Cherry Bud Workshop 2006
*Building Models from Data*
Keio University, Yokohama, JAPAN

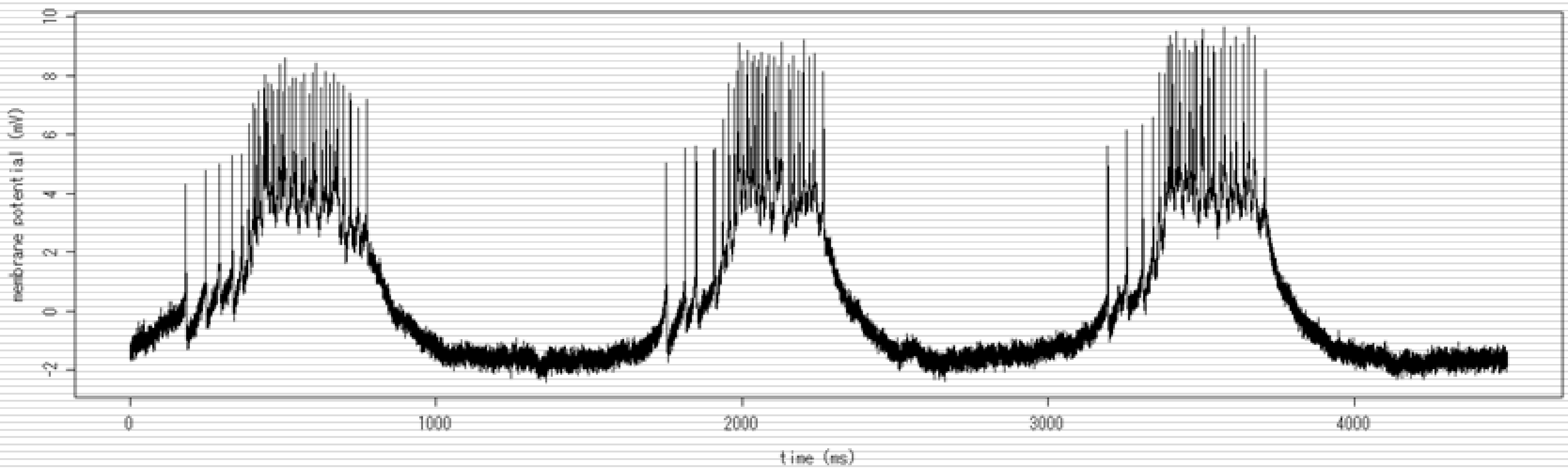
Integrative Mathematical Sciences
統合数理科学

# Data Modelling of Neuron Membrane Potential

Hideyasu SHIMADZU
Ritei SHIBATA
Toshinobu SHIMOI
Kotaro OKA
(Keio University, JAPAN)

KEIO UNIVERSITY
CALAMVS GLADIO FORTIOR
1858

# Neuron membrane potential




First part of observed data

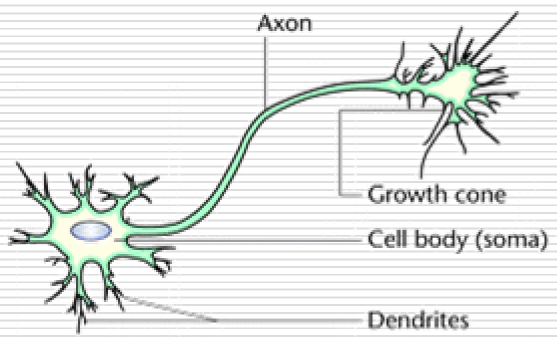
# Outline

- Background
  - Action potential
- Data
- Ideas for data modelling
  - Instantaneous change, gradual change
- Model
- Conclusion

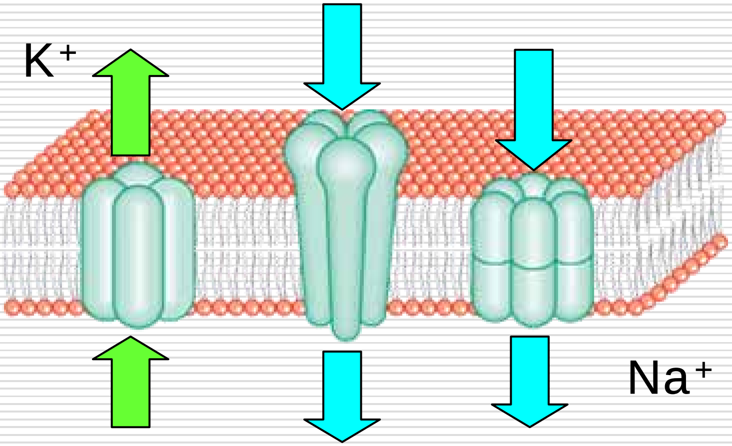
# Overview

- Membrane potential change are caused by exchange of ions across the neuron membrane.

**Neuron**

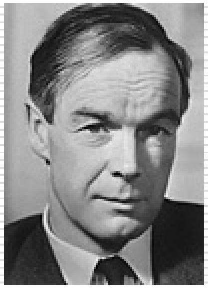



**Membrane**



# Hodgkin-Huxley model

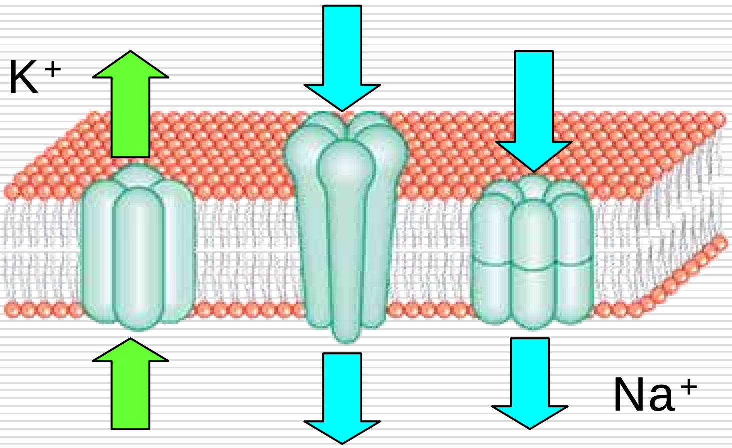
(Hodgkin & Huxley 1952)

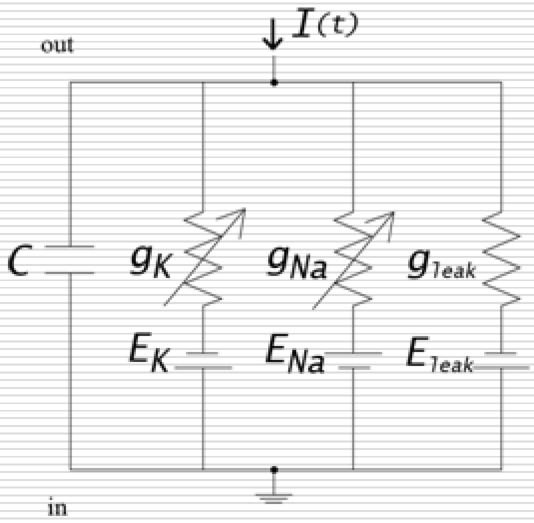


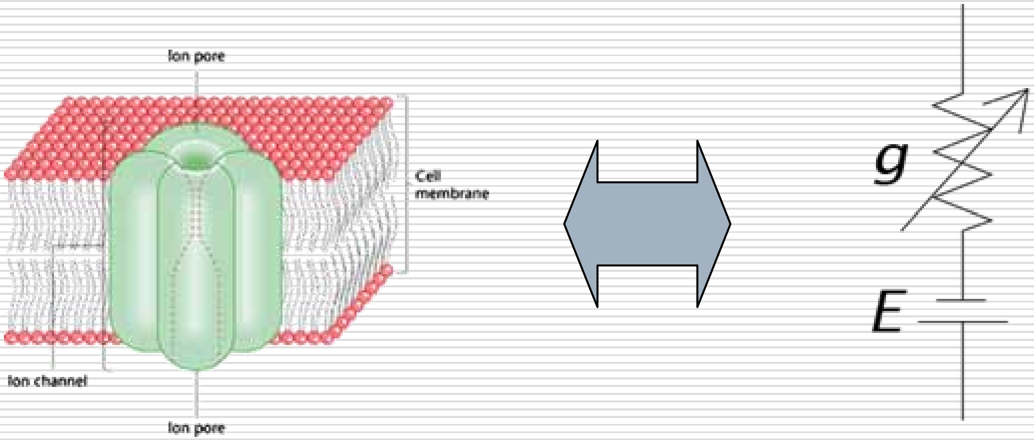
http://nobelprize.org/

## Their ideas:

- Membrane ⟺ Condenser;
- Ion channel ⟺ Resistance and Battery.



