Financial Risk Control : Theory and Practice

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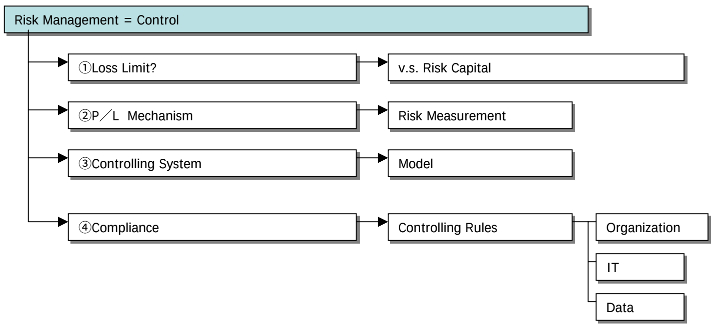
Index of Topics

1． Financial Risk Management : Quantitative Method

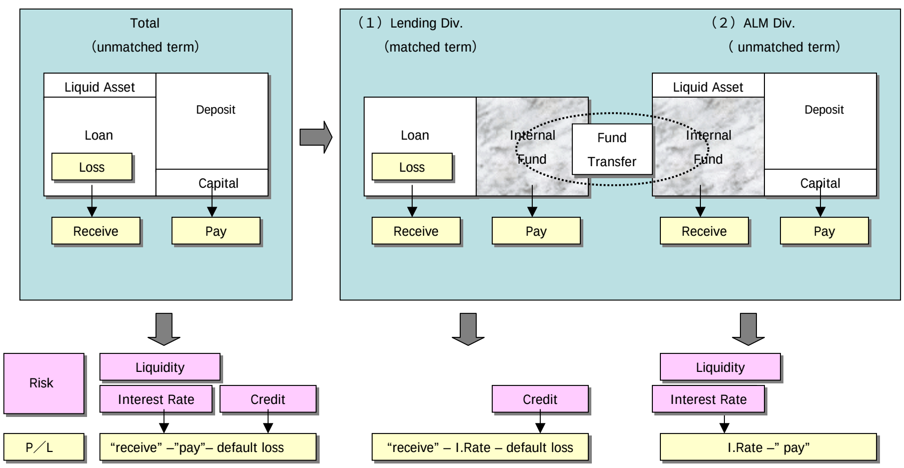
■Objective

A． Loss < Confidence Limit

B． Profit V.S. Risk

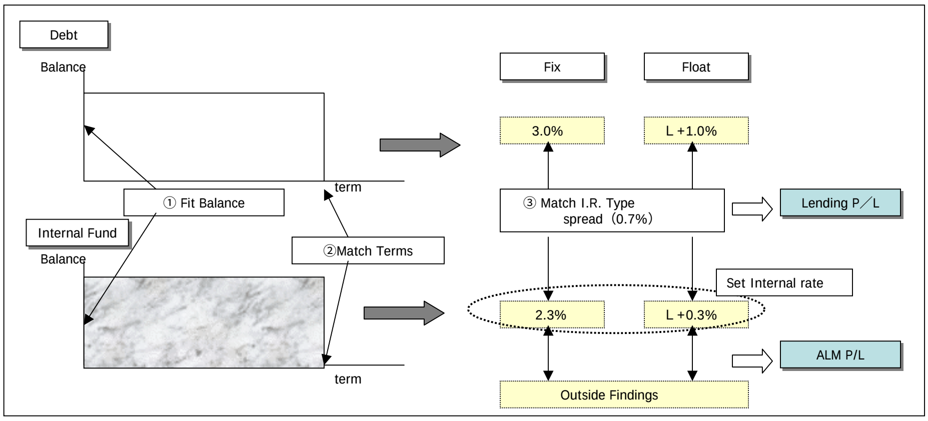


2． Fund Transfer System (Banking Operation Breakdown)

