Models for Dependent Credit Risks and Their Calibration

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Components of Credit Risk

- Arrival risk: Uncertainty whether a default will occur or not. Measured by the probability of default, within a given time horizon, usually one year.
- Timing risk: Uncertainty about the time of default.
- Exposure risk: Relatively clear for loans or bonds (face value, market value), greater uncertainty in the credit reinsurance business as primary insurers might have successfully decreased credit lines in advance.
- Recovery risk: Uncertainty about the size of the loss w.r.t. the exposure. Historical data show a large variability of recovery rate, depending on collateral, seniority of the bond, etc. Specified by conditional distribution of recovery rate given default occurred.

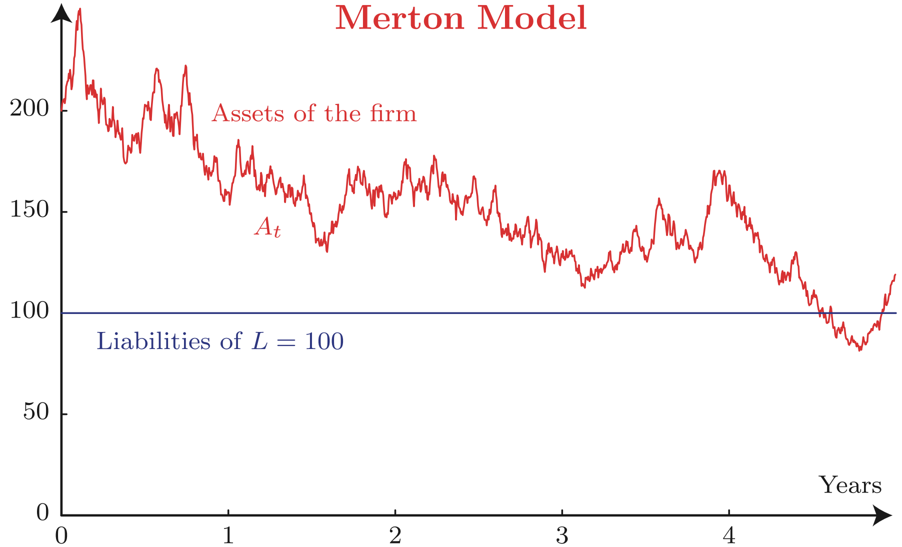
Components of Credit Risk (Cont.)

- Rating transition risk: Risk of changing market price of a defaultable security due to a changed perception of the market towards the timing or recovery risk (without an actual default already happening).
  It often happens together with an up- or down-rating of the creditworthiness by a rating agency.
- Default correlation risk: Risk of several obligors defaulting together; leads to substantial losses even in well diversified portfolios. Defaults in investment-grade rating classes are rare, hence it is hard to collect data to estimate the dependence of defaults.

Classification of Credit Risk Models

- Firm-value (or structural) models
  Pioneered by Black & Scholes (1973), Merton (1974)
  Industry models: Portfolio Manager (by KMV),
  CreditMetrics (RiskMetrics Group)
- Intensity-based (or reduced-form) models
  Jarrow & Turnbull (1995),
  Jarrow, Lando & Turnbull (1997),
  Lando (1996, 1998), Duffie & Singleton (1999)
- Actuarial models
  Mixture models, CreditRisk$^+$ (CS Financial Products)
- Macroeconomic models
  Industry: CreditPortfolioView (McKinsey & Company)

Merton Model



Exponential Brownian motion $A_t = 200 \exp(0.3 W_t)$.
Default occures when $A_t < L$.

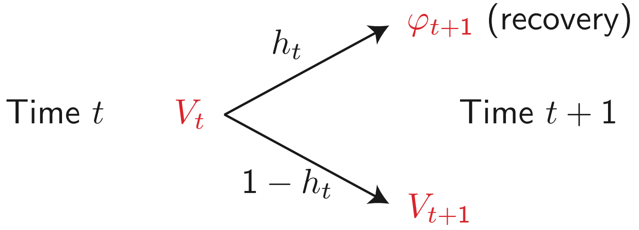
Discrete-Time Motivation for Intensity Models

Time $0 \quad 1 \quad \dots \quad t \quad t+1 \quad t+2 \quad \dots \quad T$

Notation

X	promised (but defaultable) payout at time T
h_t	conditional probability at time t for default during the period $[t, t+1]$
r_t	continuously compounded, default-free interest rate for the period $[t, t+1]$
φ_{t+1}	random recovery at $t+1$ in case of default during $[t, t+1]$
$\mathbb{E}_{\mathbb{Q}}[\cdot \mid \mathcal{F}_t]$	conditional expectation under $\mathbb{Q}$ given all the information $\mathcal{F}_t$ up to time t

Evolution of Market Value V_t



Recursion formula

$$V_t = h_t \, e^{-r_t} \, \mathbb{E}_{\mathbb{Q}}[\varphi_{t+1} \mid \mathcal{F}_t] + (1 - h_t) \, e^{-r_t} \, \mathbb{E}_{\mathbb{Q}}[V_{t+1} \mid \mathcal{F}_t]$$

with terminal value $V_T = X$.
An explicit formula for V_0 by backward induction is available but complicated to evaluate.