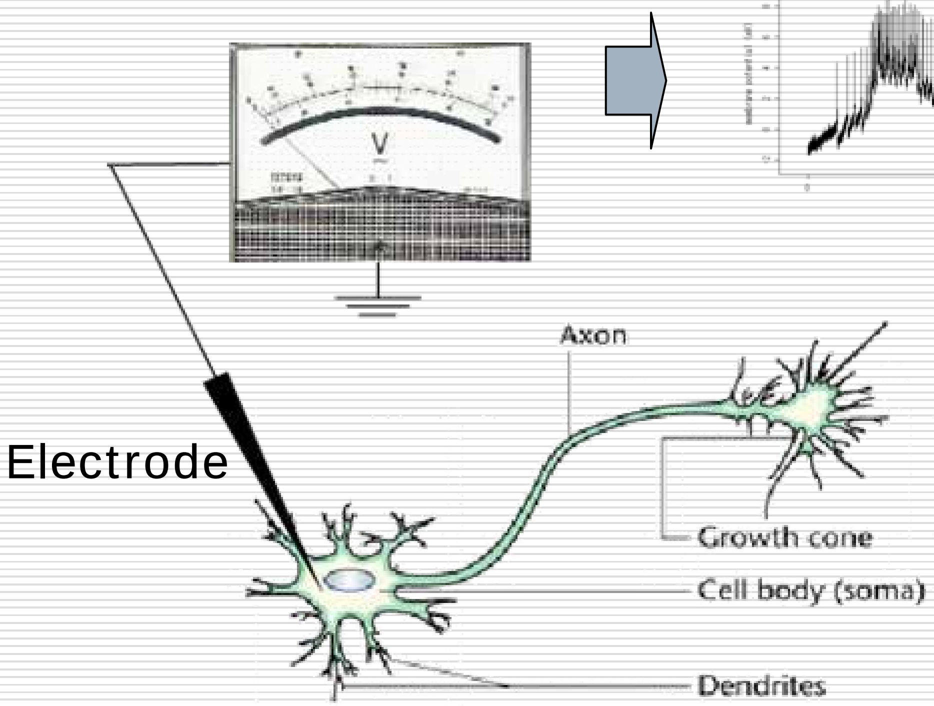


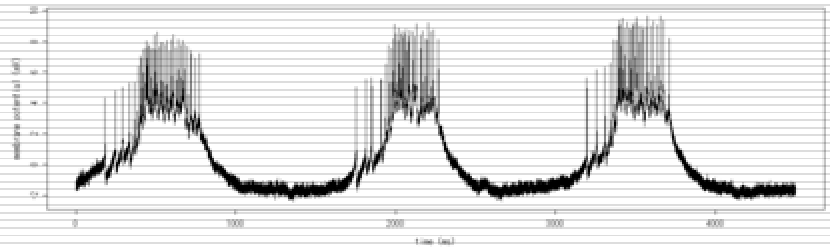

$$
\begin{aligned}
C\,\frac{dv(t)}{dt} &= I(t) - g(v(t),t)\,(v(t)-E)\\
\frac{dg(v(t),t)}{dt} &= \alpha(v(t))\,(1-g(v(t),t)) - \beta(v(t))\,g(v(t),t)\,.
\end{aligned}
$$

## Membrane potential changes conductance

Izhikevich (2003, 2004), Rose and Hindmarsh (1989), Wilson (1999) etc.

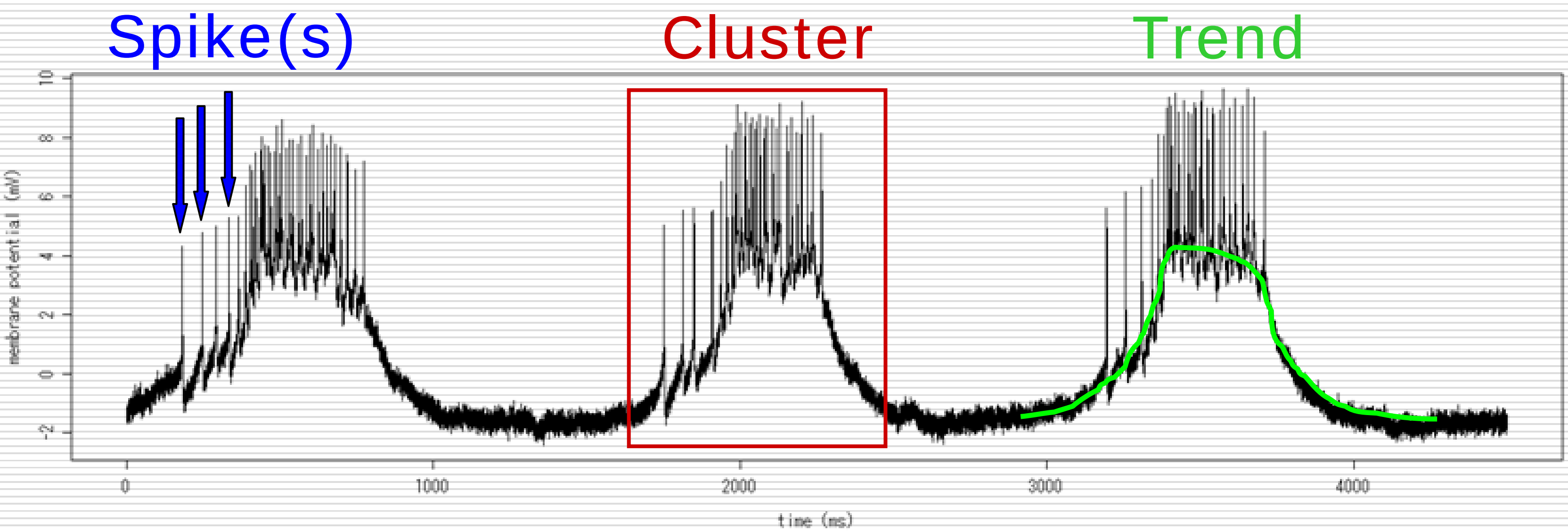
# Data



- Earthworm’s neuron
- Membrane potential
- 879000 observations (44 s per 0.05 ms)

