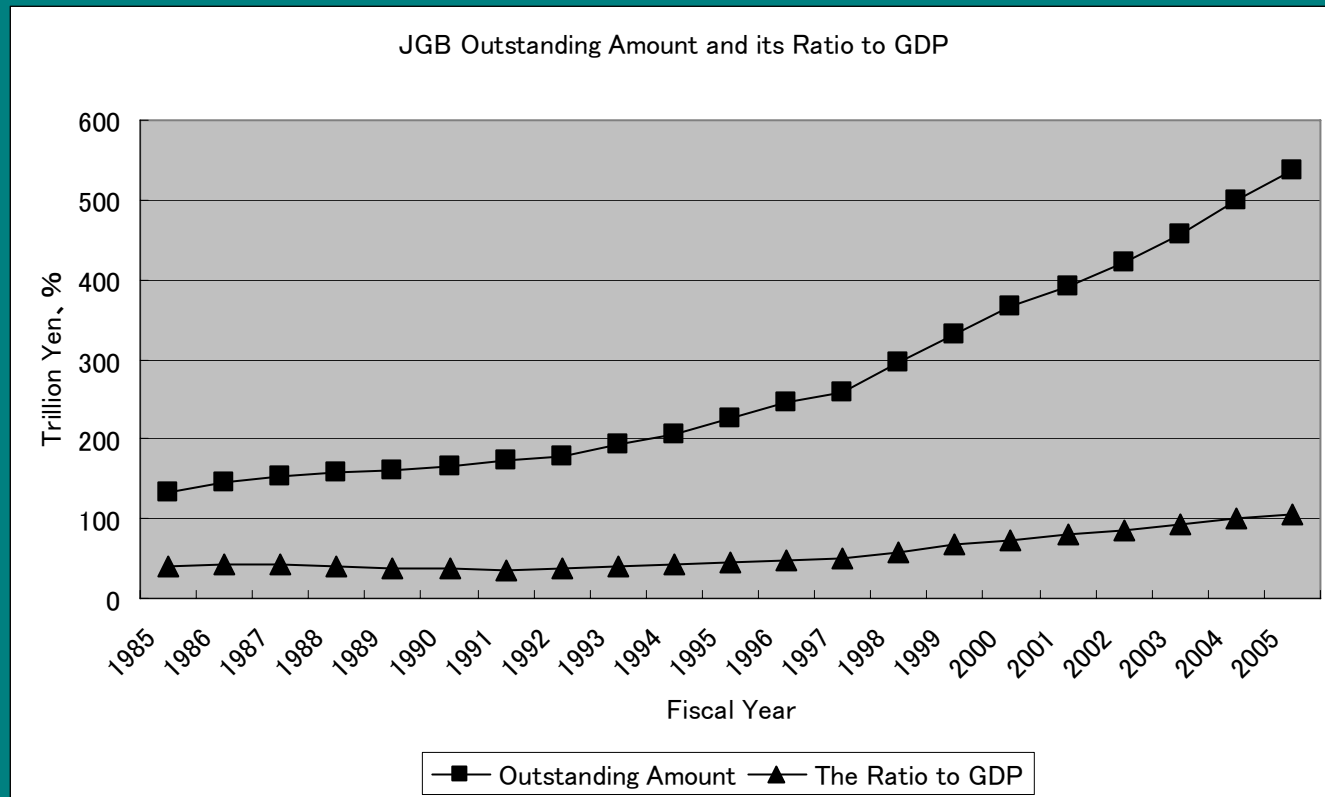


# When did the JGB market become efficient?

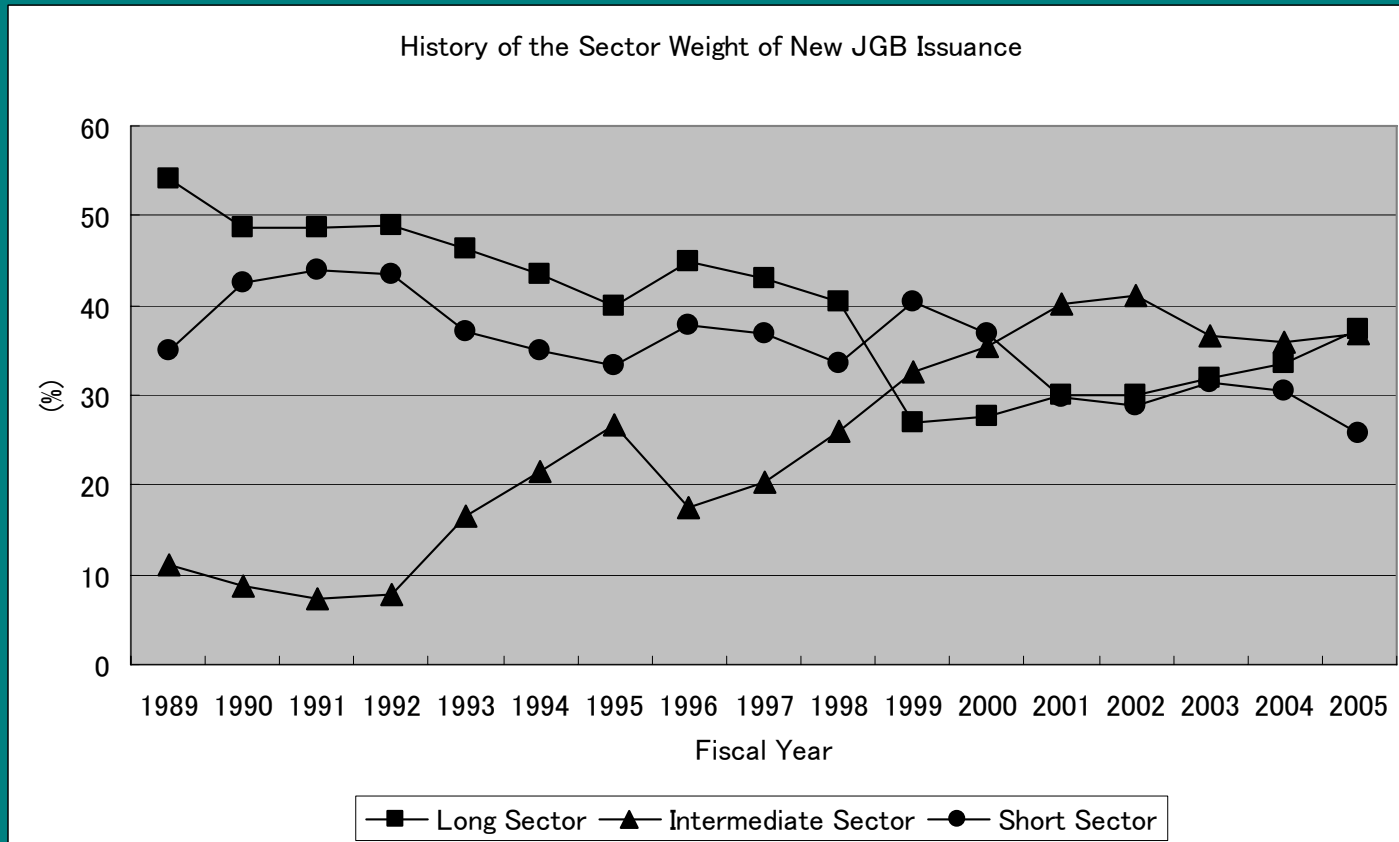
Cherry Bud Workshop 2007  
The 21<sup>st</sup> Century COE Program at Keio University

University of Electro-Communications  
Koichi Miyazaki  
(Co-worked with Satoshi Nomura)