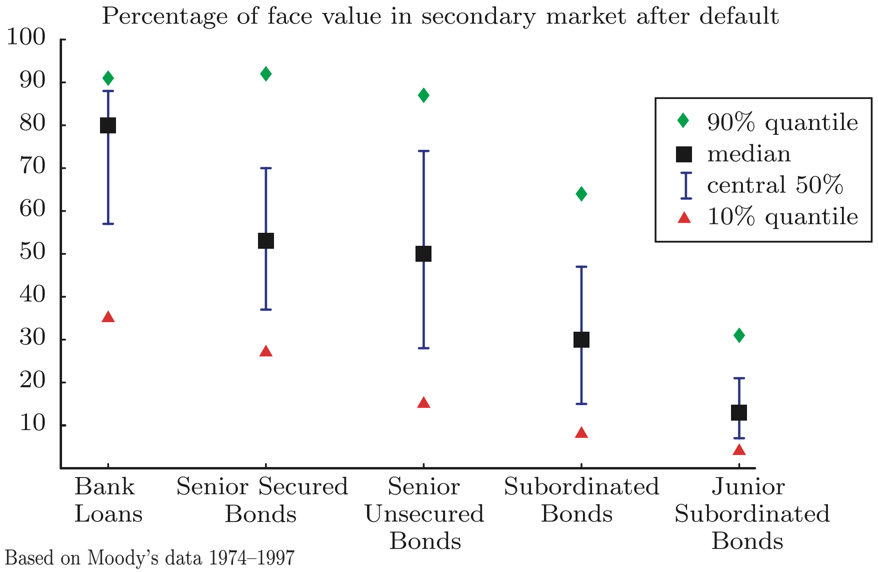
Different Assumptions for the Recovery

Recovery of face value: $\varphi_t = 1 - L_t$
The creditor receives a (possible random) fraction of the face value 1 immediately upon default.

Recovery of treasury: $\varphi_t = (1 - L_t) P(t, T)$
The creditor receives a (possible random) fraction of a corresponding default-free government bond.

Recovery of market value (RMV):
$\mathbb{E}_{\mathbb{Q}}[\varphi_{t+1} \mid \mathcal{F}_t] = (1 - L_t) \, \mathbb{E}_{\mathbb{Q}}[V_{t+1} \mid \mathcal{F}_t]$
The expected recovery is a (random) fraction of the expected market value in case of no default.

Distribution of Recovery by Seniority



Based on Moody's data 1974–1997

Transition to Hazard and Loss Rates

With recovery of market value

$$V_t = \underbrace{\{(1-h_t)\,e^{-r_t} + h_t\,e^{-r_t}(1-L_t)\}}_{=:\,e^{-R_t}}\,\mathbb{E}_{\mathbb{Q}}[V_{t+1}|\mathcal{F}_t]$$

with $e^{-R_t} = (1-h_tL_t)e^{-r_t} \approx e^{-(r_t+h_tL_t)}$, because

$$\Big(1-\frac{h_t}{n}L_t\Big)^n \to e^{-h_tL_t} \qquad n\to\infty.$$

n is the number of subdivisions per period. Hence

$$V_0 = \mathbb{E}_{\mathbb{Q}}\big[e^{-(R_0+\cdots+R_{T-1})}X\big].$$

RiskLab Project: Intensity-Based Non-Parametric Default Model for Residential Mortgage Portfolios

Swiss banks hold over 500 billion CHF in mortgages.
Data for 73 683 obligors of Credit Suisse used.
Default intensity tested for dependence on
– Regional unemployment rates,
– Fixed- or variable-rate mortgage product,
– Interest-rate changes,
– Divorce rates,
– Regional real estate price indices,
– Time-lags until default.
Reference: Paper (38 pages) by Enrico De Giorgi
http://www.risklab.ch/Papers.html#RMSRMMLP

Introduction to CreditRisk$^+$, Features

- Developed by Credit Suisse Financial Products.
- Actuarial model for the aggregation of credit risks.
- Based on the Poisson approximation of individual defaults and the divisibility of the Poisson distribution.
- Takes random exposures/recovery rates into account.
- Probability generating function ϕ_X of the credit portfolio loss X is available in closed form.
- Distribution of X can be calculated from ϕ_X with a numerically stable algorithm.

Project CreditRisk$^+$

- Background: 2. Basel Capital Accord of the Basel Committee on Banking Supervision ("Basel II")
- Research and development cooperation of
  - Research Group Financial and Actuarial Mathematics
  - Austrian Central Bank (OeNB)
  - Austrian Financial Market Authority (FMA)
- Aim: Supervision of credit risk in the portfolio of all ($\geq$ 900) Austrian banks
- Large single credit risks are reported individually
- Efficient method and numerically stable algorithm to calculate risk of credit portfolio
- Java implementation (Mag. Severin Resch)

Time Series versus Default Modelling

Time series modelling (e.g. exchange rates)

- Collect data for a long time (e.g. CHF/US-\$).
- Assume stationarity of stochastic behaviour, fit a suitable model (random walk, GARCH, etc.) and make predictions about the future.

Default modelling

- Observing a firm until today doesn't give a default observation ($\rightarrow$ observation bias).
- Solution: Observe a group of firms, draw conclusions for a specific firm.
- Problems: Relevance of data for the specific firm? When are firms similar w.r.t. creditworthiness?

