Motion texture modelling and multi-parameter auto-models

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Plan

« Building models from data »

I. ***Data :*** **motion computation, motion textures**

II. ***Models :*** **mixed-state auto-models**

III. **More generally: multi-parameter auto-models**

IV. **Back to the data : some experiments**

I. Data : motion computation from videos

- **Current challenges of video analysis in computer vision**
  - Automated analysis via « dynamic content »
  - Applications: videos indexing, video summarizing ...

a). Motion computation

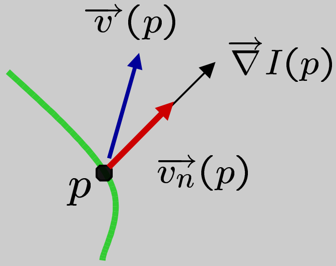
- **Happening : dynamic content conveys much information**
- **Goal*:* *Representation & modelisation* of motion mesures**
  - to cope with a large diversity of dynamic contents from videos
  - suited for detection, classification or recognition of dynamic contents

**** Warning : Quite almost all sample images are under copyright ****

More details

- *I(p,t)* = intensity value at pixel *p* and time *t*
- A video = {*I(p,t), p*∈*S, t=1,...,T*}
- Unkown: vector field of motion {*v(p,t)*}
  - Optical flow: only part of information recovered
  - Inverse problem with missing information: non unique theory
  - Here: we choose the *measurable*, normal projections

$$\overrightarrow{v_n}(p) = -\frac{I_t(p)}{\|\overrightarrow{\nabla} I(p)\|} \cdot \frac{\overrightarrow{\nabla} I(p)}{\|\overrightarrow{\nabla} I(p)\|}$$



b). Scales of motions

- **1. Global motions**
  - Camera, zoom, ...
  - Model: parametric 2D polynomials
    - Standard choice :

$$\mathbf{w}_\theta(p) = \begin{pmatrix} a_1 + a_2 x + a_3 y \\ a_4 + a_5 x + a_6 y \end{pmatrix}$$

with $p = (x, y)$ et $\theta = (a_1, a_2, a_3, a_4, a_5, a_6)$

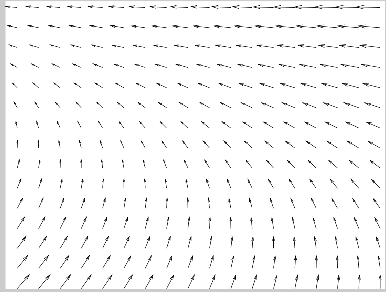
  - References:

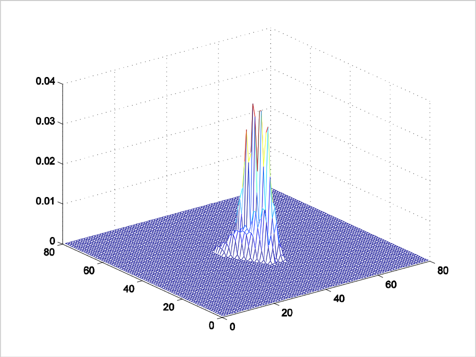
Odobez J.M. and P. Bouthemy, 1995. *IEEE. Trans. Pattern Analysis Mach. Intelligency.*

Global motions

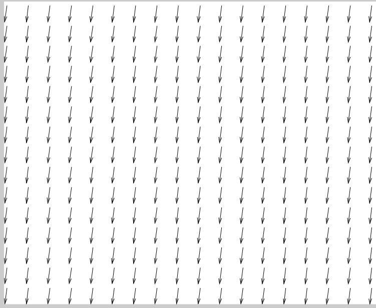
$$\mathbf{w}_{\theta_t}(p)$$

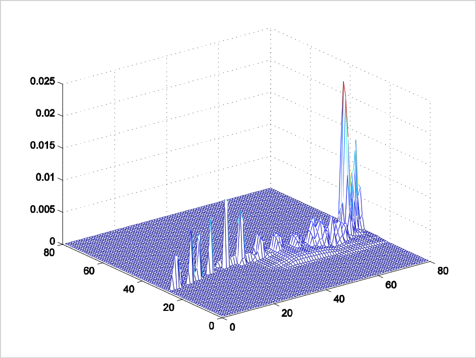
Zoom
+
travelling





Up
&
Down





Scale of motions

2. **Local motions** (scene depedent)

