Quantifying Regulatory Capital for Operational Risk: Utopia or Not?

Paul Embrechts
ETH Zürich
and
London School of Economics
(www.math.ethz.ch/∼embrechts)

Based on joint work with V. Chavez-Demoulin, H.J. Furrer, R. Kaufmann and G. Samorodnitsky

Contents

A. The New Accord (Basel II)

- **1988**: Basel Accord (Basel I): minimal capital requirements against credit risk, one standardised approach, Cooke ratio
- **1996**: Amendment to Basel I: market risk, internal models, netting
- **1999**: First Consultative Paper on the New Accord (Basel II)
- **to date**: CP3: Third Consultative Paper on the New Basel Capital Accord (`www.bis.org/bcbs/bcbscp3.htmcp3`)
- **2004**: Revision: (final) version
- **2006–2007**: full implementation of Basel II ([13])

Basel II: What is new?

- **Rationale** for the New Accord: More flexibility and risk sensitivity
- **Structure** of the New Accord: Three-pillar framework:

❶ Pillar 1: minimal capital requirements (risk measurement)

❷ Pillar 2: supervisory review of capital adequacy

❸ Pillar 3: public disclosure

- Two options for the measurement of credit risk:
  - Standard approach
  - Internal rating based approach (IRB)

- Pillar 1 sets out the minimum capital requirements (Cooke Ratio):

$$\frac{\text{total amount of capital}}{\text{risk-weighted assets}} \geq 8\%$$

- MRC (minimum regulatory capital) $\stackrel{\text{def}}{=}$ 8% of risk-weighted assets

- Explicit treatment of operational risk

Operational Risk:

The risk of losses resulting from inadequate or failed internal processes, people and systems, or external events.

Remark: This definition includes legal risk, but excludes strategic and reputational risk.

- **Notation**: C_{OP}: capital charge for operational risk

- **Target**: $C_{\mathsf{OP}} \approx 12\%$ of MRC (down from initial 20%)

- **Estimated total losses** in the US (2001): $50b

- **Some examples**

  - 1977: Credit Suisse Chiasso-affair
  - 1995: Nick Leeson/Barings Bank, £1.3b
  - 2001: Enron (largest US bankruptcy so far)
  - 2002: Allied Irish, £450m

B. Risk measurement methods for OP risks

Pillar 1 regulatory minimal capital requirements for operational risk:
Three distinct approaches:

❶ Basic Indicator Approach

❷ Standardised Approach

❸ Advanced Measurement Approaches (AMA)

Basic Indicator Approach

- Capital charge:

$$\boxed{C_{\mathsf{OP}}^{\mathsf{BIA}} = \alpha \times GI}$$

- $C_{\mathsf{OP}}^{\mathsf{BIA}}$: capital charge under the Basic Indicator Approach

- GI: average annual gross income over the previous three years

- $\alpha = 15\%$ (set by the Committee based CISs)

Standardised Approach

- Similar to the BIA, but on the level of each business line:

$$\boxed{C_{\mathsf{OP}}^{\mathsf{SA}} = \sum_{i=1}^{8} \beta_i \times GI_i}$$

$\beta_i \in [12\%, 18\%]$, $i = 1, 2, \ldots, 8$

- 8 business lines:

Corporate finance (18%)	Payment & Settlement (18%)
Trading & sales (18%)	Agency Services (15%)
Retail banking (12%)	Asset management (12%)
Commercial banking(15%)	Retail brokerage (12%)

- Some remarks from the proposal:
  - A bank must be able to demonstrate that its approach captures potentially severe "tail" loss events
  - A one year holding period and a 99.9% confidence interval (like internal ratings-based approach for credit risk)
  - Independent model validation
  - $RC = EL + UL$, however ...
  - Sufficient granularity
  - Addition of risk measures
  - Correlation effects
  - Use of internal data, external data, scenario analysis and driving factors
  - Repetitive versus non-repetitive losses
  - Minimum 5-year (initially 3-year) internal loss observation period
  - De minimis gross loss threshold, e.g. €10 000
  - From partial to full AMA

Advanced Measurement Approaches (AMA)

- Allows banks to use their internally generated risk estimates
- Preconditions: Bank must meet qualitative and quantitative standards before being allowed to use the AMA
- Risk mitigation via insurance possible ($\leq 20\%$ of $C_{\text{OP}}^{\text{SA}}$)
- Incorporation of risk diversification benefits allowed
- "Given the continuing evolution of analytical approaches for operational risk, the Committee is not specifying the approach or distributional assumptions used to generate the operational risk measures for regulatory capital purposes."
- Examples: - **AMA1**: Internal measurement approach
  - **AMA2**: Loss distribution approach