Credit Ratings for Bonds

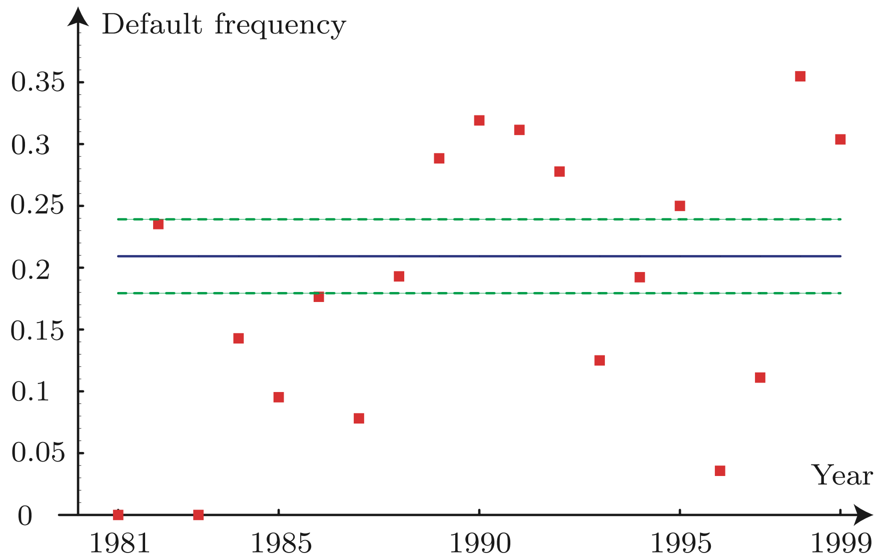
A credit rating is a current opinion of an obligor's overall financial capacity (its creditworthiness) to pay its financial obligations.

Standard & Poor's Investor Services

Investment grade: AAA, AA, A, BBB
Speculative: BB, B, CCC, CC, C (D = Default)
AA–B: + = above, − = below average in rating class

Moody's Investor Services

Investment grade: Aaa, Aa, A
Speculative: Baa, Ba, B, Caa, Ca, C
Aa–B: 1 = above, 2 = at, 3 = below average in rating class



Empirical default frequencies l_i/n_i of the CCC-rated firms with ML-estimate $\hat{p}$ and asymptotic 95%-confidence interval.

Overview of Mixture Model and Calibration

- Form homogeneous groups characterized by their credit rating (homogeneity/statistical significance).
- Given a group's default probability, the individual defaults of its members are independent Bernoulli.
- Use beta distributed random variables for the default probabilities of each group.
- Preserve strict monotonicity of default probabilities according to credit rating (hierarchical dependence structure, Dirichlet distribution for default probab.).
- Calibrate to Standard & Poor's data using the expectation–maximization algorithm.

Exchangeable Sequences

Definition: Random variables $X_1, \dots, X_n$ are called exchangeable, if for every permutation π of $1, \dots, n$

$$(X_1, \dots, X_n) \stackrel{\text{dist.}}{=} (X_{\pi(1)}, \dots, X_{\pi(n)}).$$

$\{X_n\}_{n\in\mathbb{N}}$ are called exchangeable, if $X_1, \dots, X_n$ are exchangeable for every $n \in \mathbb{N}$.

Remarks:

- i. i. d. $\Longrightarrow$ exchangeable $\Longrightarrow$ stationary $\Longrightarrow$ identically distributed
- exchangeable $\not\Longrightarrow$ independent
  Take $X_n \triangleq Y$ for all $n \in \mathbb{N}$ with arbitrary Y or $(X_1, X_2) \triangleq (Y, -Y)$ with symmetric Y.

Exchangeability and Correlation

- Let $X_1, \dots, X_n \in L^2$ be exchangeable. We get with $\sigma^2 = \mathrm{Var}(X_i)$ and $\varrho = \mathrm{Corr}(X_i, X_j)$ for $i \neq j$

$$0 \le \mathrm{Var}(X_1 + \cdots + X_n) = n\sigma^2 + n(n-1)\varrho\sigma^2 \Longrightarrow \varrho \ge -1/(n-1).$$

- Equality is possible with normally distributed $X_1, \dots, X_n$, for example.
- $\{X_n\}_{n\in\mathbb{N}} \subset L^2$ exchangeable $\Longrightarrow$ $\mathrm{Cov}(X_i, X_j) \ge 0$ for $i \neq j$.
- No restrictions on positive correlations.

Structure of Exchangeable Sequences

De Finetti's Theorem: Let $\{X_n\}_{n\in\mathbb{N}}$ be exchangeable. Then there is a σ-field conditional on which the sequence is i. i. d.

Corollary: Let $\{X_n\}_{n\in\mathbb{N}}$ be exchangeable, taking as values the unit vectors $e_1, \dots, e_{k+1}$ in $\mathbb{R}^{k+1}$. Then there exists a random probability distribution $P = (P_1, \dots, P_{k+1})$ such that, for all $l = (l_1, \dots, l_{k+1})$ in $\mathbb{N}_0^{k+1}$ with $l_1 + \cdots + l_{k+1} = n$,

$$\mathbb{P}(X_1 + \cdots + X_n = l \,|\, P_1, P_2, \dots, P_{k+1}) \stackrel{\text{a.s.}}{=} \frac{n!}{l_1!\, l_2! \dots l_{k+1}!} P_1^{l_1} P_2^{l_2} \dots P_{k+1}^{l_{k+1}}.$$