Approximation by locally smoothing the normal projections *{v_n(p,t)}*

[Irani *et al.* 92], [Odobez et Bouthemy 94]

$$v_{res}(p,t) = \frac{\sum_{q \in \mathcal{F}(p)} \|\nabla I(q,t)\|^2 \cdot |v_n^{res}(q,t)|}{N_{\mathcal{F}(p)} \cdot \max\left(G_m^2, \frac{1}{N_{\mathcal{F}(p)}} \sum_{q \in \mathcal{F}(p)} \|\nabla I(q,t)\|^2\right)}$$

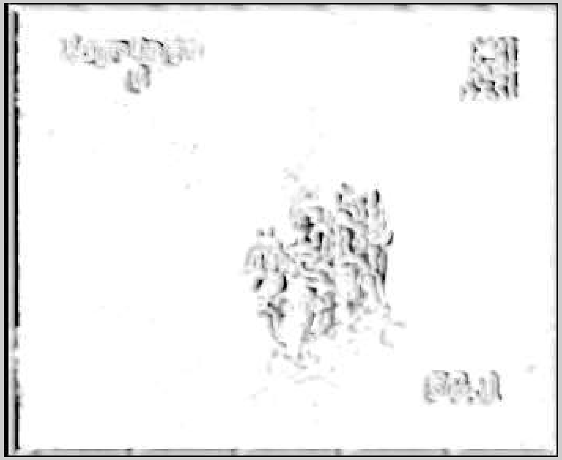
with

$$v_n^{res}(p,t) = -\frac{DFD_{\widehat{\theta}_t}(p)}{\|\nabla I(p,t)\|}$$

$$DFD_{\widehat{\theta}_t}(p) = I(p + w_{\widehat{\theta}_t}(p), t+1) - I(p,t)$$

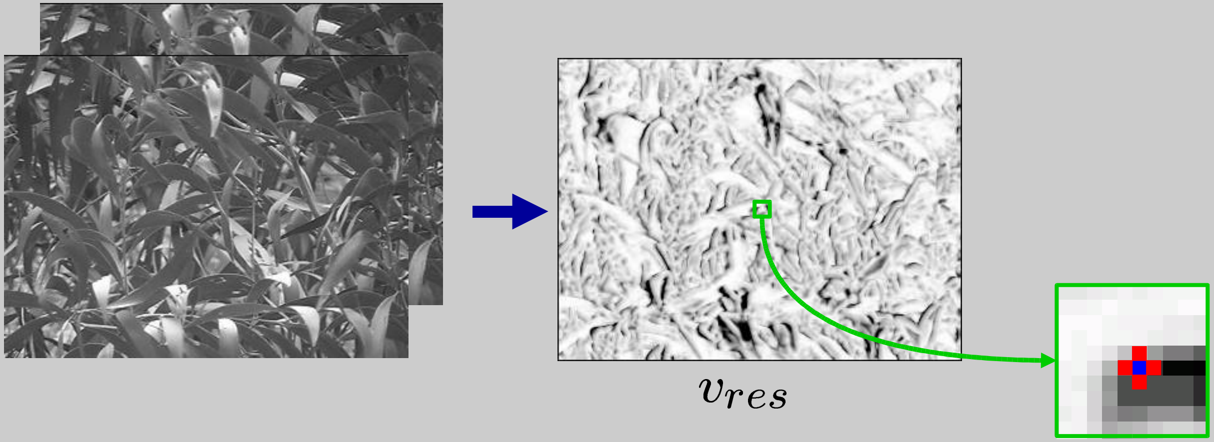
Local motions

Example:



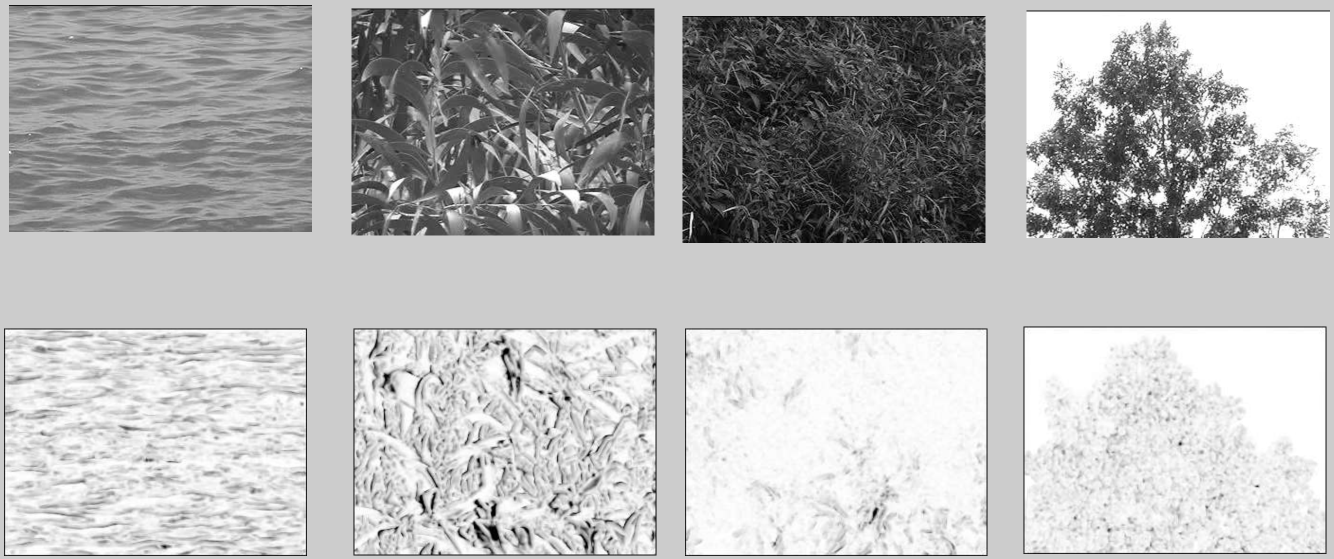
c). Focus: modelling of motion textures

- Images as well as motions are *spatially homogeneous*
- Modelling of spatial dependence



Motion textures: examples

➢



Aims:

- spatial modelling of these motion measurements
- Possibly: sptio-temporal modelisation (dynamic spatial models)

IV. Experiements with Gaussian mixed-state auto-models

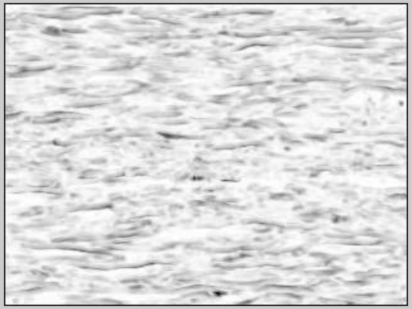
- **Isotropy & anisotropy**

  - Trees sequence

Modèle complet	a	b	c_1	c_2
	-5.8049	3.0435	3.0568	2.9541

Modèle isotrope	a	b	c
	-5.7813	3.0441	3.0000

  - Sea-waves sequence

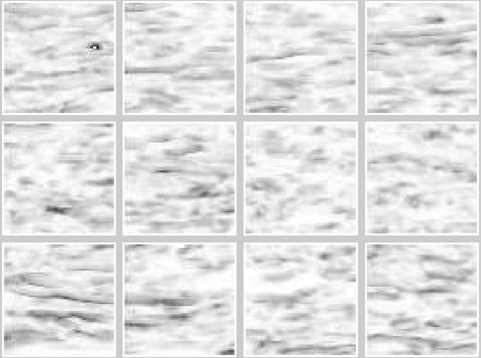


Modèle complet	a	b	c_1	c_2
	-7.9412	0.3697	5.7920	1.4219

Experiments:

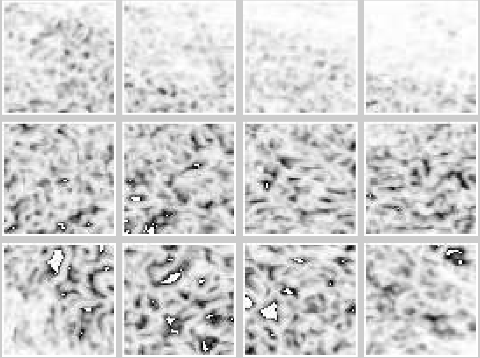
- **Spatial stationarity**

  - Sea-Waves sequence



	a	b	c_1	c_2
B_1	-9.30205	0.29694	5.87898	2.13676
B_2	-9.39952	0.32878	5.46864	2.72478
B_3	-9.04816	0.34152	7.23150	1.14051
B_4	-9.60199	0.32897	7.33581	1.36095
B_5	-8.91001	0.37100	5.65410	2.04668
B_6	-7.35726	0.39963	5.74128	1.10768
B_7	-7.57434	0.43946	5.24632	1.71629
B_8	-7.47818	0.58794	5.08877	1.85792
B_9	-8.30468	0.36271	6.36993	1.18090
B_{10}	-7.61364	0.30174	6.41590	0.70918
B_{11}	-8.86299	0.28625	7.59329	0.65160
B_{12}	-8.87836	0.32870	5.83942	1.85034
Écart-type	0.8220	0.0830	0.8403	0.6203

  - River sequence



	a	b	c_1	c_2
B_1	-10.02623	0.36189	4.76893	3.48537
B_2	-8.39303	0.44575	5.71362	1.93230
B_3	-7.10429	0.69088	3.51732	3.46525
B_4	-5.69048	0.87768	3.22945	2.40213
B_5	17.13015	0.11962	21.95125	10.82028
B_6	8.17958	0.11417	13.38513	7.91897
B_7	8.27053	0.10669	13.18007	8.31032
B_8	8.18484	0.12814	13.56948	8.01166
B_9	-11.78903	0.11195	9.79198	1.06964
B_{10}	8.04156	0.07512	13.68093	7.89088
B_{11}	-3.50982	0.09938	11.51522	-4.15503
B_{12}	-12.91980	0.11302	4.74714	5.13039
Écart-type	10.1118	0.2690	5.6741	4.1530

Some conclusions

- Random field modelling for motion measurements is just *starting*.
- Mixed-state observations are frequent,
  - Need adapted solutions: time series, Markov chains for these ?
  - Can avoid the use of a hidden process
- New: multi-parameter auto-models
  - In particular, provide a rigourous solution for mixed-state observations
- Further researches:
  - An estimation theory
  - Dynamic mixed-state auto-models :
    - *Motion measurements,* ***daily rainfall data , ...***

Motion texture modelling and multi-parameter auo-models (Part II.)

Jian-feng Yao

Joint works with P. Bouthemy , G. Piriou *and* C. Hardouin

Cherry Bud Workshop 2006, **Building Models from Data**

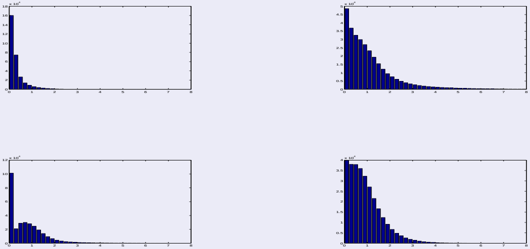
Outline Of Part II.

II. **Motion measurements are mixed**

III. **More generally: multi-parameter auto-models**

IV. **Back to the data: some experiments**

Measurements from motion textures: histograms



Top to bottom, and left to right: grass, foliage, trees and sea-waves.

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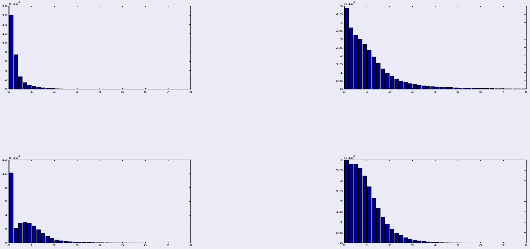

Top to bottom, and left to right: grass, foliage, trees and sea-waves.

