Measuring the Effectiveness of Marketing Activities and Baseline Sales from POS Data using Bayesian State Space Models

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Business environment in Japan

The uncertainty is increasing !!

- Globalization of the economy
- Development of the information technology
- Change in customer's sense of value
- Deregulation

Main aim

- To develop the methodology for predicting daily and baseline sales of individual items by considering specific factors

- The prediction is very useful for
  - inventory management
  - marketing strategy planning
  - manpower planning
  - production planning

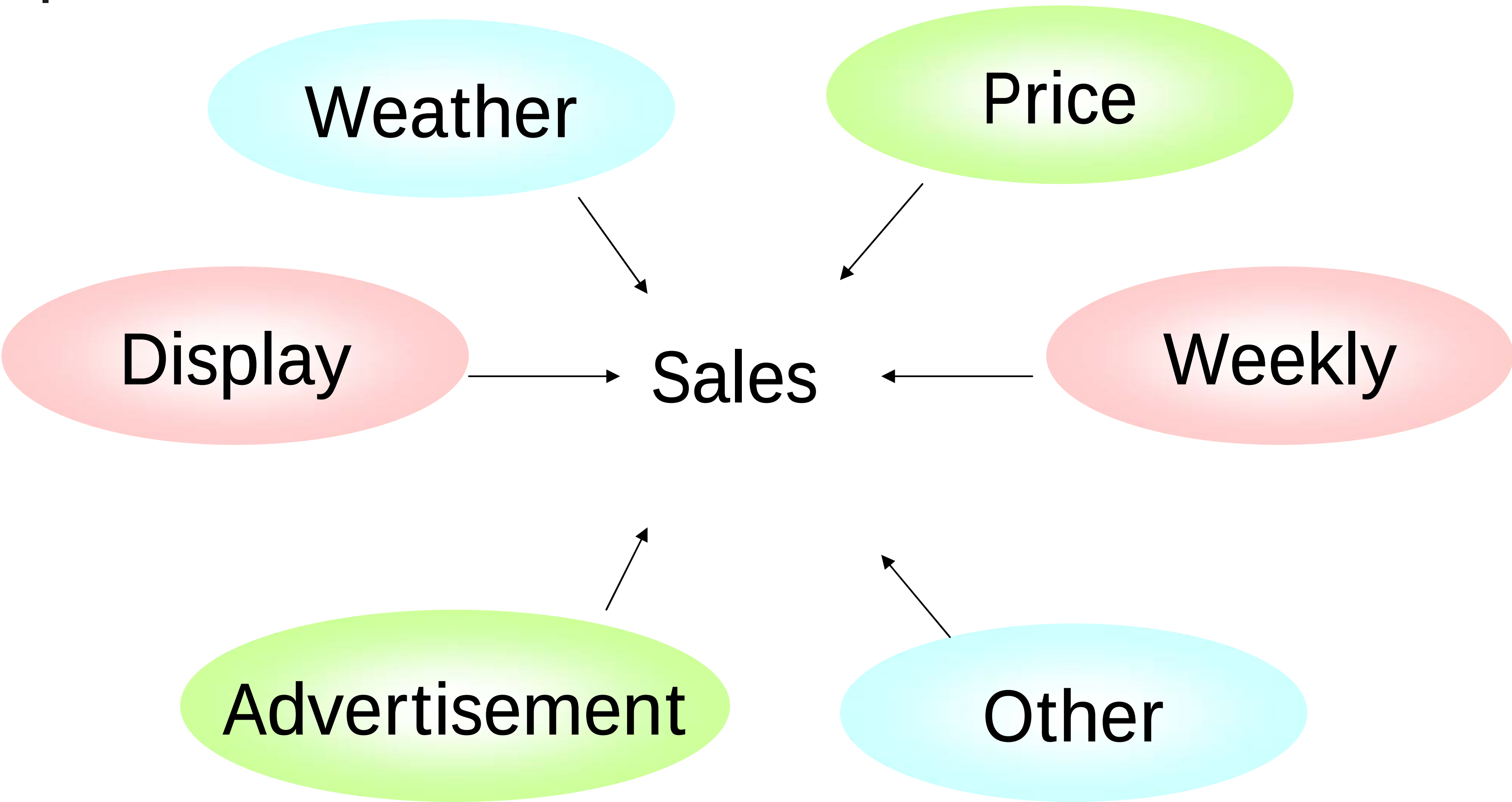
Agenda

- Data description
- Model description
- Bayesian MCMC Estimation
- Model evaluation
- Application results
- Conclusion