Standard & Poor's Data

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
A-firms	470	464	445	456	497	576	523	517	578	604
defaults	0	2	0	0	0	1	0	0	0	0
B-firms	85	155	150	178	207	287	353	408	409	372
defaults	2	5	7	6	13	24	13	17	13	31
CCC-firms	15	17	19	21	21	17	64	57	52	47
defaults	0	4	0	3	2	3	5	11	15	15

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	...
A-firms	627	711	781	894	1101	1163	1172	1184	1207	
defaults	0	0	0	1	0	0	0	0	1	
B-firms	295	228	238	336	388	424	463	683	873	
defaults	37	17	5	10	16	11	15	30	64	
CCC-firms	61	54	48	26	28	28	27	31	79	
defaults	19	15	6	5	7	1	3	11	24	

Dependence between Groups

General notation:
k homogeneous groups of $n_1, \dots, n_k$ companies of ratings $r_1 \prec r_2 \prec \cdots \prec r_k$, where

$$r_i \prec r_j \iff \text{rating } r_i \text{ better than } r_j$$

Problem:
Corresponding random default probabilities $\bar{P}_1, \bar{P}_2, \dots, \bar{P}_k$ should satisfy $\bar{P}_1 \le \bar{P}_2 \le \cdots \le \bar{P}_k$.

Considered solutions:

- Multi-colour urn scheme for defaults
- Iterative urn scheme for defaults

Pólya's Multi-Colour Urn Scheme for Defaults

- Urn with originally $b_1, \dots, b_{k+1}$ balls of colours $1, \dots, k+1$. Set $b \triangleq b_1 + \cdots + b_{k+1}$.
- Draw balls randomly from urn and set $X_n = (X_{n,1}, \dots, X_{n,k+1}) \triangleq e_j$ if the n^{th} ball has colour j. Here e_j is the j^{th} unit vector in $\mathbb{R}^{k+1}$.
- After each draw return the ball with c others of the same colour.
- If the n^{th} company has rating r_j, then it defaults if and only if $X_n \in \{e_1, \dots, e_j\}$.
  (Smaller colours represent bad luck, colour $k+1$ means no default whatever the rating.)

Multi-Colour Urn Scheme (cont.)

- Conditional probability for colour j in n^{th} draw

$$\begin{aligned} Q_{n,j} &= \mathbb{P}(X_n = e_j \,|\, X_1, \dots, X_{n-1}) \\ &= \frac{\#\{\text{balls of colour } j \text{ in urn after } n-1 \text{ draws}\}}{\#\{\text{balls in urn after } n-1 \text{ draws}\}} \\ &= \frac{b_j + c\sum_{i=1}^{n-1} X_{i,j}}{b + (n-1)c} \end{aligned}$$

- If $c \ge 0$, then $Q_n = (Q_{n,1}, \dots, Q_{n,k+1})$, $n \in \mathbb{N}$, is a measure-valued martingale with

$$P = (P_1, \dots, P_{k+1}) \triangleq \lim_{n\to\infty} Q_n = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} X_i \quad \text{a.s.}$$

Exchangeability for Multi-Colour Urn Scheme

For $n \in \mathbb{N}$ and $d_1, \dots, d_n \in \{e_1, \dots, e_{k+1}\}$ set $l = (l_1, \dots, l_{k+1}) \triangleq \sum_{i=1}^{n} d_i$. Here l_j is the number of balls of colour j within the first n draws. Then

$$\mathbb{P}(X_1 = d_1, \dots, X_n = d_n)$$
$$= \frac{\prod_{j=1}^{k+1} \prod_{i=0}^{l_j-1} (b_j + ic)}{b(b+c)(b+2c)\dots(b+(n-1)c)}.$$

$\{X_n\}_{n \in \mathbb{N}}$ is exchangeable. By de Finetti's theorem,

$$\mathbb{P}(X_1 + \dots + X_n = l \,|\, P_1, P_2, \dots, P_{k+1})$$
$$= \frac{n!}{l_1!\, l_2! \dots l_{k+1}!} P_1^{l_1} P_2^{l_2} \dots P_{k+1}^{l_{k+1}}.$$

Appearance of the Dirichlet Distribution

With $\alpha_j \triangleq b_j/c$ for $c > 0$ and $\alpha \triangleq \alpha_1 + \dots + \alpha_{k+1}$

$$\mathbb{E}[P_1^{l_1} P_2^{l_2} \dots P_{k+1}^{l_{k+1}}] = \frac{\Gamma(\alpha)}{\Gamma(\alpha+n)} \prod_{j=1}^{k+1} \frac{\Gamma(\alpha_j + l_j)}{\Gamma(\alpha_j)},$$

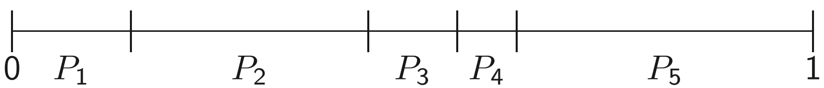
where $n \triangleq l_1 + \dots + l_{k+1}$. For these moments, P has to be Dirichlet distributed. The density of $D_{k+1}(\alpha_1, \dots, \alpha_{k+1})$ is

$$f_{k+1}(p_1, \dots, p_{k+1}) = \Gamma(\alpha) \prod_{j=1}^{k+1} \frac{p_j^{\alpha_j - 1}}{\Gamma(\alpha_j)}$$

with the constraint $p_1 + \dots + p_{k+1} = 1$.

