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History of Western Philosophy from the quantum theoretical point of view; [Ver. 3]

by

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History of Western Philosophy from the quantum theoretical point of view; [Ver. 3]

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Abstract: Recently I proposed "quantum language" (or, "measurement theory", "the linguistic Copenhagen interpretation of quantum mechanics", "the quantum mechanical worldview"), which is not only characterized as the metaphysical and the linguistic turn of quantum mechanics but also as the linguistic turn of Descartes=Kant epistemology. And further, I believe that quantum language is the only scientifically successful theory in dualistic idealism. Hence, this turn encourages us to understand Western philosophies (Parmenides, Plato, Descartes, Locke, Berkeley, Hume, Kant, Wittgenstein, etc.) within the framework of quantum language. This is done in this paper. That is, I show that most unsolved philosophical problems in dualism are solved within the framework of quantum language.

Preface: Did philosophy make progress?

Quantum language (i.e., QL (=MT = measurement theory), the linguistic Copenhagen interpretation of quantum mechanics, the quantum mechanical worldview) proposed by myself is a language that is inspired by the Copenhagen interpretation of quantum mechanics, and moreover, it has a great power to describe classical systems as well as quantum systems. My lecture for graduate students in the faculty of science and technology in Keio university has been continued, with a gradual improvement, for about 20 years. The contents of my lecture are roughly as follows.

Assertion 0.1 [The location of quantum language in the history of world-description (*cf.* ref.[43, 66])]

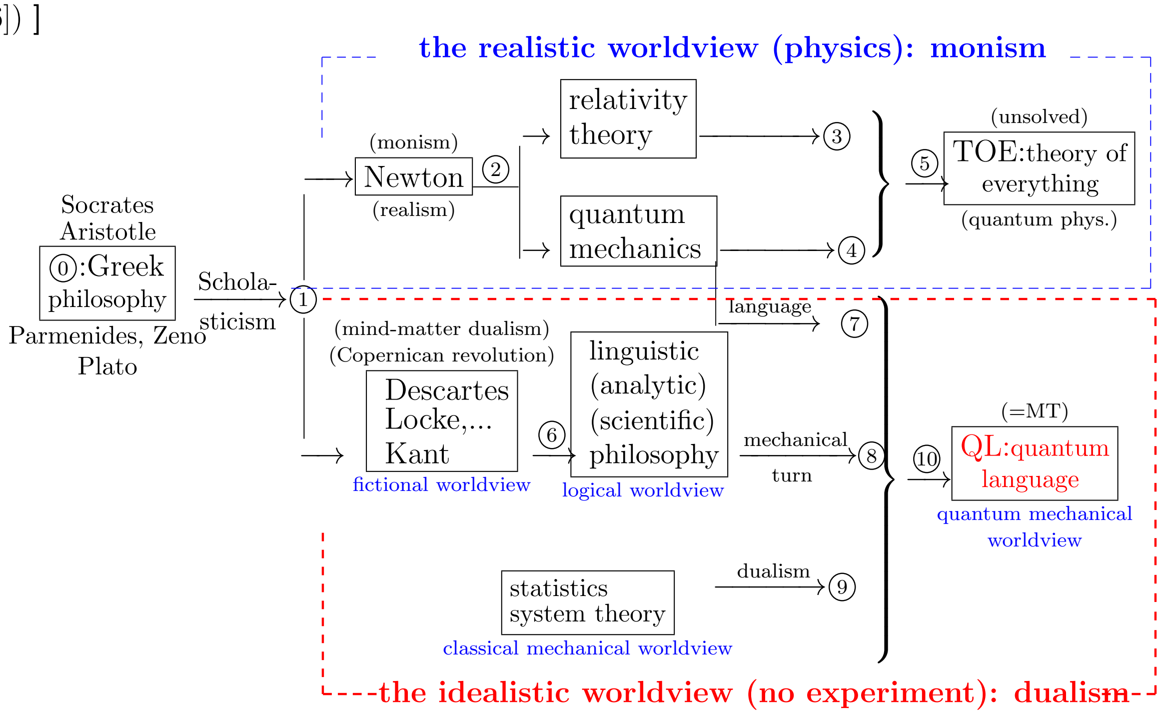

Figure 0.1: The history of the world-descriptions

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The part of quantum theory [2 - 7 - 10] and statistics[9 - 10] in the above were already studied in the following:

(A) ref. [66]: S. Ishikawa, "Linguistic Copenhagen interpretation of quantum mechanics: Quantum Language [Ver. 5]", Dept. Math. Keio University, 2019, KSTS/RR-19/003, 473 pages
(`http://www.math.keio.ac.jp/academic/research_pdf/report/2019/19003.pdf`)

Therefore, in this text I devote myself to the part [0 - 1 - 6 - 8 - 10], which is almost equal to the history of western philosophy. This part [0 - 1 - 6 - 8 - 10] was not detailed in my lecture, but it will be a good preparation to read (A) (since (A) may be too hard). That is, this paper is written such as it can be read without the mathematical preparation.

I think that the following is one of the most fundamental problems of western philosophy:

The progress problem (B)

(B) Do the time series [0 - 1 - 6 - 8] mean the progress of philosophy?

In this paper I assert that

The answer (C) of the progress problem (B)

(C_1) If "to make progress" is defined by "to come near quantum language" (i.e., "becoming more and more like quantum language" we can say that the time series [0→1→6→8→10] can be regarded as progress, or more precisely,

$$
\boxed{\text{Plato}} \xrightarrow[\text{progress}]{\text{Cogito turn (dualism)}} \overset{\text{mind-matter dualism}}{\boxed{\text{Descartes=Locke}}} \xrightarrow[\text{progress}]{\text{Copernican revolution}} \overset{\text{transcendental idealism}}{\boxed{\text{Kant}}}
$$

$$
\xrightarrow[\text{progress}]{\text{linguistic turn}} \overset{\text{linguistic philosophy}}{\boxed{\begin{matrix}\text{Saussure}\\ \text{Wittgenstein}\end{matrix}}} \xrightarrow[\text{progress (quantification)}]{\text{mechanical turn (scientific turn)}} \underset{\text{quantitative argument is possible}}{\overset{\text{the quantum mechanical worldview}}{\boxed{\text{Quantum language}}}} \tag{0.1}
$$

The name of "scientific turn" is due to the fact that quantitative argument is possible in quantum language and that it is only quantum language which scientists need in above-mentioned. Also, it should be noted that quantum language is more useful than statistics. Thus, we conclude that

(C_2) a scientific perfection of dualism and idealism is realized by quantum language

which is surely one of the most important assertions in western philosophy. Here, in this paper, it suffices to consider that "idealism"≈"linguistic theory"≈"metaphysics" (*cf.* Definition 1.8)

It is sure:

- If anyone believes that he has found a theory that goes beyond ⑩, he will certainly want to talk about the landscape [⓪ - ① - ⑥ - ⑧] seen from the theory.

Hence I wanted to do that too. This is my motive of writing this paper.

It should be noted that most unsolved problems in the history of western philosophy are caused by insufficient understanding of idealism and dualism. Thus,

- if the (C_2) is true (i.e., if quantum language is overwhelmingly powerful compared to other immature philosophies), I can expect that most unsolved problems in dualism can be solved in the framework of quantum language.

In this text, I will show that the following unsolved philosophical problems are easily solved as corollaries of the (C_2):

(D) **The list of my answers for philosophical unsolved problems**

- What is probability (or, measurement, causality) ? *cf.* Sec. 1.1.1)
- Zeno paradox (Flying arrow), (*cf.* Sec. 2.4.2)
- Zeno paradox (Achilles and a tortoise), (*cf.* Sec. 2.4.3)
- the measurement theoretical understanding of Plato's allegory of the sum , (*cf.* Sec. 3.3.2)
- Plato's Idea theory≈Zadeh's fuzzy theory≈Sausuure's linguistic theory (*cf.* Sec. 3.5.3)
- Syllogism holds in classical systems, but not in quantum systems (*cf.* Sec. 4.3.3)
- Only the present exists (*cf.* Sec. 6.1.2)
- What is the problem of universals? (*cf.* Sec. 6.5.1)
- What is Geocentrism vs. Heliocentrism? After all, the worldviewism (*cf.* Sec. 7.4.2)
- Two (scientific or non-scientific) interpretations of I think, therefore I am .(*cf.* Sec. 8.2.2)
- Leibniz-Clark correspondence (i.e., what is space-time?), (*cf.* Sec. 9.3)
- The problem of qualia (*cf.* Sec. 9.5.1)
- Brain in a vat argument (*cf.* Sec. 9.5.2)
- The solution of Hume's problem of induction (*cf.* Sec. 9.7.1)
- Grue paradox cannot be represented in quantum language (*cf.* Sec. 9.7.2)
- What is causality? (*cf.* Sec. 10.3)
- What is Peirce's abduction? (*cf.* Sec. 11.3.1)
- Five-minute hypothesis (*cf.* Sec. 11.4.1)
- McTaggart's paradox (*cf.* Sec. 11.4.2)
- quantitative representation of "Signifier" and "signified" (*cf.* Sec. 11.5.3)
- A scientific understanding of Wittgenstein's picture theory (*cf.* Sec. 11.6.2)
- Wittgenstein's paradox (*cf.* Sec. 11.6.3)
- Flagpole problem, (*cf.* Sec. 11.7.1)
- Hempel's raven paradox (*cf.* Sec. 11.8)
- the mind-body problem (i.e., How are mind and body connected?), (*cf.* Sec. 11.9.4)

(♯) Also, for the solutions of unsolved problems in quantum mechanics, statistical mechanics, statistics and probability theory, see ref. [66]). Particularly, I think that the following two are important:

- the discovery of Heisenberg's uncertainty relation (Sec. 4.3 in ref. [66])
- The clarification of the projection postulate (i.e., the wavefunction collapse) (Sec. 11.2 in ref. [66])

Lastly I should add the following:

- this paper can be read without the knowledge of quantum theory since the mathematical arguments were left to ref. [66] mentioned in (A): KSTS/RR-19/003 (2019); 473 p (`http://www.math.keio.ac.jp/academic/research_pdf/report/2019/19003.pdf`)

- this paper is the revised version of ref. [61]: *History of Western Philosophy from the quantum theoretical point of view* [Ver. 2], Research Report (Department of mathematics, Keio university, Yokohama), (KSTS-RR-17/004, 2017, 132 pages) (`http://www.math.keio.ac.jp/academic/research_pdf/report/2017/17004.pdf`) Roughly speaking, I say that

  (∗) this [Ver. 3] = [Ver. 2] + refs. [68, 70, 71]

- My specialty is not philosophy but mathematical physics (i.e., quantum mechanics and mathematics). Thus this paper may have many mistakes and misconceptions. However, I have a confidence in only the insistence (C_2):*a scientific perfection of dualism and idealism is realized by quantum language.* Thus I hope that readers will improve this paper based on the (C_2).

I believe that philosophy is for amateur philosophy enthusiasts. Therefore, I have written this book so that readers can read it without prior knowledge. Also, readers are recommended to go reading this paper fast, and further, to advance reading [66][2].

November 2020
Shiro Ishikawa

[2] For the further information concerning quantum language, see my home page: `http://www.math.keio.ac.jp/~ishikawa/indexe.html`

Contents

Chapter 1

The outline of quantum language (=measurement theory)

Readers don't need any prior knowledge to read this paper. It is sufficient to read the overview of quantum language (and the linguistic Copenhagen interpretation) described in section 1.1 of this chapter. However, if you are a graduate student, I recommend you to read this paper and "quantum language (ref. [66])" together.

In Section 1.2, I remark that

($\sharp_1$) roughly speaking, western philosophy and the linguistic Copenhagen interpretation are similar.

Further, in Section 1.3, I explain the worldview (= world description), which is classified by the following four,

($\sharp_2$)
- the realistic worldview($\approx$ physics),
- the fictional worldview ($\approx$ western philosophy),
- the logical worldview ($\approx$ analytic, linguistic philosophy),
- the mechanical worldview ($\approx$ statistics, quantum language).

as seen in Assertion 0.1 in Preface.

1.1 The overview of quantum language(=measurement theory)

1.1.1 * Axiom 1 (What is probability (or, measurement) ?) and Axiom 2 (What is causality ?)

The idea of quantum language (also known as "measurement theory") is inspired and structured by quantum mechanics, in which microphenomena are analyzed. Quantum language is a language, by which we cannot only describe quantum mechanics but also most sciences (e.g., economics, psychology, engineering, etc.). However, it should be noted that quantum language is not almighty, for example, the theory of relativity is beyond the description of quantum language.

The framework of measurement theory(=quantum language) is simple, that is, it is composed with two axioms (Axiom 1 concerning measurement and Axiom 2 concerning causal relation) and the manual to use Axioms 1 and 2 (called the linguistic Copenhagen interpretation). That is, (*cf.* refs [42, 66]),

$$\overset{\text{(=quantum language)}}{\boxed{\text{measurement theory}}} = \overset{\text{[Axiom 1]}}{\boxed{\text{measurement}}} + \overset{\text{[Axiom 2]}}{\boxed{\text{causal relation}}} + \underset{\text{[the manual to use Axioms 1 and 2]}}{\boxed{\text{linguistic Copenhagen interpretation}}} \tag{1.1}$$

Although it is not needed to read Axioms 1 and 2 for reading this paper, we, for completeness, mention them as follows. (For the mathematical foundations of Axioms 1 and 2, see Section 1.5 (Appendix: the mathematical foundations of quantum language) later.

Axiom 1 [measurement] (in Section 1.1)

(This will be explained more precisely in Section 1.5)

With any system S, a basic structure $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$ can be associated in which measurement theory of that system can be formulated. When the observer takes a measurement of an observable (or, by a measuring instrument) $\mathsf{O}{=}(X, \mathcal{F}, F)$ for a system $S_{[\rho]}$ i.e., a system S with a state ρ), the probability that a measured value x $(\in X)$ obtained by the measurement belongs to Ξ $(\in \mathcal{F})$ is given by $\rho(F(\Xi))(\equiv {}_{\mathcal{A}^*}(\rho, F(\Xi))_{\mathcal{N}})$.

And

Axiom 2 [causality] (in Section 1.1)

(This will be explained more precisely in Section 1.5)

Let T be a **tree** (i.e., semi-ordered tree structure). For each $t(\in T)$, a basic structure $[\mathcal{A}_t \subseteq \mathcal{N}_t \subseteq B(H_t)]$ is associated. Then, the **causal chain** is represented by a **sequential causal operator** $\{\Phi_{t_1,t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}\}_{(t_1,t_2)\in T^2_{\leqq}}$.

Here, note that

(A) **the above two axioms are kinds of spells (i.e., incantation, magic words, metaphysical statements), and thus, it is impossible to verify them experimentally. What we should do is not to understand the two, but to learn the spells (i.e., Axioms 1 and 2) by rote.**

In this sense, quantum language is metaphysics. Therefore,

(B) **The formation of quantum languages depends on human marvelous language ability**

♠**Note 1.1.** If metaphysics did something wrong in the history of science, it is because metaphysics attempted to answer the following questions seriously in ordinary language:

($♣_1$) What is ◯◯ (e.g., ◯◯ = measurement, probability, causality, space-time, etc.) ?

Although the question ($♣_1$) looks attractive, it is not productive. What is important is *to create a language* to deal with the keywords. So we replace ($♣_1$) by

($♣_2$) How should ◯◯ (e.g., ◯◯ =measurement, probability, causality) be used ?

The problem ($♣_1$) will now be solved in the sense of ($♣_2$). If there are some failure in the history of philosophy, philosophers failed to propose a suitable language. It should be noted that Newton's success is due to the proposal of "the language called Newtonian mechanics". Recall that Newton mechanics does not answer the question "What are mass, force, acceleration?", but "How should they be used?" or equivalently, "How do they relate to each other?". That is, "[mass] · [acceleration] = [force]." Thus, I think that Newton's most important accomplishment is the following three (which are equivalent):

($\flat_1$) making physics called Newtonian mechanics.

($\flat_2$) proposing the worldview called Newtonian mechanics.

($\flat_3$) making the language called Newtonian mechanics.

where these are equivalent: i.e., ($\flat_1$)≈($\flat_2$)≈($\flat_3$).

Similarly I think that the greatest task of philosophy is the following three: from the idealistic point of view,

($\sharp_1$) making a phirosophy called □ □

($\sharp_2$) proposing a worldview called □ □

($\sharp_3$) making a language called □ □

where these are equivalent: i.e., ($\sharp_1$)≈($\sharp_2$)≈($\sharp_3$).

Thus we can answer the following problem from the quantum linguistic point of view:

Problem 1.1. Scientifically please answer to the problem:

- $\boxed{\text{What is measurement, probability, causality?}}$

[Answer]:

As mentioned in Note 1.1, the question: "What is measurement, probability, causality?"

should be interpreted as the question: "How should the terms: measurement, probability, causality be used?". Since measurement, probability, causality are key-words of quantum language (i.e., Axioms 1 and 2), what is important is to use Axioms 1 and 2 appropriately. To this end, I recommend readers to read ref. [66] and this text carefully. □

♠**Note 1.2.** The above problem is the most difficult unsolved problem in the history of philosophy. The readers may be surprised at how easily this can be solved. I think many readers already know how powerful it is to create a proper language as the great success of Newtonian mechanics. In addition, recall that Newtonian mechanics never answer to "What are forces, accelerations, and masses?". What Newtonian mechanics answers to is "How are used the terms: forces, accelerations, and masses?"

1.1.2 Linguistic Copenhagen interpretation (or in short, Linguistic interpretation)

1.1.2.1 Descartes figure (the linguistic Copenhagen interpretation)

In the previous section, we introduced the outline of quantum language. Here, note that

(C_1) *the axiom is a kind of spells (i.e., incantation, magic words, metaphysical statement), so that, it is impossible to verify it experimentally.*

And thus, we say:

(C_2) Since quantum language is a language, it may be difficult to use it well at first. We need to make practice, and will master it only by trial and error.

♠**Note 1.3.** In Mermin's book [85], he said

- If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be **"Shut up and calculate"**

This means that "What is the Copenhagen interpretation? is unsolved problem. Thus, I assert that the linguistic Copenhagen interpretation is the true Copenhagen interpretation (*cf.* ref. [66]).