# History of JGB Outstanding Amount and its Ratio to GDP



# History of the Sector Weight of New JGB Issuance

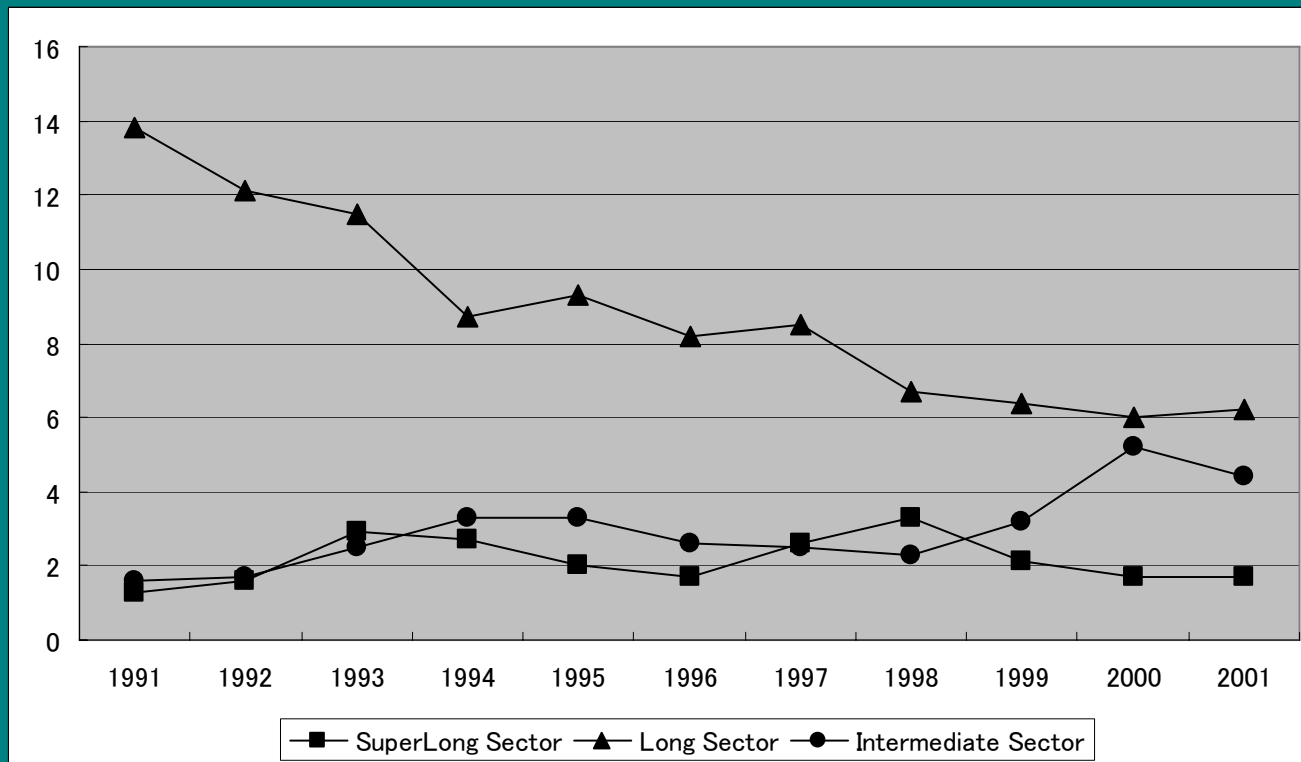


# Preceding Research

- Efficiency~Liquidity is usual perception
- Annual Trading Frequency
- Bid/Ask Spread
- Existence Rate of Quoted Price
- Price Impact in the Execution
- Amount of the order around current execution price (板の厚さ)

# Annual Trading Frequency

- Annual Trading Frequency = Annual Trading Volume ÷ Fiscal Year End Outstanding Amount



# Bid/Ask Spread

In Year 1998:

- Benchmark: 0.5BP
- Maturity Over 10Y: 2~4BP
- Maturity 7~10Y: 0.5~1BP
- Maturity 2~7Y: 2BP
- Maturity Less Than 2Y: 2~4BP

