Discussion on structural default risk modeling for implementation

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Default risk modeling

Our approach

Empirical Approach

• Z-score model (discriminant analysis)
• Logistic regression
• Decision tree
• Artificial Neural Network

Aim:
Improve default prediction based on accounting information

Stochastic Approach

• Structural approach
-Merton's model
-First-passage-time model

• Reduced-form approach
-Stochastic hazard rate model
-Transition matrix model

Aim:
Calculate default term structure and the value of defaultable contingent claim

Outline of this talk

- Introduce two models
  - First hitting time approach based on a filtering model (incomplete information modeling)
  - Merton-type approach that features retained earnings and that uses forecast data about sales and incomes (joint work with H. Yamauchi)

First hitting time approach based on a filtering model

c.f.) H. Nakagawa, "A Filtering Model on Default risk", J.Math.Sci.Univ.Tokyo 8 (2001), 107-142.

Filtering model

Structural approach originated by Merton

- Merit
  - Specification of default is acceptable from a financial viewpoint.
- Demerit
  - Implementation is difficult.
  - If possible, the outputs can hardly be satisfactory for practical use.

Filtering model

Motivation

Duffie-Lando(2001) ``Term Structure of Credit Spreads with Incomplete Accounting Information,'' Econometrica.

- Propose a structural model based upon imperfect information
- Represent the hazard rate in terms of volatility and conditional density of the firm's value process.

Kusuoka(1999) ``A Remark on default risk Models,'' Adv. Math. Econ.

- Propose a filtering model including the information of whether the default occurred or not

Aim: Achieve the specific representation of hazard rate in the framework of a filtering model

Filtering model

"Main system"

$$dX_t = dB_t + \mu(t, X_t)dt, \ \ X_0 = x_0 > 0,$$

"Default time" : $\tau = \inf\{t \in [0,T] | X_t \leq 0\}$

"Observation"

$$dY_t = \sigma_1(t, Y_t)dW_t + b_1(t, X_{t\wedge\tau}, Y_t)dt,$$

B_t and W_t are independent standard Brownian motions.

Filtering model

"Filtration"

$$\mathcal{G}_t^Y = \bigcap_{u>t} \sigma\{Y_s, s \le u\}, \qquad \mathcal{F}_t = \bigcap_{u>t} (\mathcal{G}_u^Y \vee \sigma\{\tau \wedge u\})$$

"Default-counting process" : $N_t = 1_{\{\tau \le t\}}$

"Hazard rate process" is defined by the $((\mathcal{G}_t^Y))$-progressively measurable process $h(t)$ such that

$$M_t = N_t - \int_0^t (1 - N_s) h(s) ds$$

is a $(P, (\mathcal{F}_t))$-martingale.

Filtering model

Under some technical conditions, the following result holds.

Theorem 1. [Duffie-Singleton, Kusuoka, etc.] Let $0 \leq t \leq s \leq T$. For $(\mathcal{G}_t^Y)$-measurable integrable random variable Z,

$$E[Z(1-N_s))|\mathcal{F}_t] = (1-N_t)E\left[Z\exp\left(-\int_t^s h(u)du\right)|\mathcal{G}_t^Y\right].$$

In particular,

$$P(\tau > s|\mathcal{F}_t) = (1-N_t)E\left[\exp\left(-\int_t^s h(u)du\right)|\mathcal{G}_t^Y\right].$$

Filtering model

Measure change : $P \longrightarrow \tilde{P}$

$$dX_t = d\tilde{B}_t, \qquad dY_t = \sigma_1(t, Y_t)d\tilde{W}_t$$

($\tilde{B}$ and $\tilde{W}$ are independent $\tilde{P}$-Brownian motions.)

Consider the density process specified by

$$\rho_t = \tilde{E}[\frac{dP}{d\tilde{P}}|\mathcal{F}_t].$$

Filtering model

Let

$$q(t) = \int_t^\infty \frac{x_0}{2\pi s^3} \exp\Big(-\frac{x_0^2}{2s}\Big) ds \quad t \in [0, T],$$

and $\lambda(s) = -q(t)^{-1} \dfrac{d}{dt} q(t)$. Then

$$\tilde{M}_t = N_t - \int_0^t (1 - N_s)\lambda(s) ds$$

is a $(\tilde{P}, \mathcal{F}_t)$-martingale.

Filtering model

Theorem 2. [N.(2001)]

ρ_t is the unique solution to the following SDE:

$$\rho_t = 1 + \int_0^t \rho_{s-}(\gamma(s)^\top d\tilde{W}_s + \kappa(s) d\tilde{M}_s),$$

where $\gamma(s)$ and $\kappa(s)$ are some $(\mathcal{F}_t)$-predictable processes. $\tilde{W}$ can be viewed as a $(\tilde{P}, (\mathcal{F}_t))$-Brownian motion.

It also follows that the hazard rate $h(t)$ under P is given by

$$h(t) = (1 + \kappa(t))\lambda(t) \quad \text{for } t < \tau.$$

Filtering model

Theorem 3. [N.(2001)]

Under the original measure P, $(\mathcal{G}_t^Y)$-hazard rate process $h(t)$ is given by

$$h(t) = \frac{\widehat{H}(t;Y)}{\widehat{K}(t;Y)}\psi(t),$$

where $\widehat{H}(t;y)$ and $\widehat{K}(t;y)$ are seen as some functionals on the space $C([0,T];\mathbf{R}^{N_2})$, and

$$\psi(t) = \frac{x_0}{\sqrt{2\pi t^3}} \exp\Big(-\frac{x_0^2}{2t}\Big)\Big(\equiv \lambda(t)q(t)\Big).$$

Filtering model

$$
\begin{aligned}
\widehat{H}(t;y) &= \int_{\mathbf{W}} \nu_{0,x_0}^{t,0}(d\theta)\mathcal{E}(t,\theta;y), \\
\widehat{K}(t;y) &= \int_0^\infty dx g(t,x_0,x)e^{\psi(t,x)} \int_W \nu_{0,x_0}^{t,x}(d\theta)\mathcal{E}(t,\theta;y), \\
\mathcal{E}(t,\theta;y) &= \exp\Big(\int_0^t F(u,\theta(u))dy(u) - \frac{1}{2}\int_0^t F(u,\theta(u))^2 du\Big),
\end{aligned}
$$

where $F(t,\theta)$ is a function represented in terms of coefficient functions of SDE for Y, $\psi(t,x)$ is a function represented in terms of drift functions of SDE for X, and $g(s,x,y)$ is a transition density of Brownian motion absorbed at zero.

$\nu_{0,x_0}^{t,x}(d\theta)$ can be seen as the law of 3-dim. Bessel bridge between $(0,x_0)$ and (t,x).

Filtering model

Some ideas for specification of the model

1. X : Log(Total assets / Total Liabilities)
   Y : Stock price

2. X: Firm's credit quality (viewed as a continuous version of rating)
   Y: Credit spread of corporate bond

How to model the relation between X and Y ?

Filtering model

Numerical illustration

(Model)

$$\begin{aligned} X_t &= x_0 + \mu_0 t + B_t, \quad x_0 > 0, \\ Y_t &= y_0 + \mu_1 \int_0^t X_{s\wedge\tau} ds + \sigma_1 W_t, \end{aligned}$$

where $\tau := \inf\{t > 0 | X_t \leq 0\}$.

For $x_0 = 1, \mu_0 = -0.5, \mu_1 = 1, \sigma_1 = 0.5$, we compute

$$E\left[\exp\left(-\int_0^1 h(u)du\right)\right],$$

where $h(t)$ is computed as the functional of Y as is given in Theorem 3.

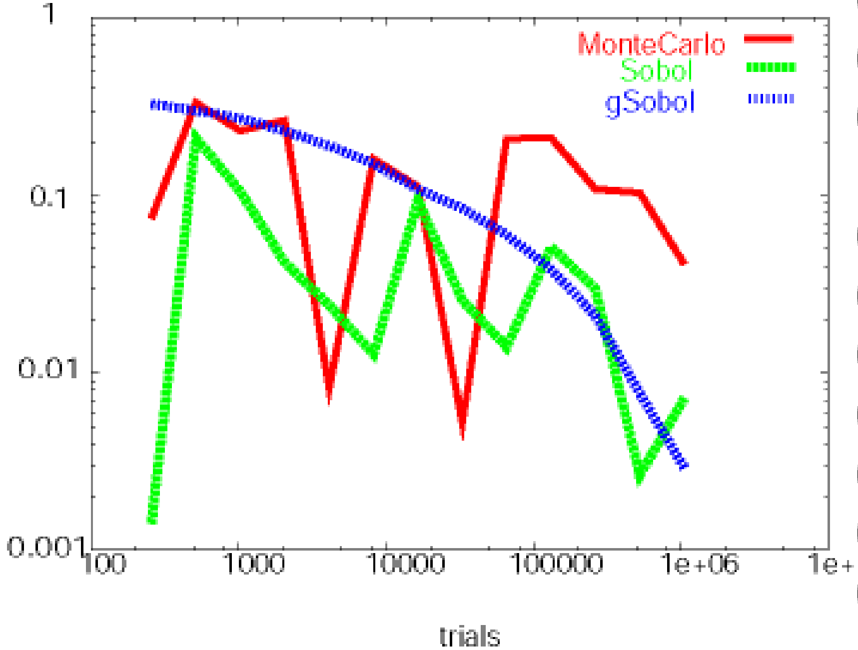
O. Ikeuchi, one of Nakagawa's students, studied this numerical computation for his master thesis.