# Words

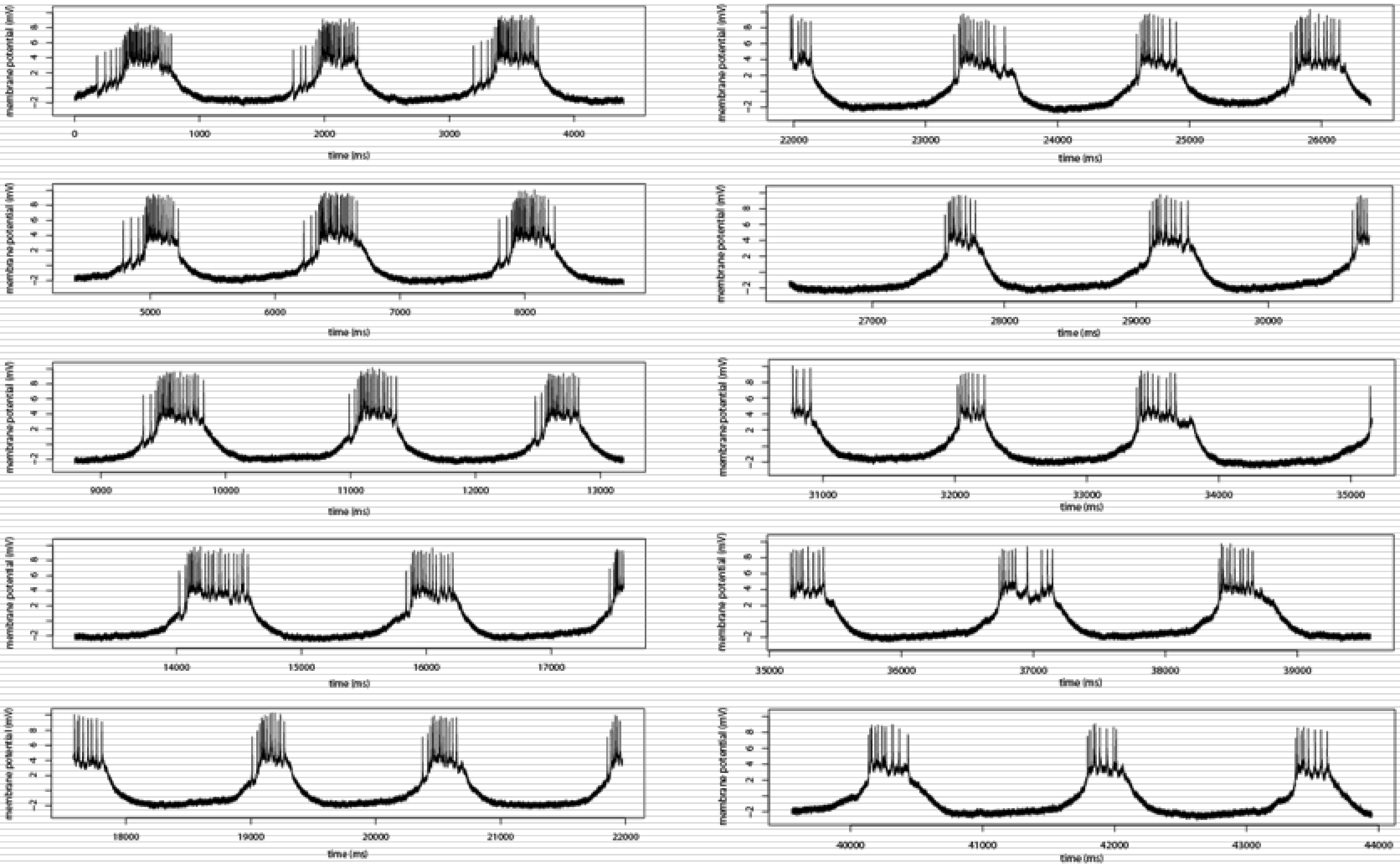



Spike(s): instantaneous jump

Cluster: dense spikes

Trend: gradual change

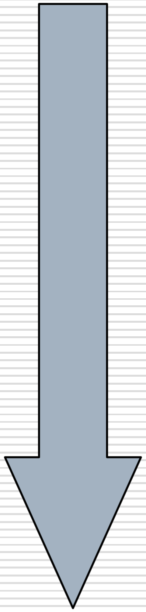
# Appearance of whole data



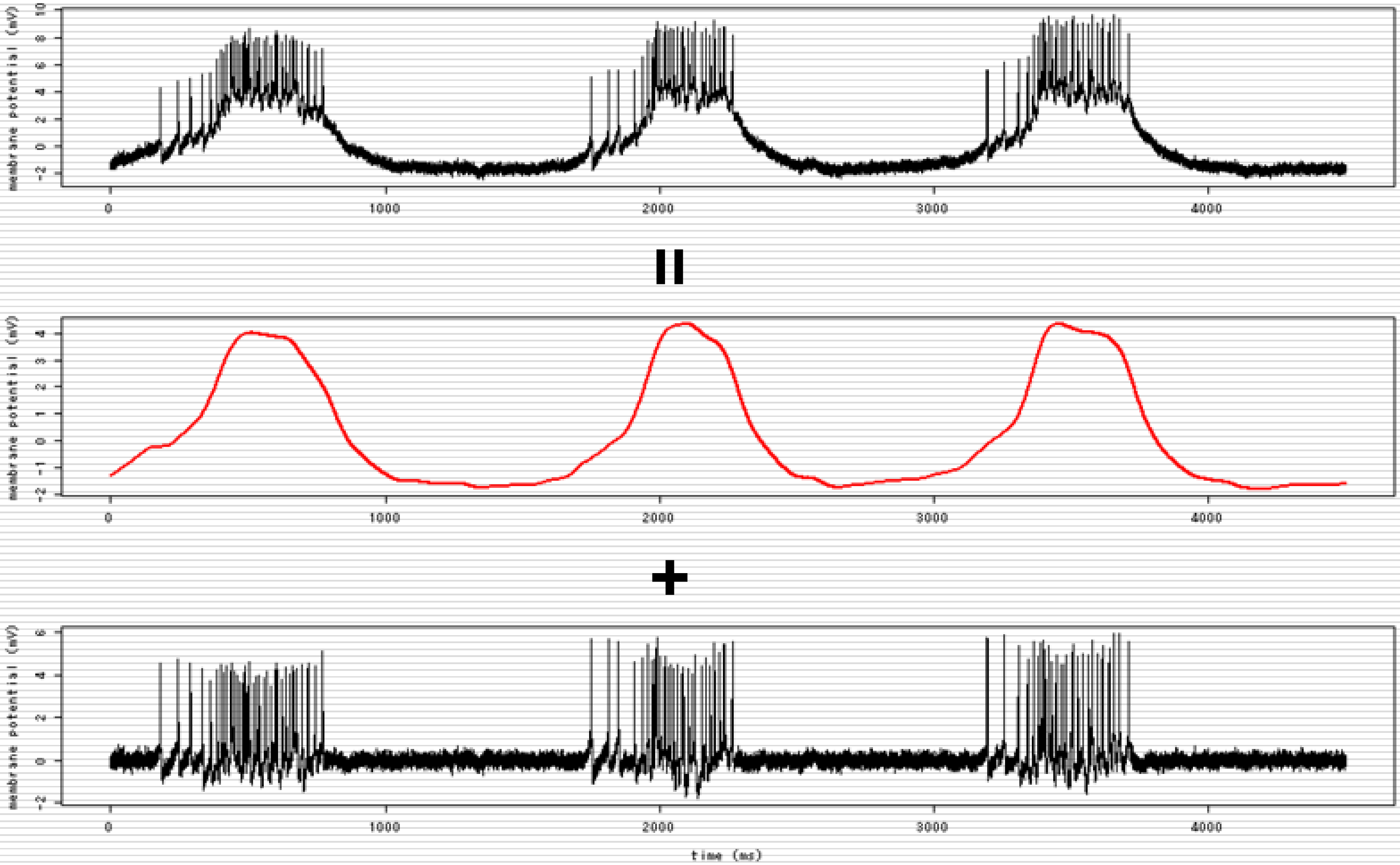
# Basic ideas for modelling

- Trend (second)
  - Dynamic model
- A spike (mili second)
  - Dynamic model
- Occurrence points of spikes (in a wink)
  - Stochastic model