# When did the JGB become efficient?

Efficient: All the JGB sectors are fairly priced in the relative basis



## Three Models to quantify the efficiency of the JGB market

### ○ Model1

Relative comparison of bond yields

### ○ Model2

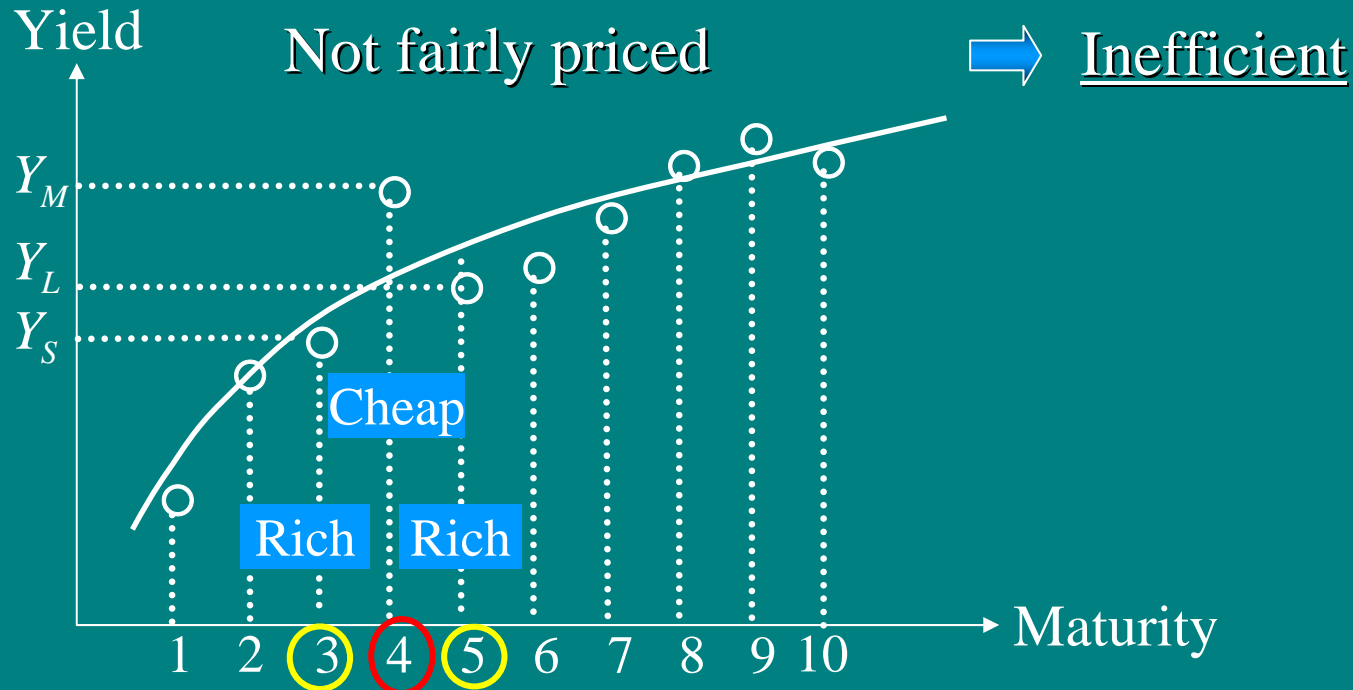
Price correction speed of the bond

### ○ Model3

Timing of the change in efficiency

# Model1

## ○ Relative comparison of bond yields

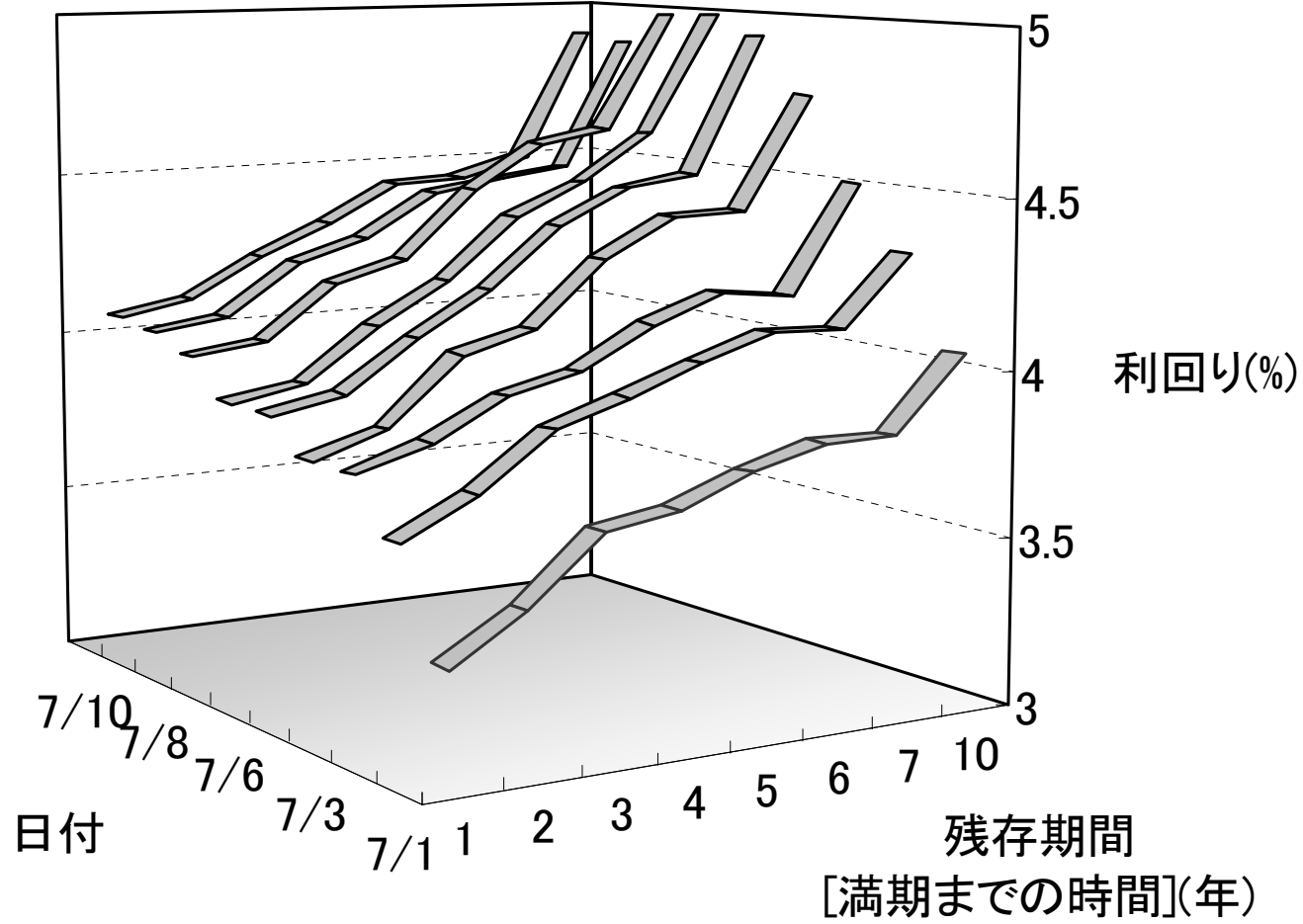


What kind of model is appropriate for the comparison?

Spline-curve, Factor Analysis, Butterfly Spread



# Yield Dynamics



# Time-series-wise relative comparison of bond yields

## ➤ Factor Analysis

The yield of each maturity bond is represented by the linear combination of factor loading (maturity specific) and the common factor (time series) .

The difference between the actual yield and the represented yield clarifies trading opportunity.

$$y_i(t) = a_{i,1} f_1(t) + a_{i,2} f_2(t) + \varepsilon_i \quad i = 1, \dots, n$$

Diagram illustrating the components of the equation:

- i-Y Yield** (points to  $y_i(t)$ )
- First Factor** (points to  $f_1(t)$ )
- Second Factor** (points to  $f_2(t)$ )
- First factor loading** (points to  $a_{i,1}$ )
- Second factor loading** (points to  $a_{i,2}$ )
- Rich/Chaep** (points to  $\varepsilon_i$ )

## ➤ Regression Based Butterfly Analysis (Industry)

# Butterfly Spread Analysis

$Y_S$ ,  $Y_M$ ,  $Y_L$ : Three different maturity bond yields from the short end

$$Y_M - Y_S = \alpha(Y_L - Y_S) + \beta + \varepsilon$$



$$\underbrace{Y_M}_{(4Y\text{yield})} - \left\{ \underbrace{(1-\alpha)Y_S}_{(3Y\text{yield})} + \underbrace{\alpha Y_L}_{(5Y\text{yield})} + \beta \right\} = \varepsilon$$

Butterfly Spread

(4Yyield) (3Yyield) (5Yyield)

Relative comparison of the yields  
based on the butterfly spreads



One evaluation model of the market efficiency

# Important remarks on empirical analysis

➤ What is the criteria of fairness in the butterfly analysis?