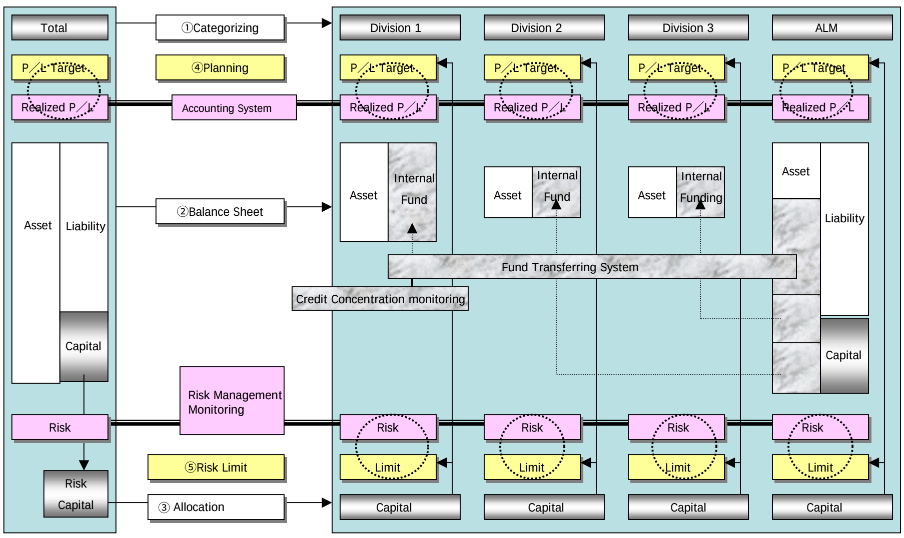


3． More on Fund Transfer System

Principal ➡ Interest



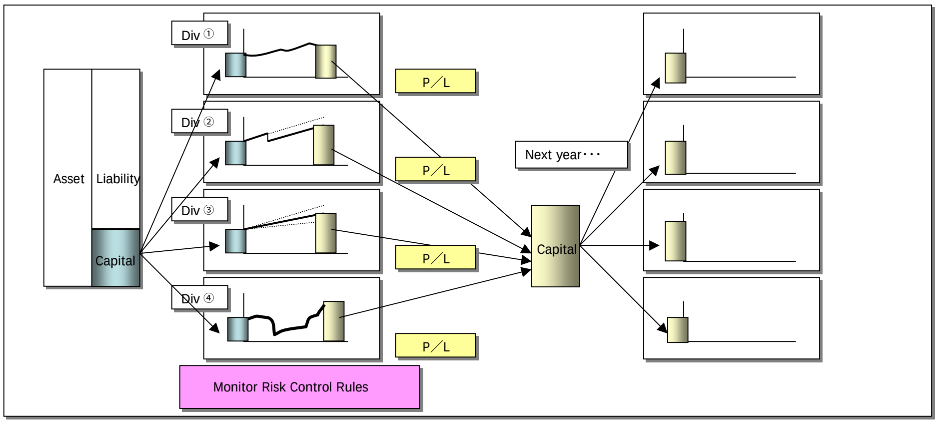
4． Management Framework



5． Management Framework ②

◯Bank is Collection of Risk Generating Bodies

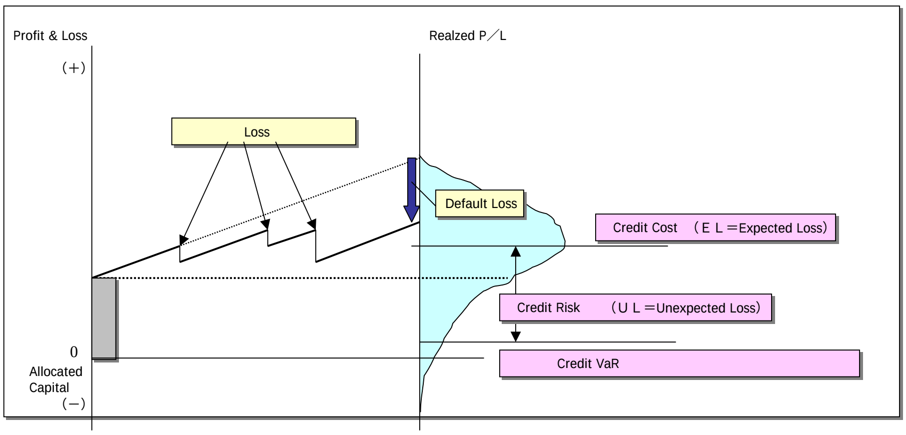
◯Allocating Capital ⇒ Controlling Risk ⇒ Piling up Profit