However,

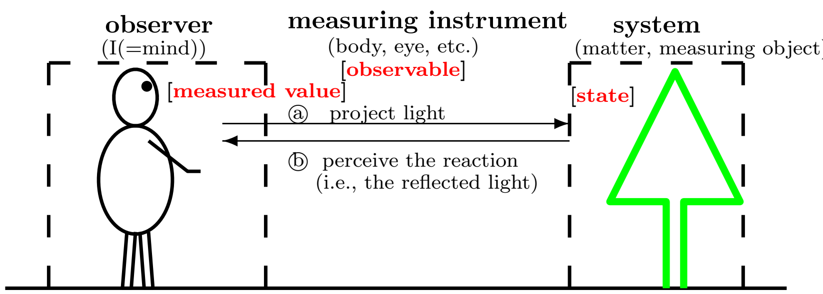
(C_3) if we want to master quantum language as quick as possible, we will need a good manual to use the axioms.

Here, we think that

(C$_4$) the linguistic Copenhagen interpretation
= manual to use spells (Axioms 1 and 2)

Since Axiom 1 includes the term "measurement", the concept of "measurement" should be, at first, understood in dualism (i.e., "observer" and "measuring object") as illustrated in Figure 1.2.

Figure 1.2. [Descartes Figure]: Image of "measurement(=ⓐ+ⓑ)" in dualism



In the figure, "measurement" is characterized as interaction between "observer" and "system"(matter or object to be measured, measuring object), composed of

(D$_1$) ⓐ projection of light onto the object (i.e., someone, not necessarily an observer, shines the light.)
ⓑ perception of the reaction of the object (i.e., the observer receives the reaction.)

However, I want to emphasize that the interaction cannot be represented by kinetic equations. Therefore,

(D$_2$) in measurement theory (= quantum language), we use the term "measurement" instead of "interaction". Therefore, we won't say the above (D$_1$) outright.

After all, we think that:

(D$_3$) there is no measured value without observer, and that measurement theory is composed of three keywords:

$$\underset{\text{(observer, brain, mind)}}{\boxed{\text{measured value}}}, \quad \underset{\text{(telescope, thermometer, eye, ear, body, polar star)}}{\boxed{\text{observable (= measuring instrument)}}}, \quad \underset{\text{(matter, measuring object)}}{\boxed{\text{state}}}.$$

In view of the above figure, it might be called "ternary relation (or, trialism)" instead of "dualism". But, following the convention, we use *"dualism"* throughout this paper.

♠**Note 1.4.** (i): The following argument, called the Bohr-Einstein debates, is extremely profound (*cf.* Sec.9.4.3 Bohr-Einstein debates).

- For example, if the voltmeter needle points to 5V, but the observer doesn't see it, it's just a physical phenomenon. Therefore, Axiom 1 (measurement) is not required. However, in the dualistic position of introducing measurement into science, Axiom 1 is essential.

Einstein is the former position (i.e., monism) and Bohr is the latter (i.e., dualism). It's the biggest scientific debate of the 20th century, but it's not yet settled. My standing point is as follows.

- These two (monism and dualism) are not to be unified. As mentioned in Assertion 0.1 in Preface, they should be compatible.

  (ii): The concept of "observable" (which can be identified with "measuring instrument") is not easy. For example, telescopes, glasses and eyes are a type of measuring instrument. A directional magnet is, of course, a measuring instrument. If so, then the polar star is also a type of measuring instrument.

////

1.1.2.2 The linguistic Copenhagen interpretation [(E_1)-(E_7)]

The linguistic Copenhagen interpretation is "the manual to use Axiom 1 and 2". Thus, there are various explanations for the linguistic Copenhagen interpretations. However, it is usual to consider that the linguistic Copenhagen interpretation is characterized by statements in Panel (E), among which the most important is

(E_4) Only one measurement is permitted.

(E):The linguistic Copenhagen interpretation

With Descartes figure 1.2 and the following (E_1)-(E_7) in mind, describe every phenomenon in terms of Axioms 1 and 2!

(E_1) Consider the **dualism** composed of "observer" and "matter (= object to be measured)", where "observer" and "matter (= measuring object)" must be absolutely separated. Figuratively speaking, "Audience should not go on stage". In addition, the measurement should not be dependent on the observer. In other words, it makes no difference if I measure it, if you measure it, or if Newton measures it.

♠**Note 1.5.** I'm not sure if the following analogy is appropriate, consider the followings:

($\sharp_1$) I measure my body temperature with a thermometer.

($\sharp_2$) I feel my body feverish.

and

($\flat_1$) The doctor measures my body temperature with a thermometer.

($\flat_2$) The doctor feels my body feverish.

In terms of measurement, ($\sharp_1$) and ($\flat_1$) are the same. On the other hand, ($\sharp_2$)and ($\flat_2$) are different. Thus, in the strictest sense, we consider that ($\sharp_2$) cannot be regarded as a measurement. However, the ($\flat_2$) seems to be a measurement. This example will help you understand that cogito proposition "I think, therefore I am" in Chapter 8.

(E_2) While "matter" is in the space-time, the observer is not. Thus, the question: "When, where and by whom was the measured value obtained?" is out of the scope of measurement theory. Thus, words such as "now," "here," and "I" should not be used in a scientific proposition. If you are going to use it, you need to be very careful. That is, there is no tense either in measurement theory or in science.

(E_3) In measurement theory, "observable(=measuring instrument≈body)" is the most important than "measured value(≈mind)" and "state(≈matter)" in (D_3).

(E_4) **Only one measurement is permitted**. Thus, the state after measurement (or, wave function collapse, the influence of measurement) is meaningless. For the virtual wave function collapse, see:

[59] S. Ishikawa, *Linguistic interpretation of quantum mechanics; Projection Postulate,* JQIS, Vol. 5, No.4 , 150-155, 2015, DOI: 10.4236/jqis.2015.54017 (http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=62464)

♠**Note 1.6.** The Schrödinger cat is the most famous paradox in quantum mechanics. However, we are not bothered by this paradox since the state after measurement is not described in quantum language.

(E_5) There is no probability without measurement. Also, the measurement cannot be measured.

(E_6) State never moves. Thus, we always use the Heisenberg's picture (and not the Schrödinger picture)

and so on.

Since it is believed that

quantum language is the final goal of dualistic idealism

(*cf.* ⑧ in Figure 0.1 in Assertion 0.1 (in Preface), it is deduced that

(E_7) maxims of the philosophers (particularly, the dualistic idealism) can be regarded as expressions in linguistic Copenhagen interpretation.

For example, Wittgenstein's "The limits of my language mean the limits of my world", "What we cannot speak about we must pass over in silence", etc. teach us the spirit of quantum language.

Some people may wonder (E_7). However, note

(F_1) Descartes=Kant philosophy and quantum language have the same purpose to establish the worldview,

and

(F_2) Descartes=Kant philosophy and quantum language have the same methodology of idealistic dualism.

Then, it is natural to consider

$$\text{maxims of philosophers} \approx \text{the linguistic Copenhagen interpretation.}$$

We assert the following.

Assertion 1.3. [descriptive power of quantum language](*cf.* ref. [66])
Roughly speaking,

- quantum language has a great descriptive power more than statistics.

Therefore, we assert that

(G_1) **quantum language is a language, by which most sciences (e.g., economics, psychology, engineering, etc.) are described**

Quantum language is a language, by which we cannot only describe quantum mechanics but also most sciences (e.g., economics, psychology, engineering, etc.). Thus, as a modified version of "statistics is the language of science", we consider that

(G_2) **Quantum language is the language of science,**
or,
to do sciences is to describe phenomena by quantum language

However, it should be noted that quantum language is not almighty, for example, the theory of relativity is beyond the description of quantum language.

♠**Note 1.7.** It is one of the roles of the linguistic Copenhagen interpretation to exclude a scientifically nonsense propositions from quantum language. As mentioned later, for example,

($\sharp_1$) Cogito proposition **"I think, therefore I am"** is not the proposition within quantum language.

In cogito proposition, we see that "observer"="I" and "object to be measured"="I", which is inconsistent with that the above (E_1). Thus, cogito proposition is not a proposition in quantum language. This will be discussed again in Proposition 8.2.

Next,

($\sharp_2$) The hypothesis that **the world was created five minutes ago** (due to B. Russell in Sec.11.4) is not within quantum language.

That is because this hypothesis is considered under the premise such that the observer's time, which is prohibited by (E_2).

KSTS/RR-20/001
November 15, 2020

1.2 The history of worldview and our purpose

1.2.1 Quantum language in the history of worldviews

We assert that, in the history of worldview, quantum language is located as follows.

Assertion 1.4. (= Assertion 0.1 in Preface) [The location of quantum language in the history of world-description (*cf.* ref.[43, 66])]

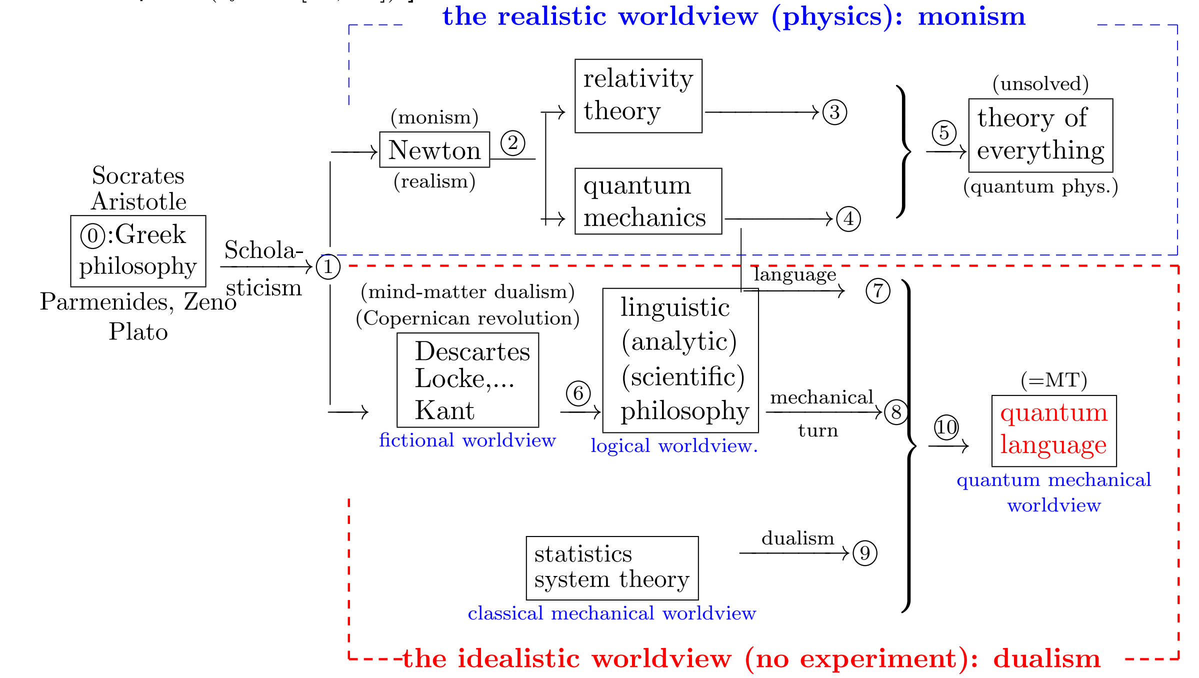


Figure 0.1: The history of the world-descriptions

Therefore, quantum language has the following three aspects:

The three aspects of quantum language

$$
= \begin{cases} ⑦\text{: the linguistic turn of quantum mechanics} \\ \qquad \text{(i.e., the true color of the Copenhagen interpretation)} \\ ⑧\text{: the final goal of dualistic idealism} \\ \qquad \text{(i.e., the linguistic and mechanical turn of the Descartes=Kant epistemology)} \\ ⑨\text{: the dualistic reconstruction of statistics} \end{cases} \tag{1.2}
$$

♠**Note 1.8.** I cannot answer the question: "What is quauntum mechanics?", since this is related to ⑤ in the above figure. However, I am convinced that our interpretation (i.e., the linguistic Copenhagen interpretation) is the true color of the Copenhagen interpretation.

1.2.2 Our purpose

Our purpose is to answer the problem "Has western philosophy made progress?", and to conclude the following Assertion 1.5.

In this paper I assert that

Has western philosophy made progress?

Assertion 1.5. (A) If "to make progress" is defined by "to come near quantum language", we can say that the time series [⓪→ ①→⑥→⑧→⑩] can be regarded as progress, or more precisely,

$$
\boxed{\text{Plato}} \xrightarrow[\text{progress}]{\text{Cogito turn (dualism)}} \overset{\text{mind-matter dualism}}{\boxed{\text{Descartes=Locke}}} \xrightarrow[\text{progress}]{\text{Copernican revolution}} \overset{\text{transcendental idealism}}{\boxed{\text{Kant}}}
$$

$$
\xrightarrow[\text{progress}]{\text{linguistic turn}} \overset{\text{linguistic philosophy}}{\boxed{\begin{matrix}\text{Saussure}\\ \text{Wittgenstein}\end{matrix}}} \xrightarrow[\text{progress}]{\text{mechanical turn (scientific turn)}} \underset{\text{quantitative argument is possible}}{\overset{\text{the quantum mechanical worldview}}{\boxed{\text{Quantum language}}}} \tag{1.3}
$$

The name of "scientific turn" is due to the fact that quantitative argument is possible in quantum language and that it is only quantum language which scientists need in above-mentioned.
Thus, we conclude that

(B) a scientific perfection of dualism and idealism[a] is realized by quantum language.

[a] Throughout this paper, it suffices to consider that "idealistic"≈"linguistic"≈"metaphysical" (*cf.* Definition 1.8)

It is sure:

(C) if someone believes that he/she finds the theory beyond ⑩, he/she certainly wants to talk about the landscape [⓪ - ① - ⑥ - ⑧] seen from the theory.

This will be done in this paper.

It should be noted that most unsolved problems in the history of western philosophy are caused by insufficient understanding of idealism and dualism. Thus, if the (B)[a scientific perfection of dualism and idealism is realized by quantum language] is true, we can expect that most unsolved problems can be solved in the framework of quantum language.

In fact, in this paper we show the solutions of the following problems

List 1.6.

(D) **The list of my answers for philosophical unsolved problems**

- What is probability (or, measurement, causality) ? *cf.* Sec. 1.1.1)
- Zeno paradox (Flying arrow), (*cf.* Sec. 2.4.2)
- Zeno paradox (Achilles and a tortoise), (*cf.* Sec. 2.4.3)
- the measurement theoretical understanding of Plato's allegory of the sum , (*cf.* Sec. 3.3.2)
- Plato's Idea theory≈Zadeh's fuzzy theory≈Sausuure's linguistic theory (*cf.* Sec. 3.5.3)
- Syllogism holds in classical systems, but not in quantum systems (*cf.* Sec. 4.3.3)
- Only the present exists (*cf.* Sec. 6.1.2)

- What is the problem of universals? (*cf.* Sec. 6.5.1)
- What is Geocentrism vs. Heliocentrism? After all, the worldviewism (*cf.* Sec. 7.4.2)

- Two (scientific or non-scientific) interpretations of I think, therefore I am .(*cf.* Sec. 8.2.2)
- Leibniz-Clark correspondence (i.e., what is space-time?), (*cf.* Sec. 9.3)
- The problem of qualia (*cf.* Sec. 9.5.1)
- Brain in a vat argument (*cf.* Sec. 9.5.2)
- The solution of Hume's problem of induction (*cf.* Sec. 9.7.1)
- Grue paradox cannot be represented in quantum language (*cf.* Sec. 9.7.2)
- What is causality? (*cf.* Sec. 10.3)
- What is Peirce's abduction? (*cf.* Sec. 11.3.1)
- Five-minute hypothesis (*cf.* Sec. 11.4.1)

- McTaggart's paradox (*cf.* Sec. 11.4.2)
- quantitative representation of "Signifier" and "signified" (*cf.* Sec. 11.5.3)
- A scientific understanding of Wittgenstein's picture theory (*cf.* Sec. 11.6.2)
- Wittgenstein's paradox (*cf.* Sec. 11.6.3)
- Flagpole problem, (*cf.* Sec. 11.7.1)
- Hempel's raven paradox (*cf.* Sec. 11.8)
- the mind-body problem (i.e., How are mind and body connected?), (*cf.* Sec. 11.9.4)

($\sharp$) Also, for the solutions of unsolved problems in quantum mechanics, statistical mechanics, statistics and probability theory, see ref. [66]). Particularly, I think that the following two are important:

- the discovery of Heisenberg's uncertainty relation (Sec. 4.3 in ref. [66])
- The clarification of the projection postulate (i.e., the wavefunction collapse) (Sec. 11.2 in ref. [66])

Since these problems are easily solved in quantum language, we can be convinced that

quantum language is the final goal of the dualistic idealism[⓪ - ① - ⑥ - ⑧]

♠**Note 1.9.** It should be noted that Einstein's success is due to the proposal of "the language called the theory of relativity". On the other hand, I think that

($\sharp_4$) *philosophers failed because they did not propose a suitable language.*

Talking cynically, we say that

($\sharp_5$) Philosophers has investigated "linguistic Copenhagen interpretation" (="how to use Axioms 1 and 2") without language (i.e., Axiom 1 (measurement) and Axiom 2 (causality)).

Therefore, in most cases many philosophers wander. However, great philosophers rarely miss the point. For example, Wittgenstein did not propose his language, but he left the maxim such that

($\sharp_2$) "The limits of my language mean the limits of my world" which is just the spirit of quantum language (as mentioned in Chapter 11).

1.3 Realistic worldview and idealistic worldview

1.3.1 The worldviewism

The **worldviewism** is

the spirit which starts from worldview

That is,

(A): The worldviewism

The worldviewism has the following form:

$$(\mathrm{A}_1)\quad \underset{\text{premise}}{\overset{\text{world is so}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{subject}}{\overset{\text{conclusion}}{\boxed{\text{practical logic } (\approx \text{ inference, how to think, calculation}), \text{ discussions, etc.}}}}$$

That is,

(A_2) **Since the world is like this, think like this!**

That is, the worldviewism is the spirit such that "Start from worldview". I think that

(A_3) It's not an exaggeration to say **"philosophy = worldviewism"**.

[**Remark**] This is not trivial. That is because the above (A) says that the worldview is greater than logic. That is,

"logic" (precisely, "practical logic") is created by a worldview.

(*cf.* Section 2.4.1 [Zeno's paradoxes], Section 4.3.2 [Aristotle's syllogism] etc.).
Arguing repeatedly in this paper, we consider that the ignoring of the worldviewism (or, relying on a bad worldview) causes that philosophy falls into a blind alley.
Slogan-wise, we say

(A_4) **Without a worldview, there is no logic (i.e., no calculation)**

(A_5) **Without a worldview, we can't have any discussion.**

- ······

♠**Note 1.10.** Without a worldview, we have paradoxes as follows.