Dirichlet Distribution: Interpretation & Simulation

$(P_1, \dots, P_{k+1}) \sim D_{k+1}(\alpha_1, \dots, \alpha_{k+1})$ gives a random partition of the unit interval. Example:



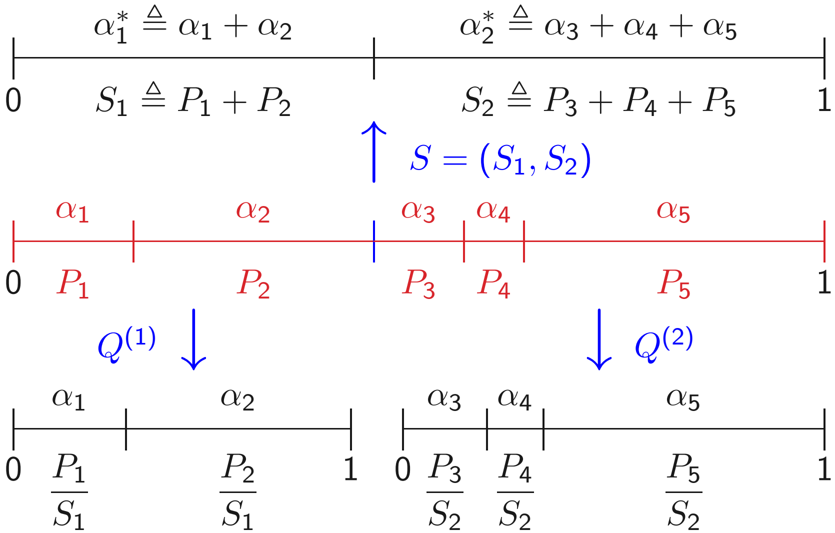
Sampling from the Dirichlet distribution

Take independent $Y_1, \dots, Y_{k+1}$ with $Y_j \sim \text{Gamma}(\alpha_j)$, i. e. $f_{Y_j}(y) = y^{\alpha_j - 1} e^{-y} / \Gamma(\alpha_j)$ for $y > 0$, and set

$$P_j \triangleq \frac{Y_j}{Y_1 + \dots + Y_{k+1}} \quad \text{for } j = 1, \dots, k+1.$$

Then $(P_1, \dots, P_{k+1}) \sim D_{k+1}(\alpha_1, \dots, \alpha_{k+1})$.

Amalgamation Property of the Dirichlet Distribution



Joint Default Probabilities in MC Urn Scheme

Consider l_i defaults within n_i firms of credit rating r_i and define $\bar{P}_i \triangleq P_1 + \cdots + P_i$ as the corresponding random default probability for $i \in \{1, \ldots, k\}$. Then

$$\mathbb{P}(N_1(n_1) = l_1, \ldots, N_k(n_k) = l_k) = \mathbb{E}\Bigl[\prod_{i=1}^{k} \binom{n_i}{l_i} \bar{P}_i^{l_i} (1 - \bar{P}_i)^{n_i - l_i}\Bigr].$$

Inserting the Dirichlet distribution for $(P_1, \ldots, P_{k+1})$ and using the binomial formula, this expectation can be expressed in terms of $k-1$ sums involving binomial numbers and the Gamma function.

Joint Default Probabilities (cont.)

$$= \binom{n_1}{l_1} \cdots \binom{n_k}{l_k} \frac{\Gamma(\sum_{j=1}^{k+1} \alpha_j)}{\prod_{j=1}^{k+1} \Gamma(\alpha_j)} \sum_{j_1=0}^{l_k} \binom{l_k}{j_1} B(\alpha_k + j_1, \alpha_{k+1} + n_k - l_k)$$

$$\times \sum_{j_2=0}^{l_{k-1}+l_k-j_1} \binom{l_{k-1} + l_k - j_1}{j_2} B(\alpha_{k-1} + j_2, \alpha_k + \alpha_{k+1} + n_{k-1} + n_k - l_{k-1} - l_k + j_1)$$

$$\cdots \times \sum_{j_{k-1}=0}^{l_2+\cdots+l_k-j_1-\cdots-j_{k-2}} \binom{l_2 + \cdots + l_k - j_1 - \cdots - j_{k-2}}{j_{k-1}}$$

$$\times B(\alpha_2 + j_{k-1}, \alpha_3 + \cdots + \alpha_{k+1} + n_2 + \cdots + n_k - l_2 - \cdots - l_k + j_1 + \cdots + j_{k-2})$$

$$\times B(\alpha_1 + l_1 + \cdots + l_k - j_1 - \cdots - j_{k-1}, \alpha_2 + \cdots + \alpha_{k+1} + n_1 + \cdots + n_k - l_1 - \cdots - l_k + j_1 + \cdots + j_{k-1}).$$

with $B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$.

Calibration of Credit Risk Mixture Model with the Expectation–Maximization Algorithm

Strategy: View default data as incomplete and use EM-algorithm to compute maximum-likelihood estimators from Standard & Poor's 2003 yearly default data, assuming independence of the $m = 22$ years.