Internal Measurement Approach

- Capital charge (similar to Basel II model for Credit Risk):

$$\boxed{C_{\mathsf{OP}}^{\mathsf{IMA}} = \sum_{i=1}^{8} \sum_{k=1}^{7} \gamma_{ik}\, e_{ik}}$$

e_{ik}: expected loss for business line i, risk type k

γ_{ik}: scaling factor

- 7 loss types: Internal fraud
  External fraud
  Employment practices and workplace safety
  Clients, products & business practices
  Damage to physical assets
  Business disruption and system failures
  Execution, delivery & process management

C. Loss Distribution Approach

- For each business line/loss type cell (i, k) one models

  $L_{i,k}^{T+1}$: OP risk loss for business line i, loss type k over the future (one year, say) period $[T, T+1]$

$$L_{i,k}^{T+1} = \sum_{\ell=1}^{N_{i,k}^{T+1}} X_{i,k}^{\ell} \quad \text{(next period's loss for cell } (i,k))$$

Note that $X_{i,k}^{\ell}$ is truncated from below

Remark: Look at the structure of the loss random variable L^{T+1}

$$L^{T+1} = \sum_{i=1}^{8} \sum_{k=1}^{7} L_{i,k}^{T+1} \quad \text{(next period's total loss)}$$

$$= \sum_{i=1}^{8} \sum_{k=1}^{7} \sum_{\ell=1}^{N_{i,k}^{T+1}} X_{i,k}^{\ell}$$

A methodological pause 1

$$L = \sum_{k=1}^{N} X_k \quad \text{(compound rv)}$$

where (X_k) are the severities and N the frequency

Models for X_k:

- gamma, lognormal, Pareto (≥ 0, skew)

Models for N:

- binomial (individual model)
- Poisson(λ) (limit model)
- negative binomial (randomize λ as a gamma rv)

- Choice of a risk measure g ($\alpha \in (0,1)$ fixed)

$$C_{i,k}^{T+1,\mathsf{OR}} = g(L_{i,k}^{T+1}) = \begin{cases} F^{\leftarrow}_{L_{i,k}^{T+1}}(\alpha) = \mathrm{VaR}_\alpha(L_{i,k}^{T+1}) \\ \mathsf{ES}(L_{i,k}^{T+1}) = E\left(L_{i,k}^{T+1} \mid L_{i,k}^{T+1} > \mathrm{VaR}_\alpha(L_{i,k}^{T+1})\right) \end{cases}$$

- VaR_α is not coherent (example)
- ES_α is coherent (modulo trivial change)

$$C^{T+1,\mathsf{OR}} = \sum_{i,k} g(L_{i,k}^{T+1}) \quad \text{(perfect correlation)}$$

- Why?
- Dependence effects (copulae)

VaR_α is in general not coherent:

- 100 iid loans: 2%-coupon, 100 face value, 1% default probability (period: 1 year):

$$X_i = \begin{cases} -2 & \text{with probability } 99\% \\ 100 & \text{with probability } 1\% \quad (\text{loss}) \end{cases}$$

- Two portfolios $L_1 = \sum_{i=1}^{100} X_i$, $L_2 = 100 X_1$

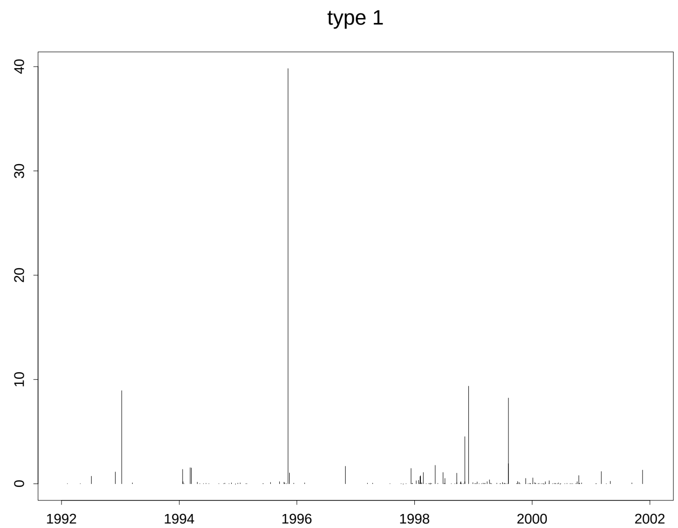
- $\underbrace{\mathrm{VaR}_{95\%}(L_1)}_{\mathrm{VaR}_{95\%}(\sum_{i=1}^{100} X_i)} > \underbrace{\mathrm{VaR}_{95\%}(100X_1)}_{\sum_{i=1}^{100} \mathrm{VaR}_{95\%}(X_i)}$ (!)