- Zeno's paradoxes (*cf.* Sec. 2.4.2)
- The solution of Hume's problem of induction (*cf.* Sec. 9.7.1)
- Grue paradox cannot be represented in quantum language (*cf.* Sec. 9.7.2)
- Wittgenstein's paradox (*cf.* Sec. 11.6.3)

////

The following questions often appear in philosophy:

Does time exist?	Does I exist?
Does position exist?	Does velocity exist?
Does beauty exist?	Does mathematics exist?
Does a particle exist?	Does motion exist?
Does God exist?	

However, these statements are ambiguous since the term "exist" (or "to be") is not determined. Thus I think as follows.

Definition 1.7. ["existence" (i.e., "to be"]
Fix a worldview. Then, we say "[X] exists", if [X] is a keyword in the worldview (or more generally, if [X] is a thing that can be expressed using these keywords)

Remark: For example, in Newtonian mechanics, "velocity", "acceleration", "force", etc. are exist. Also, if we see Christianity as a kind of worldview, we say that God exists.

We begin with the following definition:

Definition 1.8.
["realism", "idealism"],["monism" , "dualism"]

(B) realism := a worldview (or, theory) that requires experimental verification.
idealism := a worldview (or, theory) that does not necessarily require experimental verification.
("idealism" is also called "linguistic theory", "non-realism" or "metaphysics" in this paper)

(C) monism := a worldview about objects
dualism := a worldview about humans and objects, (i.e., a theory with "observers" and "measuring objects")

Though the above is quite rough, it is sufficient to read this paper. I think that there is no strict definition in philosophy (not mathematics). And the more rigorous we try to say it, the more imprecise it becomes.

As seen in Assertion 1.4, we have the following classification of philosophies:

Classification 1.9. [(D): Classification of worldviews]

$$
\begin{cases}
(\flat_1) : \underset{\text{(monism)}}{\text{the realistic worldview}} : \text{Aristotle, Archimedes, Galileo, Newton, Einstein} \\[2ex]
(\flat_2) : \underset{\text{(dualism)}}{\text{the idealistic worldview}} \begin{cases} (\flat_{21}):\ \text{the fictional worldview } (\approx \text{ western philosophy}) \\ \qquad \text{Plato, Descartes, Locke, Kant} \\ (\flat_{22}):\ \text{the logical worldview } (\approx \text{ analytic, scientific phil.}) \\ \qquad \text{Frege, Saussure, Russell, Wittgenstein...} \\ (\flat_{23}):\ \text{the mechanical worldview} \\ \qquad \text{statistics, quantum language} \end{cases} \\
\quad \text{Throughout this paper, we explain that} \\
\quad \boxed{(\flat_{21}):\ \text{Plato}} \xrightarrow[\text{turn}]{\text{cogito}} \boxed{(\flat_{21}):\ \text{Descartes}} \xrightarrow[\text{revolution}]{\text{Copernican}} \boxed{(\flat_{21}):\ \text{Kant}} \\
\qquad \xrightarrow[\text{turn}]{\text{linguistic}} \boxed{(\flat_{22}):\ \text{Wittgenstein}} \xrightarrow[\text{turn}]{\text{mechanical}} \boxed{(\flat_{23}):\ \text{QL}} \\[2ex]
(\flat_3) \text{Others (Thinking Tip, etc.) : Darwin's theory of evolution, Hegel's dialectic, etc.} \\[2ex]
(\flat_4) \text{mathematics}
\end{cases}
$$

////

I think mathematical logic in $(\flat_{22})$ should be classified in $(\flat_4)$, but for the sake of convenience, in this text I sometimes think that mathematical logic (or, set theory) belongs to $(\flat_{22})$. This is not generally a mistake. This is because, for example, when we use the word "set" in everyday language (or, in the naive set theory (*cf.* [25])), it is naturally interpreted as "a collection of things".

♠**Note 1.11.** Lord Kelvin (1824–1907) said

($\sharp$) Mathematics is the only good metaphysics.

The $(\flat_2)$, $(\flat_3)$ and $(\flat_4)$ are metaphysics, as they does not require experimental verification. For example, note that Darwin's theory of evolution has not been settled by experimental verification. Thus, if we believe the above ($\sharp$), we must consider $(\flat_2)$, $(\flat_3)$ and $(\flat_4)$ as bad theories. However, I think that at least quantum language (which is not mathematics) is a good metaphysics. Hence, I think Kelvin is wrong.

♠**Note 1.12.** For example, it is certain that

($\sharp_1$) there is no physics without the world

However,

($\sharp_2$) mathematics itself is not related to the world. In an extreme case, there may exist mathematics without the world

Therefore, it is natural to consider that mathematics is not a kind of worldview. Truth be told, I think

($\sharp_3$) mathematical theory (including logic, set theory, category theory) has nothing to do with worldview.

However, for example, consider the term: "set", which is the term in ordinary language as well as mathematics. Thus, some may comes up with the idea (which may be called the set theoretical worldview) of describing the world through the naive set theory (and not the axiomatic set theory). This set theoretical worldview is rather general in science. In Chap. 11, we will see that this worldview triggers the flagpole problem and Hemple's raven paradox.

I also think of symbolic logic as a tool for constructing mathematics, but some people may have the idea (which may be called a logical worldview) that modal logic can be used to describe the world. Thus, the relationship between logic and philosophy varies somewhat from researcher to researcher. Also, it is sure that many philosophers consider that logic is one of field of philosophy. Therefore, in spite that I believe in ($\sharp_3$), in this paper,

($\sharp_4$) **logic (or, set theory) is regarded as a worldview called the logical worldview.**

In Chapter 11. we will see that the logical worldview was not very successful.

////

More precisely, let us examine the above (D):

($\flat_1$) **the realistic worldview (= physics)**

$$\underset{\text{premise,law}}{\overset{\text{The world is ruled by Newtonian mechanics}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{discussion}}{\overset{\text{Heliocentrism is true}}{\boxed{\text{discussions, calculation, practical logic, properties}}}}$$

and

($\flat_{21}$) **the fictional worldview ($\approx$ western philosophy)**

$$\underset{\text{(fictional)premise}}{\overset{\text{invisible man exists}}{\boxed{\text{worldview (literary truth, pseudo- truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{don't steal a glance}}{\boxed{\text{ethics, practical logic, discussions}}}}$$

and

($\flat_{22}$) **the logical worldview ($\approx$ Analytic philosophy)**

$$\underset{\text{Axiomatic logic}}{\overset{\text{Describe the world by symbolic logic, set theory}}{\boxed{\text{worldview (symbol logic, set theory)}}}} \xrightarrow{\text{therefore}} \underset{\text{theorem}}{\overset{A \to B,\ \ B \to C, \text{ therefore, } A \to C}{\boxed{\text{calculation, discussions, properties}}}}$$

and

($\flat_{23}$) **the mechanical worldview (= the classical and quantum mechanical worldview)**

$$\underset{\text{language inspired by mechanics}}{\overset{\text{Describe the world by (mechanical) language}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{discussions}}{\overset{\text{If an apple falls, it has a cause.}}{\boxed{\text{calculation, practical logic,properties}}}}$$

1.3.2 The realistic worldview(physics) ($\flat_1$)

(E): The realistic worldview (physics) ($\flat_1$)

The realistic worldview has the following form:

$$(E_0)\qquad \underset{\text{law}}{\overset{\text{world is so}}{\boxed{\text{realistic worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{discussion}}{\overset{\text{Such phenomena happen}}{\boxed{\text{scientific properties}\cdot\text{calculation}}}}$$

Hence,

(E_1) [world is so] is a law, truth, (which should be verified by experiment),

(E_2) [such phenomena happen] is a consequence

As examples of the realistic worldview, we see that

Newtonian mechanics, electromagnetism, theory of relativity, etc.

For example,

$$(F)\qquad \underset{\text{premise}\cdot\text{law}}{\overset{\text{Newtonian mechanics}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{discussion}}{\overset{\text{Heliocentrism is true (\textit{cf.} Chap. 7)}}{\boxed{\text{discussions, calculation, practical logic, properties}}}}$$

Note that the realistic worldview (=physics) is the most authorized.

1.3.3 The idealistic worldview ($\flat_2$)

In what follows, consider the idealistic worldview ($\flat_2$) in

1.3.3.1 The fictional worldview (Wester philosophy) ($\flat_{21}$)

Our main theme of the preprint is the following fictional worldview:

(G): The fictional worldview (western philosophy) ($\flat_{21}$)

The fictional worldview (in Plato's way of telling philosophy) is as follows.

$$(G_0)\qquad \underset{\text{preface, introduction, (fictional)premise, expedient}}{\overset{\text{world is so}}{\boxed{\text{fictional worldview (literary truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{you should do so}}{\boxed{\text{practical logic, ethics, morals}}}}$$

Therefore,

(G_1) [world is so] is secondary,

(G_2) [you should do so] is main theme

For example,

$$(\mathrm{H}_1)\quad \underset{\text{(fictional)premise}}{\overset{\text{invisible man exists}}{\boxed{\text{fictional worldview (literary truth, pseudo- truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{don't steal a glance}}{\boxed{\text{discussions, properties}}}}$$

and

$$(\mathrm{H}_2)\quad \underset{\text{premise,fiction}}{\overset{\text{ultimate beauty exists}}{\boxed{\text{fictional worldview (literary truth, pseudo- truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main assertion}}{\overset{\text{she is an embodiment of beauty.}}{\boxed{\text{ethics, morals,aesthetic sense}}}}$$

The fictional worldview forms the main current of western philosophy such as

Plato, Descartes, Locke, Hume, Kant,

etc. □

1.3.3.2 The logical worldview (analytic philosophy) ($\flat_{22}$)

(I_1): The logical worldview (axiomatic logic) ($\flat_{22}$)

As mentioned in Note 1.12, I think:

(I_1) **the logical worldview (or, set theoretical worldview)**

$$\underset{\text{Axiomatic logic}}{\overset{\text{Describe the world by symbolic logic, set theory}}{\boxed{\text{worldview (symbol logic, set theory)}}}} \xrightarrow{\text{therefore}} \underset{\text{science, science philosophy}}{\overset{\text{understandable propositions (}\textit{cf.}\text{ Remark 1.10)}}{\boxed{\text{scientific discussions}}}}$$

Main philosophers (mentioned in this paper) are as follows.

Boole, Boole, Frege, Peirce, Russell, Wittgenstein, Hempel, Popper,...

though their positions are not unanimous.

Remark 1.10. The above (I_1) is motivated by the following:

$$(\mathrm{I}_2)\quad \underset{\text{Axioms}}{\overset{\text{Describe mathematics by logic, set theory}}{\boxed{\text{axiomatic logic, set theory}}}} \xrightarrow{\text{therefore}} \underset{\text{theorem}}{\boxed{\text{mathematics}}}$$

This is the greatest achievement of mankind, which was completed by Cantor, Russell, Zermelo=Fraenkel, etc. Thus, many philosophers might consider that

(J) (I_2) is the greatest, hence, (I_1) is also promising.

This optimistic outlook, I believe, prompted the birth of analytic philosophy.

However, I am somewhat skeptical of the logical (and set theoretical) worldview (I_1) since "mathematics" and "our world" are completely different.

But these (I_1) and (I_2) alone are too small for their research area. Therefore, the research area was expanded as follows.

(I_3) Think rationally, carefully and logically, or use words correctly! (i.e., analyze language !)

(This is similar to Bacon's assertion, *cf.* Sec. 7.2.) Thus, in this paper I assume that the term: "logical worldview" has three aspects (I_1), (I_2) and (I_3).

♠**Note 1.13.** The term: "logical worldview" has three aspects. (I_1), (I_2) and (I_3). I think that (I_2) is the greatest, but (I_1) is doubtful. In Chap. 11, we will see that scientific philosophy (e.g., the flagpole problem and Hemple's raven paradox) highlighted the weakness of the logical (and set theoretical) worldview (I_1). (In this sense, scientific philosophy is a good philosophy in (I_1).) And thus, I proposed the quantum mechanical worldview (= quantum language). However, I think that the (I_3) may be the worst in the above if "logically" means "like mathematical logic (i.e., like symbolic logic)". It should be noted that

($\sharp_1$) Good mathematicians are not always able to consider the world logically.

($\sharp_2$) I. Newton could consider the world logically, in spite that he didn't know symbolic logic.

I think that it is only in mathematics that mathematical logic is all-powerful. That is, I think that Bacon=Descartes' "rationally" may be sounder than "logically" in (I_3) (i.e., analytic philosophy).

♠**Note 1.14.** As mentioned in Chapter 11, Dr. Hawking said in his best seller book [23]:

- *However, in the nineteenth and twentieth centuries, science became too technical and mathematical for the philosophers, or anyone else except a few specialists. Philosophers reduced the scope of their inquiries so much that Wittgenstein the most famous philosopher this century, said "The sole remaining task for philosophy is the analysis of language." What a comedown from the great tradition of philosophy from Aristotle to Kant!*

I agree to him. And thus, I may not appreciate analytic philosophy. In this paper, I think that one of the main purposes of western philosophy (from Plato to Kant) is to clarify dualistic idealism. To be honest, I have no clear answer on "What is the purpose of analytic philosophy?" However analytic philosophy (or, linguistic philosophy) is important since it bridges between the fictional worldview and the quantum mechanical worldview. That is,

$$\underset{\text{transcendental idealism}}{\overset{\text{the fictional worldview}}{\boxed{\text{Kant: (dualism)}}}} \xrightarrow{\text{linguistic turn}} \overset{\text{linguistic philosophy}}{\boxed{\text{Saussure, Wittgenstein}}} \xrightarrow{\text{mechanical turn}} \underset{\text{linguistic idealism}}{\overset{\text{the quantum mechanical worldview}}{\boxed{\text{Quantum language: (dualism)}}}} \tag{1.4}$$

♠**Note 1.15.** Readers should not confuse two logics (i.e., axiomatic logic and practical logic) such that

$$\text{(K)} \begin{cases} (K_1)\text{: axiomatic logic (i.e., mathematical logic) in } (I_1) \text{ or } (J_1) \\ \qquad\qquad \text{propositional logic, predicate logic, modal logic, temporal logic,} \\ \qquad\qquad \text{intuitionistic logic, quantum logic, etc.} \\ (K_2)\text{: practical logic in (A)} \quad \text{(i.e., logic generated from worldview, or, inference)} \\ \qquad\qquad \text{e.g., the logic of Newton mechanics, statistical inference in statistics, etc.} \end{cases}$$

Recall the worldviewism (A), i.e.,

$$(\mathrm{A}_1)\ (=(\mathrm{A}))\qquad \underset{\text{premise}}{\overset{\text{world is so}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{subject}}{\overset{\text{conclusion}}{\boxed{\text{discussions, calculation (= practical logic), properties}}}}$$

That is, **"logic" (precisely, "practical logic") is created by a worldview.** As mentioned in (A), we see:

- Newtonian mechanics has the logic specific to Newtonian mechanics.
- Statistics has the logic specific to statistics (i.e., statistical inference).
- quantum language has the logic specific to quantum language.
- ......

Slogan-wise, we say

- **Without a worldview, there is no logic**
- **Without a worldview, there is no discussion**
- ......

I have an opinion that philosophers should be more interested in practical logic than in mathematical logic. Also,

(L) I don't think the mathematical achievements of the logical worldview (e.g., modal logic, temporal logic, etc.) were very successful as philosophy.

That is because these were powerless in solving unsolved philosophical problems, compared to quantum language (*cf.* List 1.6).

♠**Note 1.16.** The following is a common refrain among scientists:

($\sharp_1$) In spite that philosophers like logic, they are illogical.

Now we can understand this paradox ($\sharp_1$) as follows. That is,

($\sharp_2$) philosophers are interested in axiomatic logic, but they don't have much training in mastering practical logic under a certain worldview (e.g., Newtonian mechanics, statistics, etc.).

In fact, philosophers are well suited to syllogism, and scientists are good at calculating differential equations.

Now we have the following question:

($\flat_1$) **Why has there been such a gap between philosophers and scientists?**

To answer this question may be our purpose of this text. That is, we think that

($\flat_2$) **philosophers have no scientific worldview, but scientists have some scientific worldviews (i.e., Newtonian mechanics, statistics, etc.).**

Then, we have the last question such as

($\flat_3$) **What kind of scientific worldview have philosophers (from Plato to Kant) continued to desire?**

We can answer this question as follows.

($\flat_4$) **What they have sought is a worldview on dualistic idealism., that is, quantum language**

The above will be explained throughout this text.

1.3.3.3 The mechanical worldview (statistics, quantum language) ($\flat_{23}$)

In the era after Kant, disciplines (i.e., mathematics, physics, medicine, chemistry, biology, economics, psychology, etc.) were born one after another, and the weight of philosophy became smaller in the whole discipline. Therefore, philosophy is no longer regarded as the king of academia. the purpose of my research is to restore philosophy through quantum language.

(M): The mechanical worldview (Statistics, quantum language) ($\flat_{23}$)

The mechanical worldview is as follows.

$$\text{(M)} \qquad \underset{\text{language, words}}{\overset{\text{language is proposed like mechanics}}{\boxed{(\text{M}_1)\text{: mechanical worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{discussions}}{\overset{\text{describe phenomena by the language}}{\boxed{(\text{M}_2)\text{: calculation, scientific properties}}}}$$

Therefore,

(M_1) [language is proposed], that is, "establish the verbal system"

(M_2) [describe phenomena by the mechanical language]

For example, it suffices to consider statistics (and moreover, quantum language). Also, recall that

$$(\text{M}_3) \quad \boxed{\text{statistics } (= \text{classical mechanical worldview})} \xrightarrow{\text{therefore}} \underset{\text{discussions}}{\overset{\text{statistical arguments}}{\boxed{\text{practical logic (i.e., calculation, inference), control}}}}$$

It was such a huge success that it has been said:

(M_4) **statistics is the language of science**

Also,

$$(\text{M}_5) \quad \boxed{\text{quantum language } (= \text{quantum mechanical worldview})} \xrightarrow{\text{therefore}} \underset{\text{discussions}}{\overset{\text{statistical arguments}}{\boxed{\text{practical logic (i.e., calculation, inference), control}}}}$$

Assertion 1.3 says that, for example,

- Economics is to describe economical phenomena in terms of quantum language (or statistics)
- psychology is to describe psychological phenomena in terms of quantum language (or statistics)

1.4 Preview of the assertions in this paper

1.4.1 Realistic worldview?, (fictional, mechanical) worldview?

Classification 1.11. [**= Classification 1.9: the realistic worldview, the fictional worldview, the logical worldview, the mechanical worldview**]]

- $(\flat_1)$: the realistic worldview (monism) : Aristotle, Archimedes, Galileo, Newton, Einstein
- $(\flat_2)$: the idealistic worldview (dualism)
  - $(\flat_{21})$: the fictional worldview ($\approx$ western philosophy)
    Plato, Descartes, Locle, Kant
  - $(\flat_{22})$: the logical worldview ($\approx$ analytic, scientific phil.)
    Frege, Saussure, Russell, Wittgenstein...
  - $(\flat_{23})$: the mechanical worldview
    statistics, quantum language

  Throughout this paper, we explain that

  $$\boxed{(\flat_{21})\text{: Plato}} \xrightarrow[\text{turn}]{\text{cogito}} \boxed{(\flat_{21})\text{: Descartes}} \xrightarrow[\text{revolution}]{\text{Copernican}} \boxed{(\flat_{21})\text{: Kant}}$$

  $$\xrightarrow[\text{turn}]{\text{linguistic}} \boxed{(\flat_{22})\text{: Wittgenstein}} \xrightarrow[\text{turn}]{\text{mechanical}} \boxed{(\flat_{23})\text{: QL}}$$

- $(\flat_3)$Others (Thinking Tip, etc.) : Darwin's theory of evolution, Hegel's dialectic, etc.
- $(\flat_4)$mathematics

////

I think mathematical logic in $(\flat_{22})$ should be classified in $(\flat_4)$, but for the sake of convenience, in this text I may think of a field of mathematics as a kind of worldview. That is because, compared to people in science, philosophers like the word "logic".