6． Lending Division P／L : Anatomy

◯Loan Loss is a Random Variable

◯PL=(Net Income Gain) – (OP. Cost) – (Loan Loss)



7. Loan Division P／L : Dynamics

$$\widetilde{W}_{Loan}(T) = W_{Loan}(0) + \int_0^T d\widetilde{W}_{Loan}(t) \qquad W_{Loan}(0) : \text{Initially Allocated Capital}$$

$$d\widetilde{W}_{Loan}(t) = \sum_{i=1}^{N} X_i \cdot \pi_i \cdot dt - C_{Loan} \cdot dt - \sum_{i=1}^{N} X_i (1-\theta_i) \cdot d\widetilde{N}_i(t) \qquad (1)$$

where $X = \sum_{i=1}^{N} X_i$: Loan Portfolio $\qquad X_i$: Indivisual Loan

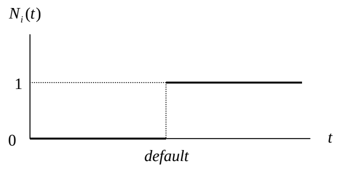
π_i : Profit Margin = Loan Rate – Internal Funding Rate

C_{Loan} : Operating Cost per time unit

θ_i : Default Recovery Rate

$\widetilde{N}_i(t)$: Default indicating Jump Process

$$\widetilde{N}_i(t) = \begin{cases} 1 & default \quad until\text{-}t \\ 0 & not-default \quad until\text{-}t \end{cases}$$



$$P[d\widetilde{N}_i(t) = \widetilde{N}_i(t+dt) - N_i(t) = 1 \mid N_i(t) = 0] = \lambda \cdot dt \qquad (2)$$

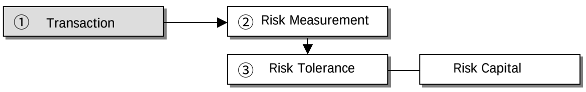
$t = 0$: Initial time, $P[\widetilde{N}_i(t) = 0 \mid N_i(0) = 0] = e^{-\lambda \cdot t} \approx 1 - \lambda \cdot t$ (3)

$$P[\widetilde{N}_i(t) = 1 \mid N_i(0) = 0] = 1 - e^{-\lambda \cdot t} \approx \lambda \cdot t$$

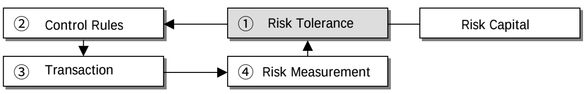
8. What is Risk Control

■Risk Control and Allocated Capital

●Risk Capital v.s. Risk allowance (A posteriori Monitoring)



●Risk Controlling Rules (A priori Control)



９．Loan Pricing Guideline : A Rule to make Profits

Expected P／L

$$E[\widetilde{W}_{Loan}(1)-W_{Loan}(0)]=\sum_{i=1}^{N}X_i\cdot\pi_i-C_{Loan}-\sum_{i=1}^{N}X_i(1-\theta_i)\cdot E[\widetilde{N}_i(1)]$$

$$=(\sum_{i=1}^{N}X_i(\pi_i-c_{Loan}(X_i))-\sum_{i=1}^{N}X_i(1-\theta_i)\cdot\lambda$$

（注）$C_{Loan}=\sum_{i=1}^{N}c_{Loan}(X_i)\cdot X_i$

$$=\sum_{i=1}^{N}X_i(\pi_i-c_{Loan}(X_i)-(1-\theta_i)\lambda) \quad (4)$$

Necessay Profit is :

For Given Γ_i :Allocated Capital, ρ :Capital Cost margin

$$X_i(\pi_i-c_{Loan}(X_i)-(1-\theta_i)\lambda)>\rho\cdot\Gamma_i \quad (5)$$

Thus necessay cost margin

$$\pi_i>c_{Loan}(X_i)+(1-\theta_i)\lambda+\rho\cdot\frac{\Gamma_i}{X_i} \quad (6)$$

a.k.a. Loan Pricing Guideline

Relationship to Credit Risk Exposure

$$\widetilde{L}=\sum_{i=1}^{N}X_i(1-\theta_i)\widetilde{N}_i(1) \qquad EL=E[\widetilde{L}]=\sum_{i=1}^{N}X_i(1-\theta_i)\lambda$$