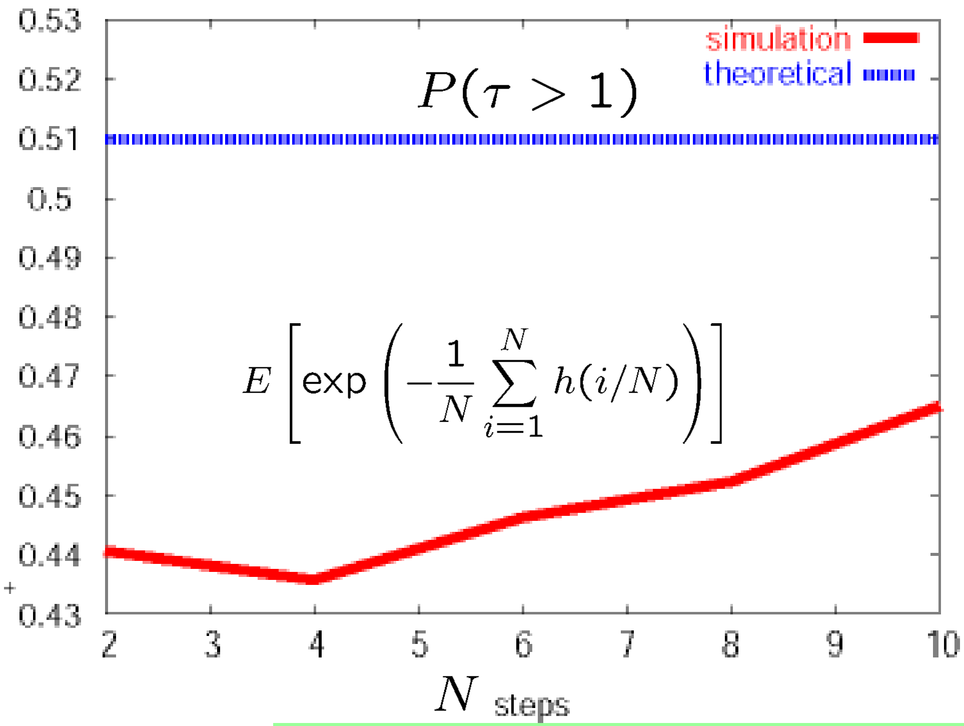
Filtering model

Discretization of [0,1] by 1/N

$$E\left[\exp\left(-\frac{1}{10}\sum_{i=1}^{10} h(i/10)\right)\right]$$

Effect of number of divisions





Standard deviation of residual
for 10 divisions between [0,1]

Average 1024 samples of Y for computing h(t)

Convergence speed is too slow to implement.

Future researches

- Relevant specification of the system and observation

- Fast and efficient numerical computation

Merton-type approach that features retained earnings and that uses forecast data about sales and incomes

(joint work with H. Yamauchi)

Merton's approach featuring RE

Motivation

- From a view of both accounting and finance, specify a dynamic measure of default risk applicable to Japanese listed companies.
- In particular, find a measure that is not so influenced by accounting manipulation and that can reflect changes of firm's financial situation as soon as possible.

Merton’s approach featuring RE

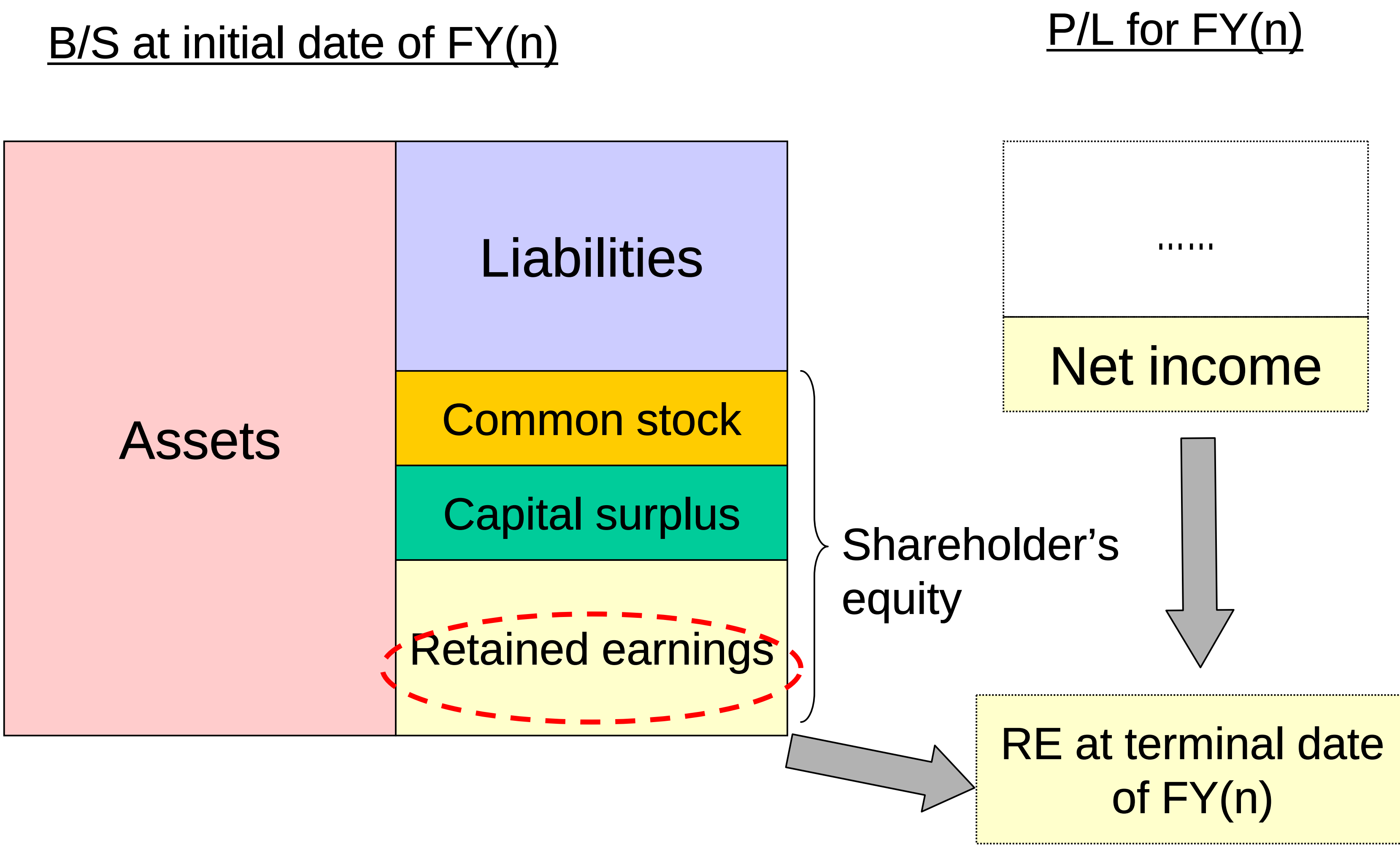
- “Retained Earnings”
  - Definition:
    - RE = Total shareholders’ equity
      - (Common stock + Capital surplus)

    (c.f. Y. Shirata, Default Prediction Model (in Japanese))
  - As financial state gets worse, RE is likely to decrease.
  - We suppose that RE / TC may be a better barometer of default warning than equity ratio or debt ratio because equity can increase by financial support from allied banks or companies even just before bankruptcy.

Merton's approach featuring RE

- Predict the distribution of terminal RE
  - Model: Normal distribution
    - terminal RE
      = initial RE + net income of the fiscal year
      (Assume that cash-out such as dividend is neglected.)
  - How we forecast the net income?
    - Assume that forecast about net sales and incomes are announced to the public (by analysts or firm itself) and are available.
    - Forecast of net income is used as the mean of normal distribution.
    - Standard deviation of net income is supposed to depend on whether past forecasts of net income are relevant.