Assertion 1.12. [**Dispute: realistic worldview vs. idealistic worldview**]
"realistic worldview? idealistic worldview?" is the biggest dispute in the history of philosophy. However, from our view-point, the two have to exist together as follows.

Table 1.1 : realistic worldview vs. idealistic worldview

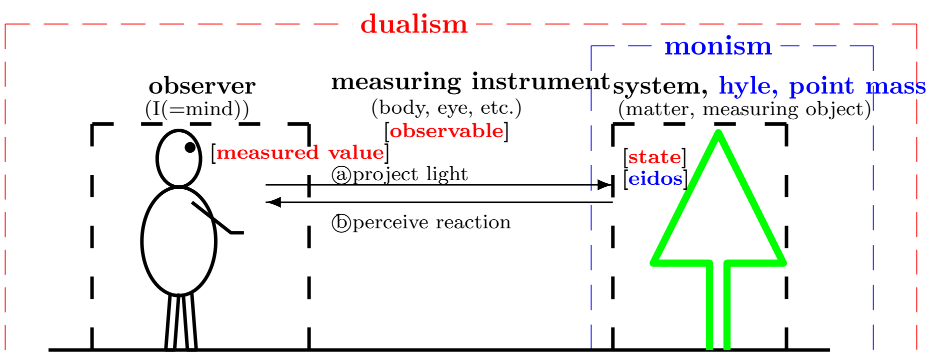
dispute ＼ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	"Nominalismus"(Ockham)	"Realismus"(Anselmus)
ⓓ: space-time	Newton	Leibniz
ⓔ: quantum theory	Einstein	Bohr

ⓐ is my fiction, ⓒ is a confusion. ⓓ is the Leibniz=Clarke correspondence (*cf.* Sec. 9.3.2), ⓔ is Bohr-Einstein debates. Quantum language is proposed as one of answers to Bohr-Einstein debates(*cf.* ref. [66]).

1.4.2 Keywords: Monism and dualism

Recall Figure 1.2 (in Section 1.1.2: The linguistic Copenhagen interpretation) below:

Figure 1.13. (Figure 1.2 in Section 1.1.2) [Descartes Figure]: Image of "measurement(=ⓐ+ⓑ)" in dualism: **dualism** and **monism**



Assertion 1.14. [The correspondence of key-words]
It is a matter of course that each worldview has the corresponding key-words. If Western philosophy (i.e., worldview) makes progress, its key-words are naturally refined and clarified. The key-word's progress of the realistic worldview [resp. idealistic worldview] is written as follows.

[The key-words of the dualism] The idealistic worldview is the mind-matter dualism, which is composed of three key-words, that is, [A](= mind), [C](= matter) and [B](= body: something connecting [A:mind] and [C:matter]). Thus, we see that:

\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
Plato	actual world	Idea	/ [Idea world]
Thomas Aquinas	universale post rem	universale ante rem	/ [universale in re]
Descartes	I, mind, brain	body	/ [matter]
Locke	mind	secondary quality	primary quality [matter]
Berkeley	mind	secondary quality	/ [God]
Kant	phenomenon	recognition	/ [thing-in-itself]
Saussure Zadeh	/	signified [signifier] fuzzy set (= membership function)	/
Wittgenstein	truth value	[proposition]	logical space [object]
statistics	sample (space)	/ [trial]	parameter [population]
quantum mechanics	measured value	observable [measurement]	state [particle]
quantum language	**measured value**	**observable** [measurement]	**state** **[system]**

[The key-words of the monism] The realistic worldview is monism, and its completed version is realized as Newtonian mechanics, whose key-words are only "point mass" and "state". Thus, we see:

\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
Aristotle	nothing	nothing	eidos [hyle]
Newton	nothing	nothing	state [point mass]

That is, we consider the following progress:

$$[\text{eidos}] \xrightarrow[\text{progress}]{\text{quantification}} [\text{state}], \qquad [\text{hyle}] \xrightarrow[\text{progress}]{} [\text{point mass}]$$

♠**Note 1.17.** (i): It is somewhat unreasonable to regard Plato's Idea theory as a type of measurement theory. Therefore, Plato and Aquinas above are doubtful, but I dare to write (*cf.* Sec. 3.3).

(ii): In mind-matter dualism, [B: medium] is the most important (*cf.* the linguistic Copenhagen interpretation (E_3) in Sec. 1.1.2). Thus, we consider that Plato's Idea theory is dualism. On the other hand, statistics lacks [B: medium]. Thus, statistics is not usually regarded as dualism but mathematical theory. However, in this paper, statistics is listed up as "incomplete dualism" in the above table.

(iii): The readers may wonder that "actual world≈mind(=human)" in Plato. However, it should be understood under the maxim: "Man is the measure of all things". Similarly, we think that "measured value≈mind(=human)" in quantum language. That is because there is no "measured value" without "mind(=brain)".

(iv): In [C: matter], the terms "state" and "system" in quantum language are always used as the form "the system with the state" (*cf.* Axiom 1 in Sec. 1.1). In the history of western philosophy, "state" and "system" were often confused, and in most cases, only one of the two has been found. Also, seeing statistics in the above table, the reader may find that understanding the difference between "parameter" and "population" is surprisingly difficult. And thus, in this paper, the difference is not emphasized.

Corollary 1.15. Of the three (i.e., "measured value", "observable", "state"), the "observable" is the most difficult to understand. Thus, Let me summarize the history of "observable" as follows.

- $$\begin{tikzcd}
\begin{array}{c}\boxed{\text{Idea}}\\ \text{Plato's idea theory}\end{array} \arrow[r, "\text{linguistic version}"] \arrow[d, "\text{congnitive}"', "\text{version}"] & \begin{array}{c}\boxed{\text{signifier, signfied}}\\ \text{Saussure's linguistic theory}\end{array} \arrow[d, "\text{quantification}"] \\
\begin{array}{c}\boxed{\text{body, secondary quality}}\\ \text{Descartes, J. Locke}\end{array} \arrow[dr, "\text{quantification}"'] & \begin{array}{c}\boxed{\text{membership function}}\\ \text{Zadeh's fuzzy theory}\end{array} \arrow[d, "\text{identification}"] \\
 & \begin{array}{c}\boxed{\text{observable (= measuring instrument)}}\\ \text{quantum language}\end{array}
\end{tikzcd}$$

Of course, Plato's Idea can be interpreted in a variety of ways. Therefore, the above figure may be my desire.

1.5 Appendix: The mathematical foundations of quantum language

Although we will explain "quantum language (= measurement theory") in this section, I may have omitted too much. For the precise explanation, see [66]. Rather, it might be recommended to skip this section.

1.5.1 Mathematical preparations

In this paper we are mainly interested in the spirit of quantum language, not the theoretical part of it. For the theoretical aspect, see ref. [66]. However, I would like to write some theoretical explanations of philosophical problems in this paper. For this, I simply add the mathematical foundations of quantum language in this section. Even if you skip this section, you will understand most of the text.
Following refs. [40, 42, 43, 66] (all our results until present are summarized in ref. [66]), we will review quantum language, which has the following form:

$$\underset{(=\text{ measurement theory})}{\boxed{\boxed{\text{Quantum language}}}} = \underset{(\text{Axiom 1})}{\boxed{\text{measurement}}} + \underset{(\text{Axiom 2})}{\boxed{\text{causality}}} + \underset{(\text{how to use Axioms 1 and 2})}{\boxed{\text{linguistic (Copenhagen) interpretation}}} \tag{1.5}$$

which asserts that "measurement" and "causality" are the most important concepts in science.

Consider an operator algebra $B(H)$ (i.e., an operator algebra composed of all bounded linear operators on a Hilbert space H with the norm $\|F\|_{B(H)} = \sup_{\|u\|_H=1} \|Fu\|_H$), and consider the triplet $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$ (or, the pair $[\mathcal{A}, \mathcal{N}]_{B(H)}$), called a *basic structure*. Here, $\mathcal{A}(\subseteq B(H))$ is a C^*-algebra, and $\mathcal{N}$ ($\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)$) is a particular C^*-algebra (called a W^*-algebra) such that $\mathcal{N}$ is the weak closure of $\mathcal{A}$ in $B(H)$.
The measurement theory (= "quantum language" = "the linguistic Copenhagen interpretation") is classified as follows.

$$\text{(A)} \quad \underset{(=\text{ quantum language})}{\text{measurement theory}} = \begin{cases} (\mathrm{A}_1)\text{: quantum system theory} & (\text{when } \mathcal{A} = \mathcal{C}(H)) \\ \\ (\mathrm{A}_2)\text{: classical system theory} & (\text{when } \mathcal{A} = C_0(\Omega)) \end{cases}$$

The Hilbert space method for the mathematical foundations of quantum mechanics is essentially due to von Neumann (*cf.* ref. [103]). He devoted himself to quantum (A_1). On the other hand, in most cases, we devote ourselves to classical (A_2), and not (A_1). However, the quantum (A_1) is convenient for us, in the sense that the idea in (A_1) is often introduced into classical (A_2).
When $\mathcal{A} = \mathcal{C}(H)$, the C^*-algebra composed of all compact operators on a Hilbert space H, the (A_1) is called quantum measurement theory (or, quantum system theory), which can be regarded as the linguistic aspect of quantum mechanics. Also, when $\mathcal{A}$ is commutative (that is, when $\mathcal{A}$ is characterized by $C_0(\Omega)$, the C^*-algebra composed of all continuous complex-valued functions vanishing at infinity on a locally compact Hausdorff space Ω (*cf.* refs. [91, 100, 108])), the (A_2) is called classical measurement theory.

Also, note that, when $\mathcal{A} = \mathcal{C}(H)$, i.e., quantum cases,

(B_1) $\mathcal{A}^*(= \mathcal{C}(H)^*) = Tr(H)$ (=trace class), $\mathcal{N} = B(H)$, $\mathcal{N}_* = Tr(H)$ (i.e., pre-dual space), thus, ${}_{Tr(H)}\big(\rho, T\big)_{B(H)} = \mathrm{Tr}_H(\rho T)$ ($\rho \in Tr(H), T \in B(H)$).

Also, when $\mathcal{A} = C_0(\Omega)$, i.e., classical cases, (B$_2$) $\mathcal{A}^*(= C_0(\Omega)^*) = \mathcal{M}(\Omega)$ i.e., "the space of all signed measures on Ω", $\mathcal{N} = L^\infty(\Omega,\nu)(\subseteq B(L^2(\Omega,\nu)))$, $\mathcal{N}_* = L^1(\Omega,\nu)$, where ν is some measure on Ω (with the Borel field $\mathcal{B}_\Omega$, thus, ${}_{L^1(\Omega,\nu)}\big(\rho, T\big)_{L^\infty(\Omega,\nu)} = \int_\Omega \rho(\omega)T(\omega)\nu(d\omega)$ $(\rho \in L^1(\Omega,\nu), T \in L^\infty(\Omega,\nu))$ (*cf.* ref. [100]).

In most cases, we devote ourselves to a compact space Ω with a probability measure ν (i.e., $\nu(\Omega) = 1$) and thus, $C_0(\Omega)$ is simply denoted by $C(\Omega)$.

Let $\mathcal{A}(\subseteq \mathcal{N} \subseteq B(H))$ be a C^*-algebra, and let $\mathcal{A}^*$ be the dual Banach space of $\mathcal{A}$. That is, $\mathcal{A}^* = \{\rho \mid \rho$ is a continuous linear functional on $\mathcal{A}$ $\}$, and the norm $\|\rho\|_{\mathcal{A}^*}$ is defined by $\sup\{|\rho(F)| \mid F \in \mathcal{A}$ such that $\|F\|_{\mathcal{A}}(= \|F\|_{B(H)}) \le 1\}$. Define the *mixed state* ρ $(\in \mathcal{A}^*)$ such that $\|\rho\|_{\mathcal{A}^*} = 1$ and $\rho(F) \ge 0$ for all $F \in \mathcal{A}$ such that $F \ge 0$. And define the mixed state space $\mathfrak{S}^m(\mathcal{A}^*)$ such that

$$\mathfrak{S}^m(\mathcal{A}^*)=\{\rho \in \mathcal{A}^* \mid \rho \text{ is a mixed state}\}.$$

Similarly , define the W^*-*mixed state* ρ_m $(\in \mathcal{N}_*)$ such that $\|\rho_m\|_{\mathcal{N}_*} = 1$ and $\rho_m(F) \ge 0$ for all $F \in \mathcal{N}$ such that $F \ge 0$. And define the W^*-mixed state space $\overline{\mathfrak{S}}^m(\mathcal{N}_*)$ such that

$$\overline{\mathfrak{S}}^m(\mathcal{N}_*)=\{\rho \in \mathcal{N}_* \mid \rho \text{ is a } W^*\text{-mixed state}\}.$$

A mixed state $\rho(\in \mathfrak{S}^m(\mathcal{A}^*))$ is called a *pure state* if it satisfies that "$\rho = \theta\rho_1 + (1-\theta)\rho_2$ for some $\rho_1, \rho_2 \in \mathfrak{S}^m(\mathcal{A}^*)$ and $0 < \theta < 1$" implies "$\rho = \rho_1 = \rho_2$". Put

$$\mathfrak{S}^p(\mathcal{A}^*)=\{\rho \in \mathfrak{S}^m(\mathcal{A}^*) \mid \rho \text{ is a pure state}\},$$

which is called a *state space.* It is well known (*cf.* ref. [100]) that $\mathfrak{S}^p(\mathcal{C}(H)^*) = \{|u\rangle\langle u|$ (i.e., the Dirac notation) $\mid \|u\|_H = 1\}$, and $\mathfrak{S}^p(C_0(\Omega)^*) = \{\delta_{\omega_0} \mid \delta_{\omega_0}$ is a point measure at $\omega_0 \in \Omega\}$, where $\int_\Omega f(\omega)\delta_{\omega_0}(d\omega) = f(\omega_0)$ $(\forall f \in C_0(\Omega))$. The latter implies that $\mathfrak{S}^p(C_0(\Omega)^*)$ can be also identified with Ω (called a *spectrum space* or simply *spectrum*) such as

$$\underset{\text{(state space)}}{\mathfrak{S}^p(C_0(\Omega)^*)} \ni \delta_\omega \leftrightarrow \omega \in \underset{\text{(spectrum)}}{\Omega}$$

In this paper, Ω and $\omega(\in \Omega)$ is respectively called a state space and a state.

In Axiom 1 later (or, more generally, in Axiom$^{(m)}$ 1) , we need the value of ${}_{\mathcal{A}^*}\big(\rho, G\big)_{\mathcal{N}}$, where $\rho \in \mathfrak{S}^m(\mathcal{A}^*)$, $G \in \mathcal{N}$. In quantum cases, we see that ${}_{\mathcal{A}^*}\big(\rho, G\big)_{\mathcal{N}} = {}_{Tr(H)}\big(\rho, G\big)_{B(H)}$. Thus, the value of ${}_{\mathcal{A}^*}\big(\rho, G\big)_{\mathcal{N}}$ is clearly determined. However, in classical cases (i.e., ${}_{\mathcal{A}^*}\big(\rho, G\big)_{\mathcal{N}} = {}_{\mathcal{M}(\Omega)}\big(\rho, G\big)_{L^\infty(\Omega,\nu)}$), we have to prepare the following definition.

(C$_1$) [Essentially continuous in general cases] An element $F(\in \mathcal{N})$ is said to be *essentially continuous at* $\rho_0(\in \mathfrak{S}^m(\mathcal{A}^*))$, if there uniquely exists a complex number α such that

- if ρ $(\in \mathcal{N}_*,\ \|\rho\|_{\mathcal{N}_*} = 1,\ \rho \ge 0)$ converges to $\rho_0(\in \mathfrak{S}^m(\mathcal{A}^*))$ in the sense of weak* topology of $\mathcal{A}^*$, that is,

$$\rho(G) \longrightarrow \rho_0(G) \ \ (\forall G \in \mathcal{A}(\subseteq \mathcal{N})),$$

then $\rho(F)$ converges to α.

(C$_2$) [Essentially continuous in quantum cases (i.e., $\mathcal{C}(H) \subseteq B(H) = B(H)$)] An element $F(\in B(H))$ is said to be *essentially continuous at* $|e\rangle\langle e|$ $(e \in H, ||e||_H = 1)$, if there uniquely exists a complex number α such that

- if ρ $(\in Tr(H),\ \|\rho\|_{Tr(H)} = 1,\ \rho \geq 0)$ converges to $|e\rangle\langle e|$ in the sense of weak* topology of $Tr(H)$, that is,

$$ {}_{Tr(H)}\big(\rho, G\big)_{\mathcal{C}(H)} \longrightarrow {}_{Tr(H)}\big(|e\rangle\langle e|, G\big)_{\mathcal{C}(H)} (= \langle e, Ge\rangle_H) \ \ (\forall G \in \mathcal{C}(H)(\subseteq B(H))), $$

then ${}_{Tr(H)}\big(\rho, F\big)_{B(H)}$ converges to α_{ω_0}.