１０．Loss Controlling Rules : Credit Risk Diversification

Minimum Debtor Count under Uniform Credibility

Uniform in 1.Principal , 2.Recovery rate , 3.iid Default

$$V[\widetilde{L}]=V[\widetilde{W}_{Loan}(1)-W_{Loan}(0)]=\sum_{i=1}^{N}\frac{X^2}{N^2}(1-\theta)^2V[\widetilde{N}_i(1)]$$

$$=\sum_{i=1}^{N}\frac{X^2}{N^2}(1-\theta)^2\lambda\cdot(1-\lambda)=\frac{X^2}{N}(1-\theta)^2\lambda\cdot(1-\lambda) \quad (7)$$

Suppose

$$UL^2=\phi^2\cdot V[\widetilde{L}]=\phi^2\cdot\frac{X^2}{N}(1-\theta)^2\lambda\cdot(1-\lambda)<W_{Loan}(0)^2 \quad (8)$$

Thus

$$N_{\min}>\phi^2\cdot\frac{X^2}{W_{Loan}(0)^2}(1-\theta)^2\lambda\cdot(1-\lambda) \quad (9)$$

１１．Loss Controlling Rules : Credit Limit

For Large Debtor, Exposure limit M_{max}

With Highest Risk Scenario

$$X = M_{max} + M_{max} + \cdots + M_{max} + 0 + \cdots + 0 \quad (10)$$

For $\exists$ Total Credit Limit X, # of Large Debtor

$$N = \frac{X}{M_{max}} \quad (11)$$

From Previous Min # Result (9)

$$\frac{X}{M_{max}} > \phi^2 \cdot \frac{X^2}{W_{Loan}(0)^2}(1-\theta)^2 \lambda \cdot (1-\lambda) \quad (12)$$

Therefore

$$M_{max} < \frac{W_{Loan}(0)^2}{X \cdot \phi^2 \cdot (1-\theta)^2 \lambda \cdot (1-\lambda)} \quad (13)$$

１２．Issues on Loan Div. Risk Control

■Agenda

(1) Loan Pricing Guideline

① Distribution of Overhead to Individual Loan

② Allocation of Risk Capital to Individual Loan (∝ Risk Contribution?)

③ Competitive Pricing against Loan Market

(2) Credit Limit

④ Uniform Credibility → Dissimilar Credibility

⑤ Uniform Recovery → Different Recovery Rate (Substantial exposure limit)