Set-up: Consider $\pi : \mathcal{X} = ([0,1]^k \times \mathbb{N}_0^k)^m \to \mathbb{N}_0^{km}$, mapping complete data $x = (x_1, \ldots, x_m)$ with i-th independent realization $x_i = (p_{i,1}, \ldots, p_{i,k}, l_{i,1}, \ldots, l_{i,k})$ of default probabilities $(P_1, \ldots, P_k)$ and default numbers $(N_1(n_{i,1}), \ldots, N_k(n_{i,k}))$ to $y = (y_1, \ldots, y_m)$ with observed defaults $y_i = (l_{i,1}, \ldots, l_{i,k})$ in year i.

EM-Algorithm, Structure of Likelihood

The likelihood $f(x\,|\,\alpha)$ of the complete data x given parameters $\alpha = (\alpha_1, \ldots, \alpha_{k+1})$ in $(0,\infty)^{k+1}$ can be written as

$$f(x\,|\,\alpha) = b(x) \exp(\langle \alpha, t(x) \rangle)/a(\alpha)$$

with statistics $t(x) = (t_1(x), \ldots, t_{k+1}(x))$, where

$$t_j(x) = \sum_{i=1}^{m} \log p_{i,j} \quad \text{for } j = 1, \ldots, k$$

and

$$t_{k+1}(x) = \sum_{i=1}^{m} \log(1 - p_{i,1} - \cdots - p_{i,k}).$$

Computations for EM-Algorithm

Maximize likelihood of observed (incomplete) data y

$$g(y \,|\, \alpha) = \int_{\pi^{-1}(y)} f(x \,|\, \alpha)\, dx,$$

obtained by integrating over the set $\pi^{-1}(y) \subset \mathcal{X}$ of all possible but unobserved default probabilities as follows:

- Start with moment estimator $\alpha^{(0)}$.

Iterate for $p = 0, 1, 2, \ldots$

- **Expectation step:** Compute

$$t^{(p)} = \mathbb{E}_{\alpha^{(p)}}[t(x) \,|\, y]$$

with explicit but long formula.

Computations for EM-Algorithm (cont.)

- **Maximization step:** Find solution $\alpha^{(p+1)}$ of system

$$\mathbb{E}_{\alpha^{(p+1)}}[t(x)] = t^{(p)}$$

with $k+1$ equations and unknowns. Note that

$$\mathbb{E}_{\alpha^{(p+1)}}[t_j(x)] = mD(\alpha_j^{(p+1)}, \alpha_\Sigma^{(p+1)} - \alpha_j^{(p+1)})$$

for $j = 1, \ldots, k+1$, where

$$D(x,y) = \frac{\Gamma'(x)}{\Gamma(x)} - \frac{\Gamma'(x+y)}{\Gamma(x+y)}, \qquad \alpha_\Sigma^{(p+1)} = \sum_{j=1}^{k+1} \alpha_j^{(p+1)}.$$

If the EM-algorithm converges to an α^*, then the derivative of the likelihood function vanishes at α^*.

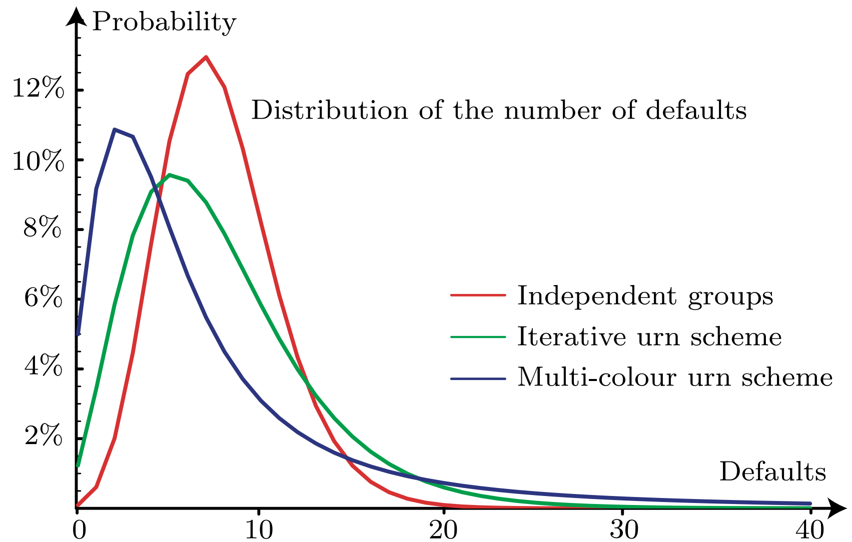
Iterative Urn Scheme for Defaults

- Urns U_i with originally b_i black and r_i red balls for $i = 1, \ldots, k$.
- Draw balls from the urns $U_1, \ldots, U_k$ and set