(C$_3$) [Essentially continuous in classical cases] An element $F(\in L^\infty(\Omega,\nu))$ is said to be *essentially continuous at* $\delta_{\omega_0}(\in \mathfrak{S}^p(\mathcal{M}(\Omega)))$ (i.e., at $\omega_0(\in \Omega)$), if there uniquely exists a complex number α_{ω_0} such that

- if ρ $(\in L^1(\Omega,\nu),\ \|\rho\|_{L^1(\Omega,\nu)} = 1,\ \rho \geq 0)$ converges to $\delta_{\omega_0}(\in \mathfrak{S}^p(\mathcal{M}(\Omega)))$ in the sense of weak* topology of $\mathcal{M}(\Omega)$, that is,

$$ \rho(G)(= \int_\Omega G(\omega)\rho(\omega)\nu(d\omega)) \longrightarrow \int_\Omega G(\omega)\delta_{\omega_0}(d\omega)(= G(\omega_0)) \quad (\forall G \in C_0(\Omega)(\subseteq L^\infty(\Omega,\nu))), \tag{1.6} $$

then $\rho(F)(= \int_\Omega F(\omega)\rho(\omega)\nu(d\omega))$ converges to α_{ω_0}.

Define $\widehat{F}$ $(\in L^\infty(\Omega))$ such that

$$ \widehat{F}(\omega) = \begin{cases} \alpha_\omega & (\text{ if } F \text{ is essentally continuos at } \omega) \\ F(\omega) & (\text{otherwise}) \end{cases} $$

Let $D \in \mathcal{B}_\Omega$. Define $\chi_D \in L^\infty(\Omega,\nu)$ such that

$$ \chi_D(\omega) = \begin{cases} 1 & (\text{ if } \omega \in D) \\ 0 & (\text{otherwise}) \end{cases} $$

Then define $\widehat{D}$ such that $\{\omega \in \Omega \mid \widehat{\chi}_D(\omega) = 1\}$

Assumption 1.16. (i): *When we consider $F(\in L^\infty(\Omega,\nu))$, we always assume that F satisfies that $F = \widehat{F}$.*
(ii): *When we consider $D(\in \mathcal{B}_\Omega)$, we always assume that D satisfies that $D = \widehat{D}$.*

The following definition is due to E.B. Davies (*cf.* ref. [13]).

Definition 1.17. [Observable] An *observable* O $=(X, \mathcal{F}, F)$ in $\mathcal{N}$ is defined as follows:

(i) [σ-field] X is a set, $\mathcal{F}(\subseteq 2^X \equiv \mathcal{P}(X)$, the power set of $X)$ is a σ-field of X, that is, "$\Xi_1, \Xi_2, ... \in \mathcal{F} \Rightarrow \cup_{n=1}^\infty \Xi_n \in \mathcal{F}$", "$\Xi \in \mathcal{F} \Rightarrow X \setminus \Xi(\equiv \{x \mid x \in X, x \notin \Xi\} \equiv \Xi^c$, i.e., the complement of $\Xi) \in \mathcal{F}$".

(ii) [Countable additivity] F is a mapping from $\mathcal{F}$ to $\mathcal{N}$ satisfying: (a): for every $\Xi \in \mathcal{F}$, $F(\Xi)$ is a non-negative element in $\mathcal{N}$ such that $0 \leq F(\Xi) \leq I$, (b): $F(\emptyset) = 0$ and $F(X) = I$, where 0 and I is the 0-element and the identity in $\mathcal{N}$ respectively. (c): for any countable decomposition $\{\Xi_1, \Xi_2, \ldots, \Xi_n, ...\}$ of Ξ $\big($i.e., $\Xi, \Xi_n \in \mathcal{F}$ $(n = 1, 2, 3, ...)$, $\cup_{n=1}^\infty \Xi_n = \Xi$, $\Xi_i \cap \Xi_j = \emptyset$ $(i \neq j)\big)$, it holds that $F(\Xi) = \sum_{n=1}^\infty F(\Xi_n)$ in the sense of weak* topology in $\mathcal{N}$.

In classical case, an observable $(X, \mathcal{F}, F)$ in $L^\infty(\Omega,\nu)$ is said to be *continuous observable*, if $F(\Xi)$ is essentially continuous on Ω $(\forall \Xi \in \mathcal{F})$.

1.5.2 Measurement theory; Axiom 1 [Measurement] and Axiom 2 [Causality]

Quantum language ([35, 66]) is composed of two axioms (i.e., Axioms 1 and 2) as follows. With any *system* S, a basic structure $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$ can be associated in which the measurement theory (A) of that system can be formulated. A *state* of the system S is represented by an element $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$ and an *observable* is represented by an observable $\mathsf{O} = (X, \mathcal{F}, F)$ in $\mathcal{N}$. Also, the *measurement of the observable* O *for the system* S *with the state* ρ is denoted by $\mathsf{M}_{\mathcal{N}}(\mathsf{O}, S_{[\rho]})$ $\Big($ or more precisely, $\mathsf{M}_{\mathcal{N}}(\mathsf{O} := (X, \mathcal{F}, F),\ S_{[\rho]})$ $\Big)$. An observer can obtain a measured value x $(\in X)$ by the measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O}, S_{[\rho]})$.

The Axiom 1 presented below is a kind of mathematical generalization of Born's probabilistic interpretation of quantum mechanics. And thus, it is a statement without reality.

Now we can present Axiom 1 in the W^*-algebraic formulation as follows.

Axiom 1 [Measurement, the probabilistic interpretation of science] (which is the same as Axiom 1 mentioned in Section 1.1)

With any system S, a basic structure $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$ can be associated in which measurement theory of that system can be formulated. When the observer takes a measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O} := (X, \mathcal{F}, F), S_{[\rho]})$ $\Big($ i.e., a measurement of an observable (or, by a measuring instrument) $\mathsf{O} = (X, \mathcal{F}, F)$ for a system $S_{[\rho]}$ i.e., a system S with a state $\rho)\Big)$, the probability that a measured value x $(\in X)$ obtained by the measurement belongs to Ξ $(\in \mathcal{F})$ is given by $\rho(F(\Xi))(\equiv {}_{\mathcal{A}^*}(\rho, F(\Xi))_{\mathcal{N}})$ if $F(\Xi)$ is essentially continuous at $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$.

This axiom gives answers to "What is probability?" and "What is measurement?". (Recall Problem 1.1.)

Remark 1.18. (i): In quantum cases (i.e., the cases that $\rho \in \mathfrak{S}^p(Tr(H)) \subseteq Tr(H)$, $F(\Xi) \in B(H)$), the probability $\rho(F(\Xi))(= {}_{Tr(H)}\big(\rho, T\big)_{B(H)} = \mathrm{Tr}_H(\rho T))$ is always defined (*cf.* (E_1)). That is, the $F(\Xi)$ is always essentially continuous at any $\rho \in \mathfrak{S}^p(Tr(H))$). On the other hand, in the classical cases (i.e., the cases that $\omega \in \Omega$, $F(\Xi) \in L^\infty(\Omega, \nu)$), it is not guaranteed that $F(\Xi)$ is essentially continuous at ω $(\in \Omega)$). Thus, put $\mathcal{F}_{\omega_0} = \{\Xi \in \mathcal{F} : F(\Xi) \text{ is essentially continuous at } \omega_0\ \}$. If $\mathcal{F}_{\omega_0} = \mathcal{F}$, the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O} := (X, \mathcal{F}, F), S_{[\omega_0]})$ makes the sample probability space $(X, \mathcal{F}, [F(\cdot)](\omega_0))$, which is usual in statistics. Thus, roughly speaking, statistics starts from "sample probability space", on the other hand, quantum language starts from "measurement".
(ii): In quantum cases, Axiom 1 is well known as Born's probabilistic interpretation of quantum mechanics. In classical case, it is natural to ask "Why does probability appear?". For the answer of this question, see Chapter 1 (Section 1.3) page 12 in ref. [66]: *Example: measurement of "Cold or Hot"*.
(iii): Axiom 1 is the quantitative realization of the spirit: "there is no science without measurements". And, we think that Axiom 1 means the probabilistic interpretation of science since it is a kind of mathematical generalization of Born's probability interpretation of quantum mechanics.

Example 1.19. [(i): projective observable] An observable $\mathsf{O} = (X, \mathcal{F}, F)$ in $\mathcal{N}$ is called a projective observable if it holds that $F(\Xi) = F(\Xi)^2$ $(\forall \Xi \in \mathcal{F})$. The following exact observable is typical in projective observables.
[(ii): Exact measurement, exact observable] Consider a basic structure $[C(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \mathcal{B}_\Omega, \nu))]$. Define the exact observable $\mathsf{O}_e = (X(= \Omega), \mathcal{F}(= \mathcal{B}_\Omega), F_e)$ in $L^\infty(\Omega, \nu)$ such that

$$[F_e(\Xi)](\omega) = 1 \ \ (\omega \in \Xi \in \mathcal{F}), \quad = 0 \ \ (\omega \notin \Xi \in \mathcal{F}).$$

Let $\omega^0 \in \Omega$. Thus, we have the exact measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_e = (X, \mathcal{F}, F_e), S_{[\omega^0]})$. Then we have the following statement

(E) Let $D(\subseteq X = \Omega)$ be any open set such that $\omega^0 \in D$. Then Axiom 1 says that the probability that the measured value $x(\in X)$ obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}_e = (X, \mathcal{F}, F_e), S_{[\omega^0]})$ belongs to D is given by 1.

This implies that $x = \omega^0$, since D is arbitrary open set such that $\omega^0 \in D$. Also, it should be noted that $F_e(\Xi)$ is not essentially continuous at ω^0 if $\omega^0 \in \partial(\Xi)$, i.e., the boundary of Ξ. Thus, $(X, \mathcal{F}, [F_e(\cdot)](\omega^0))$ is not always a probability space. However, note that there exists a probability space $(X, \mathcal{F}, \mu)$ such that $[F_e(\Xi)](\omega^0) = \mu(\Xi)$ $(\forall \Xi(\in \mathcal{F})$ such that $\omega^0 \notin \partial(\Xi)$), though the uniqueness is not guaranteed. The probability space $(X, \mathcal{F}, \mu)$ is often called the sample space of the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_e = (X, \mathcal{F}, F_e), S_{[\omega^0]})$

Next, we explain Axiom 2. Let $[\mathcal{A}_1, \mathcal{N}_1 \subseteq B(H_1)]$ and $[\mathcal{A}_2, \mathcal{N}_2 \subseteq B(H_2)]$ be basic structures. A continuous linear operator $\Phi_{1,2} : \mathcal{N}_2$ (with weak* topology) $\to \mathcal{N}_1$(with weak* topology) is called a *Markov operator,* if it satisfies that (i): $\Phi_{1,2}(F_2) \geq 0$ for any non-negative element F_2 in $\mathcal{N}_2$, (ii): $\Phi_{1,2}(I_2) = I_1$, where I_k is the identity in $\mathcal{N}_k$, $(k = 1, 2)$. In addition to the above (i) and (ii), we assume that $\Phi_{1,2}(\mathcal{A}_2) \subseteq \mathcal{A}_1$ and $\sup\{\|\Phi_{1,2}(F_2)\|_{\mathcal{A}_1} \mid F_2 \in \mathcal{A}_2$ such that $\|F_2\|_{\mathcal{A}_2} \leq 1\} = 1$.

It is clear that the dual operator $\Phi_{1,2}^* : \mathcal{A}_1^* \to \mathcal{A}_2^*$ satisfies that $\Phi_{1,2}^*(\mathfrak{S}^m(\mathcal{A}_1^*)) \subseteq \mathfrak{S}^m(\mathcal{A}_2^*)$. If it holds that $\Phi_{1,2}^*(\mathfrak{S}^p(\mathcal{A}_1^*)) \subseteq \mathfrak{S}^p(\mathcal{A}_2^*)$, the $\Phi_{1,2}$ is said to be deterministic. If it is not deterministic, it is said to be non-deterministic. Also note that, for any observable $\mathsf{O}_2 :=(X, \mathcal{F}, F_2)$ in $\mathcal{N}_2$, the $(X, \mathcal{F}, \Phi_{1,2}F_2)$ is an observable in $\mathcal{N}_1$.

Definition 1.20. [Sequential causal operator; Heisenberg picture of causality] Let $(T, \leq)$ be a tree like semi-ordered set such that "$t_1 \leq t_3$ and $t_2 \leq t_3$" implies "$t_1 \leq t_2$ or $t_2 \leq t_1$". The family $\{\Phi_{t_1,t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}\}_{(t_1,t_2) \in T^2_{\leqq}}$ is called a **sequential causal operator**, if it satisfies that

(i) For each t $(\in T)$, a basic structure $[\mathcal{A}_t \subseteq \mathcal{N}_t \subseteq B(H_t)]$ is determined.

(ii) For each $(t_1, t_2) \in T^2_{\leqq}$, a causal operator $\Phi_{t_1,t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}$ is defined such as $\Phi_{t_1,t_2}\Phi_{t_2,t_3} = \Phi_{t_1,t_3}$ $(\forall (t_1, t_2), \forall (t_2, t_3) \in T^2_{\leqq})$. Here, $\Phi_{t,t} : \mathcal{N}_t \to \mathcal{N}_t$ is the identity operator.

Now we can propose Axiom 2 (i.e., causality). (For details, see ref. [66].)

> **Axiom 2**[Causality] (which is the same as Axiom 2 mentioned in Section 1.1)
>
> Let T be a **tree** (i.e., semi-ordered tree structure). For each $t(\in T)$, a basic structure $[\mathcal{A}_t \subseteq \mathcal{N}_t \subseteq B(H_t)]$ is associated. Then, the **causal chain** is represented by a **sequential causal operator** $\{\Phi_{t_1,t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}\}_{(t_1,t_2) \in T^2_{\leqq}}$.

When parameters t_1, t_2 $(t_1 < t_2)$ are regarded as time, we usually consider that a causal operator $\Phi_{t_1,t_2} : \mathcal{N}_{t_2} \to \mathcal{N}_{t_1}$ represents "causality". Thus, this axiom gives an answer to "What is causality?". That is, we consider that, if Axiom 2 is used in the quantum linguistic representation of a phenomenon, causality exists in the phenomenon.

1.5.3 Mixed measurement theory; Axiom$^{(m)}$ 1 [Mixed measurement] and Axiom 2 [Causality]

Measurement theory is classified as follows:

(F) measurement theory

$$
= \begin{cases} (\mathrm{F}_1)\text{: (pure) measurement } (= \text{Axiom 1} + \text{Axiom 2}) \text{ in Sections 1.1.1 or 1.5.2} \\ \qquad \text{this corresponds to "Fisher's statistics} \\ \\ (\mathrm{F}_2)\text{: mixed measurement } (=\text{Axiom}^{(m)} \text{ 1} + \text{Axiom 2}) \\ \qquad \text{this corresponds to "Bayesian statistics} \end{cases}
$$

Let us explain the mixed measurement theory. Axiom 2 and the linguistic Copenhagen interpretation are common in both (pure) measurement theory and mixed measurement theory. Thus, it suffices to explain Axiom$^{(m)}$ 1 (mixed measurement) as follows.

Mixed measurement is composed of two axioms (i.e., Axiom$^{(m)}$ 1 and Axiom 2) as follows. With any *system* S, a basic structure $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$ can be associated in which the mixed measurement theory (F_2) of that system can be formulated. A *mixed state* of the system S is represented by an element $\rho_m (\in \mathfrak{S}^m(\mathcal{A}^*))$ and an *observable* is represented by an observable $\mathsf{O} = (X, \mathcal{F}, F)$ in $\mathcal{N}$. Also, the *mixed measurement of the observable* O *for the system* S *with the mixed state* ρ_m is denoted by $\mathsf{M}_{\mathcal{N}}(\mathsf{O}, S_{[*]}(\rho_m))$ $\big($ or more precisely, $\mathsf{M}_{\mathcal{N}}(\mathsf{O} := (X, \mathcal{F}, F),\ S_{[*]}(\rho_m))$ $\big)$. An observer can obtain a measured value x $(\in X)$ by the measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O}, S_{[*]}(\rho_m))$.

The Axiom$^{(m)}$ 1 presented below is a kind of mathematical generalization of Born's probabilistic interpretation of quantum mechanics. And thus, it is a statement without reality.

Now we can present Axiom$^{(m)}$ 1 in the W^*-algebraic formulation as follows.

Axiom$^{(m)}$ 1 [Mixed measurement, the probabilistic interpretation of science]

With any system S, a basic structure $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$ can be associated in which measurement theory of that system can be formulated. When an observer takes a mixed measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O} := (X, \mathcal{F}, F),\ S_{[*]}(\rho^m))$ $\Big($ i.e., a measurement of an observable (or, by a measuring instrument) $\mathsf{O} = (X, \mathcal{F}, F)$ for a system $S_{[*]}(\rho^m)$ i.e., a system S with a mixed state ρ^m $\Big)$, the probability that a measured value x $(\in X)$ belongs to Ξ $(\in \mathcal{F})$ is given by $\rho(F(\Xi)) (\equiv {}_{\mathcal{N}_*}(\rho^m, F(\Xi))_{\mathcal{N}})$ if $F(\Xi)$ is essentially continuous at $\rho (\in \mathfrak{S}^m(\mathcal{A}^*))$.

1.5.4 Several results derived from the linguistic Copenhagen interpretation

1.5.4.1 Parallel measurement

As the further explanation of parallel measurement in the linguistic Copenhagen interpretation (E_4) in Sec. 1.1.2, we have to add the following definition.