Observations are mixed :

- A prominent peak at the origin:

  ⟶ regions without motion
- a continuous component

  ⟶ actual motion in the images

Problem :

- Spatial model for motion mixed-state motion measurements

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- Solved as a Gaussian specification of a general setting

Construction of a random field

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- Its probability distribution $\mu(dx)$ to be constructed

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- For a *site* i, let

$$\mu_i(x_i|\cdot) = \mu_i(x_i|x_j, j \neq i) = \text{p.d.f of } X_i \mid \{X_j = x_j, j \neq i\} \ ,$$

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- 2D-Markovian construction:
  1. Specify the family $\{\mu_i(x_i|\cdot)\}$;
  2. Check their consistency

Besag's auto-models (1974)

- Assume the positivity condition,

$$\mu(dx) = P(x)dx \ , \qquad P(x) = Z^{-1} \exp Q(x) \ ,$$

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- Construction based on:

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$$Q(x) = \sum_{i \in S} G_i(x_i) + \sum_{\{i,j\}} G_{ij}(x_i, x_j) \; .$$

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[B] For each site i, the conditional distribution $\mu_i(x_i|\cdot)$ belongs to a one parameter exponential family:

$$\log \mu_i(x_i|\cdot) = A_i(\cdot)B_i(x_i) + C_i(x_i) + D_i(\cdot) \ , \ A_i(\cdot) \in \mathbb{R}, \ B_i(x_i) \in \mathbb{R}.$$

Theorem

[Besag, 1974] Assume the random field probability distribution μ and its energy function $Q(x)$ satisfy Conditions **[A]-[B]**. *Then, there are for all $i, j \in S$, $i \neq j$, a family of real constants $\alpha_i \in \mathbb{R}$ and β_{ij} such that*

$$A_i(\cdot) = \alpha_i + \sum_{j \neq i} \beta_{ij} B_j(x_j) \; . \tag{1}$$

Consequently the set of potentials is given by

$$G_i(x_i) = \alpha_i B_i(x_i) + C_i(x_i) \; , \tag{2}$$

$$G_{ij}(x_i, x_j) = \beta_{ij} B_i(x_i) B_j(x_j) \; . \tag{3}$$

- "Nicely" applied to Gaussian, exponential, Gamma and Poisson schemes
  auto-normal, auto-exponential, auto-Poisson models

- Unfortunately, this result inapplicable to mixed-state data!
- Reason: motion measurements belong to a exponential family with more parameters!

Need extensions!

A Gaussian mixed-state distribution

- A model Z for **one** motion measure:
  - With probability p, $\quad Z = 0$
  - With probability $1 - p$, $\quad Z = |\mathcal{N}(0, \sigma^2)|$ with density

$$g_s(x) = \frac{2}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} = g_s(0) e^{-sx^2}, \quad s = (2\sigma^2)^{-1}.$$

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- Take the reference measure on $E = \{0\} + (0,\infty)$:

$$m(dx) = \delta_0(dx) + \lambda(dx)\ ,$$

where $\delta_0 =$ Dirac measure, $\quad \lambda =$ Lebesgue

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- Therefore, Z has a density w.r.t. $m(dx)$,

$$f_\theta(x) = p\delta(x) + (1-p)\delta^*(x)g_s(x).$$

Exponential family with 2 parameters!