**Time scale**

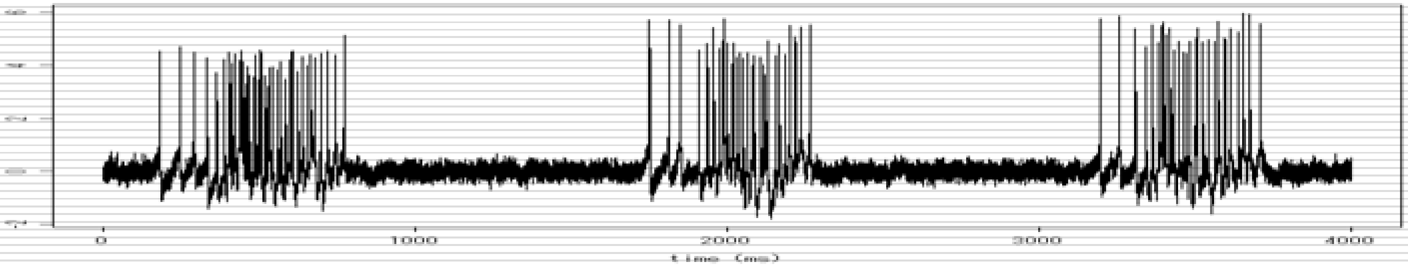


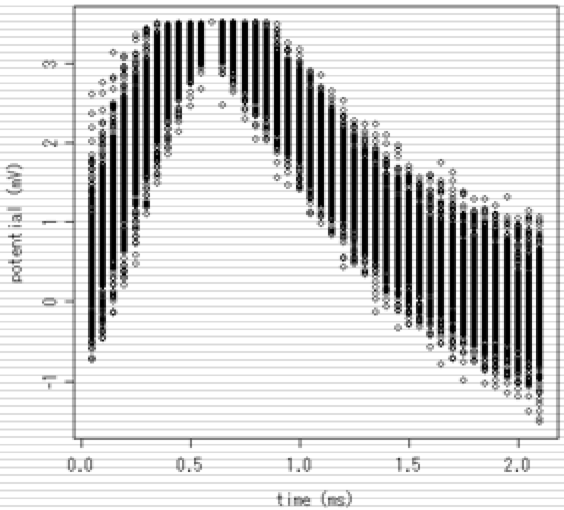
# Trend



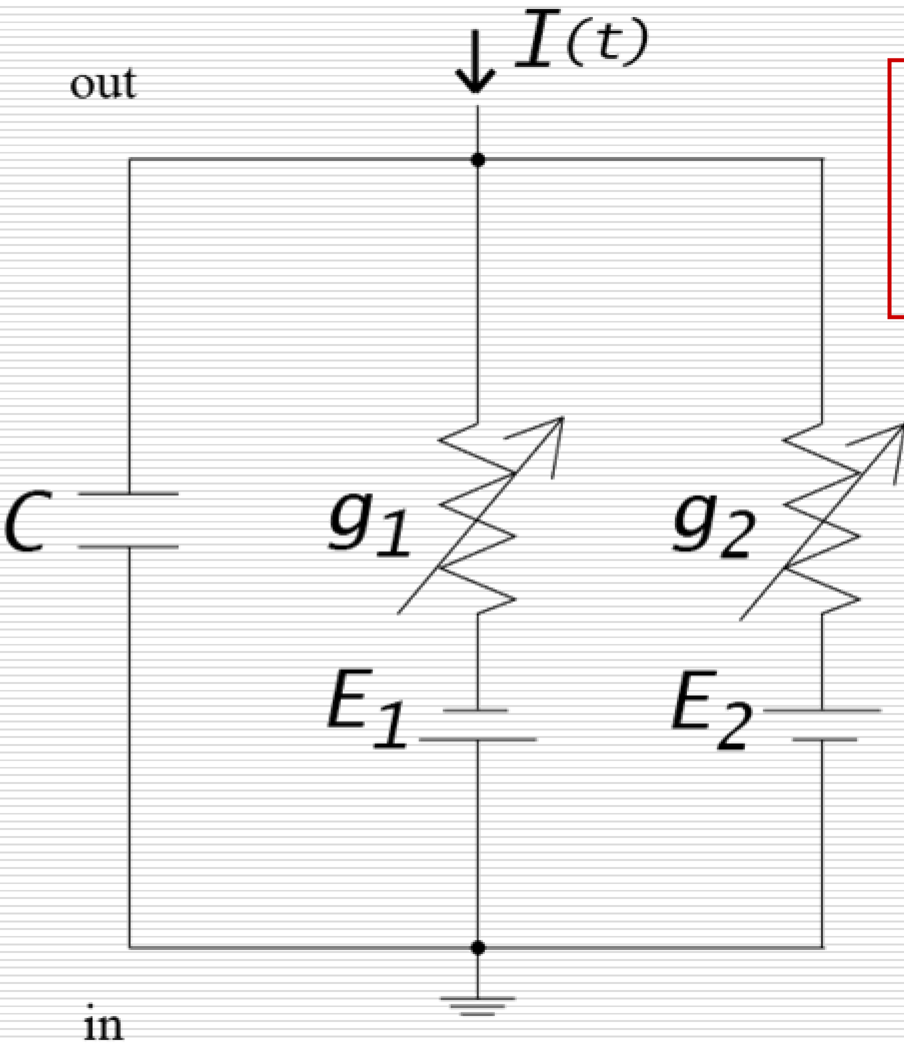
membrane potential (mV)
0
1000
2000
3000
4000
=
membrane potential (mV)
0
1000
2000
3000
4000
+
membrane potential (mV)
0
1000
2000
3000
4000
time (ms)

# Spike



# Electric circuit model




$$C\frac{dv(t)}{dt} = I(t) - g_1(v(t) - E_1) - g_2(v(t) - E_2)$$

where,

$v(t)$ : Membrane potential [V];

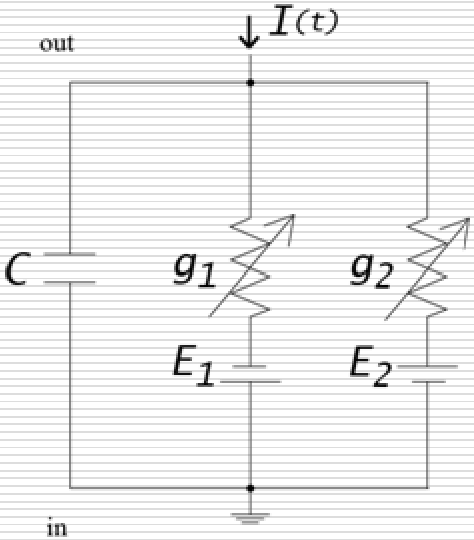
$I(t)$ : Synaptic current [A];

$C$ : Capacitance [F];

$g_i$ : Conductance [S];

$E_i$ : Battery [V].

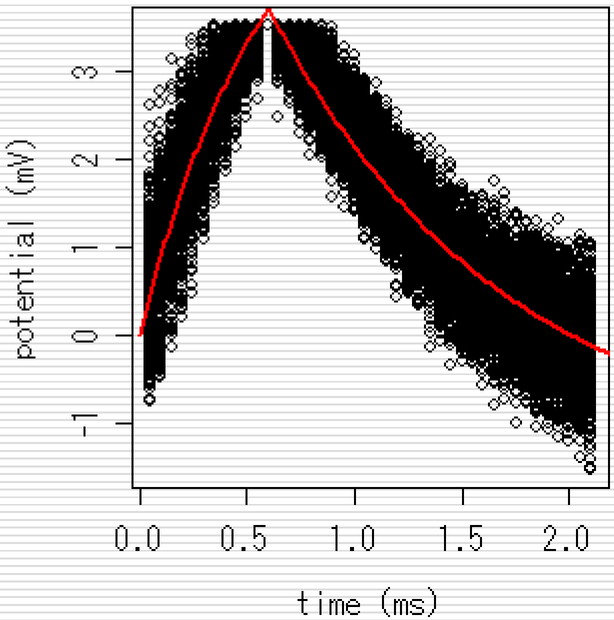
$$C\frac{dv(t)}{dt} = I(t) - g_1(v(t) - E_1) - g_2(v(t) - E_2)$$