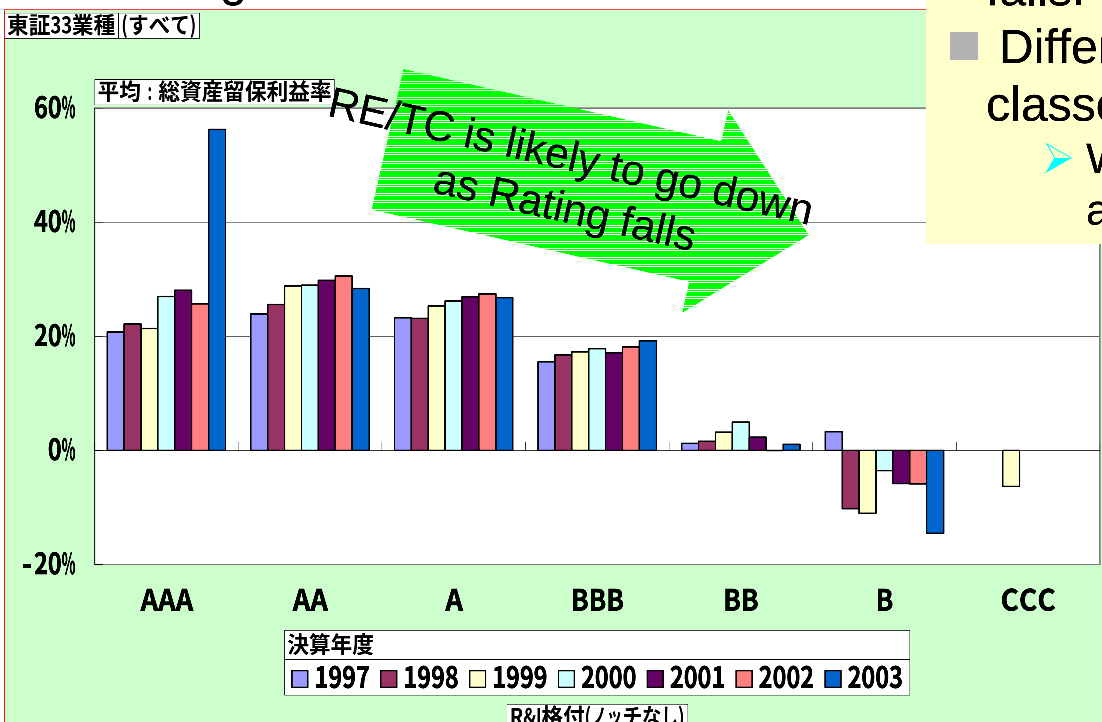
Merton’s approach featuring RE



Inspection on RE/TC

- 1st sec. of TSE, all industries with account day in March

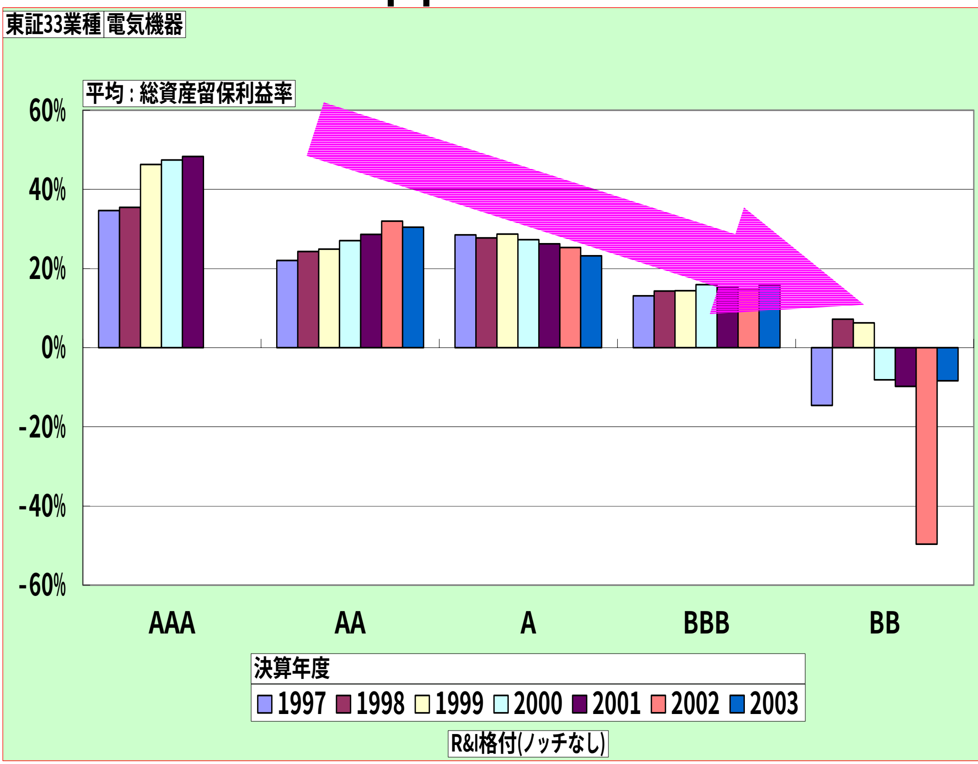
R&I ratings



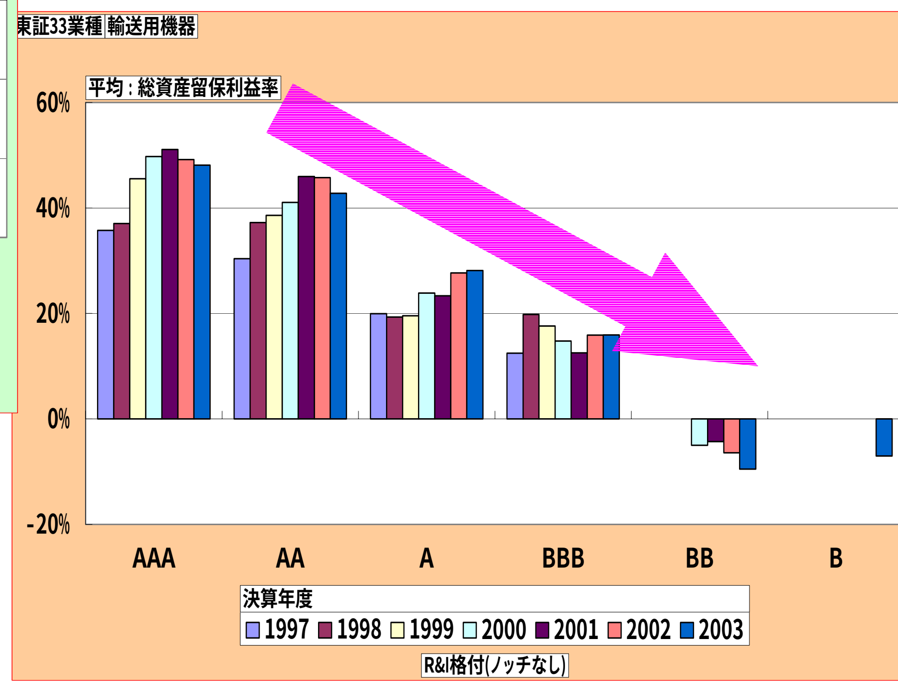
- RE/TC is likely to go down as Rating falls.
- Difference between AAA and AA classes are not distinct
  - We wonder if the power companies are included in these classes