Data description

- Daily unit sales for 10 foods
  - Cold beverages
  - Coffee
  - Tea beverages
  - Cup noodle
  - Rice balls
  - Sandwiches
  - Packed lunches
  - Jelly
  - Yogurt

Data description



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- To develop the methodology for predicting daily and baseline sales of individual items by considering specific factors

- The prediction is very useful for
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Model description

- A general state space model (Kitagawa, 1987) enables us to conduct flexible time series modeling.
- Observation model

$$y_t \sim f(y_t \mid F_t, h_t, \ldots, h_1)$$

- System model

$$h_t \sim f(h_t \mid F_t, h_{t-1}, \ldots, h_1)$$

Model description

- Observation model

Daily unit of sales
= Baseline sales
+ Sales affected by environmental effect
+ Sales obtained from marketing promotion

Model description

- Observation model (Poisson model)

Daily unit sales

$$y_t \sim Poisson(\lambda_t \mid \mathbf{x}_t)$$

$$\mathbf{x}_t = (w_t, r_t, p_t, d_t, a_t)'$$

$$\lambda_t = h_t + \beta_1 w_t + \beta_2 r_t + \beta_3 p_t + \beta_4 d_t + \beta_5 a_t$$

Baseline unit sales Weather effect Price promotion effect

Weekly effect Display promotion effect Advertisement effect

Model description

- System model (q-th order trend model)

$$\Delta^q h_t = \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2)$$

- Example

$$(q=1): h_t = h_{t-1} + \varepsilon_t$$

$$(q=2): h_t = 2h_{t-1} - h_{t-2} + \varepsilon_t$$

Model description

- Each of the 10 foods follow the Poisson model.

$$y_{jt} \sim Poisson(\lambda_{jt} \mid \mathbf{x}_{jt}), \quad j = 1, \ldots, 10$$

- We consider that the daily unit sales are mutually dependent on each other.

$$\begin{cases} \Delta^q h_{jt} = \varepsilon_{jt}, \ \ \varepsilon_{jt} \sim N(0, \sigma_j^{\ 2}), \quad j = 1, \ldots, 10 \\ Cov(\varepsilon_{j,t}, \varepsilon_{k,t}) = \sigma_{jk} \end{cases}$$

Model description

- Summarizing the above specification leads to the following observation equation and the system equation:

Observation equation

$$y_{jt} \sim f(y_{jt} \mid h_{jt}; \mathbf{x}_{jt}, \boldsymbol{\beta}_j)$$

System equation

$$\mathbf{h}_t \sim f(\mathbf{h}_t \mid \mathbf{h}_{t-1}, \ldots, \mathbf{h}_{t-q}, \Sigma), \ \Sigma = (\sigma_{ij})$$

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Bayesian estimation via Markov chain Monte Carlo (1/2)

- A set of n independent observations
- Likelihood function

$$
\begin{aligned}
L(D_n|\boldsymbol{\theta}, X_n) &= \prod_{t=1}^{n} f(\boldsymbol{y}_t|F_{t-1}, \boldsymbol{x}_t, \boldsymbol{\theta}) \\
&= \prod_{t=1}^{n} \left[\prod_{j=1}^{p} f(y_{jt}|F_{t-1}, \boldsymbol{x}_{jt}, \boldsymbol{\theta}) \right] \\
&= \prod_{t=1}^{n} \left[\prod_{j=1}^{p} \int \underbrace{f(y_{jt}|h_{jt}; \boldsymbol{x}_{jt}, \beta_j)}_{\text{Observation model}} \underbrace{f(\boldsymbol{h}_t|\boldsymbol{h}_{t-1}, \ldots, \boldsymbol{h}_{t-q}; \Sigma)}_{\text{System model}} d\boldsymbol{h}_t \right]
\end{aligned}
$$

Bayesian estimation via Markov chain Monte Carlo (2/2)

