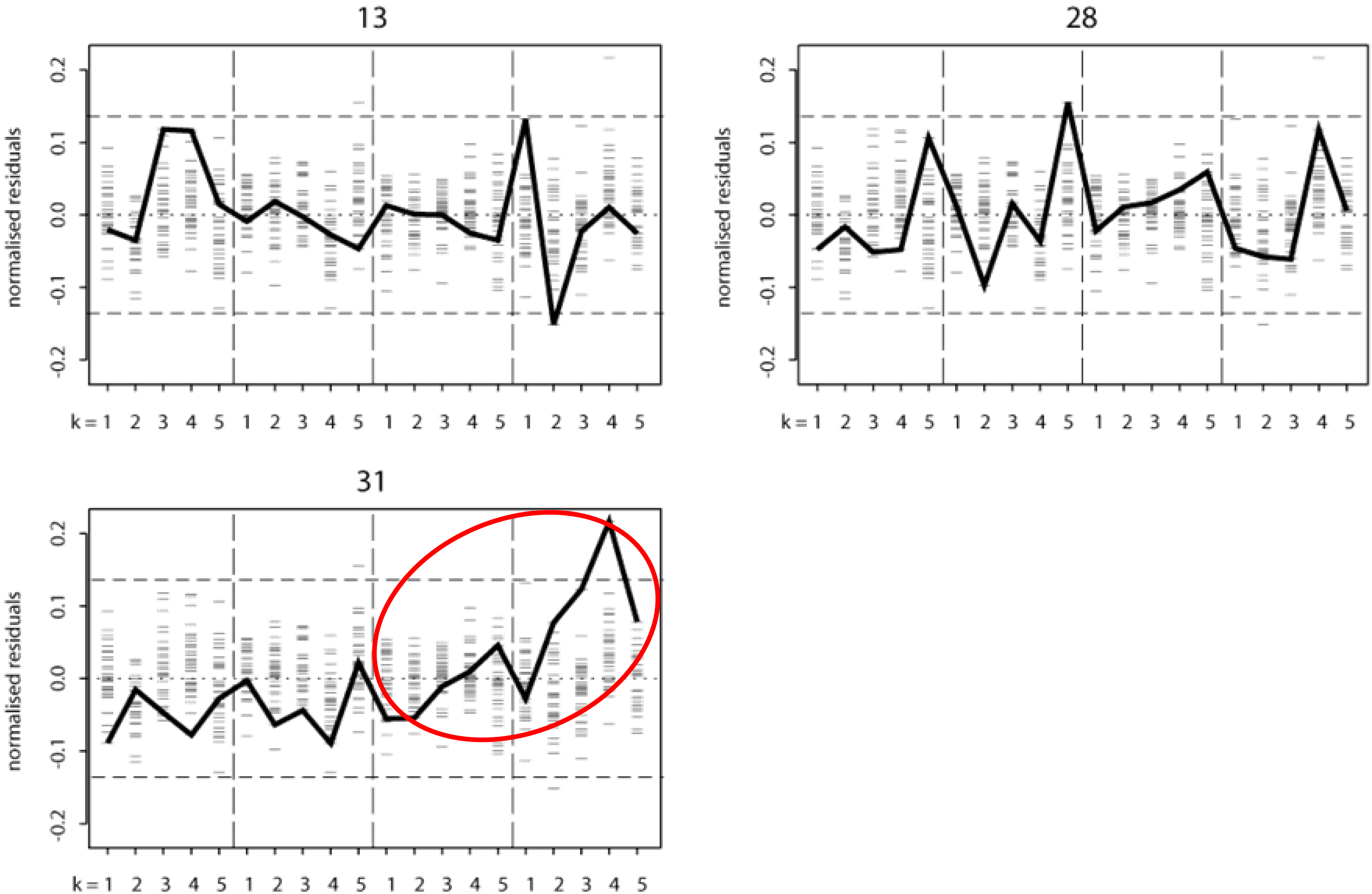
Outlying swimmers



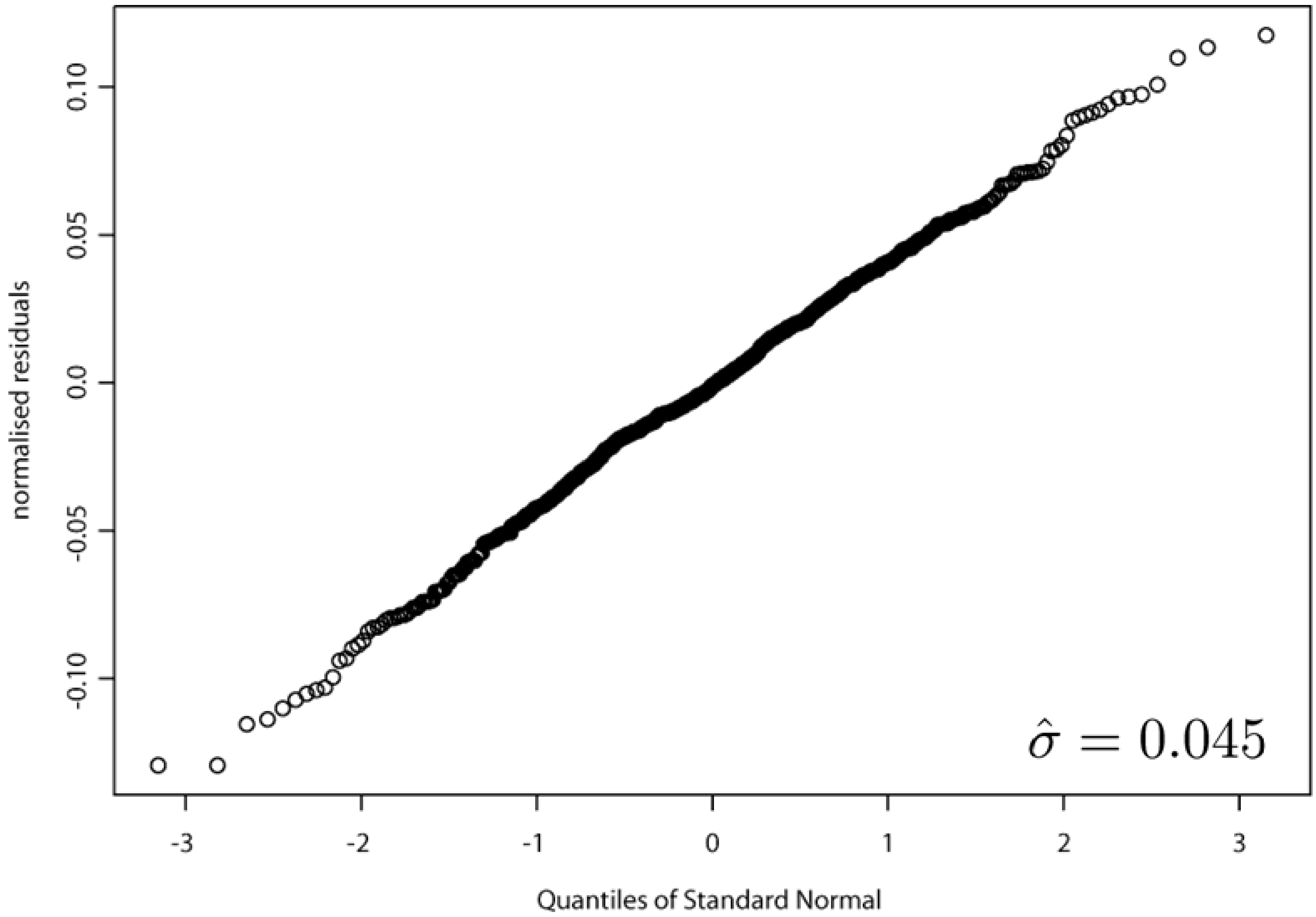




Outlying swimmers



Normal Q-Q plot



Conclusion

Our model recommends to:

- Swimmers and trainers
  - Individual parameters are of use to distinguish between swimmers
  - Residual plot shows strength/weakness of each swimmer
- Researchers in Race analysis
  - Develop data collection methods
  - Need more observation points, especially, around change points
- Researchers in Biomechanics
  - Improve the measuring method of drag and propulsion in competitive swimming

References (1)

1. Amar, J., 1920. *The human motor*. George Routledge & Sons, London.
2. Arellano, R., Brown, P., Cappaert, J., Nelson, R., 1994. Analysis of 50-, 100-, and 200-m freestyle swimmers at the 1992 Olympic games. *Journal of Applied Biomechanics* 10, 189-199.
3. Chengalur, S., Brown, P., 1992. An analysis of male and female Olympic swimmers in the 200-meter events. *Canadian Journal of Sport Sciences* 17, 104-109.
4. Craig, A., Pendergast, D., 1979. Relationship of stroke rate, distance per stroke, and velocity in competitive swimming. *Medicine and Science in Sports* 11, 278-283.
5. Craig, A., Skehan, P., Pawelczyk, J. Boomer, W., 1985. Velocity, stroke rate, and distance per stroke during elite swimming competition. *Medicine and Science in Sports* 17, 625-634.
6. Ikuta, Y., Okuno, K., Matsui, K., Terada, A., Honbu, Y., Ishikawa, M., Wakayoshi, K., Nomura, T., 1998. Relationship between control of swimming velocity and stroke rate - Suggestion from result of 100m and 200m Free-style events -. *Japanese Journal of Sport Methodology* 12, 1-8.