Inspection on RE/TC

Electric Appliances

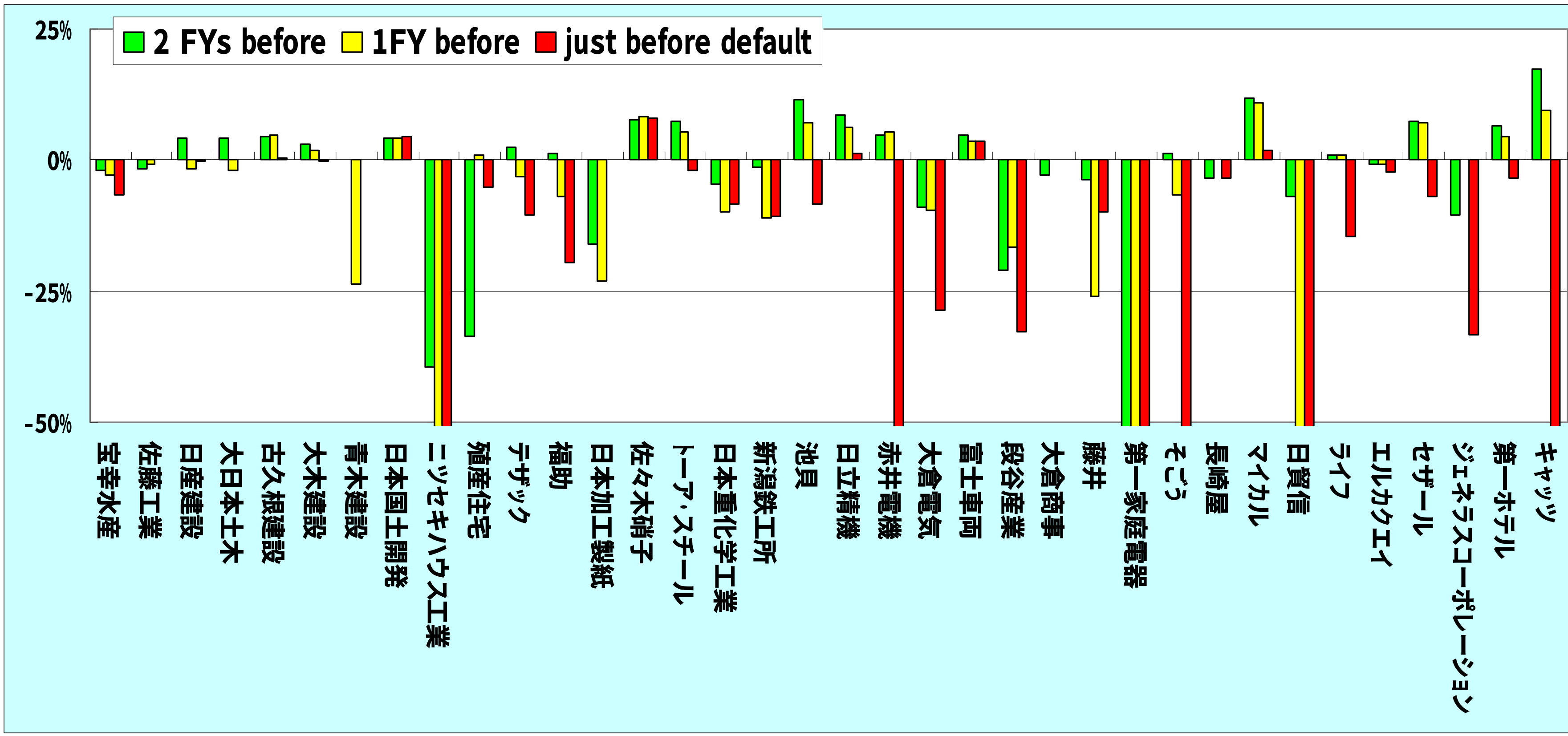


Transportation Equipments



Inspection on RE/TC

- Bankrupt companies

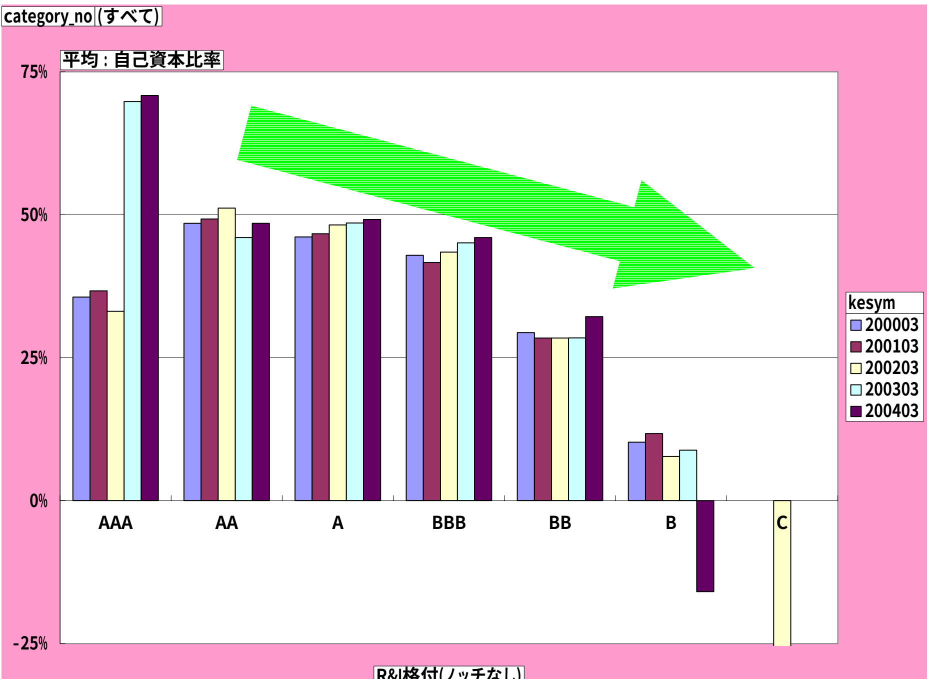


- RE/TC tends to decline a few years before bankruptcy and to be negative just at bankruptcy.

⇨ We should watch for default if RE is negative.

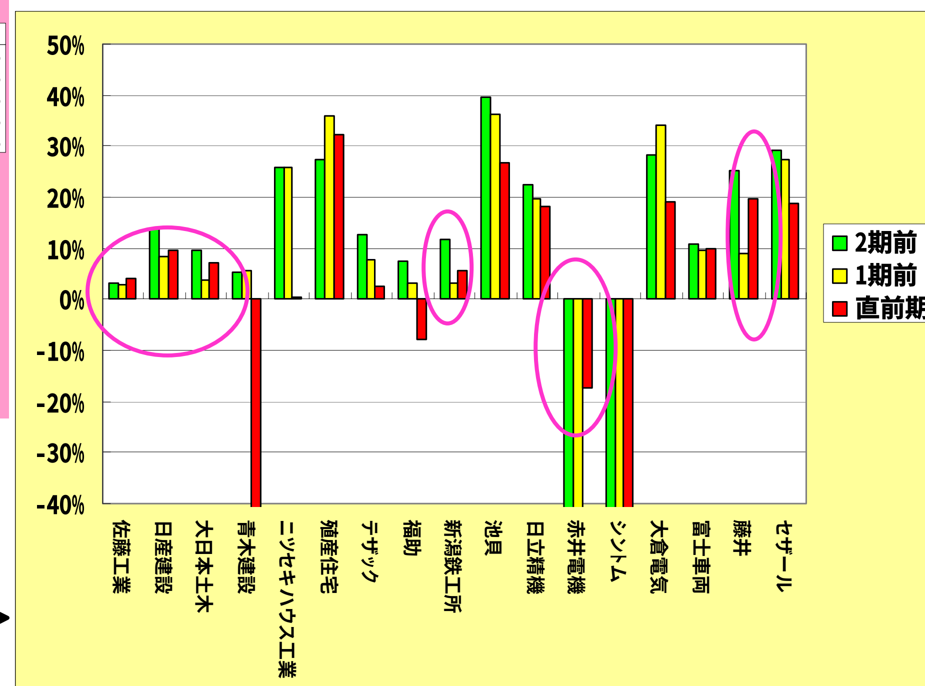
Inspection on Equity Ratio

- Rating-wise
  - ER has a tendency similar to RE/TC
- Bankrupt companies
  - ER increased even just before bankruptcy, probably owing to financial support from allied banks or companies.