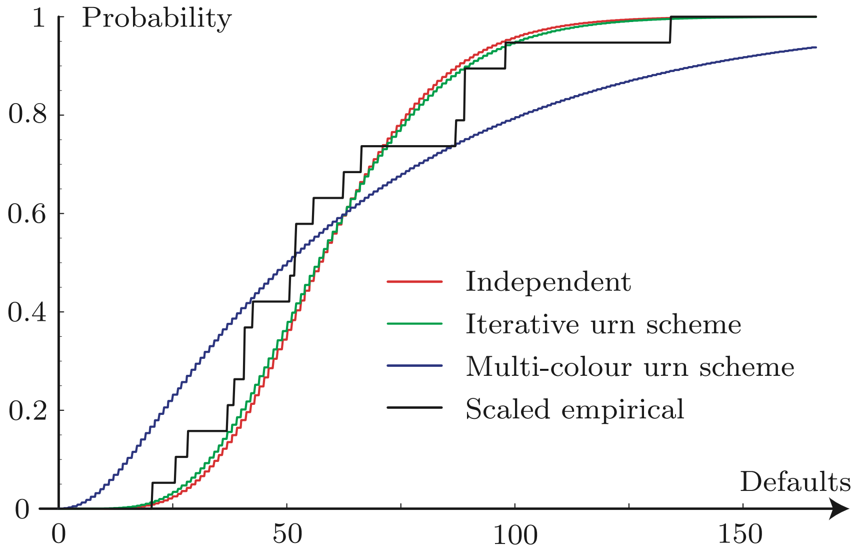
$$X_{i,n} \triangleq \begin{cases} 0 & \text{if } n^{\text{th}} \text{ ball from urn } U_i \text{ is black,} \\ 1 & \text{if } n^{\text{th}} \text{ ball from urn } U_i \text{ is red.} \end{cases}$$

- After each draw from urn U_i return the ball with c_i others of the same colour.
- If the n^{th} company has credit rating r_i, then it defaults if and only if

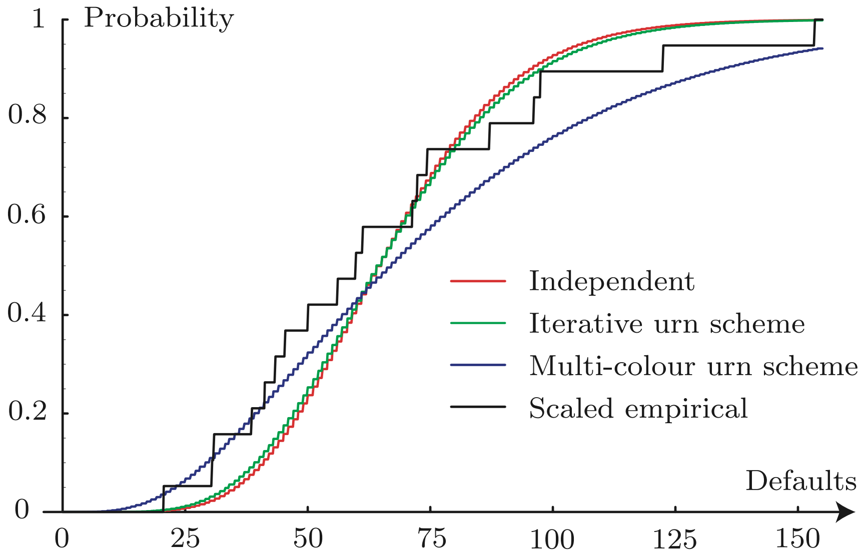
$$X_{1,n} + \cdots + X_{i,n} \geq 1.$$



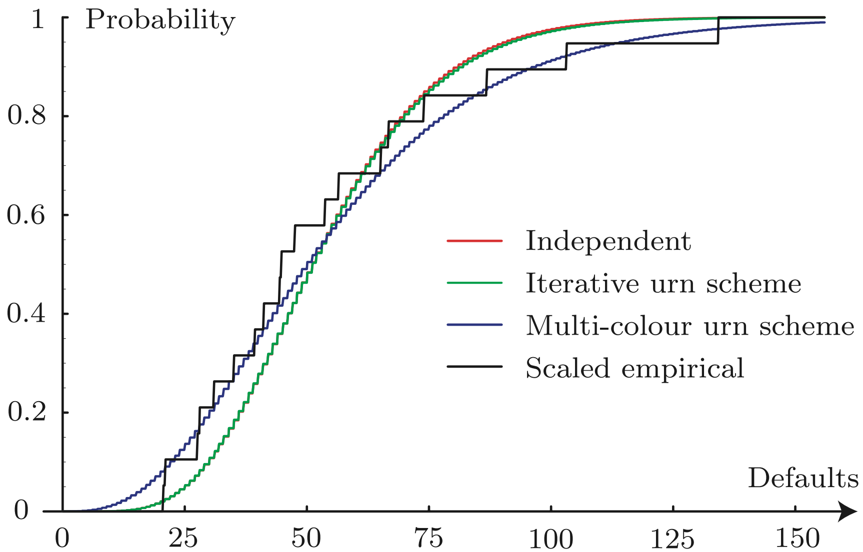
Portfolio of 27 808 AA-, 4 832 A- and 753 BBB-rated firms. Every group has 2 expected defaults.