Rewriting:

$$\begin{aligned} f_\theta(x) &= p\delta(x) + (1-p)\delta^*(x)g_s(x) \\ &= \exp\left[-\delta^*(x)\log\frac{p}{(1-p)g_s(0)} - sx^2 + \log p\right] \\ &= \exp\left[\langle\theta, B(x)\rangle + \log p\right] \end{aligned}$$

with

$$\theta = (\theta_1, \theta_2)^T = \left(\log\frac{(1-p)g_s(0)}{p}, s\right)^T, \quad B(x) = (\delta^*(x), -x^2)^T .$$

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$$

- Name: *positive mixed-state Gaussian distribution*

III. Multi-parameters auto-models

Assume

- The positivity condition for the random field

$$\mu(dx) = P(x)dx \ , \qquad P(x) = Z^{-1} \exp Q(x) \ .$$

[A] The spatial dependence is pairwise only

$$Q(x) = \sum_{i \in S} G_i(x_i) + \sum_{\{i,j\}} G_{ij}(x_i, x_j) \ .$$

[B2] For each site i, the conditional distribution $\mu_i(x_i|\cdot)$ belongs to a *multi-parameter exponential family*:

$$\log \mu_i(x_i|\cdot) = \langle A_i(\cdot), B_i(x_i) \rangle + C_i(x_i) + D_i(\cdot) \ , \ A_i(\cdot) \in \mathbb{R}^d, \ B_i(x_i) \in \mathbb{R}^d.$$

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[C] The family of sufficient statistics $\{B(x_i)\}$ is *regular* in the sense that

$$\text{for all } i \in S, \quad \text{Span}\{B_i(x_i), x_i \in E\} = \mathbb{R}^d \,.$$

Theorem

[Hardouin and Y.,2005] Assume that the random field probability distribution μ *of (6) and its energy function* $Q(x)$ *satisfy Conditions* **[A]-[B2]-[C]**. *Then, there are for all* $i, j \in S,\ i \neq j$, *a family of vectors* $\alpha_i \in \mathbb{R}^d$ *and a family of* $d \times d$ *matrices* β_{ij} *satisfying* $\beta_{ij} = \beta_{ji}^T$, *such that*

$$A_i(\cdot) = \alpha_i + \sum_{j \neq i} \beta_{ij} B_j(x_j)\ . \tag{4}$$

Consequently the set of potentials is given by

$$G_i(x_i) = \langle \alpha_i, B_i(x_i) \rangle + C_i(x_i)\ , \tag{5}$$

$$G_{ij}(x_i, x_j) = B_i^T(x_i) \beta_{ij} B_j(x_j)\ . \tag{6}$$

- Such a model is called *a multi-parameter auto-model*
- Condition **[C]** is specific to the multi-parameter case

Mixed-state auto-models for motion textures

- Start by assuming

$$\mu_i(x_i|\cdot) \in \quad \text{family of mixed-state Gaussian} \quad \{f_{\theta_i(\cdot)}(x_i)\}$$

where

$$\theta_i(\cdot) = \theta_i(x_j, j \neq i).$$

That is,

$$\log \mu_i(x_i|\cdot) = \langle \theta_i(\cdot), B(x_i) \rangle + \log p_i(\cdot) \ , \quad B(x) = (\delta^*(x), -x^2).$$

Mixed-state auto-models for motion textures

- by the Theorem, there are a family of vectors $\alpha_i = (a_i, b_i) \in \mathbb{R}^2$ and 2×2 matrices

$$\beta_{ij} = \begin{pmatrix} c_{ij} & d_{ij} \\ d_{ij}^* & e_{ij} \end{pmatrix} ,$$

satisfying $\beta_{ij} = \beta_{ji}^T$, such that

$$\theta_i(\cdot) = \alpha_i + \sum_{j \neq i} \beta_{ij} B(x_j) .$$

Moreover, the associated energy function is given by

$$Q(x_1, \ldots, x_n) = \sum_{i \in S} \left[a_i \delta^*(x_i) - b_i x_i^2 \right] + \sum_{\{i,j\}} (\delta^*(x_i), -x_i^2) \beta_{ij} (\delta^*(x_j), -x_j^2)^T .$$

Back to the data

A simple specification with the 4NN system

- Sites $S = \{1, \dots, n\} = [1, M] \times [1, N]$
- each i has 4 neighbours

$$\{i_e = i + (1, 0), i_o = i - (1, 0), i_n = i + (0, 1), i_s = i - (0, 1)\}$$

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- The model reduced to 4 parameters

$$a, \; b, \; c_1, \; c_2$$

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- Estimation by the pseudo-likelihood method Besag, (1975); Guyon (1989).