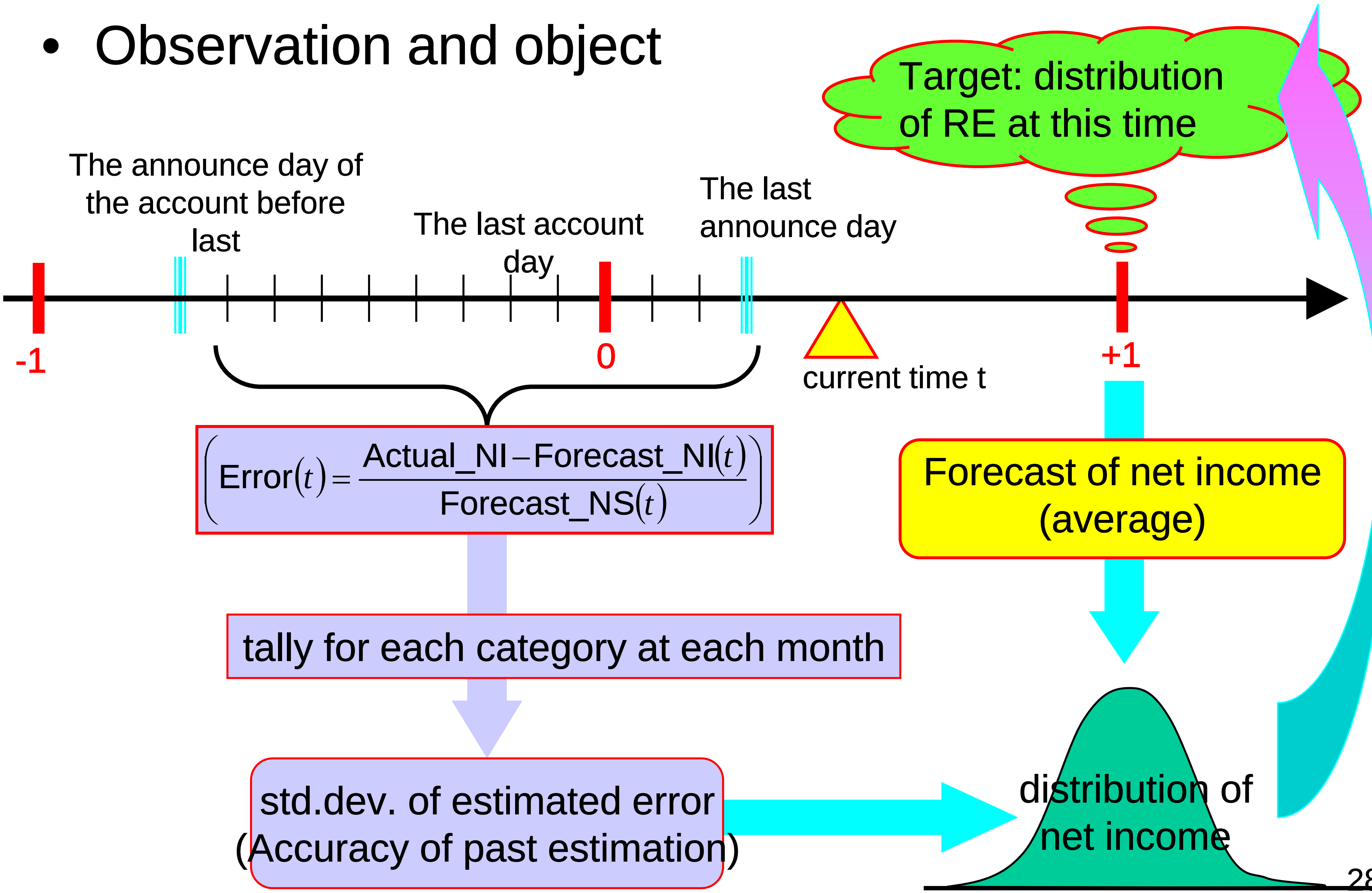
Rating -wise

bankrupt companies →



Illustration

- Observation and object



Procedure for warning probability

① Calculate RE at each month

- RE(t) = “Total equity - (Common stock + Capital surplus)” obtained from the newest B/S available at time t
  - If the fact that capital stock or capital surplus is reduced at the account day comes to light, and the amount of capital reduction is added to RE only at the month.
    - In order to compare with the initial RE, adjustment is restricted to the month.
  - Warning probability is expected to increase owing to the decline of RE by the amount of capital reduction.
    - The reduction of capital stock or capital surplus is a negative-suprising news to the investors.
- temporarily-adjusted RE(t)
  = RE(t)＋reduced common stock
  + reduced capital surplus

Procedure for warning probability

②Calculate the rate of prediction error of post-tax profit

■ Prediction error

$$= \frac{\text{Actual net income - Forecast of net income}}{\text{Forecast of net sales}}$$

- e.g.) Now = Nov.2004, the last account day = Mar. 2004,
  the last announce day = Jun. 2004,

$$\text{Error(t;Mar.2003)} = \frac{\text{Act\_NI(Mar.2003)} - \text{Fore\_NI(t;Mar.2003)}}{\text{Fore\_NS(t;Mar.2003)}},$$

$$(\text{t} = \text{Nov.2002,...,May.2003})$$

Fore_NI(t;T) means the value of net income fixed in T estimated in t.

Procedure for warning probability

③ Compute the standard deviation of the rate of estimation error of net income

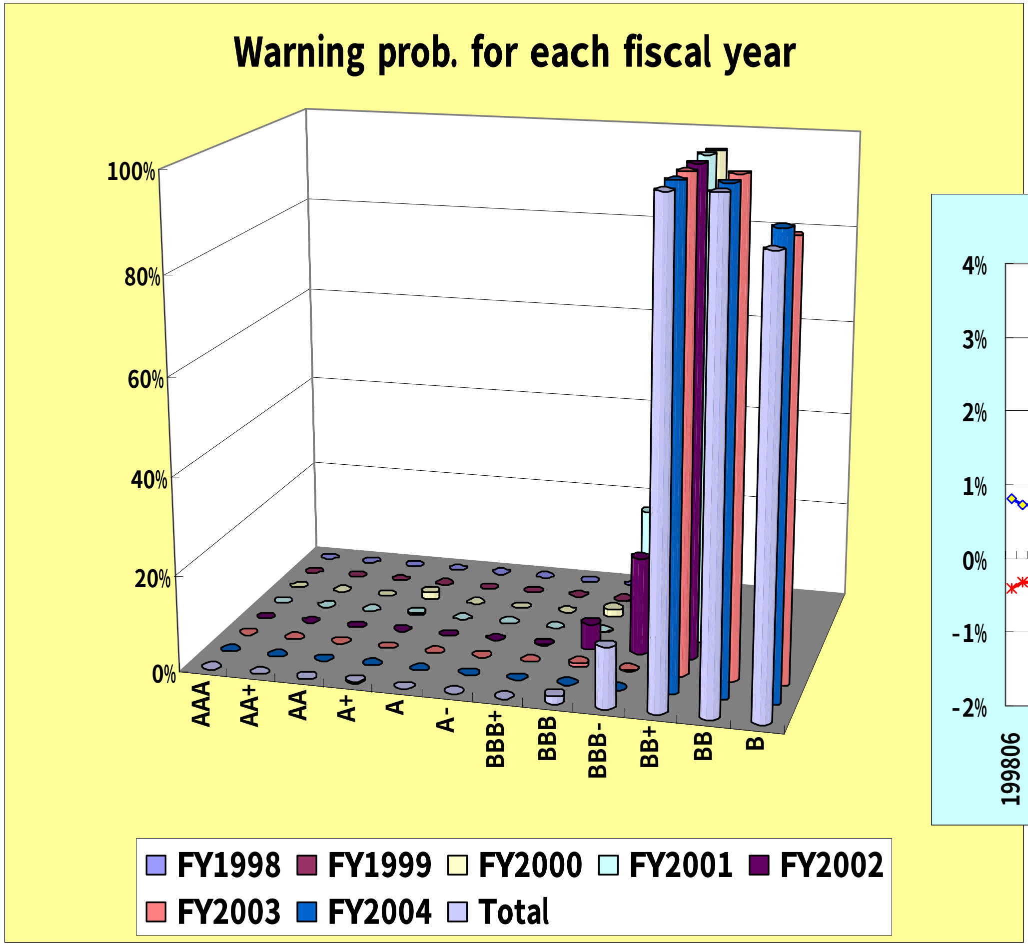
- Substitute the standard deviation of estimation error for each industry for each individual firm's standard deviation of estimation error

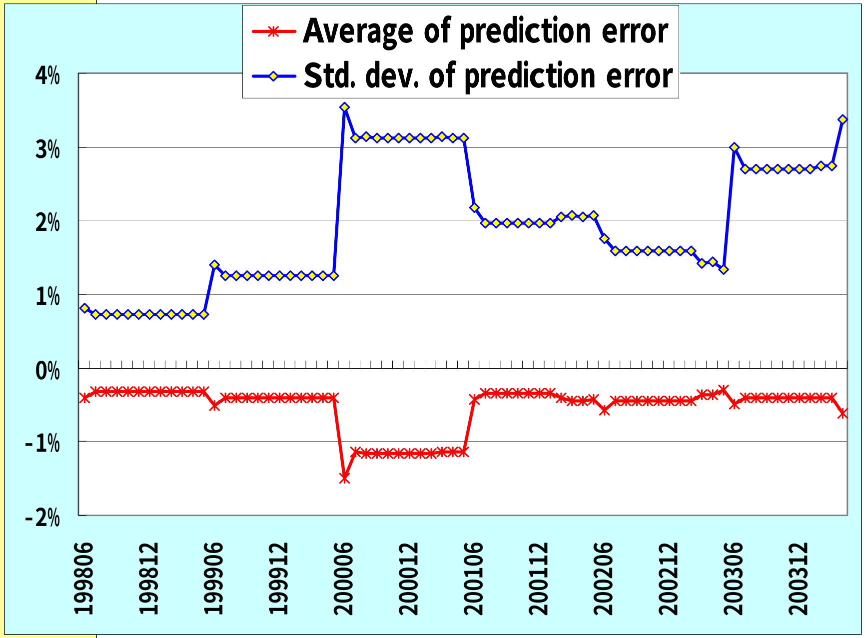
④ Compute the warning probability

- Specify the warning probability as the probability that the retained Earnings becomes negative at the next accounting day.
- Suppose that the net income at the next period follows a normal distribution.
  - Standard deviation is supposed to decrease as time approaches the announced day.
- RE at the next account day
  = the last RE + forecasted net income

Result：Transportation Equipments

- The warning probability increases sharply for the classes under BB+.