⑥ Independent Default event → Correlation Effect

○Conglomerate Effect

○ Geographic, Industry sector

(3) Other

⑦ Credit Rating Migration

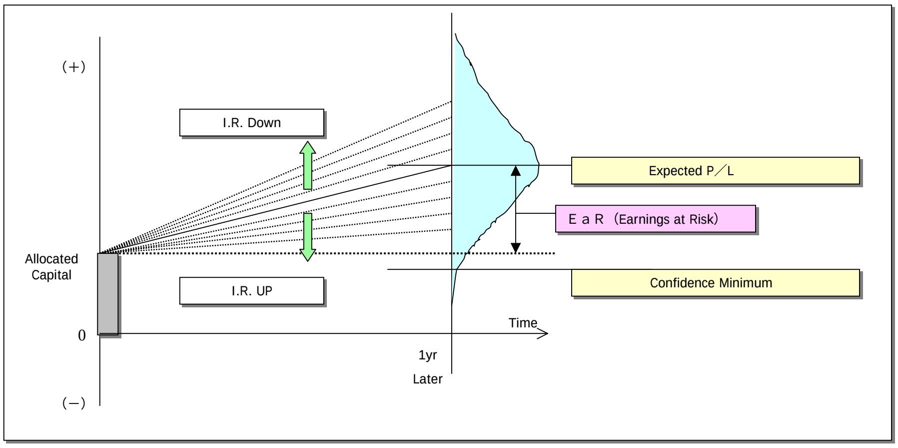
⑧ Regulated Capital issue

⑨ Restriction on Total Lending amount

１３．ALM Division P／L : Anatomy

■**ALM Division Consolidates**

- ◯Liquidity Risk
- ◯Interest Rate Fluctuation Risk



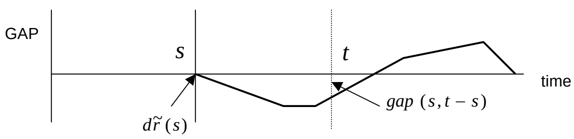
１４．ALM Division P／L : Dynamics

$$\widetilde{W}_{ALM}(T) = W_{ALM}(0) + \int_0^T d\widetilde{W}_{ALM}(t) \qquad W_{ALM}(0) : \text{Initial Capital}$$

$$d\widetilde{W}_{ALM}(t) = A(t)p_0(t)dt - L(t)q_0(t)dt - C_{ALM}dt - \left(\int_0^t gap(s,t-s)d\tilde{r}(s)\right)dt \qquad (14)$$

Where

- $A(t)$: Asset in given single Period $[t,t+dt]$
- $p_0(t)$: Average Interest Rate for the Asset
- $L(t)$: Liability in given single Period $[t,t+dt]$
- $q_0(t)$: $L(t)$ Average Interest Rate for the Liability
- C_{ALM} : Operating Cost Per time unit
- $gap(s,\tau)$: GAP at s , expected A／L interest rate sensitivity at τ years later
- $d\tilde{r}(s)$: Interest Rate Fluctuation during $[s,s+ds]$



１５．Profit making Condition

Annual P／L

$$\widetilde{W}_{ALM}(1) - W_{ALM}(0) = \int_0^1 (A(t)p_0(t) - L(t)q_0(t) - C_{ALM})dt + \int_0^1 \left(\int_0^t gap(t-s)d\tilde{r}(s)\right)dt \quad (15)$$

Assume : $gap(s, t-s) = gap(t-s)$ (16)

Interest Rate Process

$$d\tilde{r}(s) = \mu \cdot ds + \sigma \cdot d\widetilde{Z}(s) \qquad d\tilde{Z}(s) \sim N(0, dt) \quad (17)$$

Profit making condition :

$$\begin{aligned} E[\widetilde{W}_{ALM}(1) - W_{ALM}(0)] &= \int_0^1 (A(t)p_0(t) - L(t)q_0(t) - C_{ALM})dt + \int_0^1 \left(\int_0^t gap(t-s)E[d\tilde{r}(s)]\right)dt \\ &= \int_0^1 (A(t)p_0(t) - L(t)q_0(t) - C_{ALM})dt + \int_0^1 \left(\int_0^t gap(t-s)\mu \cdot ds\right)dt \\ &> \rho \cdot W_{ALM}(0) \end{aligned}$$

(18)

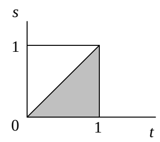
１６．Loss Controlling Rules : GAP Limit

Annual Variance :

$$V[\widetilde{W}_{ALM}(1) - W_{ALM}(0)] = E\left[\left(\int_0^1 \left(\int_0^t gap(t-s)\cdot\sigma\cdot d\widetilde{Z}(s)\right)dt\right)^2\right] \quad (19)$$

Gap limit is defined as $|gap(t-s)| \le M$

$$\int_0^1 dt \int_0^t gap(t-s)\cdot\sigma\cdot d\tilde{Z}(s) = \int_0^1 d\widetilde{Z}(s)\int_s^1 gap(t-s)\cdot\sigma\cdot dt \quad (20)$$