- Hence the well-diversified portfolio L_1 gets a higher (VaR-)risk charge than the very concentrated, "all eggs in one basket" portfolio L_2

- This cannot happen when $(X_1, \ldots, X_d)$ has a multivariate normal (or more generally, elliptical) distribution

- Link to Operational Risks: skewness

D. Some data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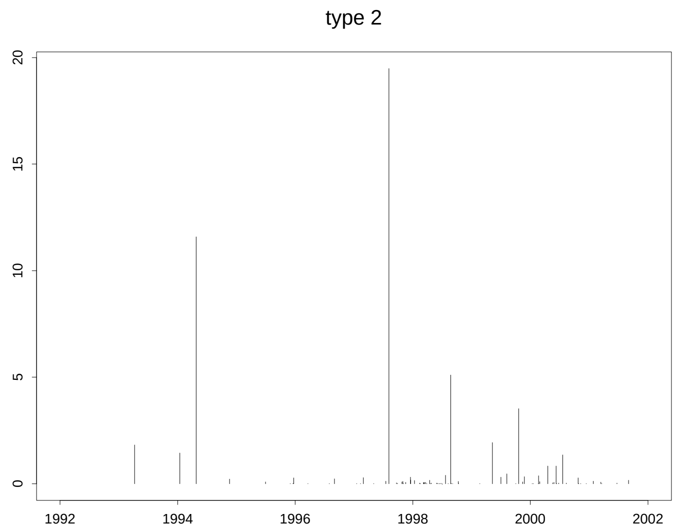
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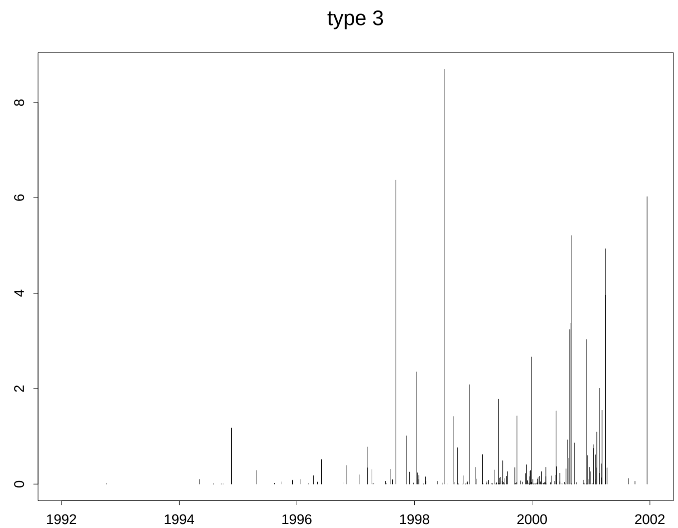
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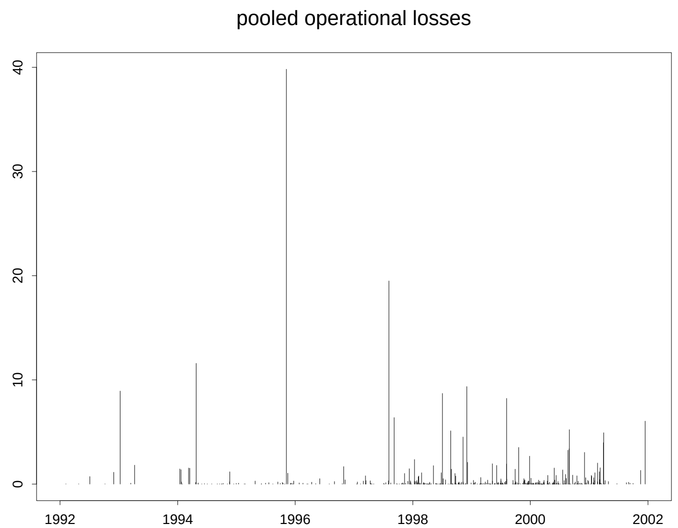
pooled operational losses
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1992
1994
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2002