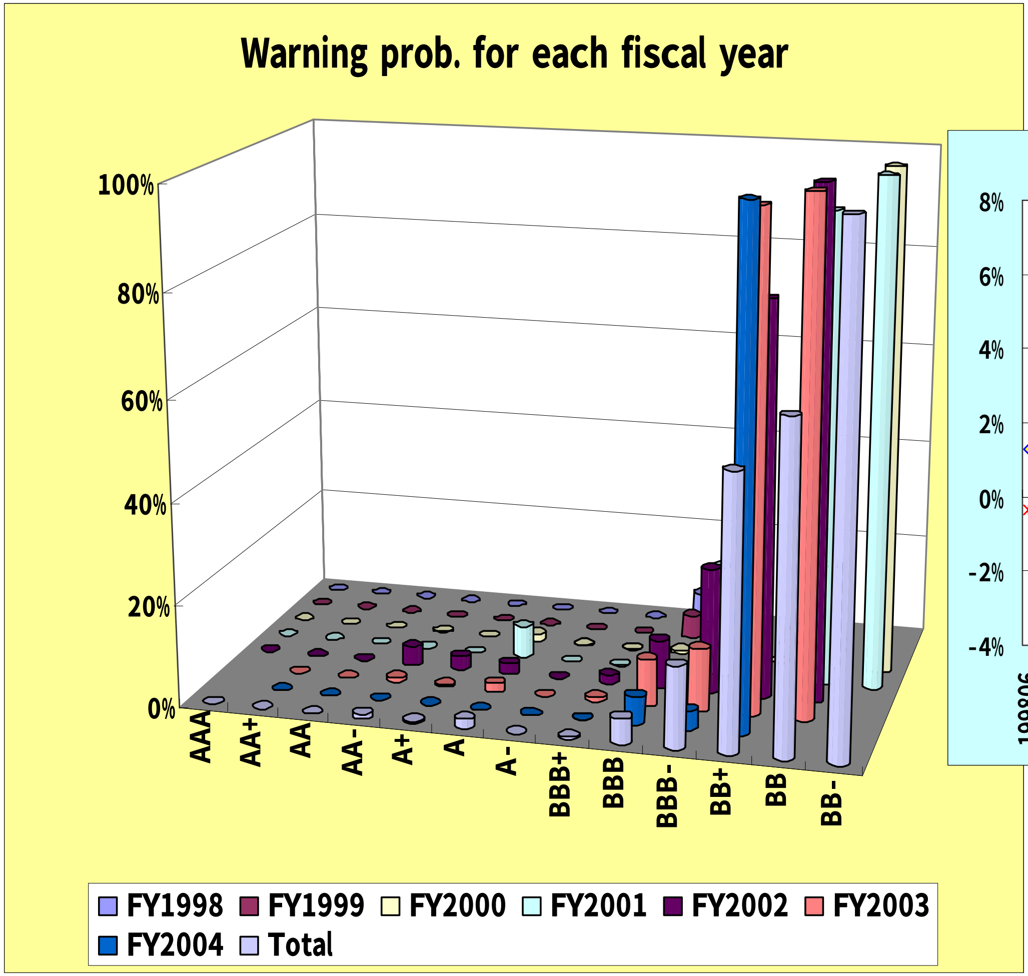


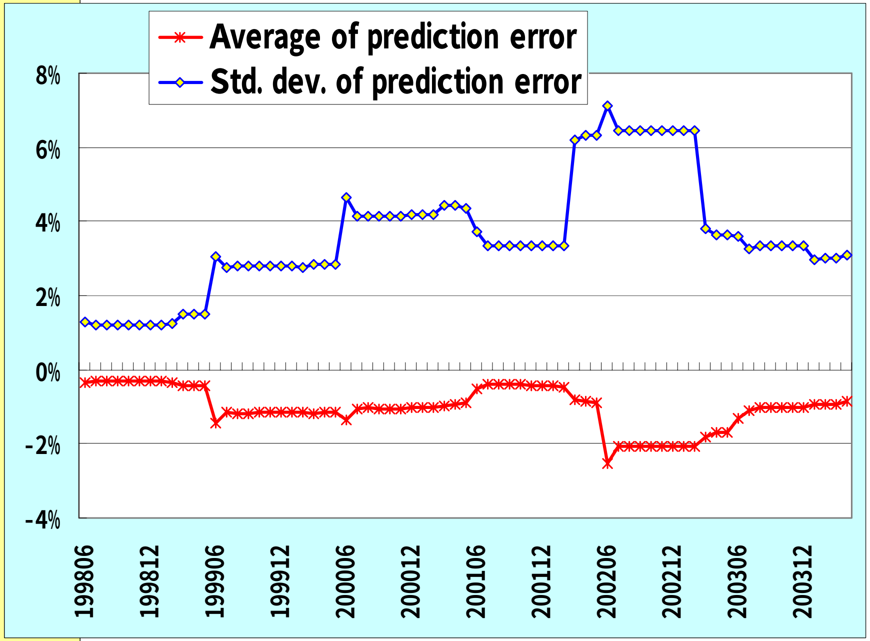


1st sec. of TSE, unconsolidated

Result：Electric Appliances

- The warning probability increases sharply for the classes under BBB.