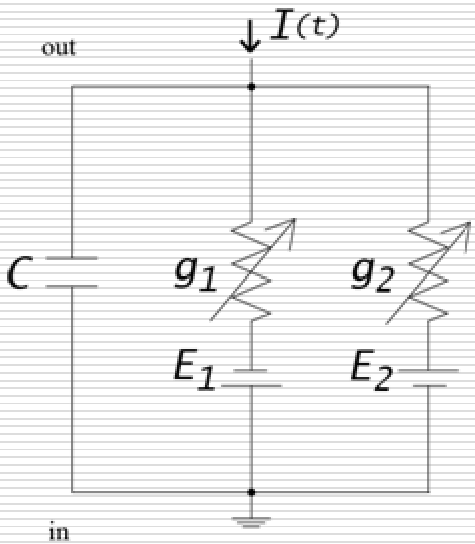



$$\begin{aligned} v(t) &= v(0)\exp\left(-\frac{G}{C}t\right) \\ &+A\left\{1-\exp\left(-\frac{G}{C}t\right)\right\} \\ &+\exp\left(-\frac{G}{C}t\right)\int^{t}\left\{I(u)\exp\left(\frac{G}{C}u\right)\right\}du, \end{aligned}$$

where $G = g_1 + g_2,\ A = \frac{g_1}{G}E_1 + \frac{g_2}{G}E_2.$

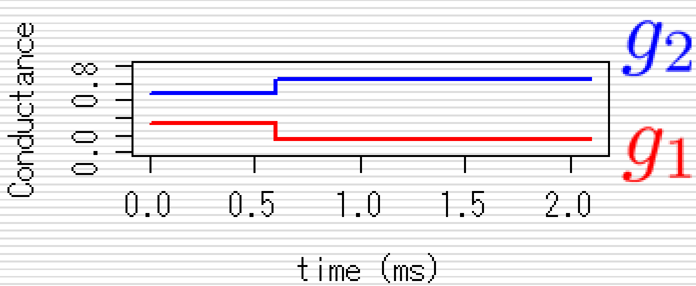
# Parameter estimation




| | Left | Right |
|---|---|---|
| $g_1$ | 0.33 | 0.14 |
| $g_2$ | 0.67 | 0.86 |

$E_1 = 70, E_2 = -10$

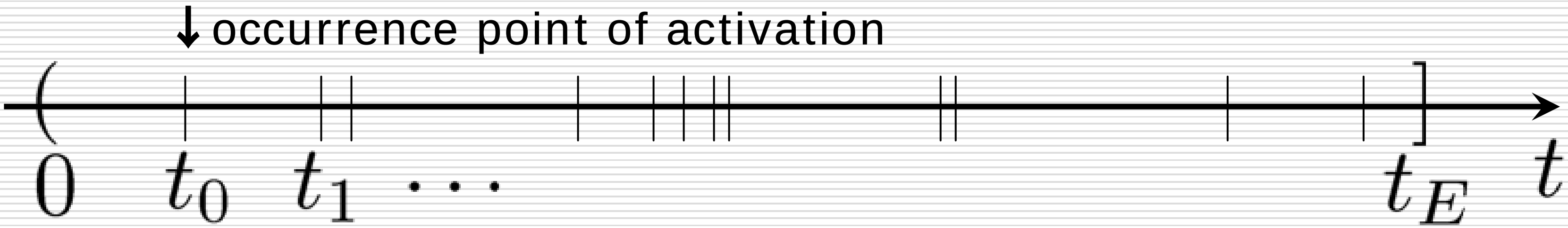



$$
\begin{aligned}
v(t) &= v(0)\exp\left(-\frac{G}{C}t\right) \\
&\quad + A\left\{1-\exp\left(-\frac{G}{C}t\right)\right\},
\end{aligned}
$$

where $G = g_1 + g_2,\ A = \frac{g_1}{G}E_1 + \frac{g_2}{G}E_2.$

# Occurrence points of spike

- Point process model (Cox and Isham 1980)

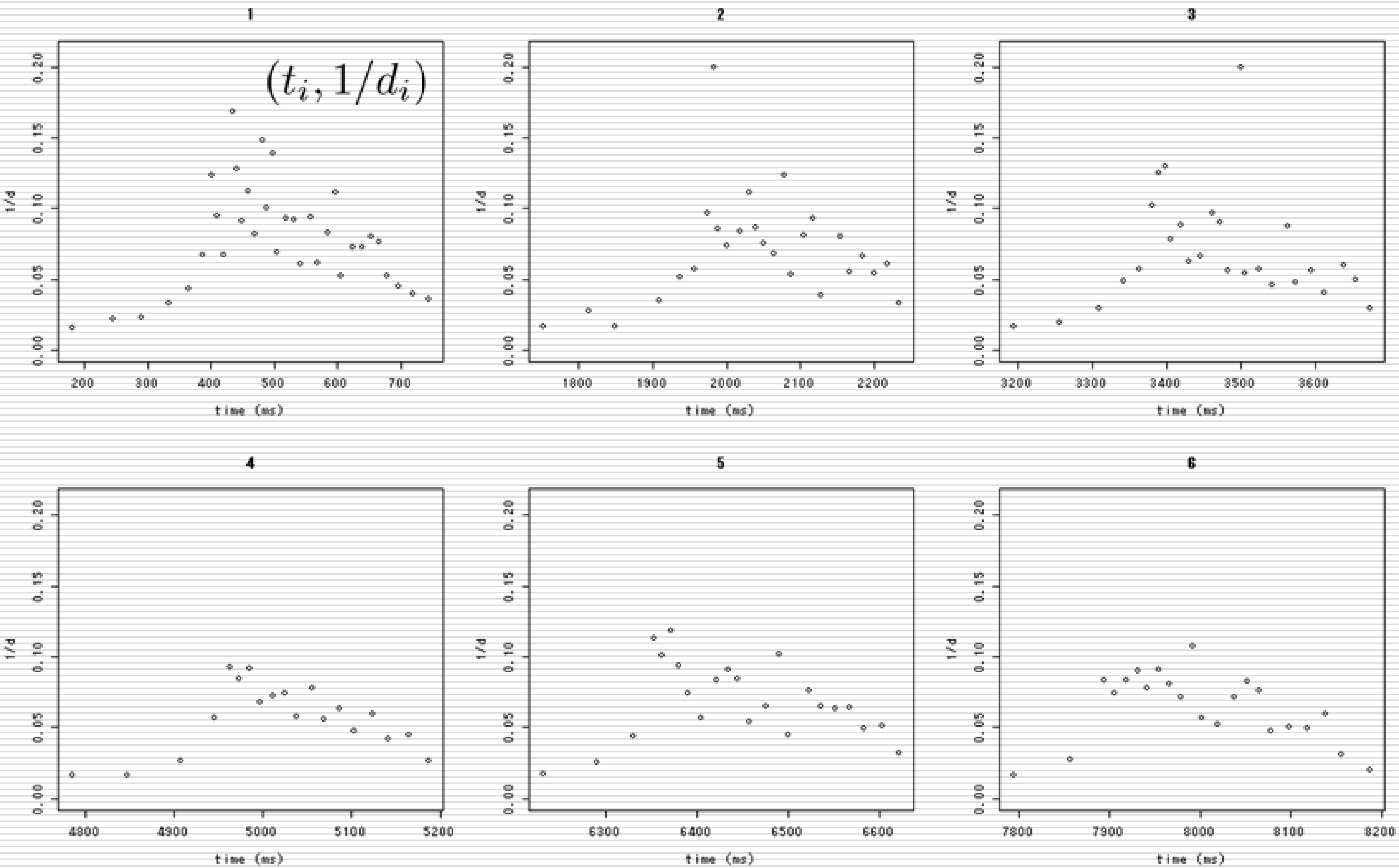



$$d_i = t_i - t_{i-1} \quad \Rightarrow \quad D \sim \mathrm{Exp}\left(\frac{1}{\lambda(t)}\right)$$