- Stylized facts about OP risk losses:
  - Loss amounts show extremes
  - Loss occurence times are irregularly spaced in time (reporting bias, economic cycles, regulation, management interactions, structural changes, . . .)
  - Non-stationarity (frequency(!), severity(?))
- Large losses are of main concern
- Repetitive versus non-repetitive losses
- **Warning flag**: observations are not in line with standard modelling assumptions

A methodological pause 2

- severity models need to go beyond the classical models (binomial, homogeneous Poisson, negative binomial: $\longrightarrow$ stochastic processes)

- as stochastic processes:

  - Poisson(λt), $\lambda > 0$ deterministic (1)
  - Poisson($\lambda(t)$), $\lambda(t)$ deterministic
    non-homogeneous Poisson, via time change $\longrightarrow$ (1)
  - Poisson($\Lambda(t)$), $\Lambda(t)$ stochastic process
    - double stochastic (or Cox-) process
    - basic model for credit risk

- a desert-island model: (NB,LN) (cover of [4])

E. The Capital Charge Problem

- Estimate $g_\alpha(L^{T+1})$ for α large

  Basel II: $g_\alpha = \mathrm{VaR}_\alpha$, $\alpha \geq 99.9\%$ (reason)

- In-sample estimation of $\mathrm{VaR}_\alpha(L^{T+1})$ for α large is difficult, if not impossible (lack of data)

- Even for nice (repetitive) data one needs a structural model: Insurance Analytics ([11])

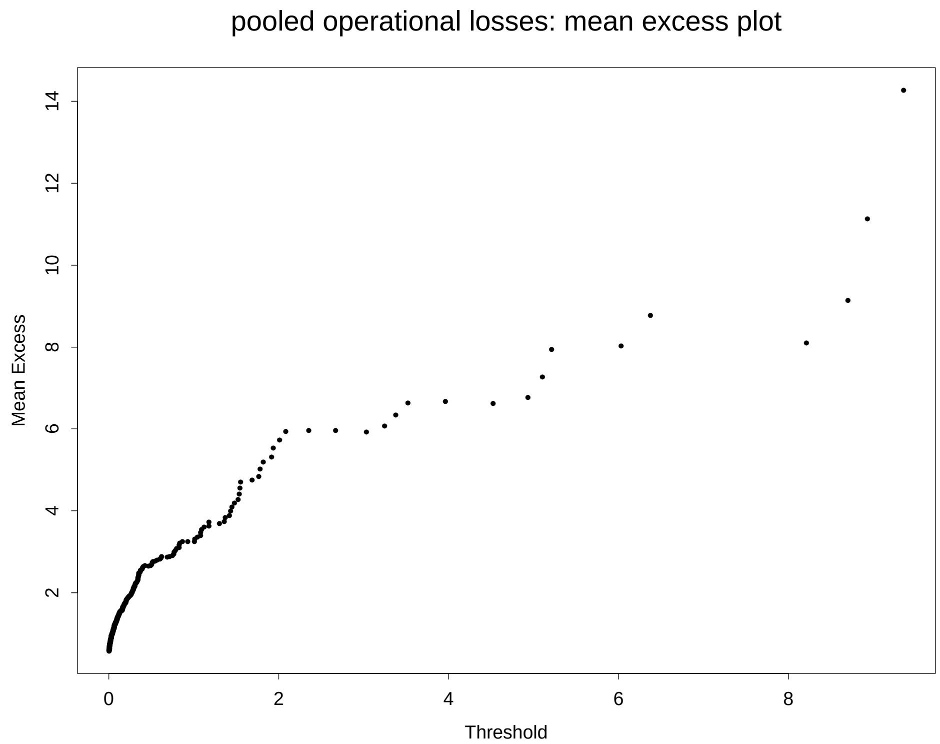
- Standard Actuarial Techniques
  - ∗ Analytic approximations (normal, translated gamma, Edgeworth, saddle-point, . . .)