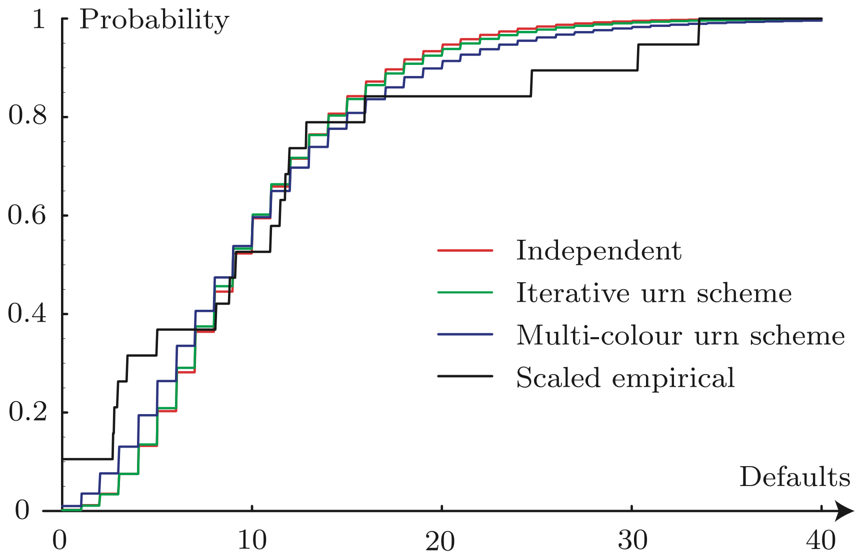
Cumulative distribution function of the total number of defaults of 1207 A-, 873 B- and 79 CCC-rated firms (1999 portfolio)



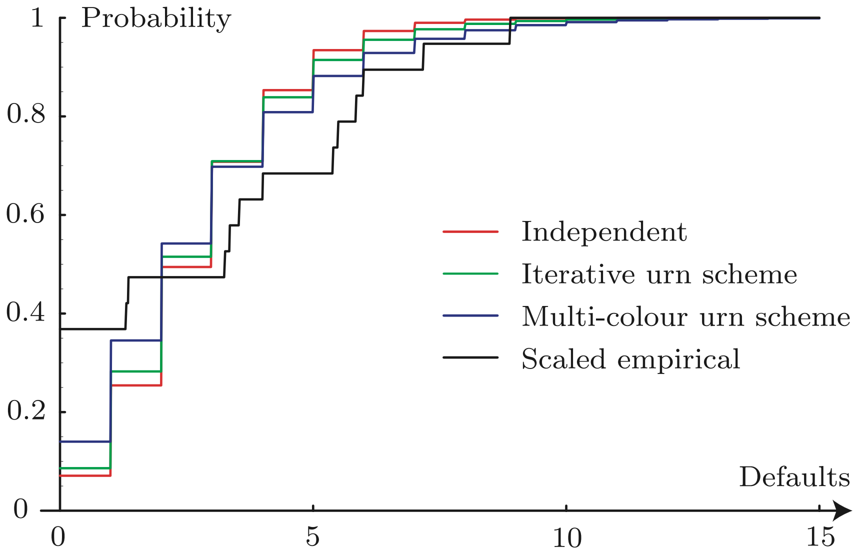
Cumulative distribution function of the total number of defaults of 773 BB-, 873 B- and 79 CCC-rated firms (1999 portfolio)



Cumulative distribution function of the total number of defaults of 1049 BBB-, 773 BB- and 873 B-rated firms (1999 portfolio)



Cumulative distribution function of the total number of defaults of 1207 A-, 1049 BBB- and 773 BB-rated firms (1999 portfolio)



Cumulative distribution function of the total number of defaults of 628 AA-, 1207 A- and 1049 BBB-rated firms (1999 portfolio)

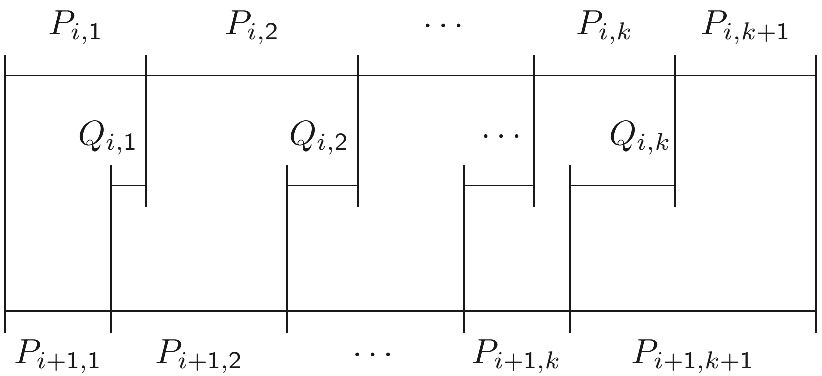
Modelling Random Rating Transitions

The random transition matrix

$$P = \begin{pmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,k} & P_{1,k+1} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,k} & P_{2,k+1} \\ \vdots & \vdots & & \vdots & \vdots \\ P_{k,1} & P_{k,2} & \cdots & P_{k,k} & P_{k,k+1} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

should satisfy $P_{i,j} + \cdots + P_{i,k+1} \leq P_{i',j} + \cdots + P_{i',k+1}$ for all $j = 1, \ldots, k+1$ and $1 \leq i \leq i' \leq k$, because a firm with the better rating $r_i \prec r_{i'}$ should have a lower probability to change to a worse ratings $j, \ldots, k+1$.
$\Longrightarrow$ complicated dependence structure

Using Dependent Dirichlet Distributions for Random Rating Transitions



Properties of the Urn Models and Conclusions

Advantages:

- Theoretical justification (exchangeability).
- Limit theorems for the chosen distributions.
- Easy to implement analytical solutions for fitting the model, even with dependence between groups.
- Numerical efficiency and stability.
- Different dependence structures possible.
- Extension to random rating transitions possible.

Problems:

- Fitting the model simultaneously for many rating classes is computationally demanding.
- Exposures and recoverables are not included.