Point process model is applied for each cluster

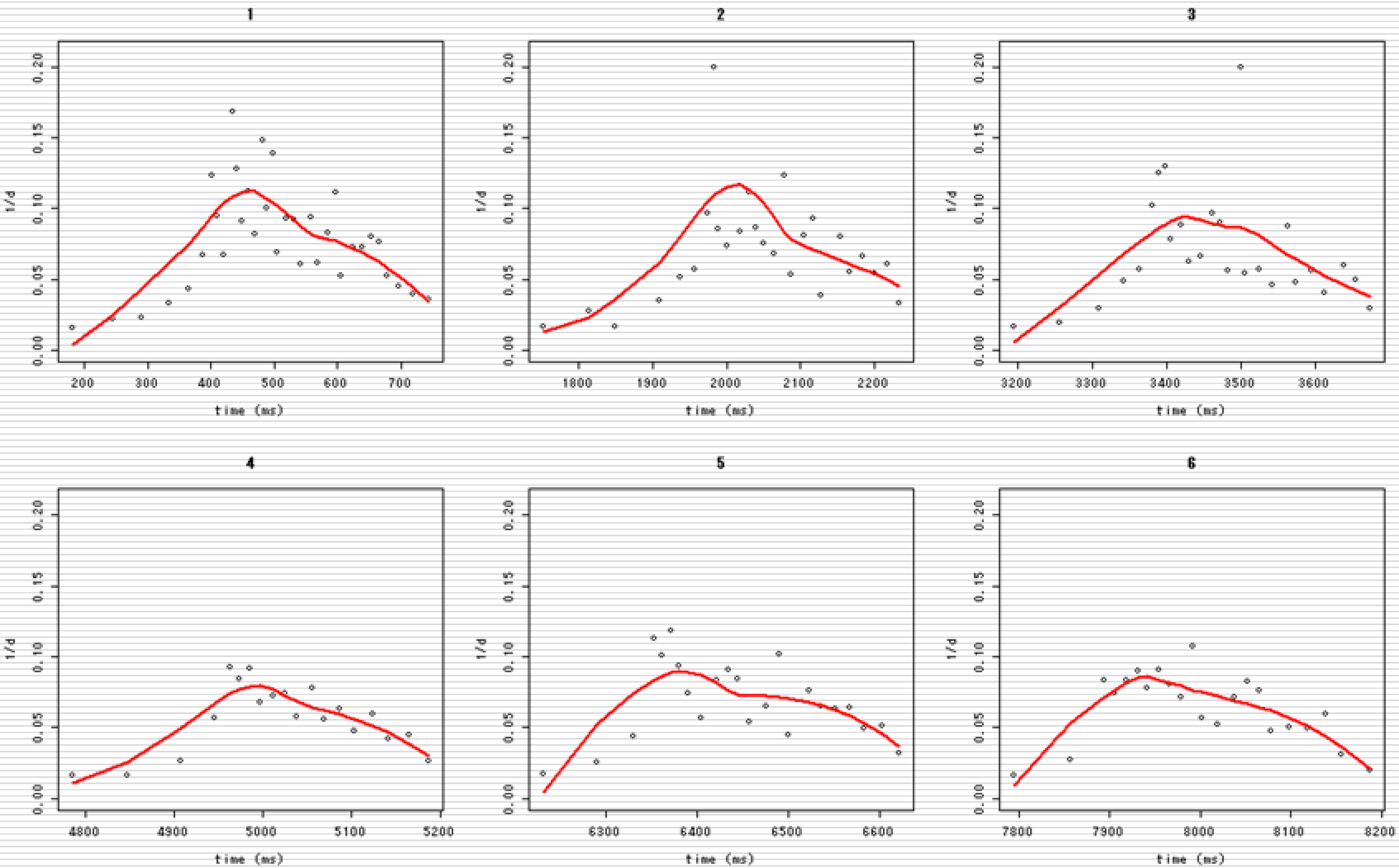
(Kass and Ventura 2001, Ventura et al. 2002 etc.)

$$d_i = t_i - t_{i-1}$$

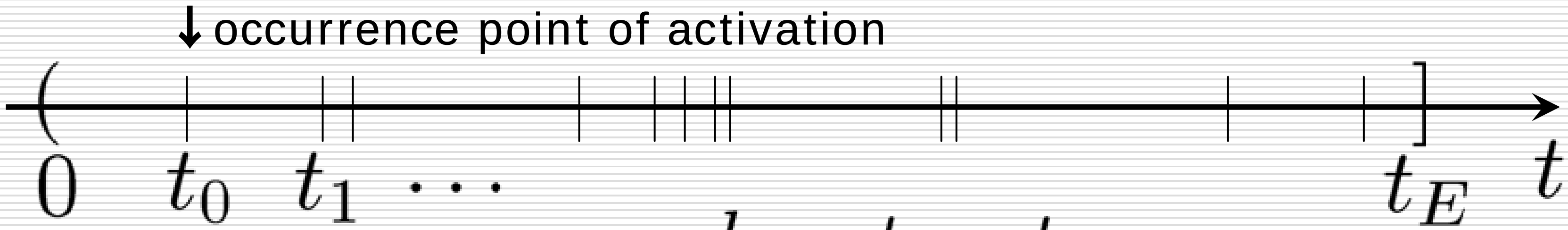


1
2
3
4
5
6
$(t_i, 1/d_i)$
1/d
time (ms)
0.00
0.05
0.10
0.15
0.20
200
300
400
500
600
700
1800
1900
2000
2100
2200
3200
3300
3400
3500
3600
4800
4900
5000
5100
5200
6300
6400
6500
6600
7800
7900
8000
8100
8200

$$d_i = t_i - t_{i-1}$$


1
2
3
4
5
6
1/d
time (ms)
0.00
0.05
0.10
0.15
0.20
200
300
400
500
600
700
1800
1900
2000
2100
2200
3200
3300
3400
3500
3600
4800
4900
5000
5100
5200
6300
6400
6500
6600
7800
7900
8000
8100
8200