- Markov chain Monte Carlo considers the state vector as model parameters

$$\mathbf{h} = (\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_n)$$

- A posterior distribution

$$\pi(\boldsymbol{\theta}, \boldsymbol{h} | D_n, X_n) \propto \pi(\boldsymbol{\theta}) \times \prod_{t=1}^{n} \prod_{t=1}^{n} f(y_{jt} | h_{jt}; \boldsymbol{x}_{jt}, \beta_j) f(\boldsymbol{h}_t | \boldsymbol{h}_{t-1}, \ldots, \boldsymbol{h}_{t-q}; \Sigma).$$

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Maximisation of the posterior mean of the expected likelihood

- Ando (2006) considered the maximization of the posterior mean of the expected log-likelihood (PMELL):

$$\eta = E_{Z_n}\left[E_{\boldsymbol{\theta}|D_n,X_n}[\log L(Z_n|\boldsymbol{\theta}, X_n)]\right]$$
$$= \int \left[\int \log L(Z_n|\boldsymbol{\theta}, X_n)\pi(\boldsymbol{\theta}|D_n, X_n)d\boldsymbol{\theta}\right] dG(Z_n|X_n),$$

- The best model is chosen by maximising η .

Ando (2006)

- The asymptotic bias is

$$
\begin{aligned}
n\hat{b} \approx\ & E_{\boldsymbol{\theta}|D_n,X_n}\left[\log\{L(D_n|\boldsymbol{\theta},X_n)\pi(\boldsymbol{\theta})\}\right] + \dim\{\boldsymbol{\theta}\}/2 \\
& - \log\{L(D_n|\hat{\boldsymbol{\theta}}_n,X_n)\pi(\hat{\boldsymbol{\theta}}_n)\} + \mathrm{tr}\left\{J_n^{-1}(\hat{\boldsymbol{\theta}}_n)I_n(\hat{\boldsymbol{\theta}}_n)\right\},
\end{aligned}
$$

where

$$
\begin{aligned}
I_n(\boldsymbol{\theta}) &= \frac{1}{n}\sum_{t=1}^{n}\left\{\frac{\partial\eta_n(\boldsymbol{y}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\frac{\partial\eta_n(\boldsymbol{y}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}'}\right\}, \\
J_n(\boldsymbol{\theta}) &= -\frac{1}{n}\sum_{t=1}^{n}\left\{\frac{\partial^2\eta_n(\boldsymbol{y}_t,\boldsymbol{\theta})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'}\right\},
\end{aligned}
$$

with $\eta_n(\boldsymbol{y}_t,\boldsymbol{\theta}) = \log f(\boldsymbol{y}_t|F_{t-1},\boldsymbol{x}_t,\boldsymbol{\theta}) + \log\pi(\boldsymbol{\theta})/n.$

Ando (2006)

- The prior is dominated by the likelihood as the sample size becomes large.

- The specified parametric models contain the true model or are close to the true model

$$\Rightarrow \quad n \times \hat{b} = \dim(\boldsymbol{\theta})$$

Bayesian predictive information criterion

- Bayesian predictive information criterion (BPIC Ando (2006)):

$$\mathbf{BPIC} = -2E_{\boldsymbol{\theta}|D_n, X_n}\left[\log L(D_n \mid \boldsymbol{\theta}, X_n)\right] + 2\dim\{\boldsymbol{\theta}\}$$

- We choose the predictive distribution that minimises the BPIC score.

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Application results

- The posterior mean of the log-likelihood and BPIC scores for each of q-th order trend models.

q	1	2	3	4
$\hat{\eta}$	-1543.081	-1507.530	-1492.693	-1483.921
BPIC	3296.162	3245.060	3235.386	3237.842
CPU time	16.75	17.54	18.28	19.09

- The CPU time (seconds) to generate 100 iterations is also reported.