Hence

$$\begin{aligned} EaR^2 = \phi^2\cdot V[\widetilde{W}_{ALM}(1) - W_{ALM}(0)] &= \phi^2\cdot E\left[\left(\int_0^1\left(\int_0^t gap(t-s)\cdot\sigma\cdot d\widetilde{Z}(s)\right)dt\right)^2\right] \\ &= \phi^2\cdot E\left[\left(\int_0^1 d\widetilde{Z}(s)\int_s^1 gap(t-s)\cdot\sigma\cdot dt\right)^2\right] \\ &= \phi^2\cdot\int_0^1 ds\cdot\left(\int_s^1 gap(t-s)\cdot\sigma\cdot dt\right)^2 \\ &\le \phi^2\cdot\int_0^1 ds\cdot\left(\int_s^1 |gap(t-s)\cdot\sigma|\cdot dt\right)^2 \\ &\le \phi^2\cdot\int_0^1 ds\cdot M^2\sigma^2(1-s)^2 = -\phi^2\cdot\frac{1}{3}M^2\sigma^2(1-s)^3\Big|_0^1 = \frac{\phi^2 M^2\sigma^2}{3} \le W_{ALM}(0)^2 \quad (21) \end{aligned}$$

$$M \le \frac{\sqrt{3}\cdot W_{ALM}(0)}{\phi\cdot\sigma} \quad (22)$$

１７. Issues on ALM Div. Risk Control

Agenda

(1) Violation of Contractual Principal Payment Assumption

→ Prepayment Modeling, Effect of Non-Maturity Deposit, e.t.c.

(2) Idiosyncratic Fluctuation of Interest Rates

→ ALM deals with variety of interest rates

→ A single GAP Auction is NOT enough

(3)Limit of Brownian Assumption

→ Cyclic／Auto-correlated behavior of Long term interest rate

→ Avoidance of negative interest rate and／or hyper-inflation

(4) Practicality of Position control

→ ALM Hedge beyond Marlcet Capacity

→ Non-tradable Risk factor , e.t.c.

１８. Trading Division P／L : Dynamics

$$\widetilde{W}_{Trade}(T) = W_{Trade}(0) + \int_0^T d\widetilde{W}_{Trade}(t) \qquad W_{Trade}(0) \text{ ; Initial Capital}$$

$$d\widetilde{W}_{Trade}(t) = \sum_{k=1}^{K} \Delta_k(t) \cdot d\tilde{x}_k(t) - C_{Trade} \cdot dt \tag{23}$$

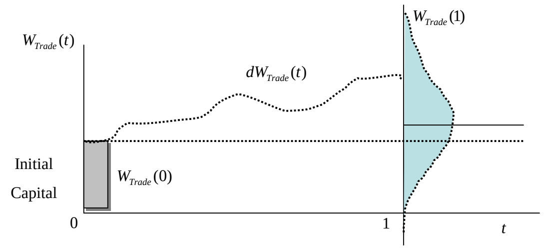
Where

$\Delta_k(t)$: Delta value of Risk Factor k during $[t, t+dt]$

$d\tilde{x}_k(t)$: Change in Risk Factor k

$$d\tilde{x}_k(t) \sim N(\mu(t)dt, \sigma^2 dt)$$

C_{Trade} : Operating Cost Per time unit



１９．Profit Making Condition

Annual P／L

$$\widetilde{W}_{Trade}(1) - W_{Trade}(0) = \sum_{k=1}^{K}\int_0^1 \Delta_k(t)\,d\tilde{x}_k(t) - C_{Trade} \qquad (24)$$

Necessity

$$E[\widetilde{W}_{Trade}(1) - W_{Trade}(0)] = \sum_{k=1}^{K}\int_0^1 \Delta_k(t)E[d\tilde{x}_k(t)] - C_{Trade}$$

$$= \sum_{k=1}^{K}\int_0^1 \Delta_k(t)\mu(t)\cdot dt - C_{Trade} \quad > \rho\cdot W_{Trade}(0) \qquad (25)$$

Requires non-zero trend, otherwise $E[d\tilde{x}_k(t)] = 0$

$$E[\widetilde{W}_{Trade}(1) - W_{Trade}(0)] = -C_{Trade} < 0 \qquad (26)$$

Good Trading strategy is essential!