# Estimation of intensity function




$$d_i = t_i - t_{i-1}$$

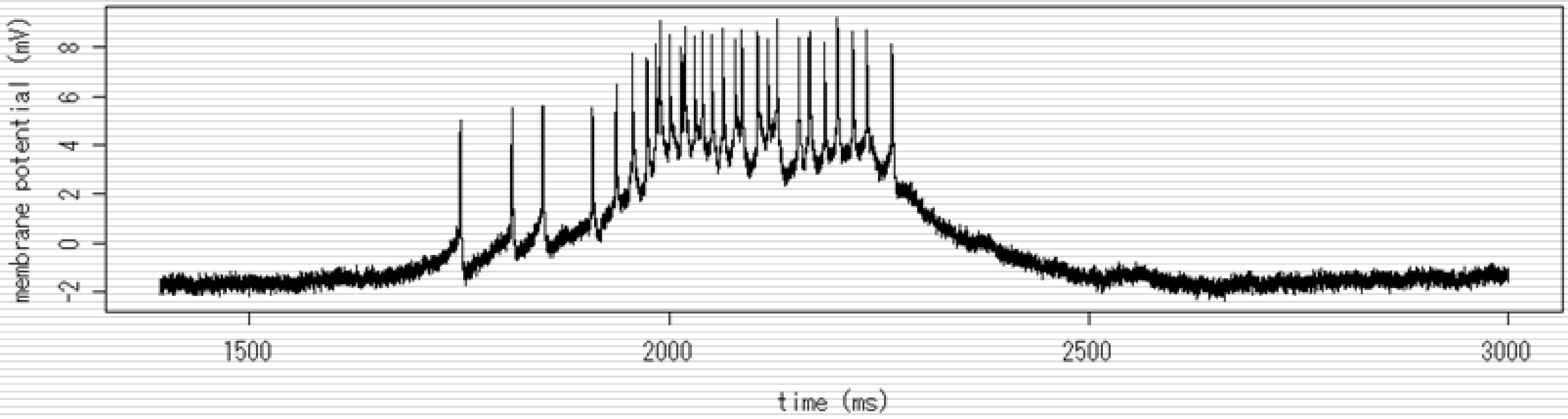
- Intensity function

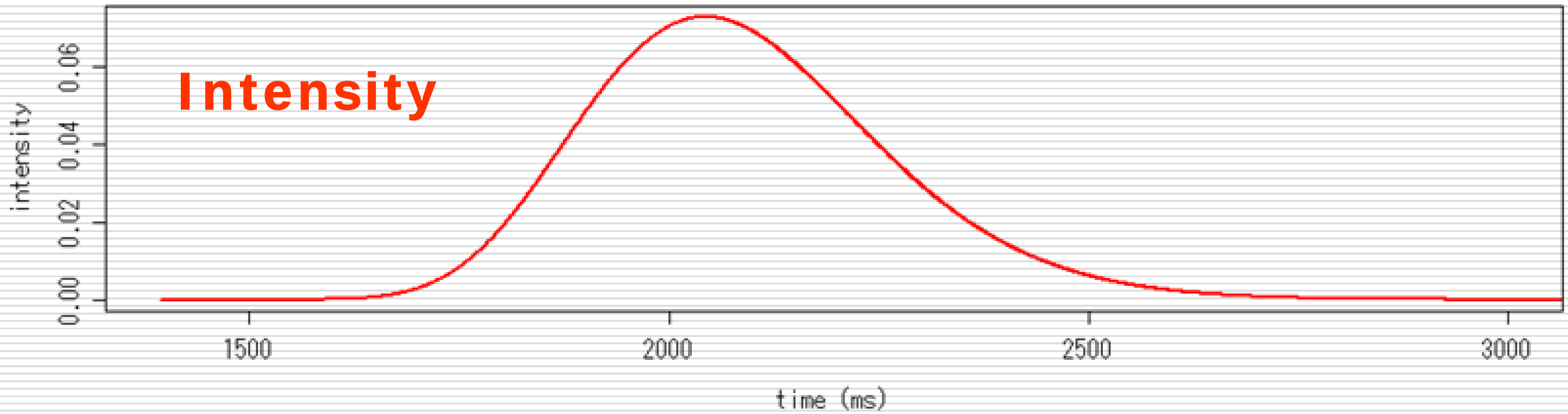
$$\lambda(t;\tau,\theta,\gamma,\kappa) = \frac{1}{\gamma\theta^{\kappa}}(t-\tau)^{\kappa-1}\exp\left(-\frac{t-\tau}{\theta}\right)$$

- Likelihood

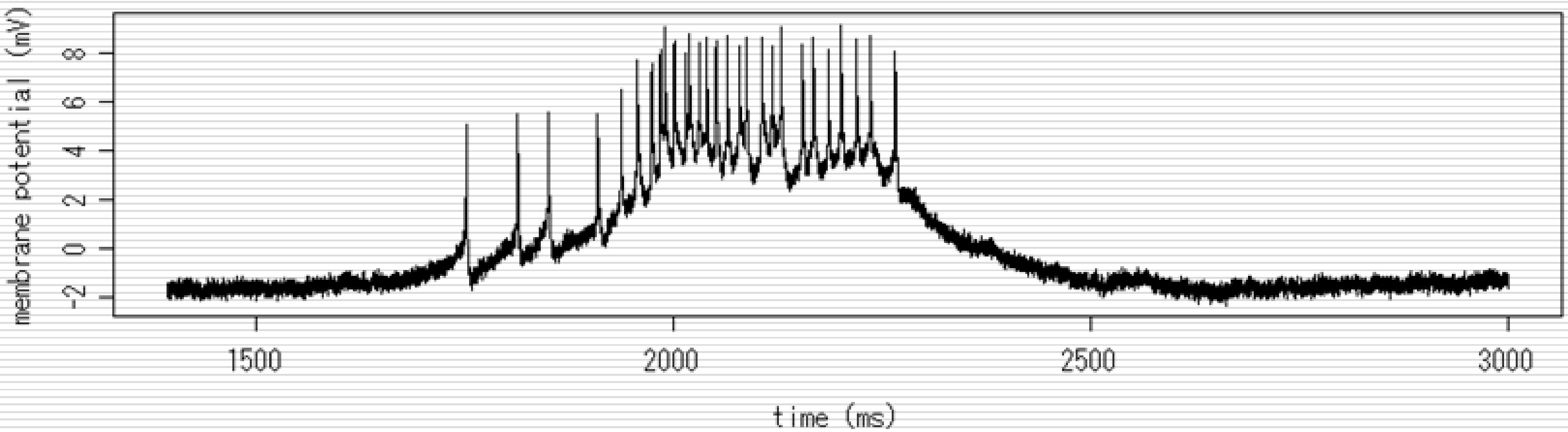
$$L(t_0,\theta,\gamma,\kappa;t_i,d_i) = \prod_{i=1}^{m}\lambda(t_i)\exp(-\lambda(t_i)\,d_i)$$

# Estimated intensity (ex. Cluster 2)

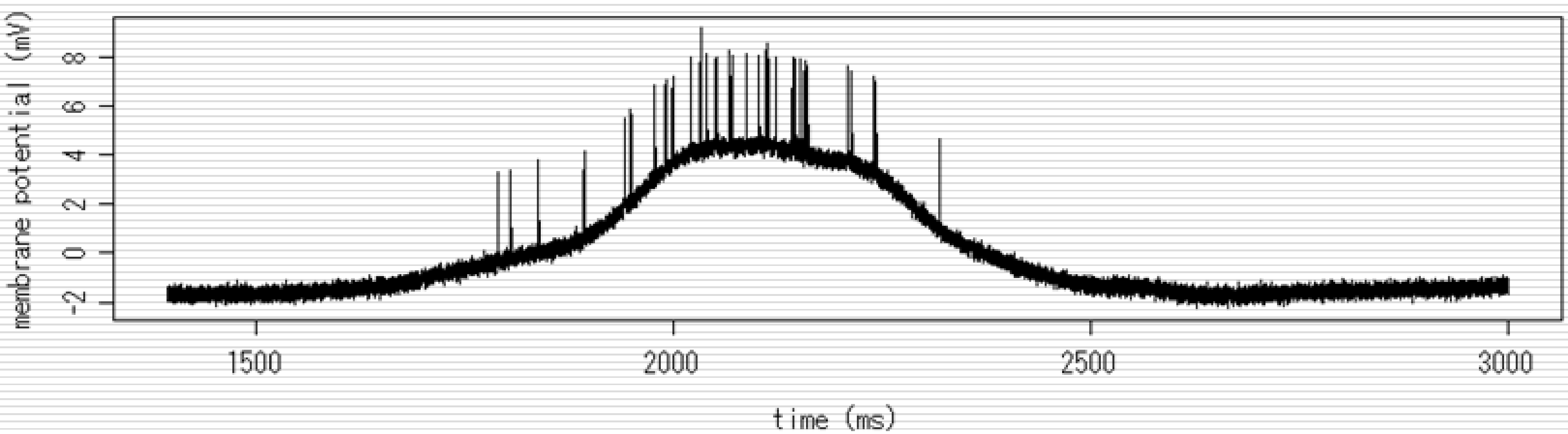


# Simulation



membrane potential (mV)
-2
0
2
4
6
8
1500
2000
2500
3000
time (ms)