Model description

- Observation model

$$y_t \sim Poisson(\lambda_t \mid \mathbf{x}_t)$$

$$\mathbf{x}_t = (w_t, r_t, p_t, d_t, a_t)'$$

$$\lambda_t = h_t + \beta_1 w_t + \beta_2 r_t + \beta_3 p_t + \beta_4 d_t + \beta_5 a_t$$

Baseline unit sales (h_t)

Weekly effect ($\beta_1 w_t$)

Weather effect ($\beta_2 r_t$)

Display promotion effect ($\beta_3 p_t$)

Price promotion effect ($\beta_4 d_t$)

Advertisement effect ($\beta_5 a_t$)

		PMs	95% CIs				PMs	95% CIs	
CB	β_{11}	0.66	[0.14	1.18]	SA	β_{61}	0.52	[0.06	0.97]
	β_{12}	7.52	[6.52	8.59]		β_{62}	0.22	[0.02	0.60]
	β_{13}	41.2	[40.2	42.3]		β_{63}	1.53	[0.17	2.56]
	β_{14}	6.38	[5.71	7.16]		β_{64}	2.36	[1.85	2.71]
	β_{15}	1.37	[0.09	2.47]		β_{65}	0.64	[0.04	1.59]
CF	β_{21}	0.58	[0.15	1.06]	P1	β_{71}	0.52	[0.07	1.28]
	β_{22}	13.2	[12.4	13.8]		β_{72}	0.99	[0.13	1.51]
	β_{23}	19.9	[19.3	20.6]		β_{73}	3.06	[1.04	4.21]
	β_{24}	4.82	[3.97	6.19]		β_{74}	17.0	[16.2	17.8]
	β_{25}	1.81	[1.10	2.68]		β_{75}	1.64	[0.98	2.40]
TB	β_{31}	0.63	[0.05	1.67]	P2	β_{81}	0.29	[0.04	0.64]
	β_{32}	9.67	[8.60	10.5]		β_{82}	0.71	[0.03	1.33]
	β_{33}	1.03	[0.21	1.59]		β_{83}	104.57	[103.6	105.4]
	β_{34}	3.15	[2.44	3.70]		β_{84}	0.45	[0.03	1.45]
	β_{35}	11.2	[10.4	12.2]		β_{85}	0.38	[0.04	0.84]
CN	β_{41}	1.01	[0.06	1.97]	JE	β_{91}	5.65	[4.47	6.89]
	β_{42}	4.61	[3.95	5.24]		β_{92}	5.65	[4.06	6.56]
	β_{43}	1.66	[1.01	2.17]		β_{93}	0.55	[0.08	1.15]
	β_{44}	9.47	[8.50	11.0]		β_{94}	49.3	[48.4	51.3]
	β_{45}	2.68	[1.27	4.12]		β_{95}	36.3	[35.0	37.8]
RB	β_{51}	1.70	[0.30	2.68]	YO	β_{101}	1.52	[0.78	2.13]
	β_{52}	16.5	[15.4	17.3]		β_{102}	14.6	[14.1	15.1]
	β_{53}	3.23	[2.60	3.88]		β_{103}	0.36	[0.04	0.84]
	β_{54}	13.1	[12.5	13.7]		β_{104}	13.5	[11.7	14.9]
	β_{55}	0.95	[0.50	1.69]		β_{105}	26.2	[25.7	26.9]