Definition 1.21. [Parallel measurement (*cf.* [66])] Though the parallel measurement can be defined in both classical and quantum systems, we, for simplicity, devote ourselves to classical systems as follows. Let $[C(\Omega) \subseteq L^\infty(\Omega.\nu) \subseteq [B(L^2(\Omega, \nu))]$ be a classical basic structure, where we assume, for simplicity, that Ω is compact space and ν is a measure such that $\nu(\Omega) = 1$ and $\nu(D) > 0$ ($\forall$open set $D \subseteq \Omega$). Consider a family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X_i, \mathcal{F}_i, F_i), S_{[\omega_i]}) \mid i = 1, 2, ..., N\}$. However, the linguistic Copenhagen interpretation (G_1) says "Only one measurement is permitted". Therefore, instead of the family of measurements, we consider the parallel measurement $\bigotimes_{i=1}^N \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X_i, \mathcal{F}_i, F_i), S_{[\omega_i]})$, which is defined by

$$
\bigotimes_{i=1}^{N} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X_i, \mathcal{F}_i, F_i), S_{[\omega_i]})
$$

$$
={\mathsf M}_{L^\infty(\times_{i=1}^N \Omega, \bigotimes_{i=1}^N \nu)}(\bigotimes_{i=1}^N {\mathsf O}_i := (\mathop{\times}_{i=1}^N X_i, \ \boxtimes_{i=1}^N {\mathcal F}_i, \ \bigotimes_{i=1}^N F_i), S_{[(\omega_1,\omega_2,\ldots,\omega_N)]})
$$

where $\times_{i=1}^N \Omega$ is the finite product compact space of Ωs, $\bigotimes_{i=1}^N \nu$ is the finite product probability of νs. Also, $\boxtimes_{i=1}^N {\mathcal F}_i$ $(\subseteq {\mathcal P}(\times_{i=1}^N X_i))$ is the infinite product σ-field, i.e., the smallest σ-field that includes

$$
\Big\{ \mathop{\times}_{i=1}^N \Xi_i | \quad \Xi_i \in {\mathcal F}_i \Big\}
$$

And further, define the observable $\bigotimes_{i=1}^N F_i$ in $L^\infty(\times_{i=1}^N \Omega, \bigotimes_{i=1}^N \nu)$ which satisfies that

$$
[(\bigotimes_{i=1}^N F_i)((\mathop{\times}_{i=1}^N \Xi_i)](\omega_1,\omega_2,\ldots,\omega_N) = \mathop{\times}_{i=1}^N [F_i(\Xi_i)](\omega_i) \quad \forall \Xi_i \in {\mathcal F}_i, \omega_i \in \Omega
$$

Then, Axiom 1 [measurement] says that

(G) the probability that a measured value obtained by the parallel measurement $\bigotimes_{i=1}^N {\mathsf M}_{L^\infty(\Omega,\nu)}$ $({\mathsf O}_i := (X_i, {\mathcal F}_i, F_i), S_{[\omega_i]})$ belongs to $\times_{i=1}^N \Xi_i$ is given by $\times_{i=1}^N [F_i(\Xi_i)](\omega_i)$, if $F_i(\Xi_i)$ is essentially continuous at ω_i $(\forall i = 1,2,\ldots,N)$.

Remark 1.22. The above finite parallel measurement can be generalized to the case that the index set Λ is infinite. That is,

$$
\begin{aligned}
&\bigotimes_{\lambda\in\Lambda} {\mathsf M}_{L^\infty(\Omega,\nu)}({\mathsf O}_\lambda := (X_\lambda, {\mathcal F}_\lambda, F_\lambda), S_{[\omega_\lambda]}) \\
=&{\mathsf M}_{L^\infty(\times_{\lambda\in\Lambda} \Omega, \bigotimes_{\lambda\in\Lambda} \nu)}(\bigotimes_{\lambda\in\Lambda} {\mathsf O}_\lambda := (\mathop{\times}_{\lambda\in\Lambda} X_\lambda, \ \boxtimes_{\lambda\in\Lambda} {\mathcal F}_\lambda, \ \bigotimes_{\lambda\in\Lambda} F_\lambda), S_{[(\omega_\lambda)_{\lambda\in\Lambda}]})
\end{aligned}
$$

The existence of the parallel measurement is guaranteed in both classical and quantum systems. Cf. § 4.2 in ref. [66]. It is not so difficult to extend the above finite parallel measurements to infinite parallel measurements for mathematicians. However, in this paper, we are not concerned with the infinite parallel measurement. That is because our concern is not mathematics but foundations of philosophy of science.

1.5.4.2 Practical logic (Implication, Syllogism)

Definition 1.23. [Image observable, marginal observable] Consider the basic structure $[{\mathcal A} \subseteq {\mathcal N} \subseteq B(H)]$. And consider the observable ${\mathsf O} = (X, {\mathcal F}, F)$ in ${\mathcal N}$. Let $(Y, {\mathcal G})$ be a measurable space, and let $\Theta : X \to Y$ be a measurable map. Then, we can define the image observable $\Theta({\mathsf O}) = (X, {\mathcal F}, F \circ \Theta^{-1})$ in ${\mathcal N}$, where $F \circ \Theta^{-1}$ is defined by

$$
(F \circ \Theta^{-1})(\Gamma) = F(\Theta^{-1}(\Gamma)) \qquad (\forall \Gamma \in {\mathcal G}).
$$

Here we add the following definition.

Definition 1.24. [Implication: *cf.* refs. [35, 66]] Consider a basic structure:

$$
[{\mathcal A} \subseteq {\mathcal N} \subseteq B(H)]
$$

Let $\mathsf{O}_1 = (X_1, \mathcal{F}_1, F_1)$ and $\mathsf{O}_2 = (X_2, \mathcal{F}_2, F_2)$ be observables in $\mathcal{N}$. Let $\mathsf{O}_{12} = (X_1 \times X_2, \mathcal{F}_1 \boxtimes \mathcal{F}_2, F_{12})$ be an observable such that $F_1(\Xi_1) = F_{12}(\Xi_1 \times X_2)$ and $F_2(\Xi_2) = F_{12}(X_1 \times \Xi_2)$ $(\forall \Xi_1 \in \mathcal{F}_1, \Xi_2 \in \mathcal{F}_2)$. Let $\rho \in \mathfrak{S}^p(\mathcal{A}^*)$, $\Gamma_1 \in \mathcal{F}_1$, $\Gamma_2 \in \mathcal{F}_2$. Then, if it holds that

$$\rho(F_{12}(\Gamma_1 \times (X_2 \setminus \Gamma_2))) = 0$$

this is denoted by

$$[\mathsf{O}_1; \Gamma_1] \underset{\mathsf{M}_{\mathcal{N}}(\mathsf{O}_{12}, S_{[\rho]})}{\Longrightarrow} [\mathsf{O}_2; \Gamma_2] \quad \text{or, equivalently,} \quad [\mathsf{O}_1; X_1 \setminus \Gamma_1] \underset{\mathsf{M}_{\mathcal{N}}(\mathsf{O}_{12}, S_{[\rho]})}{\Longleftarrow} [\mathsf{O}_2; X_2 \setminus \Gamma_2]$$

That is because the probability that a measured value $(x_1.x_2)$ obtained by the measurement $\mathsf{M}_{\mathcal{N}}(\mathsf{O}_{12} := (X_1 \times X_2, \mathcal{F}_1 \boxtimes \mathcal{F}_2, F_{12}), S_{[\rho]})$ belongs to $\Gamma_1 \times (X_2 \setminus \Gamma_2)$ is equal to 0.

Remark 1.25. [Syllogism] Using Definition 1.24, we showed that

- Syllogism always holds in classical systems (*cf.* ref. [35])
- Syllogism does not always hold in quantum systems (*cf.* ref. [47], § 8.7 in ref. [66])

1.5.4.3 Practical logic (Inference; Fisher's maximum likelihood method)

We begin with the following notation:

Notation 1.26. $[\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}, S_{[*]})]$*:* Consider a measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}{:=}(X, \mathcal{F}, F),\ S_{[\omega_0]})$ formulated in the basic structure $[C(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu))]$. Here, note that

(H$_1$) in most cases that the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}{:=}(X, \mathcal{F}, F),\ S_{[\omega_0]})$ is taken, it is usual to think that the state $\omega_0 (\in \Omega)$ is unknown.

That is because

(H$_2$) the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}, S_{[\omega_0]})$ may be taken in order to know the state ω_0.

Therefore, when we want to stress that we do not know the state ω_0, the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}{:=}(X, \mathcal{F}, F),\ S_{[\omega_0]})$ is often denoted by $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}{:=}(X, \mathcal{F}, F),\ S_{[*]})$

Theorem 1.27. [Inference; Fisher's maximum likelihood method (*cf.* ref. [38] or §5.2 in ref. [66]] For simplicity, assume that X is finite set. Assume that the measured value $x (\in X)$ is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}{:=}(X, 2^X, F),\ S_{[*]})$. Then, the unknown state $[*]$ can be inferred to be $\omega_0 (\in \Omega)$ such that

$$[F(\{x\})](\omega_0) = \max_{\omega \in \Omega} [F(\{x\})](\omega)$$

Proof. It is an easy consequence of Axiom 1 (*cf.* §5.2 in ref. [66]). □

Remark 1.28. [Inference and Control *cf.* §5.2 in ref. [66]] The inference problem is characterized as the reverse problem of measurements. That is, we consider that

(I$_1$) $\text{(state } \omega_0\text{, observable } \mathsf{O}) \xrightarrow[\text{measurement (Axiom 1)}]{\mathsf{M}_{L^\infty(\Omega,\nu)}\ (\mathsf{O}:=(X, 2^X, F),\ S_{[\omega_0]})} \text{measured value } x_0$

On the other hand

(I$_2$) (measured value x_0, observable O) $\xrightarrow[\text{inference (reverse Axiom 1)}]{\mathsf{M}_{L^\infty(\Omega,\nu)}\ (\mathsf{O}{:=}(X, 2^X, F),\ S_{[*]})}$ state ω_0

Thus, (I$_1$) and (I$_2$) are in reverse problem.
Also, we note, from the mathematical point of view, that inference problem (I$_3$) and control problem (I$_4$) are essentially the same as follows.

(I$_3$) [Inference problem; statistics]: when measured value x_0 is obtained, infer the unknown state ω_0

and

(I$_4$) [Control problem; dynamical system theory]: Settle the state ω_0 such that measured value x_0 will be obtained!

Thus, we think, from the theoretical point of view, that statistics and dynamical system theory are essentially the same. Thus, we consider that statistics (= dynamical system theory) is the mathematical representation of classical mechanical worldview. On the other hand, quantum language is regarded as the mathematical representation of quantum mechanical worldview.

Chapter 2

Ancient Greek philosophy (before Socrates)

Readers can start reading from this chapter (i.e., you can skip Chapter 1).

In Ancient Greek philosophy (before Socrates), the phase "the arche (= the first thing of all things) is ◯◯" is standard. Here "◯◯" is, for example, as follows.

Thalēs······water	Democritus······atom
Pythagoras······number	Hērakleitos······motion, fire
Parmenides······logic, motion	Zeno······logic, motion

In this chapter, we first point out that Parmenides' claim is very similar to the Copenhagen interpretation of quantum mechanics. Next we examine Zeno's paradoxes (the flying arrow, Achilles and the tortoise) from a quantum linguistic perspective and present a new view of Zeno's paradoxes. And we conclude that

- The confusion about Zeno's paradoxes is due to the lack of the worldviewism (called the motion function method) and not the mathematical problem concerning geometric series.

2.1 Thalēs (BC.640 - BC.546)

2.1.1 Thalēs: the first philosopher: " the arche (= the first thing of all things) is water"

Every race had its own "mythology". Myths are the literature that explains the world by reason of God. Myths have been handed down from generation to generation from our ancestors by oral and written word.

There is a great deal of maritime trade, and many ethnic groups come together to interact, but it is rare to find a region that is not united by force of arms because it is not a vast plain. This situation was realized on the eastern Mediterranean coast (now called Greece, Turkey, Syria, and Israel (Egypt)) around 1000 BC. In this region, various civilizations and cultures have merged to create a new culture. Particularly noteworthy was the realization of unification not by force of arms, but by culture. In other words, "philosophy as a synthesis of several myths" and "the alphabet as a synthesis of several letters" were born. That is,

$$(A)\begin{cases}\text{integration of several myths} & \Longrightarrow \text{philosophy} \\ \text{integration of the several characters} & \Longrightarrow \text{Phoenician alphabet} \Longrightarrow \text{alphabet}\end{cases}$$

The alphabet is a phonetic alphabet because it was created with the intention of being a common letter between different ethnic groups. Egypt had an advanced civilization (such as the pyramids), but it was so unified in thought that philosophy did not develop.

In Aristotle's book "Metaphysics," Thales Is called the "first philosopher".

For example, to the question "why does an earthquake occur?" Most myths will say "because God is angry". Thalēs appeared in Miletus in ancient Greece and said:

(B) The arche (= the first principle of all things) is water. Therefore, the earthquake is caused by the vibration of the water

This may be childish, but is an explanation that does not brought out the "God" . This is the reason why Aristotle said Thalēs was the "the first philosopher."

♠**Note 2.1.** S. Weinberg (1933 -), An American physicist, won a Nobel Prize of physics in 1979, said in his book [105] "To explain the word; The discovery of modern science" as follows:

($\sharp$) *It seems to me that to understand these early Greeks, it is better to think of them not as physicists or scientists or even philosophers, but as* **poets***.*

I agree with him. As mentioned earlier, I believe that the main purpose of the ancient Greek philosophers was not to pursue truth but to provide a common topic (i.e., a common myth, a universal myth) in order to avoid conflict between different peoples. This paper, however, may be more radical. As far as to describing the world (not ethics), Plato, Descartes, Kant, etc. all should be regarded as poets or fiction writers.

♠**Note 2.2.** Who is the first philosopher? Of course there may be a lot of opinions for this question. As mentioned above, Aristotle said Thalēs was the "the first philosopher." Also, A.N.Whitehead (1861 - 1947) said that

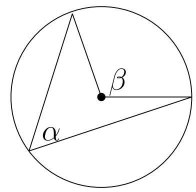
($\sharp$) **Western philosophy is characterized as a series of footnotes to Plato**

Although I do not know Whitehead's intension, I want to think that this means "Plato is the first philosopher", which is the same as the spirit of this paper mentioned later.

2.1.2 Thalēs' ability at math

By the statement:"The first principle of all things is water", we cannot judge Thalēs' knowledge, However, the following is known as Thalēs' theorem, which shows his ability of math:

Theorem 2.1. [central angle $= \beta$, inscribed angle$= \alpha$] $\Longrightarrow \beta = 2\alpha$



Proof. It suffices to draw the additional line through the center. □

♠**Note 2.3.** When Thales visited Egypt, there is a story that the king of Egypt was impressed by Thales' measuring the height of the pyramid in the way of measuring called triangulation. But, I think it is unreliable. Three great pyramids in the Egyptian Giza desert (deceased persons are Khufu, Khafre, Menkaure) erecting time of is the 2500 B.C. Thus I guess that the triangulation was known in those days (2000 years before Thalēs) in Egypt. If so, Thalēs' theorem should be doubt whether it is due to Thalēs. However, even as a true prover was unknown, the ability of mathematics at the time (i.e., the discovery of the concept of "proof") should be surprising. This led to Euclid's "Elements" (due to Euclid (275 BC - 330 BC)).

2.2 Pythagoras (BC.582 - BC.496)

2.2.1 The mathematical ability of Pythagorean religious organization

Pythagoras was a leader in the mathematics study group, which may be regarded as a kind of religious organization called Pythagorean religion. As the mathematical achievements, the followings are known:

- the discovery of irrational numbers, the Pythagorean theorem, the construction of a regular pentagon

and so on.

Theorem 2.2. Finding of irrational members, e.g., $\sqrt{2}$ is an irrational number.

Theorem 2.3. (Pythagorean theorem): In $\triangle$ABC, the followings are equivalent:

($\sharp_1$) $\angle \mathrm{A} = 90^\circ$

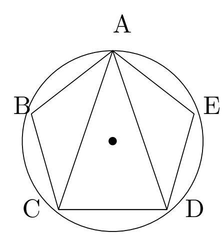
($\sharp_2$) $\mathrm{AB}^2 + \mathrm{CA}^2 = \mathrm{BC}^2$

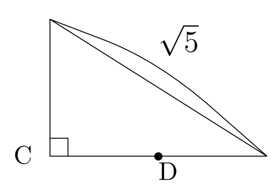
Construction 2.4. the construction of a regular pentagon

Explanation: In a regular pentagon as shown in the figure below (left), put AB = BC = CD = DE = EA = 1. Then, we see

$$\mathrm{AC} = \mathrm{AD} = \frac{1+\sqrt{5}}{2}$$

Hence, it suffices to construct $\frac{1+\sqrt{5}}{2}$. By the Pythagorean theorem, $\sqrt{5}(= \sqrt{1^2+2^2})$ can be constructed as follows (the figure below (right)). Thus, we easily get $\frac{1+\sqrt{5}}{2}$.





□

♠**Note 2.4.** It could have been something extra, but I wrote the explanation of the construction of a regular pentagon for beginners. Even university students in the department of mathematics sometimes don't know this.

♠**Note 2.5.** The above two (the discovery of irrational numbers and the Pythagorean theorem) are one of the most important discoveries in mathematics. If the following episodes are true, we can trust his mathematical ability.

($\sharp_1$) Pythagoras was killing the disciple, who found the irrational number, in order to hide the existence of irrational numbers.

($\sharp_2$) When Pythagoras discovered the Pythagorean theorem, he celebrated it, offered the sacrifice of the bull.

The two theorems, about 2000 years from the originally discovered to the scientific revolution (Descartes' Analytical Geometry, etc.; 17th century), not been used even once with an essential meaning. Nevertheless, their importance had been recognized in Pythagorean organization. This suggests a very high mathematical level of Pythagorean organization. Even if I had been a member of the Pythagorean school, I don't think I would have been able to recognize the importance of these two theorems as worthy of ($\sharp_1$) and ($\sharp_2$) above.

♠**Note 2.6.** (i): Pythagoras is said to have known that the earth was round. At the time, astronomy might be believed to be a part of mathematics. Greek astronomy develops in the following way.

$$(\flat_1) \qquad \underset{\text{BC582-496}}{\overset{\text{the earth is round}}{\boxed{\text{Pythagoras}}}} \longrightarrow \underset{\text{BC400-347}}{\overset{\text{geocentrism 1}}{\boxed{\text{Eudoxus}}}} \longrightarrow \underset{\text{BC384-322}}{\overset{\text{geocentrism 2}}{\boxed{\text{Aristotle}}}} \longrightarrow \underset{\text{AD83-168}}{\overset{\text{geocentrism 3}}{\boxed{\text{Ptolemaeus}}}}$$

(Although there are several geocentrism k $(= 1, 2, 3)$, the explanation is omitted.)

where Eudoxus(BC.400 – BC.347) is a Greek astronomer and mathematician (called the greatest of ancient Greek mathematicians), a student of Plato. He proposed a kind of geocentrism and the method of exhaustion (as the quadrature). It is said that many of his theorems are written in Euclid's Elements.

Even in ancient China which had the great culture, the prevailing belief was that the Earth was flat and square, while the heavens were round, until the introduction of European astronomy in the 17th century

(ii): Also, ancient Greek mathematics should be praised. That is,

$$(\flat_2) \qquad \underset{\text{BC582-496}}{\overset{\text{Pythagorean theorem}}{\boxed{\text{Pythagoras}}}} \longrightarrow \underset{\text{BC400-347}}{\overset{\text{method ofexhaustion}}{\boxed{\text{Eudoxus}}}} \longrightarrow \underset{\text{BC330-275}}{\overset{\text{Elements}}{\boxed{\text{Euclid}}}} \longrightarrow \underset{\text{BC287-212}}{\overset{\text{quadrature}}{\boxed{\text{Archimedes}}}}$$

The above two ($\flat_1$) and ($\flat_2$) are the greatest achievements in ancient Greek philosophy.