✘ The butterfly spread is not corrected when it stays within Bid/Offer spread

➡ Introduce some thresholds

➤ How to remove the effects of yield level and market volatility?

$$|\varepsilon| = \gamma Y_M + \delta \sigma + \xi \quad \Rightarrow \quad \text{Use } \xi$$

✘ When the coefficients are not statistically significant, we may use butterfly spread itself.

Count # of the business days that the butterfly spread exceed the threshold quarterly basis

# Model2

- Focus on the correction speed of the richness/cheapness



Once the butterfly spread exceeds the threshold, its dynamics follows:

$$d\varepsilon_t = -\kappa\varepsilon_t dt + \sigma_{SP} dW_t$$

$\kappa$  : Mean-reversion parameter       $\sigma_{SP}$  : Volatility

✂ **Half lives of the butterfly spread:**  $\tau = \frac{1}{\kappa} \ln 2$



Another evaluation model of the market efficiency

# Model3

- The model to capture the timing of the change in efficiency

Model1 : Quantify # of the butterfly spreads that exceed the threshold quarterly basis.

Model2 : Quantify the average half-lives of the butterfly spreads (Dates) quarterly basis.

$$Y_t \sim \begin{cases} POI(\lambda) & t = 1, \dots, k & \lambda : \text{Intensity in Inefficient} \\ POI(\phi) & t = k + 1, \dots, n & \phi : \text{Intensity in Efficient} \end{cases}$$

$n$  : # of data,  $k$  : **the timing of the change**



Markov Chain Monte Carlo Method (MCMC)

# MCMC

$(\lambda, \phi)$  Priors :  $\lambda \sim \text{Gamma}(a_0, b_0), \phi \sim \text{Gamma}(c_0, d_0)$

$k$  Prior : discrete uniform distribution on  $1, 2, \dots, n$ .



$(\lambda, \phi)$  Posteriors :  $\lambda \sim \text{Gamma}(a_1, b_1), \phi \sim \text{Gamma}(c_1, d_1)$

Where,  $a_1 = a_0 + \sum_{t=1}^k y_t, b_1 = b_0 + k, c_1 = c_0 + \sum_{t=k+1}^n y_t, d_1 = d_0 + (n - k)$

$k$  Posteriors :

$$\pi(k|\lambda, \phi, y) = \frac{w_k}{\sum_t w_t}, \quad w_k = \lambda^{a_1-1} \phi^{c_1-1} \exp\{-(\lambda - \phi)k\}$$

# Method of the Empirical Analysis

## Model1

### STEP1-1:

In each quarter, derive  $\xi$  based on the daily data

### STEP1-2:

In each quarter, count # of days that the butterfly spread  $\xi$  exceeds the threshold.

## Model2

### STEP2-1:

Derive  $\xi$  as in STEP1-1

### STEP2-2:

Estimate the mean-reversion parameter of  $\xi$

### STEP2-3:

Compute the averages of the half-lives  $\tau$  for all butterfly trade combination

## Model3



Estimate the timing of the change in market efficiency



# Data and the Setting

- Data periods

Jan.4.1989~Mar.31.2005  
(4Q1988~4Q2004)