References (2)

7. Karpovich, P. V., 1933. Water resistance in swimming. *Research Quarterly* 4, 21-28.
8. Kjendlie, P., Stallman, R. K., Gundersen, J., 2004. Adults have lower stroke rate during submaximal front crawl swimming than children. *European Journal of Applied Physiology* 91, 649-655.
9. Kolmogorov, S., Duplishcheva, O., 1992. Active drag, useful mechanical power output and hydrodynamic force coefficient in different swimming strokes at maximal velocity. *Journal of Biomechanics* 25, 311-318.
10. Matsui, T., Terada, A., Tatesada, E., Honbu, Y., Ikuta, Y., Wakayoshi, K., Nomura, T., 1997. Changes in swimming velocity and stroke variables in 5m intervals during competitive swimming race -Comparisons between elite and sub-elite swimmers in 200-m freestyle race of Japanese championship-. *Japanese Journal of Sport Methodology* 11, 87-93.
11. Okuno, K., Horinouchi, T., Naitou, K., Ikuta, Y., Wakayoshi, K., Nomura, T., 2003. A study on elite swimmer's race pattern of competition swimming in individuals medley events. *Annual Report of Physical Education* 35, 87-92.
12. Takagi, H., Shimizu, Y., Kodan, N., 1999. A hydrodynamic study of active drag in swimming. *JSME International Journal Series B* 42, 171-177.
13. Toussaint, H., de Groot, G., Savelberg, H., Vervoorn, K., Hollander, P., van Ingen, Schenau G., 1988. Active drag relates to velocity in male and female swimers. *Journal of Biomechanics* 21, 435-438.

Thank you for kind attention.
Comments and suggestions welcomed!

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