2.2.2 The arche (= the first principle of all things) is number

The main spirit of Pythagorean religious organization is "the first principle of all things is number".

Now let us explain the following principle (called Pythagoreanism in this paper):

(A):Pythagoreanism

The first principle of all things is number.

That is, "Describe the world using mathematics!"

The phrase: "The arche (= first principle) of all things is ◯◯" is a fashion in those days. Note that "water", "fire" etc. are visible, but "number" is not.

♠**Note 2.7.** After about 2000 years from Pythagoras, Galileo was talking about a similar thing. That is, *the universe is written in the language of mathematics.*

In fact, Galileo wrote the universe in the language of mathematics.

♠**Note 2.8.** As mentioned in Note 2.6, Pythagoras is said to have known that the earth was round. It may be a joke, but apparently he thought the earth was round because the most beautiful shape was a sphere. It is natural that the pure Pythagoreanism such as

($\sharp$) The world is written in only the language of mathematics.

is not true. I believe that the world and mathematics are not directly related (though religious people might not think so). If it can be written in only the language of mathematics, it is just mathematics. However, this pure Pythagoreanism has carried over to the present day and is inhibiting the healthy development of our worldview (e.g., see "The theory of probability" in [66], or "Analytic philosophy" in Chap. 11).

Hence we have the following problem (i.e., the problem of worldview), which is the main problem in this paper:

Problem 2.5. The problem of worldview is as follows.

- A scientific worldview has the form: "scientific worldview= mathematics + α". If so,

$$\text{what is } \alpha?$$

As mentioned later, let us say here conclusion now. For example, α is "motion", "causal relation", "probability", "measurement", etc. From the quantum theoretical point of view, that is, in this paper, we devote ourselves to "measurement (Axiom 1) and causality (Axiom 2) (*cf.* Sec.1.1).

♠**Note 2.9.** Possibly in this time, distinction of mathematics and science was not clear. What is mathematics? We had to wait for Cantor's set theory and axiomatic set theory (by Zermelo and Fraenkel) (circa 1900 A.D.) before we would know a definite answer to this question.

Pythagoreanism blossomed for the first time in the early modern times (i.e., the scientific revolution due to Galileo=Newton mechanics and Bernoulli's law of large numbers, etc.*cf.* Chapter 7 later).

In this paper, we consider the following as a genealogy of dualism:

- $\boxed{\text{Plato}} \to \boxed{\text{Descartes}} \to \boxed{\text{Kant}} \to \boxed{\text{Wittgenstein}}$

However, it can be called only mysteriousness that Pythagoreanism has not blossomed in the main stream of western philosophy. As emphasized later, our assertion is

$$\boxed{\text{dualism}} + \boxed{\text{mathematics}} \longrightarrow \boxed{\text{Quantum language}}$$

that is, we assert that

- **Pythagoreanism blossoms as quantum language in western philosophy**

if the following is accepted:

- $\boxed{\text{Plato}} \to \boxed{\text{Descartes}} \to \boxed{\text{Kant}} \to \boxed{\text{Wittgenstein}} \xrightarrow[\text{quantification}]{\text{Pythagoreanism}} \boxed{\text{Quantum languge}}$

2.3 Hērakleitos and Parmenides

2.3.1 Hērakleitos(BC.540 - BC.480)

Hērakleitos said the following.

(A):Hērakleitos(BC.540 - BC.480)

The arche (= the first principle of all things) is fire

And, further,

Everything flows.

Although "Everything flows" and "You cannot step into the same river twice" are interesting, everyone may be able to say similar thing.

Hence, in this paper, we interpret "Everything flows" as follows.

(B) "motion" is the most fundamental key-word in science.

If so, we can relate the (B) to Parmenides.

2.3.2 Parmenides(BC.515 - unknown)

In the same period of Hērakleitos, Parmenides said the exact opposite of words of Hērakleitos. That is,

(C):Parmenides(BC.515 - unknown)

Parmenides said:

(C_1) Everything does not change. There is no motion and no change. Time does not exist. There exists only "one", and not "many".

Also,

(C_2) We should not rely on our senses to understand the world, but should think logically with reason. Even if it appears to be moving, it's just that the human being has the sense to see it. It does not guarantee the existence of the movement.

(Notice): Remark the similarity between (C_1) and the linguistic Copenhagen interpretation (*cf.* Sec. 1.1.2). Also, in case of quantum mechanics, its object is too small, is not seen. Thus, we cannot rely on the sense, but only calculation. We can completely consent to Parmenides' assertion (C_2) in case of quantum mechanics.

♠**Note 2.10.** Thus, I may have an opinion such that

(♯) It is not too much to say that Parmenides and Kolmogorov are the founders of the Copenhagen interpretation of quantum mechanics.

(where Kolmogorov is the founder of the modern theory of probability (*cf.* ref. [76]). It is known that Kolmogorov's extension theorem is the most fundamental in the theory of probability.) That is because

($\flat_1$) Parmenides says that there exists only "one", and not "many", there is no motion.

($\flat_2$) Linguistic Copenhagen interpretation says that only one measurement is permitted.

($\flat_3$) Kolmogorov's extension theorem says that only one probability space is permitted.

Roughly speaking, I think that ($\flat_1$)=($\flat_2$)=($\flat_3$).

If Parmenides didn't think motion was important, there's no way he could say "motion doesn't exist". Thus, I consider that Parmenides believed in the importance of motion. Parmenides' assertion is similar to Hērakleitos', that is,

(D) "motion" is the most fundamental key-word in science.

The following (E_1) and (E_2) are my fiction about the difference between Hērakleitos and Parmenides:

(E_1) Since Hērakleitos said "The first principle of all things is fire", he seems to premise the realistic world. Thus, his motion is similar to the motion in physics.

(E_2) Parmenides might study the abstract motion in the linguistic worldview. For example, his motion is "vegetable growth", "increase of the population", "economic growth", "Achilles' motion" and so on.

♠**Note 2.11.** Many would think the following.

($\sharp_1$) Hērakleitos is ordinary, and thus understandable, hence, scientific, therefore, realistic.

($\sharp_2$) Parmenides is ridiculous, and thus incomprehensible, hence, philosophical, therefore, idealistic (= linguistic).

Therefore, as mentioned in Assertion 1.12, I consider that

Realistic worldview vs. idealistic worldview

dispute ╲ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	"Nominalismus"(Ockham)	"Realismus"(Anselmus)
ⓓ: space-time	Newton	Leibniz
ⓔ: quantum theory	Einstein	Bohr

2.3.3 Motion function method as a worldview

As mentioned in the previous section, Pythagoras said "The arche (= the first principle of all things) is number", Hence,

(F) Mathematics is indispensable to describe the world. However, we need words to connect mathematics and the world.

And further, we want to consider the following fiction:

(G) As an influential candidate of the key-words, Parmenides (and Hērakleitos) thought of "motion"

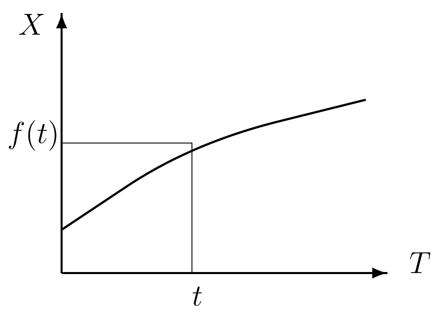
As one of the mechanical worldviews, we introduce the following "motion function method", which is assumed to be due to Parmenides in this paper though the true discoverer cannot be specified, (*cf.* Note 2.14).

(H): (Scientific linguistic) motion function method (due to Parmenides)

Principle 2.6. [The motion function method]

Let T be time axis, and let X be space axis. A function $f : T \to X$ is called a motion function.
Then, the motion function method (in the mechanical worldview) is proposed as follows:

(H_1) **"motion" should be described by the motion function $f : T \to X$.**



♠**Note 2.12.** In the above, we should note that

($\sharp_1$) "Moving feeling" is erased since the above graph is fixed.

If it is so, as Parmenides says, we think that

($\sharp_2$) if we devote ourselves to logic or mathematics without relying on the sense, then we cannot look at "motion".

Also, the motion function method belongs to the realistic worldview as well as the linguistic worldview. In fact, it is easy to see that the motion function method can be derived from Newton mechanics. However, in this paper we usually consider that it belongs to the linguistic worldview.

($\sharp_3$) For the derivation of motion function method from quantum language, see ref. [41], or Chap. 14 in ref.[66].

The motion function method is easy, and it is usually studied in elementary school as follows.

Problem 2.7. An A spot and a B spot are 1400 meters away. Amy left the A spot for a B spot by 80 m per minute. Betty left the B spot for A spot at the same time by 60 m per minute. How many minutes later will Amy and Betty meet?

[**Answer**] Amy's motion function $f_A : \mathbb{R}(\text{time axis}) \to \mathbb{R}(\text{one dimensional space axis})$ is defined by $f_A(t) = 60t$, Betty's motion function $f_B : \mathbb{R}(\text{time axis}) \to \mathbb{R}(\text{one dimensional space axis})$ is defined by $f_B(t) = 1400 - 80t$, Thus, solving $f_A(t) = f_B(t)$, we see

$$60t = 1400 - 80t \quad \text{then,} \quad t = 10$$

Hence, after 10 minutes later, the two meet. □

♠**Note 2.13.** Some may think that to consider two motion functions f_A and f_B is not consistent with Parmenides' saying: *there exists only "one" and not "many"*(*cf.* Sec.1.1.2: linguistic Copenhagen interpretation). However, if so, it suffices to consider the following motion function:

(♯) $(f_A, f_B) : \mathbb{R}(\text{time axis}) \to \mathbb{R}^2(\text{two dimensional space axis})$

♠**Note 2.14.** (a): Note that the motion function method is derived from quantum language (*cf.* ref. [39, 40, 66]). Namely,

$$\text{quantum language} \xrightarrow[\text{derivation}]{} \text{dynamical system theory} \xrightarrow[\text{derivation}]{} \text{motion function method}$$

That is,

(♯) **the motion function method is one aspects of the quantum mechanical world-view (i.e., quantum language)**

Although I do not know, from the historical point of view, the discoverer of the motion function method, I want to assume that Parmenides is the main character, since he was a teacher of Zeno (*cf.* next section). Strictly speaking, the discovery might not be in Ancient Greece since the complete understanding of the concept of "function" is after Leibniz. However, we think that the spirit of the motion function method was understood by Pythagoras, Aristotle, Archimedes, etc.

(b): Of course, the above "motion function method" is incomplete and temporary. The motion function $f : T(\text{time}) \to X(\text{space})$ is not sufficient without the answers to the questions "What is time?" and "What is space?" (the Leibniz=Clarke correspondence (*cf.* Sec. 9.3.2). For the quantum linguistic answers to this questions, see ref. [66].

♠**Note 2.15.** As the scientific worldviews before Newtonian mechanics, the most important is

"the motion function method" and "Archimedes' principle of leverage and buoyant force"

Some may have a question such as

- Why isn't the importance of the motion function method emphasized?
  Why can't the discoverer of the motion function method be specified?

Although I have no clear answer, I consider as follows:

- The realistic worldview (i.e., physics) was usually discovered by one genius, for example, Archimedes, Newton, Maxwell, Einstein and so on. On the other hand, the mechanical worldview is discovered by plural persons. For example, the discoverer of the theory of probability (e.g., Pascal, J. Bernoulli, Laplace, Kolmogorov, etc.) cannot be specified. Probability theory was imperceptibly formed by a number of people. In this paper (*cf.* ref. [66]), we want to regard quantum theory, discovered by Heisenberg, Schrödingerand Born, as a kind of mechanical worldview (and not realistic worldview).

2.4 Zeno(BC490 - BC430): The Motion Paradox

In this section, from the quantum linguistic point of view, we study Zeno's paradoxes, the oldest paradox in science.

Pay attention to the following.

- All the arguments in this section are my opinion, not the common.

Therefore, I invite readers to read this section from a critical perspective.

2.4.1 What is Zeno's paradoxes? Without a worldview, we cannot say anything

Zeno is a disciple of Parmenides. Thus, Zeno's Paradoxes may a collaboration between the two.

Although Zeno's paradox has some types (i.e., "flying arrow", "Achilles and a tortoise", "dichotomy", "stadium", etc.), I think that these are essentially the same problem. And I think that the flying arrow expresses the essence of the problem exactly and is the best masterpiece in Zeno's paradoxes.

Now we present Zeno's paradoxes (i.e., flying arrow) as follows:

Please taste the literary pleasure.

Paradox 2.8. [**Zeno's paradoxes: The literature-like antinomy**]

The literature-like proof of [**Flying arrow is at rest**]

- Consider a flying arrow. In any one instant of time, the arrow is not moving. Therefore, if the arrow is motionless at every instant, and time is entirely composed of instants, then motion is impossible. □

The literature-like proof of [**Flying arrow is not at rest**]

- We have to accept that an arrow passes there. However, "to pass there" is not equivalent "to exist there". What is "to pass there"? "To pass there" is both "to exit there" and "not to exist there". Therefore, flying arrow is not at rest.

□

Now we can answer the question "What is Zeno's paradoxes?". That is, we consider that

(A) **Zeno's paradoxes say "Don't trust on 'logic' unconditionally", that is, "Start from a worldview and not logic".**

since Paradox 2.8 shows that "antinomy" happens in the logic in ordinary language. If so, we have to obey the worldviewism in Sec. 1.3.1. that is,

- First declare the worldview, and discuss the world in the worldview. Namely,

$$\underset{\text{premise}}{\overset{\text{world is so}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{subject}}{\overset{\text{conclusion}}{\boxed{\text{discussions, calculation, practical logic, properties}}}}$$

Seeing the literature-like proof in Paradox 2.8, we naturally have the question:

- **Under what kind of worldview is the above literature-like proof presented ?**

Therefore, to solve Zeno's paradoxes is to solve the following problem

Problem 2.9. Propose a certain worldview, in which Zeno's paradoxes (e.g., Flying arrow) can be discussed.

[Answer]; This is answered in Answer2.10 below.
Slogan-wise, we say

(B) **"Without a worldview, there is no logic (or precisely, practical logic)".**

Again see "the worldviewism (A)" in Sec. 1.3.1.

♠**Note 2.16.** The above is not to be taken for granted. As mentioned in Chap. 11 later, the spirit of analytic philosophy is "Start from logic", though the great tradition of philosophy from Plato to Kant was always based on the worldviewism (*cf.* Chaps. 3-10). Thus, I am somewhat skeptical of analytic philosophy since I believe that the above (B) is the fundamental spirit of philosophy.

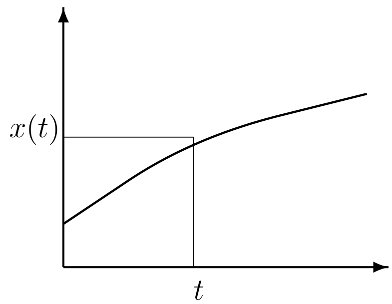
2.4.2* The solution⋆ about Zeno's paradoxes (e.g., Flying arrow) in the motion function method

If we obey the motion function method (in the mechanical worldview), we can easily solve Zeno's paradoxes (e.g., Flying arrow) as follows.

Answer 2.10. [**An answer to Problem 2.9(scientific answer)**] Under the motion function method (*cf.* Section 2.3.3) in the mechanical worldview, we discuss "Flying arrow" as follows.

- Consider the motion function $x(t)$, that is, for each time t, the position $x(t)$ of the arrow is corresponded. It is obvious that
  - "for each time t, the position $x(t)$ of the arrow is corresponded" do not imply that the motion function $x(t)$ is a constant function.

  Therefore, the arrow is not necessarily at rest.



$\square$

♠**Note 2.17.** Recall that we were confused in Paradox2.8[Zeno's paradoxes]. However, we could easily solve it in Answer 2.10. Thus we should be surprised at the power of the motion function method. If a certain worldview is determined, Zeno's paradoxes can be solved. Thus, the motion function method is not necessarily determined uniquely. For example, it is a good exercise to solve Zeno's paradoxes under Newtonian mechanics or the theory of relativity.

♠**Note 2.18.** As mentioned in Section 2.3.2, Parmenides' assertion is similar to the linguistic Copenhagen interpretation. Therefore, we think that the first step of the worldview was formed by Parmenides who belongs to the (quantum) mechanical worldview. Also, Hērakleitos is located in Paramenides' opposite side. Thus we see the following. (*cf.* Classification 1.11 [the classification of philosophers]):

We consider the following classification of philosophers.

$$(\flat) \begin{cases} (\flat_1)\text{: the realistic worldview (physics)} \\ \quad \text{Hērakleitos, Aristotle, Archimedes, Galileo, Newton, Einstein}, \cdots \\ \\ (\flat_{21})\text{: the fictional worldview (Western philosophy)} \\ \quad \text{Plato, Scholasticism, Descartes, Locke, Leibniz, Berkeley, Hume, Kant, Husserl} \\ \\ (\flat_{22})\text{: the logical worldview} \\ \quad \text{Boole, Frege, Peirce, Saussure, Russell, Wittgenstein, Hempel, Popper} \\ \\ (\flat_{23})\text{: the mechanical worldview (statistics, quantum language)} \\ \quad \text{Parmenides, Zeno, J. Bernoulli, statistics (e.g., Fisher), quantum language} \end{cases}$$

Since Zeno is a student of Parmenides, it is natural to consider that Parmenides and Zeno are in the same position. Also, Pythagoras has no position in the above since we decide that mathematics is not a kind of worldview.

2.4.3 Appendix: The discussion about Zeno's paradoxes (e.g., Achilles and a tortoise]) in the motion function method

The idea of associating Zeno's paradox ("Achilles and a tortoise") with an infinite series misses the essence of Zeno's paradox, because Parmenides and Zeno's interest should be a worldview. That is, "Flying arrow" is the most important paradox in Zeno's paradoxes.

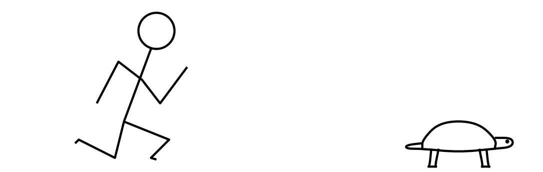
However, since geometric series' method is most famous, I add "Achilles and a tortoise" as an appendix (*cf.* ref. [41], or Chap. 14 in ref.[66]).

Readers should also taste the literary pleasure in the following.

Paradox 2.11. [Zeno's paradoxes(the literature-like answer)]
[Achilles and a tortoise]

Zeno's paradox (Achilles and a tortoise) is as follows.