- JGB Yields

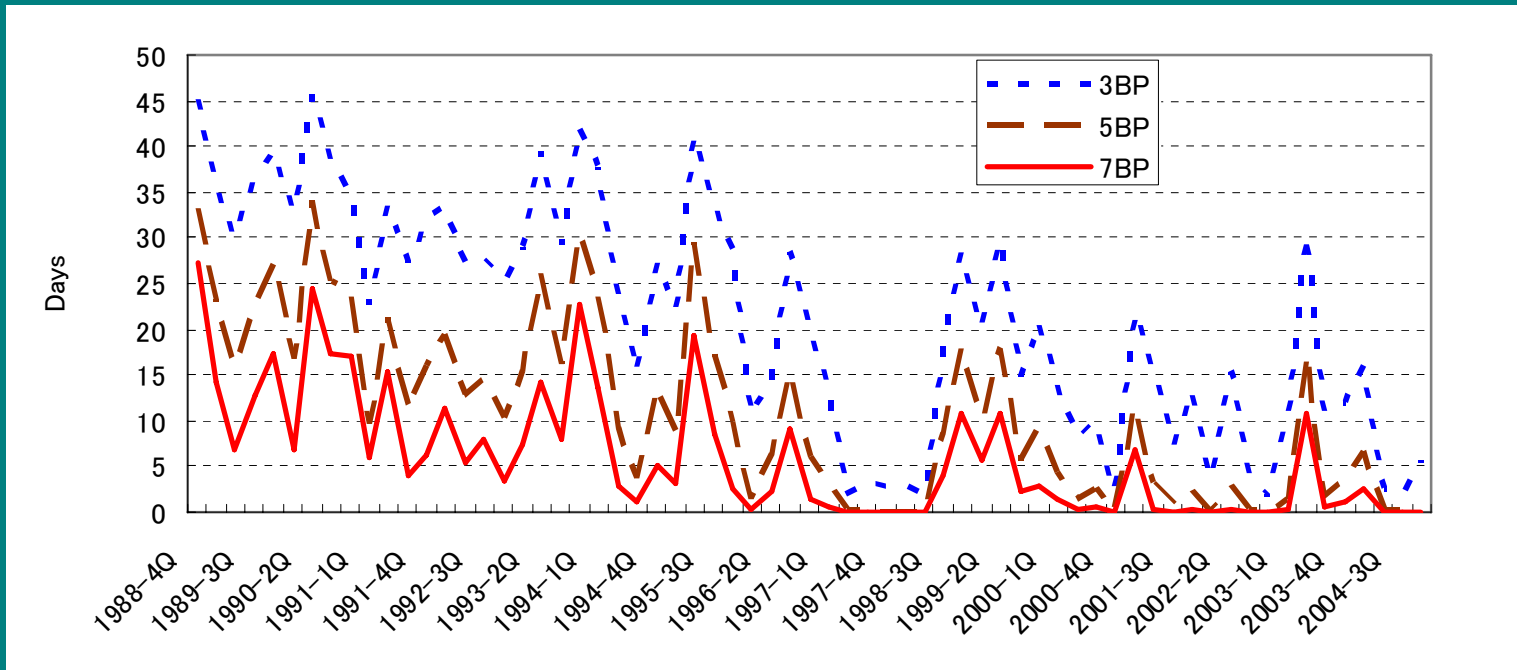
2Y · 5Y · 7Y · 10Y · 20Y JGB Yields

(10 kinds of butterfly spreads)

- Threshold Values

3BP · 5BP · 7BP (Three kinds of settings)