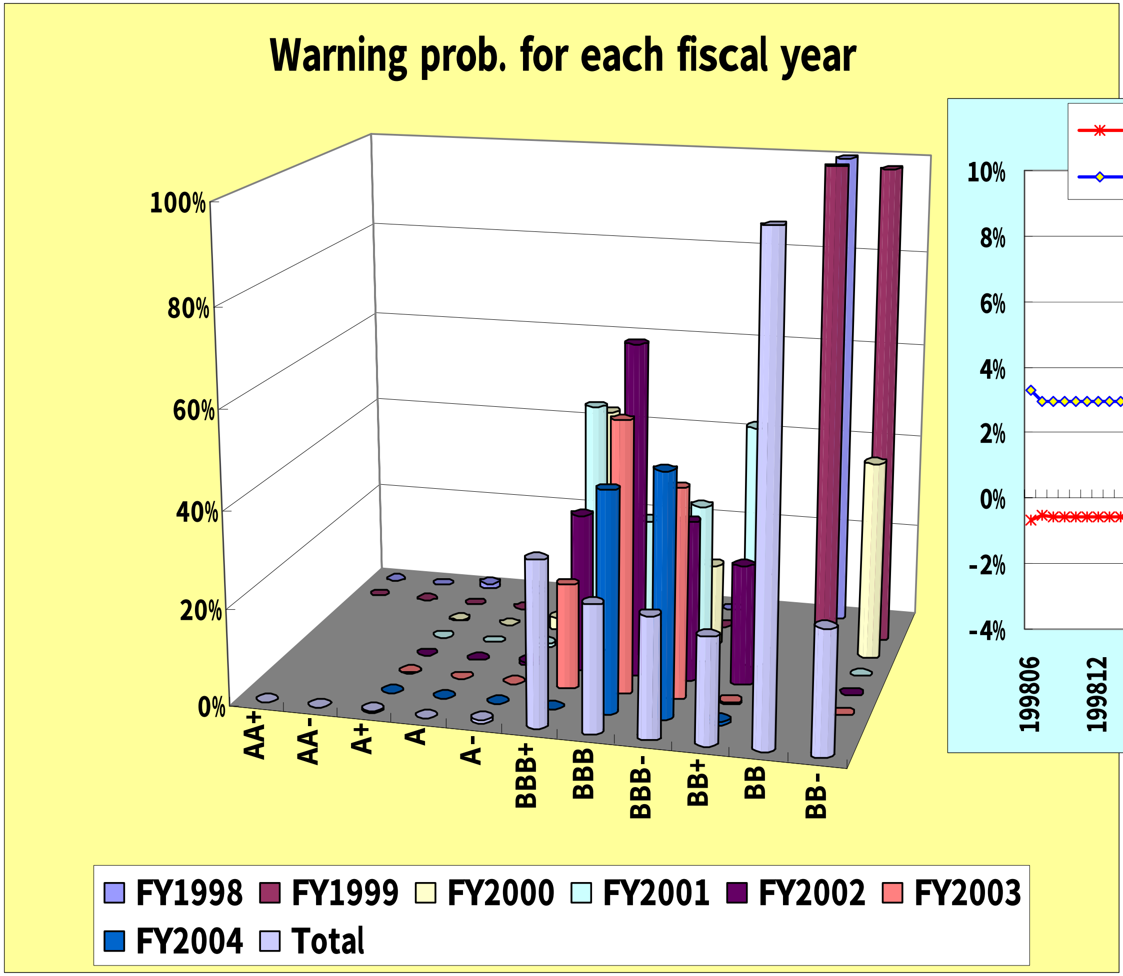


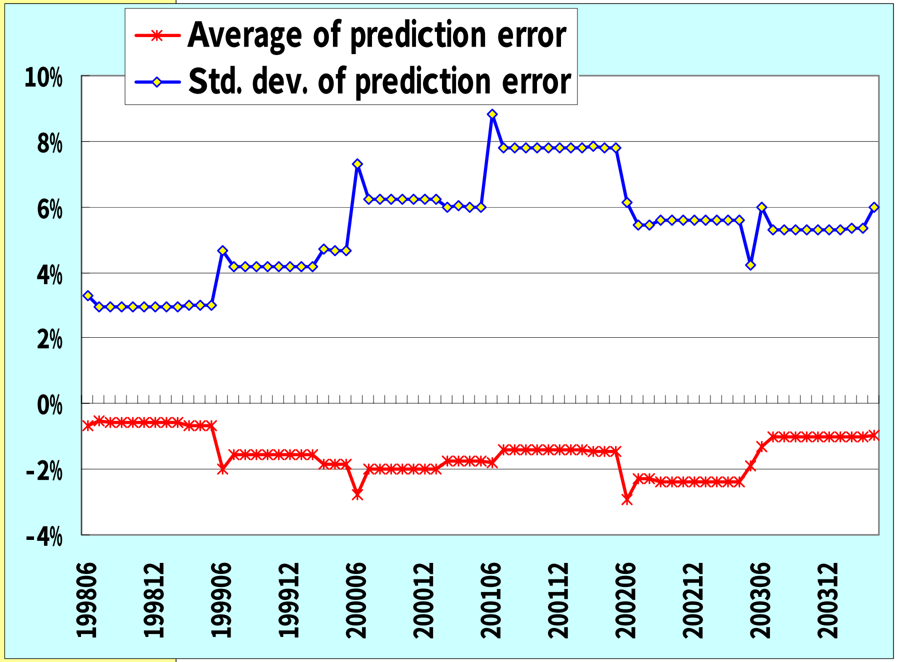


1st sec. of TSE, unconsolidated

Result：Iron & Steel

- The warning probability for the class BBB is relatively high.

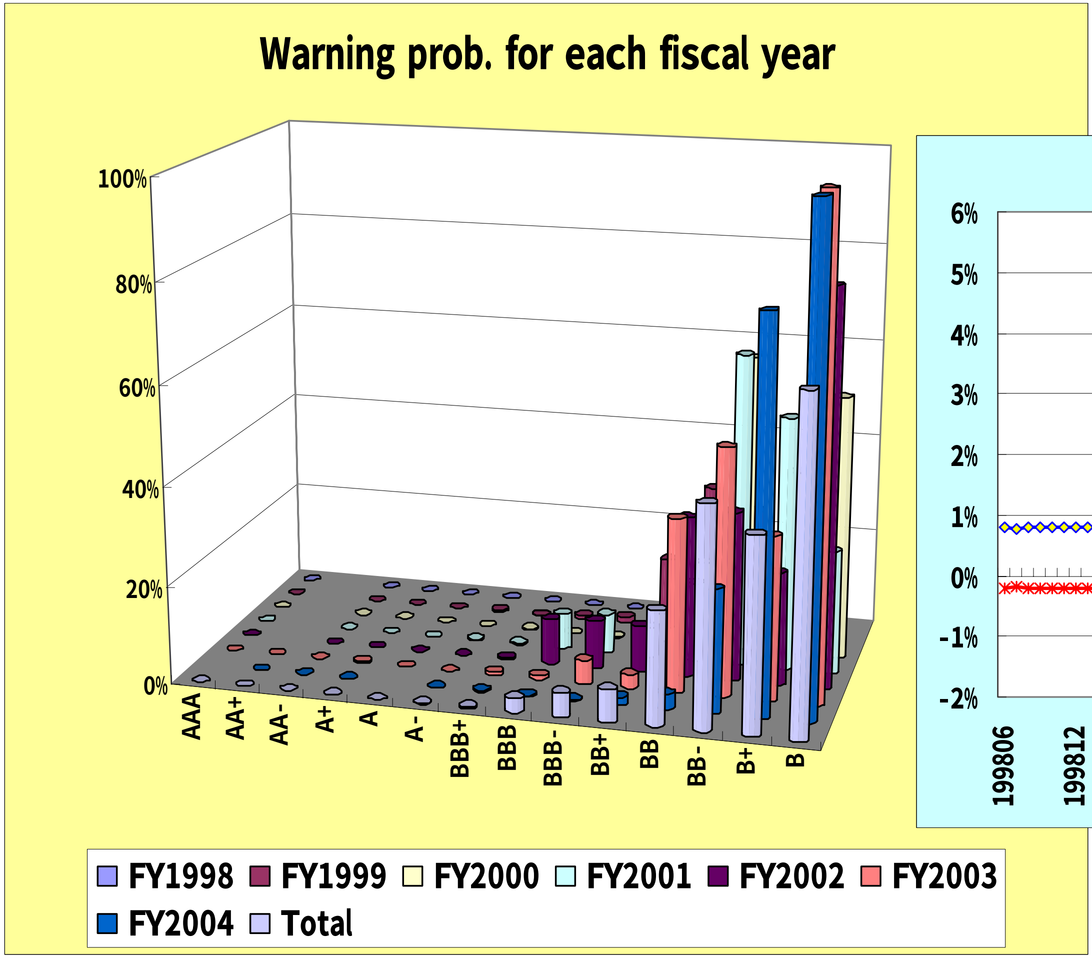


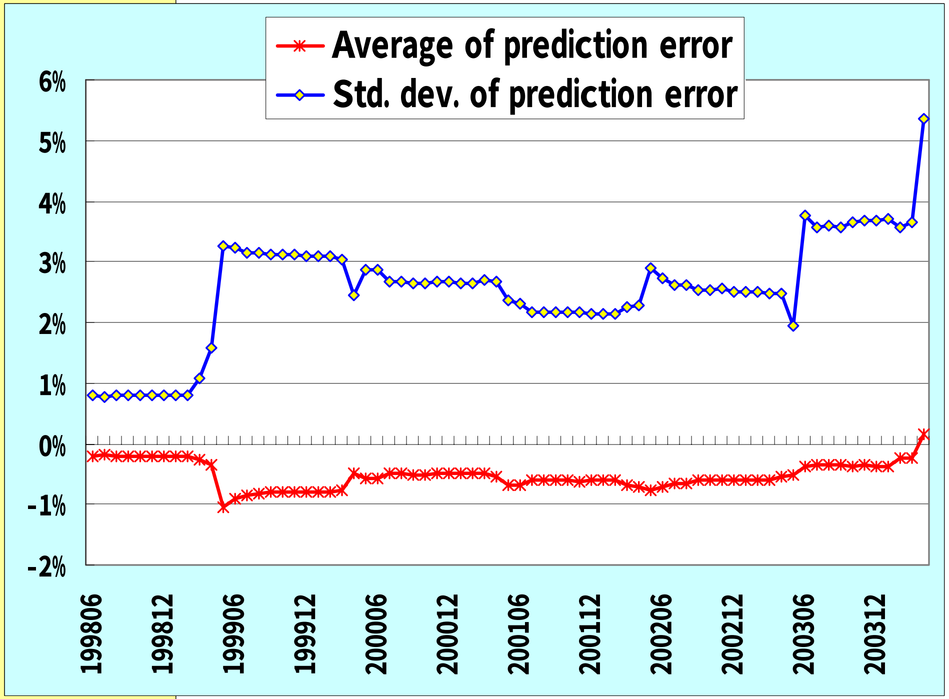


1st sec. of TSE, unconsolidated

Result：Retail Trade

- The warning probability increases sharply under the class BB.