２０．Loss Controlling Rules : Position Limit

Assume $d\tilde{Z}(t) = \left(d\tilde{Z}_1(t),\cdots,d\tilde{Z}_K(t)\right)$ distributed multi-dim. Normal with constant parameters

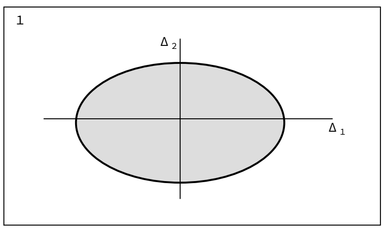
Then $$d\widetilde{W}_{Trade}(t) \sim N\left(\sum_{k=1}^{K}\Delta_k(t)\mu_k(t)dt, \vec{\Delta}'(t)\cdot\Sigma\cdot dt\cdot\vec{\Delta}(t)\right) \qquad (27)$$

$\vec{\Delta}(t)$: Delta vector

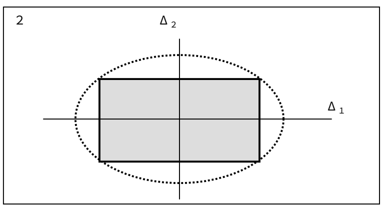
VaR limit →

$$\Delta'(t)\cdot\Sigma\cdot\Delta(t)\cdot dt \le \frac{W_{Trade}(0)^2}{\phi^2}\cdot dt \qquad (28)$$

VaR limit



Delta limit



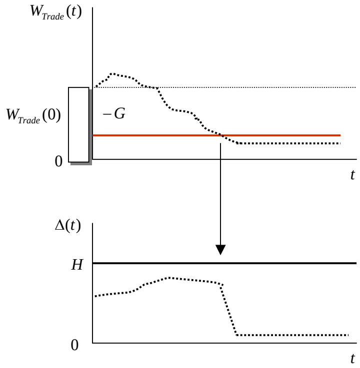
２１．Loss Controlling Rules : Loss Cut Rules

Delta limit is NOT Sufficient

←Loss Fluctuation Too Large

Loss Cut :

- Close the Position
- Reset Delta limit
- Re-Allocate Capital



Analysis Requires Probability of Loss-Cut (Absorbing Barrier)

$$d\widetilde{W}_{Trade}(t) = H\sigma \cdot d\tilde{Z}(t) \qquad d\tilde{Z}(t) \sim N(0, dt) \quad (29)$$

$$\Pr[\tau(G) > t] = \Phi\left(\frac{G}{\sigma \cdot H\sqrt{t}}\right) - \Phi\left(\frac{-G}{\sigma \cdot H\sqrt{t}}\right) \quad (30)$$

$$1-\Pr[\tau(G)>1] = 1-\Phi\left(\frac{G}{\sigma \cdot H\sqrt{t}}\right) + \Phi\left(\frac{-G}{\sigma \cdot H\sqrt{t}}\right) = 2\Phi\left(\frac{-G}{\sigma \cdot H\sqrt{t}}\right) \quad (31)$$

H : Possition Delta limit $\tau(G)$: Hitting time of the wall $(-G)$

２２．Issues on Trading Div. Risk Control

Agenda

(1) Limitation of Delta (Linear effect) control

→ Gamma, Vega (Quadratic or Higher) Effects

(2) Non-Normal Case

(3) Position Limit

(①Var limit) No Paticular limit on each delta

② limit on individual delta $|\Delta_k(t)| \le M_k$

→ Largest area(volume)capture

③ limit on sum of deltas M_k

$|\Delta_1(t) + \cdots + \Delta_K(t)| \le M$

← e.s. Principal Component Analysis on I.R.

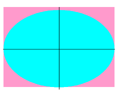
(4) Effective Loss Cut Rules

→ Cut Level

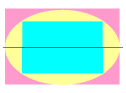
→ Closing Lag + Assc. Risk

→ Reopen Policy, e.t.c.

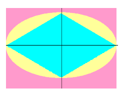
① Var limit



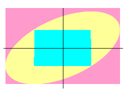
② Uncorrelated



③ Delta-sum limit



② Correlated



２３．Summary : Profit v.s. Risk Control

【Issues】
- ○ Risk diversification Effect
- ○ Capital Allocation
- ○ Performance Measure

- ○ Bank is Collection of Risk generating Bodies
- ○ Building Profit on Allocated Capital via Controlling Risk
- ○ Appraisal at END of Accounting Term（ＲＡＰＭ＝Risk Adjusted Performance measurement）
- ○ Avoid Bankruptcy within A Confidence Level