# Average # of days for which all kinds of butterfly spread $\varepsilon$ exceeded each threshold value

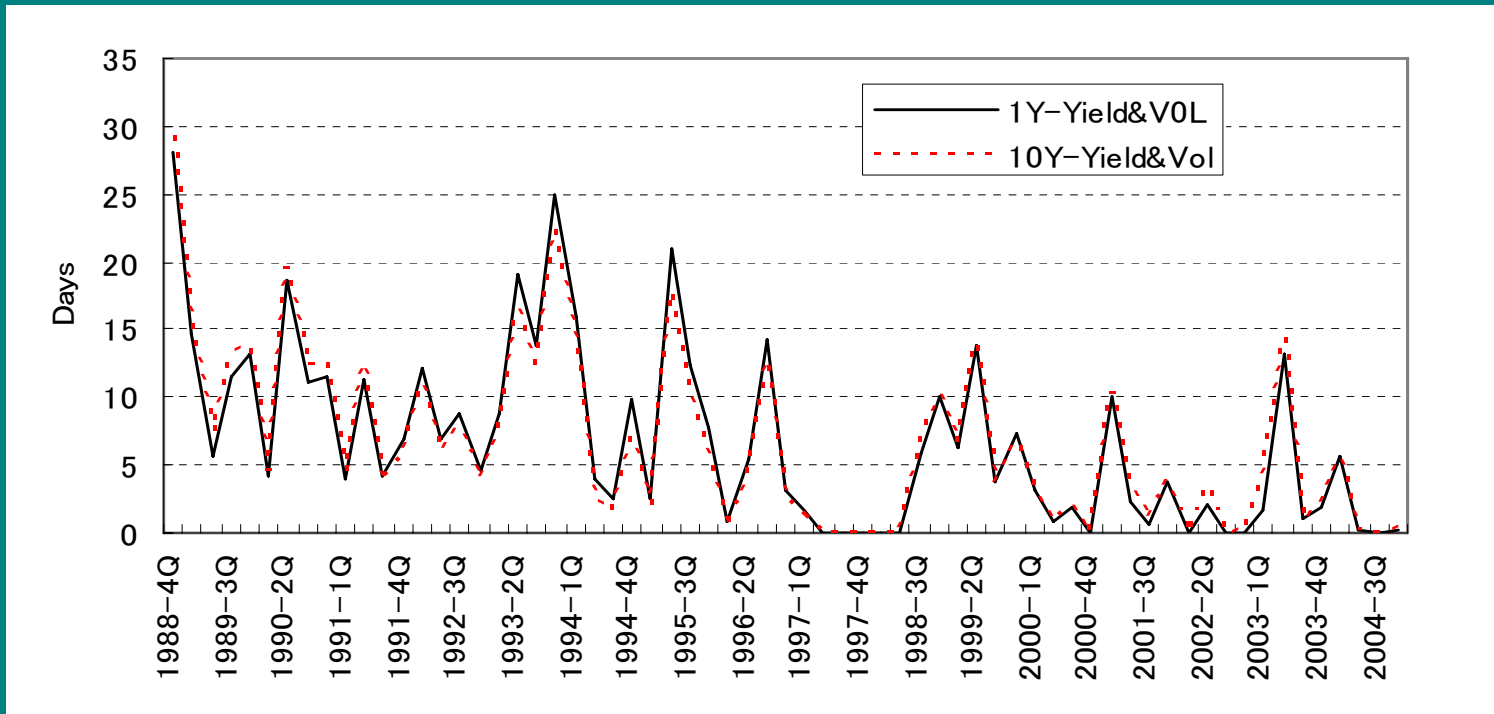


Decreased in late 1990s

# The effects of yield level and volatility

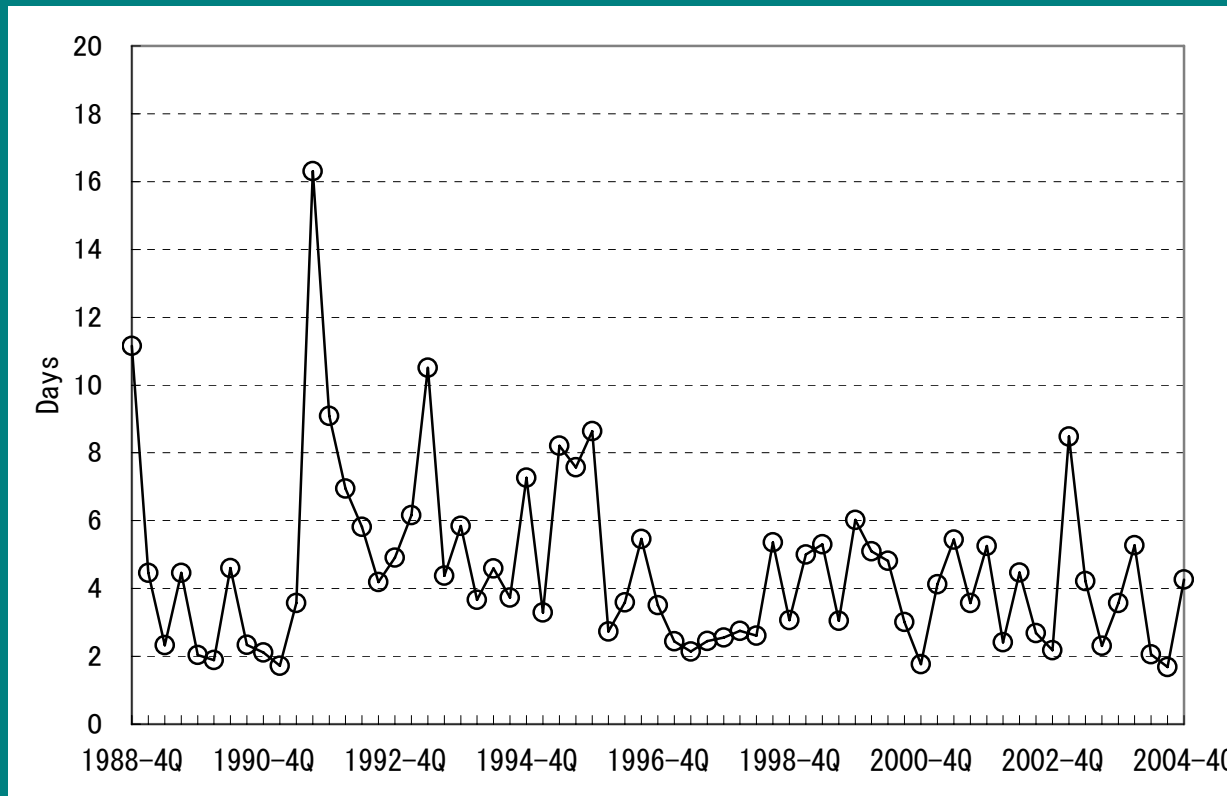
	1-Year Yield		Volatility		R <sup>2</sup>	10-Year Yield		Volatility		R <sup>2</sup>
	$\gamma$	t-value	$\delta$	t-value		$\gamma$	t-value	$\delta$	t-value	
"2-5-7"	0.29	19.3	0.024	15.4	0.18	0.46	23.8	0.021	13.8	0.21
"2-5-10"	0.46	21.0	0.025	11.1	0.16	0.64	22.4	0.023	10.2	0.17
"2-5-20"	0.52	19.4	0.028	10.1	0.14	0.63	17.5	0.028	10.0	0.12
"2-7-10"	0.40	20.5	0.023	11.4	0.16	0.56	22.0	0.021	10.5	0.17
"2-7-20"	0.52	18.1	0.022	7.5	0.11	0.58	15.0	0.023	7.8	0.09
"2-10-20"	0.40	18.9	0.022	10.2	0.13	0.48	16.9	0.023	10.1	0.12
"5-7-10"	0.29	20.3	0.015	10.6	0.15	0.43	23.3	0.013	9.3	0.17
"5-7-20"	0.32	18.7	0.016	9.3	0.13	0.45	19.7	0.015	8.5	0.14
"5-10-20"	0.33	21.4	0.023	14.1	0.18	0.47	22.9	0.021	13.2	0.20
"7-10-20"	0.26	19.1	0.017	12.2	0.15	0.41	22.9	0.015	10.8	0.18

# Average # of days for which all kinds of butterfly spread exceeded each threshold value



Decreased in late 1990s

# Averages of the half-lives for all butterfly trade combinations



Decreased in late 1990s

# Parameter estimation results for Model3 based on the Model1 results $\varepsilon$ : all the quarters

	Threshold Value	Mean	S.D.	95%Low	95%High
$\lambda$	3BP	31.2	0.34	30.6	31.9
	5BP	18.5	0.26	18.0	19.0
	7BP	10.8	0.21	10.4	11.2
$\phi$	3BP	12.3	0.19	12.0	12.7
	5BP	4.5	0.11	4.3	4.8
	7BP	2.3	0.08	2.2	2.5
$k$	3BP	30.3	0.03	30.3	30.4
	5BP	29.2	0.03	29.1	29.2
	7BP	27.8	0.05	27.7	27.9