Application results

- Observation model

$$y_t \sim Poisson(\lambda_t \mid \mathbf{x}_t)$$

$$\mathbf{x}_t = (w_t, r_t, p_t, d_t, a_t)'$$

$$\lambda_t = h_t + 1.7w_t + 16.5r_t + 3.2p_t + 13.1d_t + 0.95a_t$$

Baseline unit sales

Weather effect

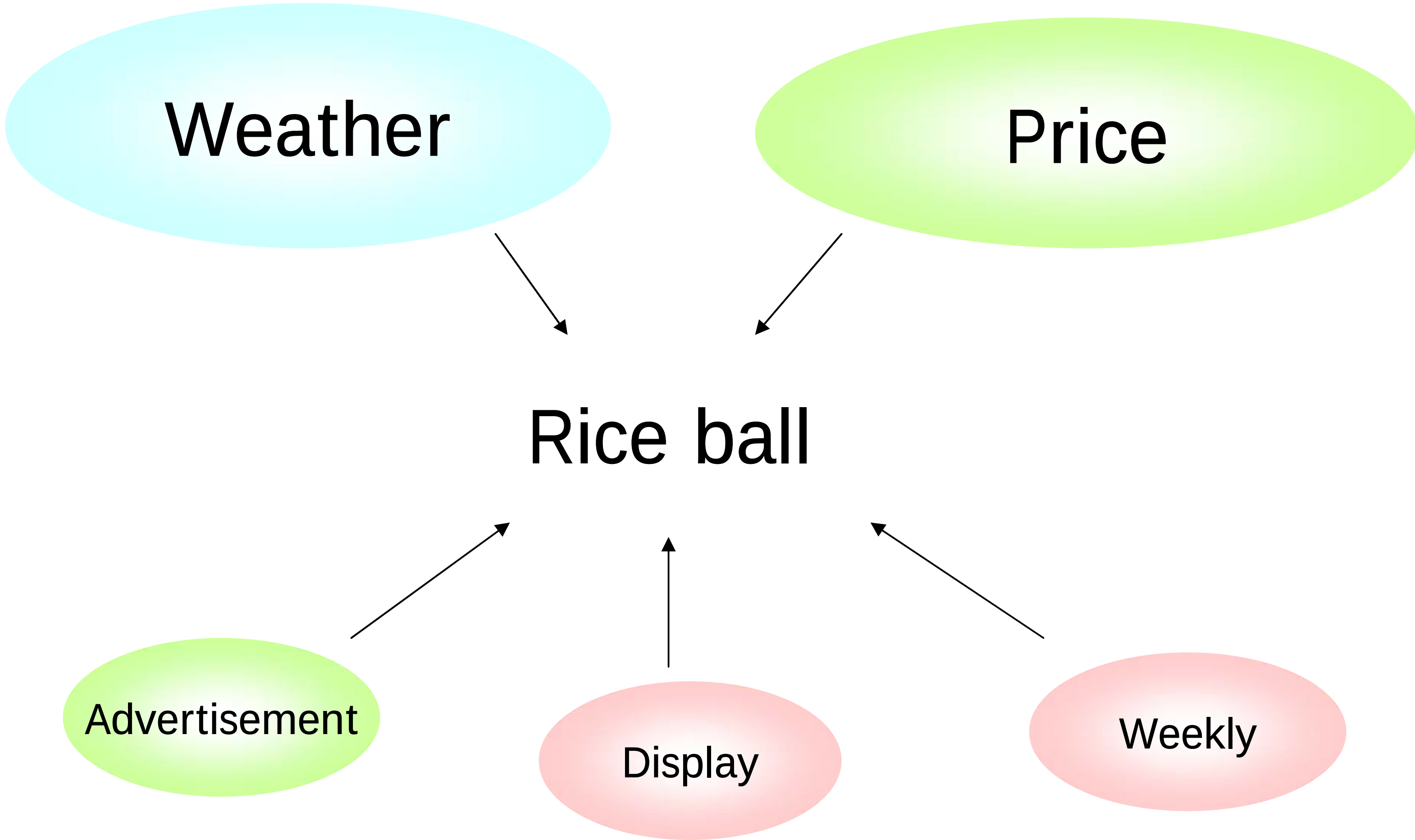
Price promotion effect

Weekly effect

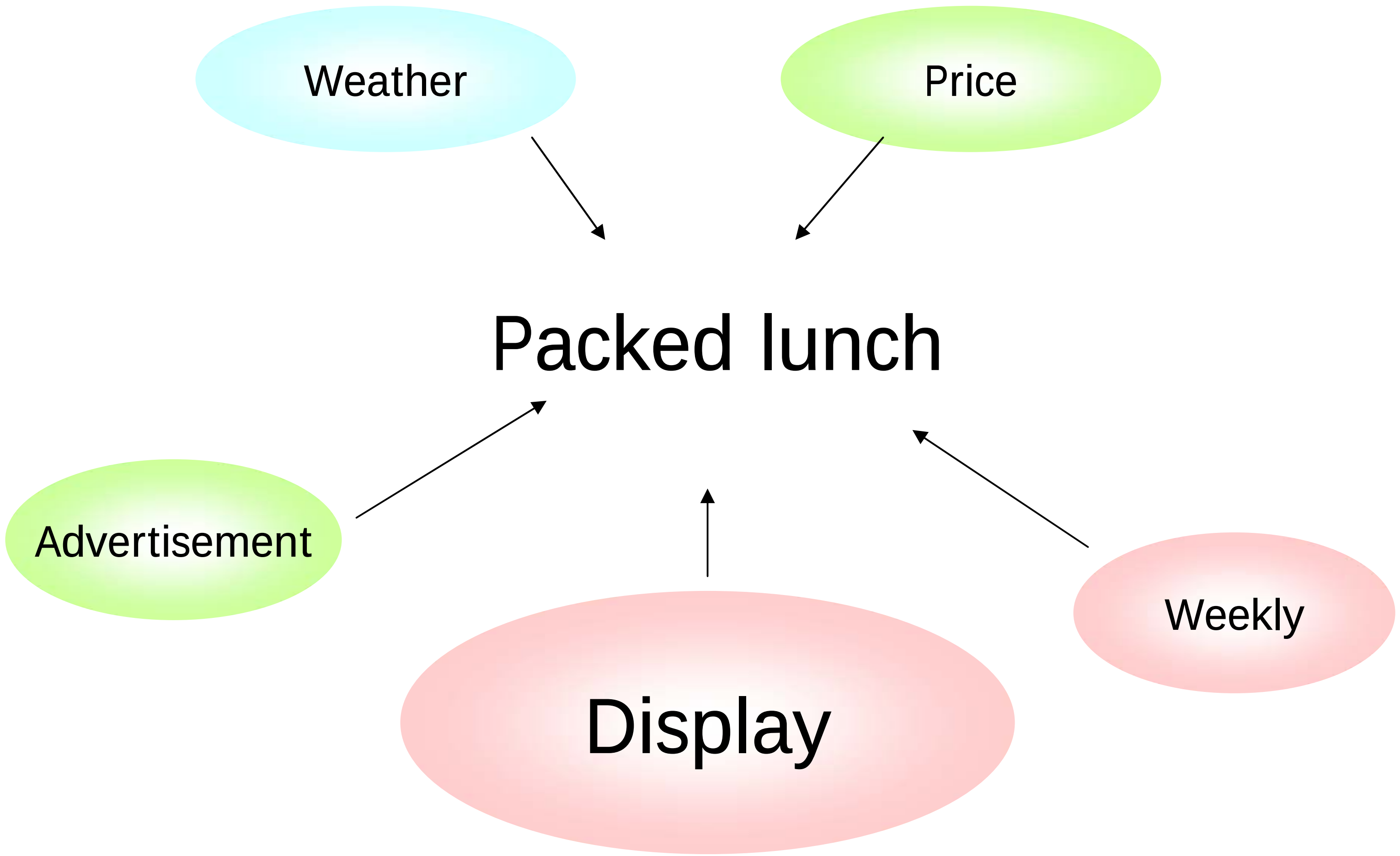
Display promotion effect

Advertisement effect

Application results



Application results



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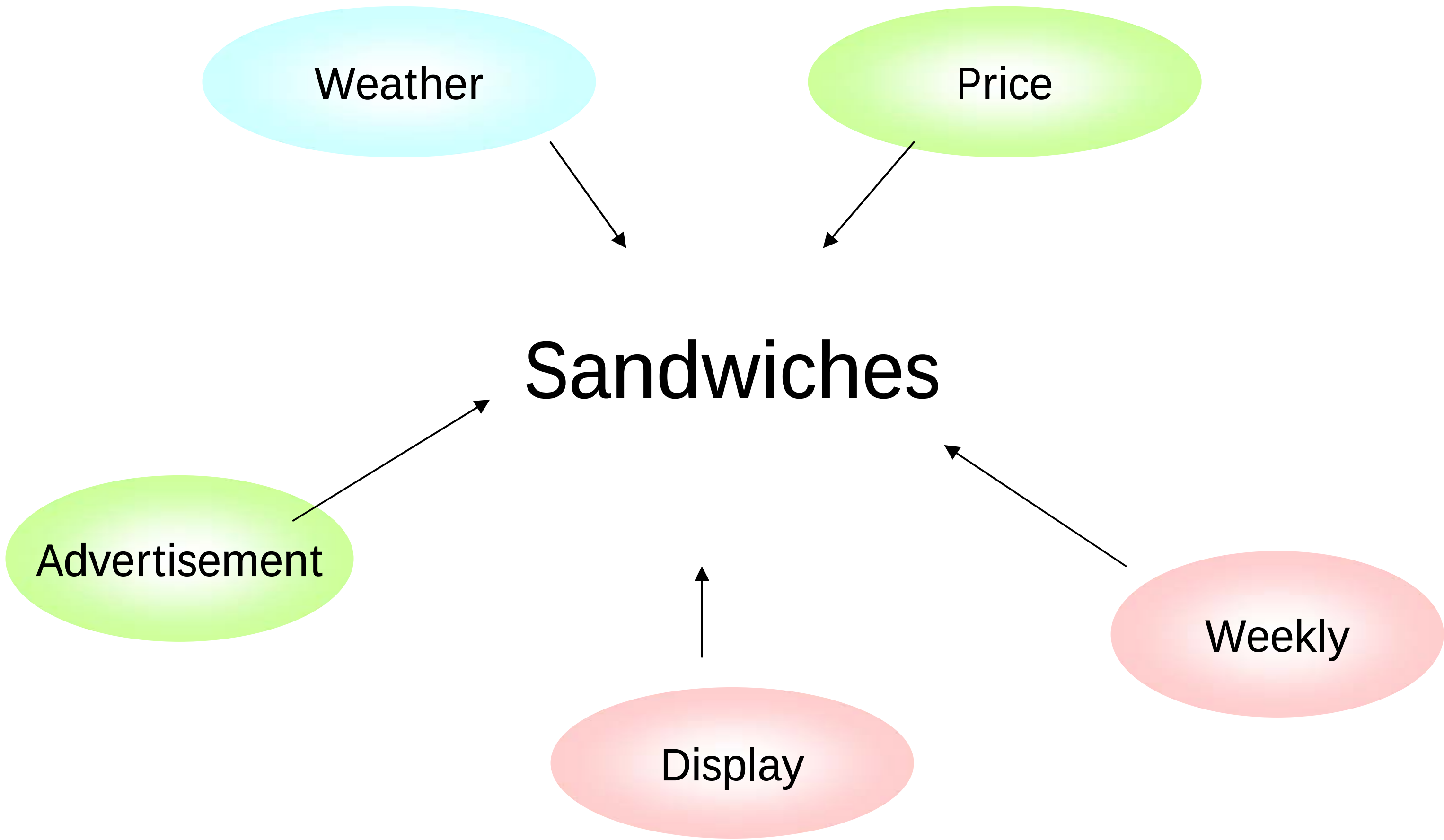

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$$\begin{cases} \Delta^q h_{jt} = \varepsilon_{jt}, \quad \varepsilon_{jt} \sim