    However: long-tailedness (Pareto, power tails)

$$P(X > x) \sim x^{-\alpha} L(x), \ x \text{ large}$$

  - ∗ Inversion methods (FFT)
  - ∗ Recursive methods (Euler-Panjer)
  - ∗ (Rare event) simulation
  - ∗ Expert system Ansatz
  - ∗ Extreme Value Theory (EVT) (confidence level $\geq 99.9\%$)
  - ∗ Ruin theory

- Back to the data



- $P(L > x) \sim x^{-\alpha} L(x), \ 1 < \alpha < 3$

- 20 – 80 rule

- one-claim-causes-ruin phenomenon ([1])

Summary

- $\alpha \simeq 1$ and heavy-tailed loss-sizes, hence extremes matter
  - Extreme Value Theory (EVT) ([8])
- adding risk measures over different risk classes, hence dependence matters
  - Copulae $(F_{\underline{X}}(\underline{x}) = C(F_1(x_1), \ldots, F_d(x_d)))$ ([9])
- complicated loss-frequencies, hence point processes matter
  - double-stochastic (or Cox) processes ([5])
- full model analytically not tractable, hence
  - rare event simulation ([3])

F. Accuracy of VaR-estimates

- **Assumptions**:
  - $L_1, \ldots, L_n$ iid $\sim F_L$
  - For some ξ, β and u large ($G_{\xi,\beta}$: GPD):

$$F_u(x) := \mathbb{P}[L - u \leq x | L > u] \sim G_{\xi,\beta(u)}(x), \; u \text{ large}$$

  - Use that: $1 - F_L(x) = (1 - F_L(u))\,(1 - F_u(x - u))\,, \quad x > u$

- Tail- and quantile estimate:

$$\boxed{\begin{aligned} 1 - \hat{F}_L(x) &= \frac{N_u}{n}\Big(1 + \hat{\xi}\,\frac{x-u}{\hat{\beta}}\Big)^{-1/\hat{\xi}}, \quad x > u \\ \widehat{\mathrm{VaR}}_\alpha &= \hat{q}_\alpha = u - \frac{\hat{\beta}}{\hat{\xi}}\Big(1 - \Big(\frac{N_u}{n(1-\alpha)}\Big)^{\hat{\xi}}\Big) \end{aligned}} \tag{1}$$

- **Idea**: Comparison of estimated quantiles with the corresponding theoretical ones by means of a simulation study ([12], [6]).

- **Simulation procedure**:

  ❶ Choose F_L and fix $\alpha_0 < \alpha < 1$, N_u ($\#$ of data points above u)

  ❷ Calculate $u = q_{\alpha_0}$ and the true value of the quantile q_α

  ❸ Sample N_u independent points of F_L above u by the rejection method. Record the total number n of sampled points this requires

  ❹ Estimate ξ, β by fitting the GPD to the N_u exceedances over u by means of MLE

  ❺ Determine $\hat{q}_\alpha$ according to (1)

  ❻ Repeat N times the above to arrive at estimates of Bias$(\hat{q}_\alpha)$ and SE$(\hat{q}_\alpha)$

  ❼ Require bias and standard error to be small $\Rightarrow$ datasize

Example: Pareto distribution with $\alpha = 2$

$u = F^{\leftarrow}(x_q)$	α	Goodness of $\widehat{\text{VaR}}_\alpha$
$q = 0.7$	0.99	A minimum number of **100 exceedances** (corresponding to 333 observations) is required to ensure accuracy wrt bias and standard error.
	0.999	A minimum number of **200 exceedances** (corresponding to 667 observations) is required to ensure accuracy wrt bias and standard error.
$q = 0.9$	0.99	Full accuracy can be achieved with the minimum number **25** of **exceedances** (corresponding to 250 observations).
	0.999	A minimum number of **100 exceedances** (corresponding to 1000 observations) is required to ensure accuracy wrt bias and standard error.

Summary

- Minimum number of observations increases as the tails become thicker ([12], [6]).
- Large number of observations necessary to achieve targeted accuracy.
- **Remember**: The simulation study was done under idealistic assumptions (iid, exact Pareto). Operational risk losses, however, typically do NOT fulfil these assumptions.

G. Conclusions

- OP risk $\neq$ market risk, credit risk
- OP risk losses resemble non-life insurance losses
- Actuarial methods (including EVT) aiming to derive capital charges are for the moment of limited use due to
  - lack of data
  - inconsistency of the data with the modelling assumptions
- OP risk loss databases must grow
- Sharing/pooling internal operational risk data? Near losses?

- Choice of risk measure: ES better than VaR
- Heavy-tailed ruin estimation for general risk processes ([10]): an interesting mathematical problem related to time change
- Alternatives?
  - Insurance. Example: FIORI, Swiss Re (Financial Institution Operating Risk Insurance)
  - Securitization / Capital market products
- OP risk charges can not be based on statistical modelling alone

▶ Pillar 2 (overall OP risk management such as analysis of causes, prevention, . . .) more important than Pillar 1

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