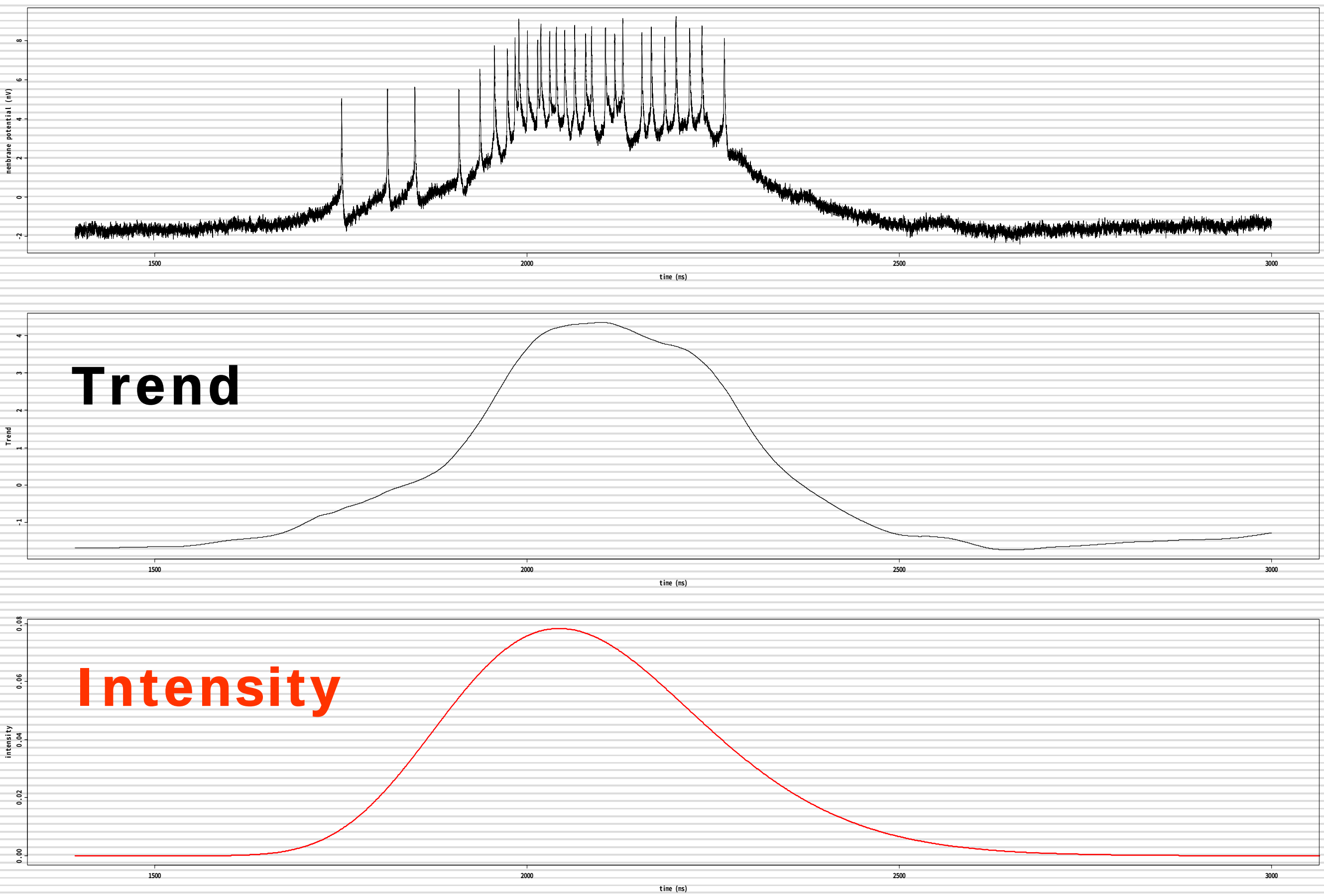



membrane potential (mV)
-2
0
2
4
6
8
1500
2000
2500
3000
time (ms)

# What the model tells us?

- Relationship between intensity and trend
- Parameter changes of intensity function among clusters

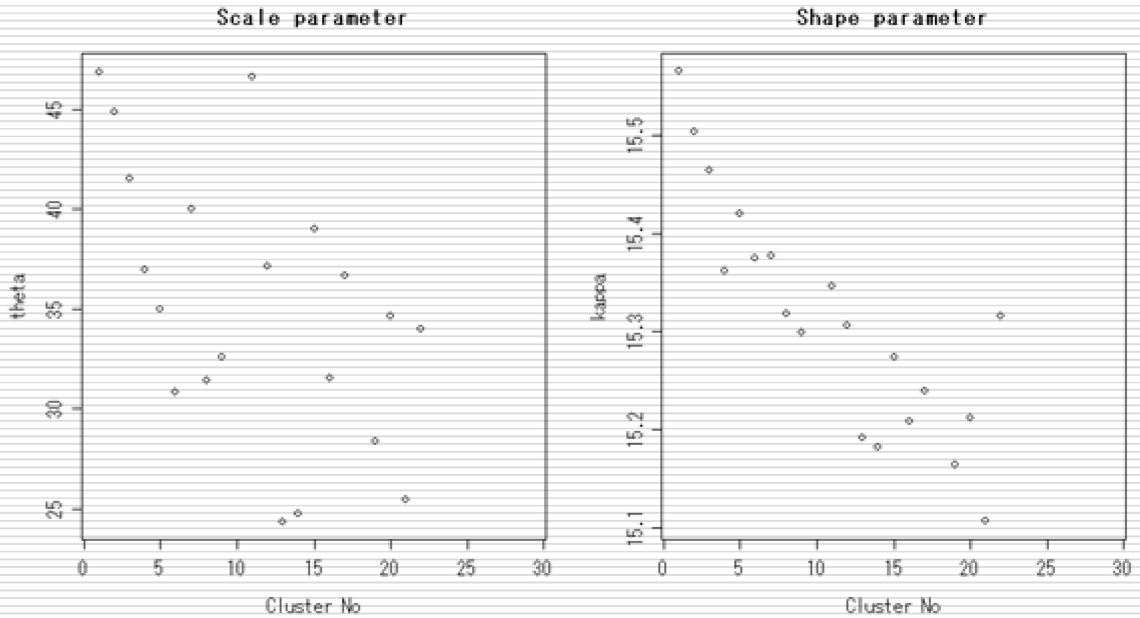
# Relationship between intensity and trend



# Parameters changes of intensity function among clusters

$$\lambda(t; t_0, \theta, \gamma, \kappa) = \frac{1}{\gamma\theta^\kappa}(t - t_0)^{\kappa-1}\exp\left(-\frac{t - t_0}{\theta}\right)$$

$\kappa$: Shape parameter, $\theta$: Scale parameter



# Future works

We just have a model for a cluster

↓

- What is the trend?
- How build a model of relationship between clusters?

# References (1)


- Cleveland and Devlin (1988). Locally Weighted Regression: An Approach to Regression Analysis by Local Fitting. *Journal of the American Statistical Association*, **83**(403):596-610.
- Cox and Isham (1980). *Point Processes*. Chapman and Hall, London.
- Hodgkin and Huxley (1952). A Quantitative Description of Membrane Current and Its Application to Conduction and Excitation in Nerve. *Journal of Physiology*, **117**:500-544.
- Izhikevich (2003). Simple model of spiking network. *IEEE Transactions on Neural Networks* **14**(6):1569–1572.
- Izhikevich (2004). Which model to use for cortical spiking neurons? *IEEE Transactions on neural networks* **15**(5):1063-1070.

# References (2)


- Ogata (1981). On Lewis' Simulation Method for Point Processes. *IEEE Transactions on Information Theory* IT-**27**(1):23-31.
- Kass and Ventura (2001). A spike-train probability model. Neural Computation, **13**: 1713-1720.
- Rose and Hindmarsh (1989). The assembly of ionic currents in a thalamic neuron I. the three-dimensional model. *Proceedings of the Royal Society of London. Series B, Biological Sciences* **237**:267–288.
- Ventura et al. (2002). Statistical analysis of temporal evolution in single-neuron firing rates. Biostatistics **3**(1): 1-20.
- Wilson, H. (1999). Simplified dynamics of human and mammalian neocortical neurons. *Journal of Theoretical Biology* **200**:375–388.

Thank you for kind attention.
Comments and suggestions are welcomed!

Hideyasu SHIMADZU
(Keio University)
shimadzu@stat.math.keio.ac.jp