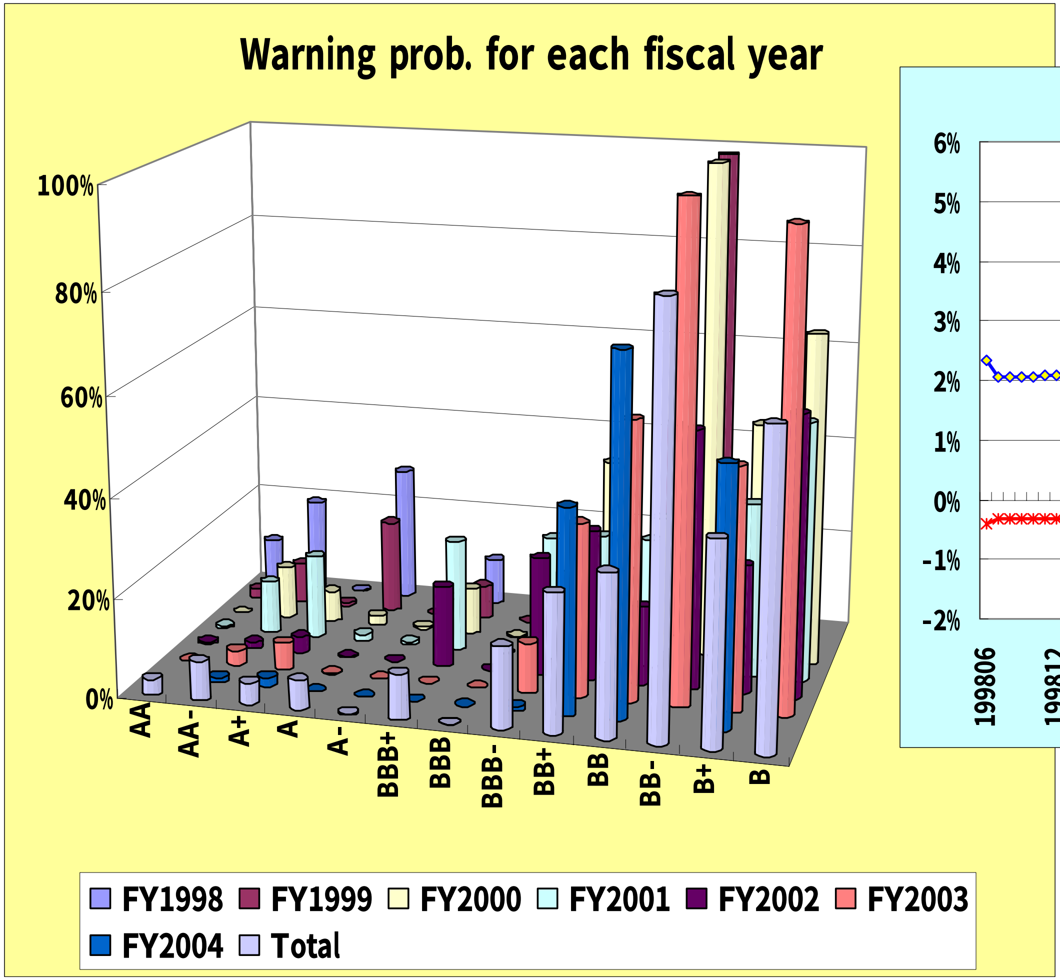


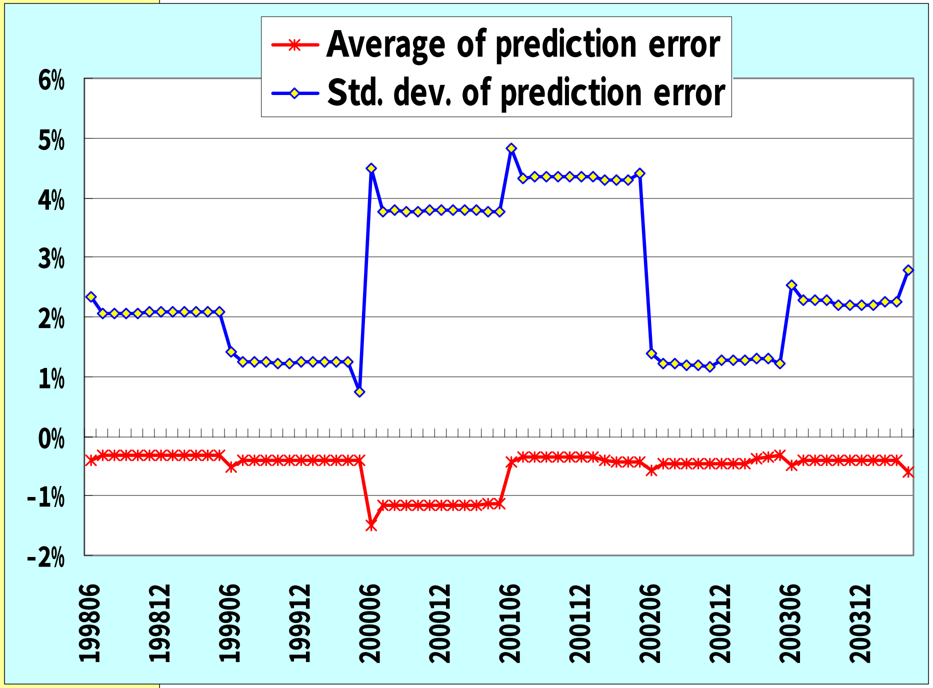


1st sec. of TSE, unconsolidated

Result：Wholesale Trade

- Some good-rated companies has relatively high warning probability.



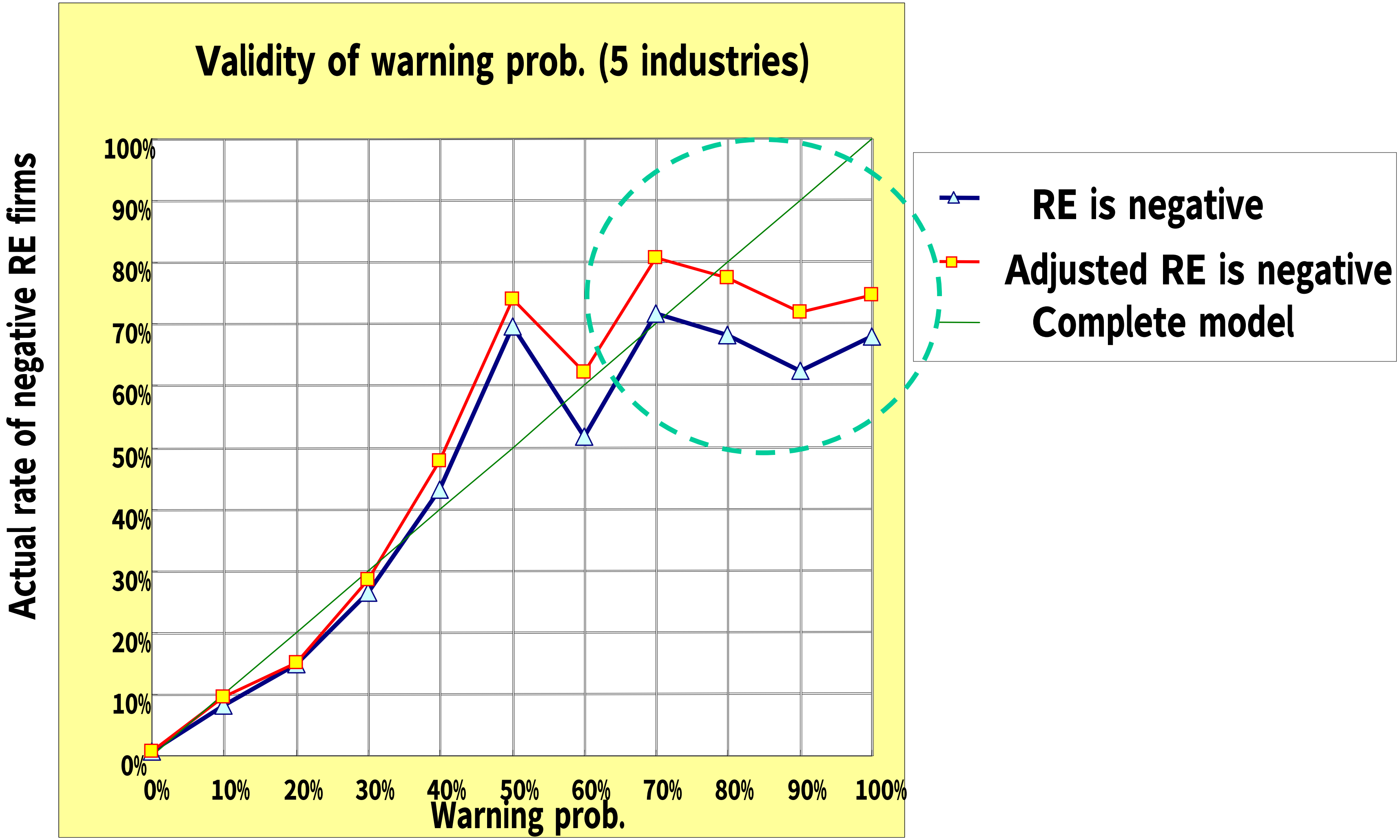


1st sec. of TSE, unconsolidated

Validity of warning probability

- Method of verification
  - Divide the warning probability by 10% pitch and tally up the proportion of the companies whose actual RE at the next accounting day is negative to all companies every class.
- Consequences
  - Not bad
  - For the class whose warning probability is above 80%, the rate of the companies whose next actual RE is negative flattens.

Validity of warning probability



Warning prob.	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
num. of samples	26967	566	304	231	299	138	116	124	182	1185	808

Future researches

- Modification of estimation bias
  - From these tentative analyses, it follows that estimation error tends to have a negative bias, which implies that analysts often give a bull prospect.

- How to combine the result with existing empirical models successfully