N(0, \sigma_j^{\ 2}), \quad j = 1, \ldots, 10 \\ Cov(\varepsilon_{j,t}, \varepsilon_{k,t}) = \sigma_{jk} \end{cases}$$

Application results

- The posterior means of correlation

	CB	CF	TB	CN	RB	SA	P1	P2	JE	YO
CB:	1.00	0.54	0.35	0.15	0.52	0.18	0.17	0.24	0.35	0.46
CF:	0.54	1.00	0.37	0.29	0.40	0.21	0.36	0.12	0.49	0.44
TB:	0.35	0.37	1.00	0.55	0.74	0.13	0.34	0.34	0.28	0.36
CN:	0.15	0.29	0.55	1.00	0.47	0.23	0.28	0.17	0.41	0.29
RB:	0.52	0.40	0.74	0.47	1.00	0.07	0.31	0.35	0.59	0.61
SA:	0.18	0.21	0.13	0.23	0.07	1.00	0.43	0.34	0.22	0.23
P1:	0.17	0.36	0.34	0.28	0.31	0.43	1.00	-0.24	0.41	0.61
P2:	0.24	0.12	0.34	0.17	0.35	0.34	-0.24	1.00	0.20	0.13
JE:	0.35	0.49	0.28	0.41	0.59	0.22	0.41	0.20	1.00	0.83
YO:	0.46	0.44	0.36	0.29	0.61	0.23	0.61	0.13	0.83	1.00

CB: cold beverages, CF: coffee, TB: tea beverages, CN: cup noodle, RB: rice balls, SA: sandwiches, P1: packed lunches (type1), P2: packed lunches (type2), JE: jelly and YO: yogurt.

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Conclusion

- The problem of identifying unobserved baseline sales, marketing promotion effects and other specific effects using POS data is considered.
- General state space model
- Bayesian MCMC estimation
- Bayesian predictive information criterion is utilized.
- The unobserved baseline sales, marketing promotion effects and other specific effects are estimated simultaneously.

Thank you for your attention