The timing of the change in market efficiency is around  $k=30$  (Q1FY1996)

# Parameter estimation results for Model3 based on the Model1 results

$\varepsilon$ : excluding Q21997-Q21998

	Threshold Value	Mean	S.D.	95%Low	95%High
$\lambda$	3BP	29.8	0.13	29.5	30.0
	5BP	17.4	0.10	17.2	17.6
	7BP	10.5	0.07	10.4	10.7
$\phi$	3BP	13.2	0.09	13.0	13.4
	5BP	5.1	0.16	4.8	5.4
	7BP	2.5	0.03	2.4	2.6
$k$	3BP	34.4	0.11	34.2	34.6
	5BP	32.3	0.32	31.7	32.9
	7BP	29.6	0.12	29.4	29.9

The timing of the change in market efficiency is around  $k=30 \sim 35$  ( $k=30$ :Q1FY1996)

# Parameter estimation results for Model3 based on the Model1 results $\xi$ : all the quarters

	Control Yield	Mean	S.D.	95%Low	95%High
$\lambda$	1Year	10.1	0.19	9.7	10.4
	10Year	9.9	0.19	9.5	10.3
$\phi$	1Year	3.2	0.10	3.0	3.4
	10Year	3.4	0.11	3.2	3.6
$k$	1Year	34.4	0.29	33.8	35.0
	10Year	32.7	0.29	32.1	33.2

The timing of the change in market efficiency is around  $k=32 \sim 35$  ( $k=30$ :Q1FY1996)



# Parameter estimation results for Model3 based on the Model1 results

$\xi$  : excluding Q21997-Q21998

	Control Yield	Mean	S.D.	95%Low	95%High
$\lambda$	1Year	9.9	0.06	9.8	10.0
	10Year	9.8	0.08	9.7	10.0
$\phi$	1Year	3.7	0.04	3.6	3.7
	10Year	4.3	0.06	4.2	4.4
$k$	1Year	35.5	0.12	35.3	35.7
	10Year	31.5	0.20	31.1	31.8

The timing of the change in market efficiency is around  $k=31 \sim 36$  ( $k=30$ :Q1FY1996)

# Parameter estimation results for Model3 based on the Model2 results $\tau$ : all the quarters

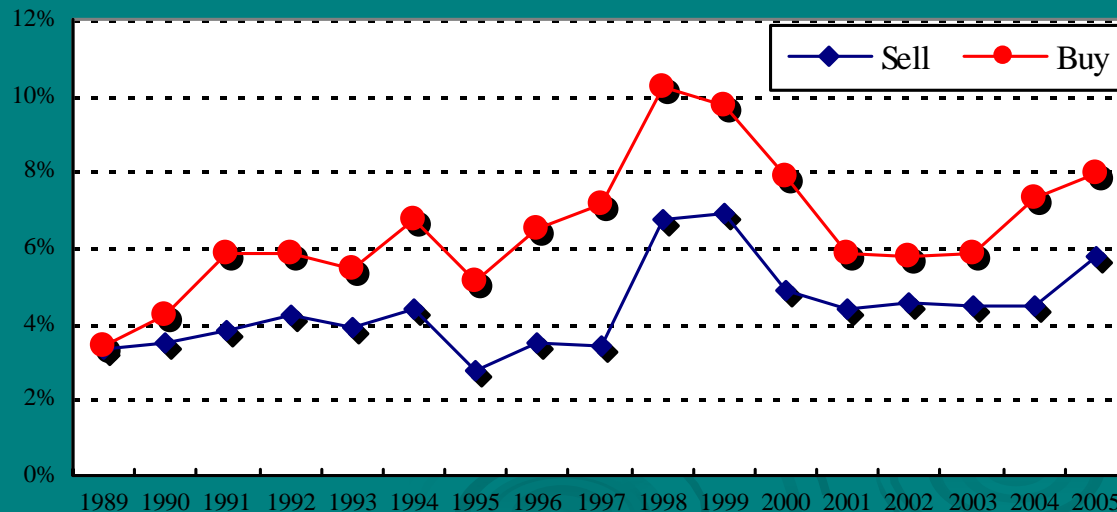
	Mean	S.D.	95%Low	95%High
$\lambda$	5.4	0.13	5.1	5.6
$\phi$	3.8	0.11	3.6	4.0
$k$	33.1	0.10	32.9	6.7

The timing of the change in market efficiency is around  $k=33$  ( $k=30$ :Q1FY1996)

# Background of the change in JGB market efficiency

- Deregulations on repo/reverse market
- Regarding the issuance, from syndication to competitive bidding
- Overseas investors participate in the market
- Increasing variety of available JGB

Percentage of overseas investors in JGB market



# Summary

○ We examined the change in the market efficiency.

We introduced three models.

Model1 : Relative comparison of bond yields

Model2 : Price correction speed of the bond

Model3: Timing of the change in efficiency



Around FY1996, the JGB market became efficient

- MoF and TSE accelerated JGB market reform. (Repo)
- The presence of overseas investors