- Consider the competition of Achilles (a quick runner) and a tortoise (a late runner). Consider the competition of Achilles and a tortoise. Achilles' starting point will be behind the turtle's starting point. Suppose that both started simultaneously. If Achilles tries to pass a tortoise, Achilles has to go to the place in which a tortoise is present now. However, then, the tortoise should have gone ahead more. Achilles has to go to the place in which a tortoise is present now further. Even if Achilles continues to do this infinite times, he will not be able to keep up with the turtle.



[The scientific answer to Zeno's paradox (Achilles and a tortoise) by the motion function method]

For example, assume that the velocity $v_q(=v)$ [resp. $v_s(=\gamma v)$] of the quickest [resp. slowest] runner is equal to $v(>0)$ [resp. γv $(0<\gamma<1)$]. And further, assume that the position of the quickest [resp. slowest] runner at time $t=0$ is equal to 0 [resp. a (>0)]. Thus, we can assume that the position $q_1(t)$ of the quickest runner and the position $q_2(t)$ of the slowest runner at time t (≥ 0) is respectively represented by the following motion function:

$$\begin{cases} q_1(t)=vt \\ q_2(t)=\gamma vt+a \end{cases} \tag{2.1}$$

Thus, it suffices to calculate formula (2.1).

Although it can be solved by various method, I present two methods as follows(i.e., (i) or (ii)):
[(i): Algebraic calculation of (2.1)]:
Solving $q_1(s_0)=q_2(s_0)$, that is,

$$vs_0=\gamma vs_0+a$$

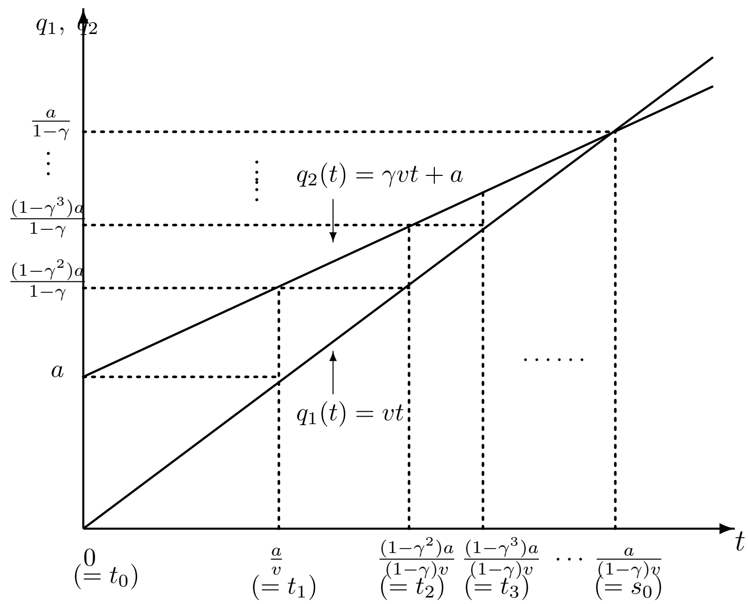
we get $s_0=\frac{a}{(1-\gamma)v}$. That is, at time $s_0=\frac{a}{(1-\gamma)v}$, the fast runner catches up with the slow runner.
[(ii): Iterative calculation of (2.1)]:
Define t_k $(k=0,1,...)$ such that, $t_0=0$ and

$$t_{k+1}=\gamma vt_k+a \quad (k=0,1,2,...)$$

Thus, we see that $t_k = \frac{(1-\gamma^k)a}{(1-\gamma)v}$ $(k = 0, 1, ...)$. Then, we have that

$$\begin{aligned}\big(q_1(t_k), q_2(t_k)\big) =& \big(\frac{(1-\gamma^k)a}{1-\gamma}, \frac{(1-\gamma^{k+1})a}{1-\gamma}\big)\\ &\to \big(\frac{a}{1-\gamma}, \frac{a}{1-\gamma}\big)\end{aligned} \tag{2.2}$$

as $k \to \infty$. Therefore, the quickest runner catches up with the slowest at time $s_0 = \frac{a}{(1-\gamma)v}$.



Graph: $q_1(t) = vt, \ q_2(t) = \gamma vt + a$

[(iii): Conclusion]: After all, by the above (i) or (ii), we can conclude that

($\sharp$) the quickest runner can overtake the slowest at time $s_0 = \frac{a}{(1-\gamma)v}$.

♠**Note 2.19.** ($\flat_1$): Note that the above (ii) [= the solution using a geometric series] is another solution of (i). Thus, there was no need to use geometric series. The point is that there is a difference in the position of whether one considers the Achilles and a turtle paradox to be a mathematical problem or a philosophical problem. Many philosophers have gotten into dead ends by confusing the two positions. Philosophers should have considered it as a problem of world description.

($\flat_2$): From the philosophical point of view, "flying arrow" is definitely better than "Achilles and a tortoise". However, as far as quizzes go, "Achilles and a tortoise" is well done. "Achilles and a tortoise" is well done in the sense that the trick is designed to make it easy for solvers to fall into it. In fact, for 2,500 years, most solvers have fallen for this trick.

♠**Note 2.20.** As mentioned in Preface, the purpose of this paper is to understand the history of western philosophy from the quantum linguistic point of view. Thus,

($\sharp_1$) We aren't interested about how Zeno considered his paradoxes.

The established theory may say that

($\sharp_2$) Zeno might study the infinite division of time in physics (and space)

However, if so, Zeno's paradoxes are the problem in physics and not philosophy. Then the problem should be entrusted to physicists. However, in this paper, we assume that Zeno's paradoxes belong to philosophy and not physics. Also, although in this section we solved Zeno's paradoxes under the motion function method, our present problem is how to characterize the motion function method in the framework of quantum language. For the answer to "What is space-time in quantum language?", see Sec. 9.3.2: (Leibniz=Clarke correspondence).

Chapter 3

The Big Three in Greek Philosophy (Socrates, Plato)

By the appearance of The Big Three in Greek Philosophy (Socrates, Plato, Aristotle), the origin of western philosophy was formed as follows.

$$
(\flat)\begin{cases}
(\flat_1): \text{the realistic worldview(Aristotle} \to \text{Newton)} \\
(\flat_2): \text{the idealistic worldview}\begin{cases}
(\flat_{21}): \text{ the fictional worldview} \\
\quad \text{the main current of western philosophy} \\
\quad \text{(Socrates, Plato, Descartes, Kant, etc.)} \\
(\flat_{22}): \text{ the mechanical worldview} \\
\quad \text{(Parmenides} \to \text{statistics, quantum language)} \\
(\flat_{23}): \text{ the linguistic philosophy} \\
\quad \text{(Aristotle} \to \text{Frege, Saussure, Wittgenstein)}
\end{cases}
\end{cases}
$$

though I think philosophers are divided on whether Aristotle belongs to the $(\flat_{23})$ or not.

3.1 Protagoras and Socrates

3.1.1 Ethics

The philosophy of worldview aimed at the following problems

(A) How is the world described? How is the world understood?
By what kind of language is the world described?

But, there is another philosophy (i.e., philosophy of ethics) different from the worldview.

Ethics,morals [How should we live?]

Many people in the sciences (no, even the liberal arts) may think that ethical philosophy is "debating skills". In fact, **Protagoras (490 BC. - 420 BC.)**, a central figure of the Sophists, preached that **"man is the measure of all things"**. He argued for relativism, which holds that there is no such thing as objective truth. In other words, he argued that only the subjective judgment of each person is essential. Since the common sense of capitalism is

"the average of subjective value" = "price"

one might say that the sophist's claim is reasonable and modern common sense.

However, **Socrates** (BC.469 - BC.399) had objected to this idea. He investigated that

(B) How should we live?

And, he clarified the following words:

(C) "goodness", "happiness", "virtue", "justice", "courage", "love" ······

That is, Socrates asserted that the investigation of the above words is also the central theme of philosophy. In the following dispute:

"relativism (rational sophists) " vs. "absolutism (a man of faith: Socrates) "

Socrates has advocated the ethical philosophy.

Note that three philosophers (Socrates (BC.469 - BC.399) , Buddha (BC.565 - BC.486), Confucius (BC.551 - BC.479) and Mozi (BC.470 - BC.390)) were contemporary, and investigated the same problem (B). In this sense, we can say that

(D_1) The mathematics of Pythagoras was unparalleled in the world. Compared to this , Socrates was common sense and mediocre.

It is a matter of course that

(D_2) If these words mentioned in the above (C) didn't spread, the human race might have been ruined. At least, we wouldn't be able to form "human society". Maybe the mankind perished.

Therefore, I cannot overemphasize the importance of ethical philosophy. Also, the philosophy of ethics is worldwide. When it isn't so, we're in trouble. That is, when it isn't so, "world peace" isn't achieved.

As emphasized throughout this paper, I believe that

- **The main theme of philosophy is ethics (and not worldview).**

Hence I agree that Socrates is called the father of philosophy.

♠**Note 3.1.** In general, Ethical philosophy does not have any truth or universality. Society of ants has "ethics and morals of ants", and apes must have "ethics and morals of apes". The ethical philosophy of the Neanderthals must have been quite different from ours. Therefore, our ancestors must have destroyed the Neanderthals. Communication and contact with civilized aliens will happen in the future. However, it is too optimistic to expect the aliens to be unconditionally friendly at this time. I remembered that the late Dr. Hawking had emphasized the same opinion.

3.1.2 Magic proposition: I know I know nothing

♠**Note 3.2.** Socrates did not leave a book. The "Sophists vs. Socrates" is Plato's fiction. Since Plato is a disciple of Socrates, it's not fair. For example. The strongest logic, **"I know that I know nothing"** goes something like this.

- Sophists: something is asserted ······
- Socrates: deny Sophists' assertion
- Sophists and Socrates: debate (called Socratic Method)
  ······ Sophists and Socrates tell eloquently, and thus, they get tired. ······
- At that time, Socrates says "Your ignorance is now revealed. **I know that I know nothing**, but you do not know that you know nothing. Thus, I am superior to you".

This is Socrates' strongest logic **"I know that I know nothing"**. If we, without sticking to an established theory, read Plato's novel which makes Socrates a main character, we may have a variety of opinions on the following issues.

- Which is playing with sophistry, Socrates or sophists?

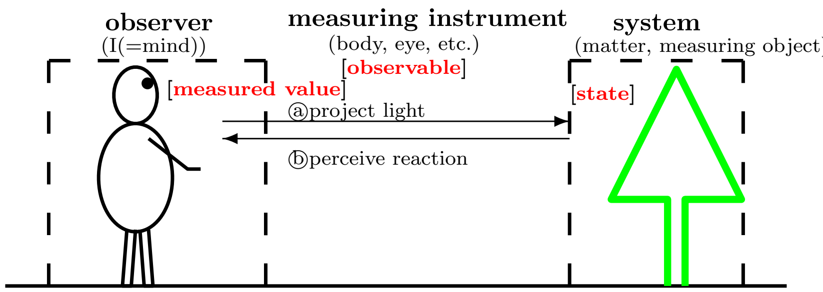
I may feel that Socrates uses more sophistry (since Socrates uses taboo statement: I know I know nothing). □

In physics, you can draw conclusions with experiments, in mathematics you can prove them, and in engineering you just need to employ something useful. However, ethical philosophy does not solve the problem in that way. So Socrates (= Plato) came up with the magic word **"I know that I know nothing"** (**self-referential proposition**, anti-Copenhagen interpretive proposition) as a way to end the discussion. To put it in an extreme way,

(E) End the discussion by saying something like a paradox that messes with the other person's brain.

Recall Figure 1.2 (in Section 1.1.2: The linguistic Copenhagen interpretation) below:

Figure 3.1. (= Figure 1.2 in Section 1.1.2) [Descartes Figure]: Image of "measurement(=ⓐ+ⓑ)" in dualism



Note that

(F) the self-referential propositions (e.g., "I know that I know nothing", "I think, therefore I am", etc.) are out of quantum language (= dualistic language).

That is because the linguistic Copenhagen interpretation says that "observer" and "measuring object" must be completely separated. However, it holds that "observer" = "I" = "measuring object" in the above self-referential propositions. Thus, the self-referential propositions are **taboo statements** in quantum language (= scientific language).

However, it is sure that the wordplay of self-referential propositions ("I know that I know nothing", "I think, therefore I am", etc.) is an important part of philosophy. In the art of argumentation, taboo propositions are often valid. Taboo propositions are so unclear in their meaning that listeners often misinterpret them as profound propositions. This is the reason that Socrates and Descartes succeeded. They were excellent sophists. A good sophist doesn't let his readers know that he is a sophist. This is natural. Because philosophy is a kind of literature.

3.2 Plato(BC.427 - BC.347)

3.2.1 The theory of Ideas – Asserted fiction –

In the binary opposition (in ethics):

(A) "relativism (rational sophists)" vs. "absolutism (a man of faith: Socrates)"

Plato, a student of Socrates, established "the theory of Ideas" as the foundation of absolutism in order to support Socrates.

If the propositions such as "Man's life is heavier than the Earth", "Love is forever", "Love always overcomes money", etc. are the objective truth, the occupation of the insurance company does not hold. However, Socrates wanted to believe so. To help Socrates, Plato proposed the occult heavenly world (i.e., the world of Idea). That is,

(B) the theory of Ideas is a reckless attempt to derive ethics (i.e., "How should we live?") from worldview (i.e., "How is the world ?"), that is,

$$\underset{\text{How is the world?}}{\boxed{\text{worldview}}} \xrightarrow[\text{therefore}]{} \underset{\text{"How should we live?}}{\boxed{\text{practical logic, ethics}}}$$

This method (= the form of philosophy) has been accepted as the standard form of "how to tell philosophy" in the history of two thousands hundreds of years.

(C): The fiction called "the theory of Ideas"

Theory 3.2. [Theory of Ideas]:
The theory of Ideas is as follows

- It cannot be said that love always overcomes money in the real world. However, there exists another world (i.e., the world of Idea), where "love always overcomes money" is believed as the objective truth. That is, there exists Idea (= the true form) in heavens. A thing existing on the ground is only the shadow.

This is the theory of Ideas.

Then, the real world is a shadow picture, hence, in the real world,

(D_1) love sometimes loses money

(D_2) We can't live on justice alone.

(D_3) Good man is sometimes unhappy.

That is, Plato wanted to say that

"love always overcomes money" is the objective truth in the world of Idea, therefore, "Believe in love!"

Whether you believe in this reasoning or not, this idea, i.e., the discovery of two key words "Idea world" and "reality world", was the beginning of "dualistic idealism" and has always been the mainstream of Western philosophy, despite the twists and turns that followed. The theory of ideas, which was supposed to be a logistic support of ethical philosophy (Socrates), became the mainstream of the philosophy of worldview.

(E) Our human DNA prefers logic (or reasoning) as if the philosophy of ethics were derived from the philosophy of worldview. That is,

"the world is so", therefore "we should live so"

namely,

$$(\mathrm{F}_1)\ \ (=(\mathrm{B})) \qquad \underset{\text{introduction·preface·fiction}}{\overset{\text{world is so}}{\boxed{\text{fictional worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{we should live so}}{\boxed{\text{practical logic, ethics·morals}}}}$$

Also, for example,

- $$\underset{\text{(fictional)premise}}{\overset{\text{invisible man exists}}{\boxed{\text{fictional worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{don't steal a glance}}{\boxed{\text{practical logic, ethics·morals}}}}$$

and

- $$\underset{\text{premise,fiction}}{\overset{\text{ultimate beauty exists}}{\boxed{\text{fictional worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{main assertion}}{\overset{\text{she is an embodiment of beauty.}}{\boxed{\text{practical logic, ethics, morals,aesthetic sense}}}}$$

This is, of course, irrational since this is a reckless attempt such that the problem "How should we live?" is answered from the objective point of view. However, the human brain does not operate on logic alone. "Logic" cannot function without some kind of sensuous common soil. Logic alone is not enough, what is important is **"logic in a common sensory soil" (= "practical logic")**.

In other word, **"without common sensory soil, logic cannot work"**, therefore **"the first thing to do is to form a common sensory soil**.

If so, Plato's method of telling philosophy (F_1) may not be reckless. That is because the (F_1) means

$$(\mathrm{F}_2) \qquad \underset{\substack{\text{introduction·preface·fiction}\\ \textbf{common sensory soil}}}{\overset{\text{world is so}}{\boxed{\text{fictional worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{we should live so}}{\boxed{\text{practical logic, Ethics·morals}}}}$$

That is, "logic" depends on "worldview". Slogan-wise, we say

(G) **"Without a worldview, there is no logic (= practical logic)".**

Recall that this slogan plays an essential role in the solution of Zeno's paradox (*cf.* Sec.2.4 (B)). Also, again see "the worldviewism (A)" in Sec. 1.3.1 and Note 1.14.

♠**Note 3.3.** Here, "logic in the common soil of the senses" has a similar meaning to "logic under a certain description of the world". In a broader sense, this "logic" is the logic usually used by ordinary people, such as the logic of Newton mechanics, the logic of a court case, the logic of politics, the logic of family life, the logic of insurance solicitation, and so on. One of the various kinds of logic is the logic of mathematical logic (= symbolic logic), which is so universal that even aliens may know it. However,

(♯) this does not mean that mathematical logic experts can master the logic of other disciplines well.

Naturally, a good understanding of economics means that one can use the logic of the common soil of economics. It is natural to expect that logic under the theory of Ideas promote our understanding of Socrates and Plato ethics.

As mentioned in Chap. 11, we have two types of logic (i.e., symbolic logic and practical logic) (*cf.* Note 1.15), i.e.,

- $\begin{cases} \text{symbolic logic (= axiomatic logic =mathematical logic)} \\ \text{practical logic (= inference)} \end{cases}$

Practical logic plays an important role in worldviews. On the other hand, symbolic logic plays an important role in mathematics. I am skeptical of analytic philosophy, in which symbolic logic and practical logic are confused. I fee that the above spirit (♯) is scarce in analytic philosophy.

♠**Note 3.4.** (i): Aesop (BC.620 - BC.510): Idea theory is similar to Aesop's fable in some ways. It is natural to be hesitant to teach morality to others face-to-face. I think Aesop was preaching ethics indirectly through the animals. Aesop's fables were already well known before the late 5th century BC. Of course, Plato must have been aware of the persuasive power of Aesop's fables.

(ii): In the book "Sapiens: A Brief History of Humankind" (*cf.* [26], 2016), Harai, the author, describes three revolutions which brought about dramatic changes to human societies.

$$\begin{cases} \text{①: the Cognitive revolution, about 70,000 years ago} \\ \text{②: the Agricultural revolution, about 11,000 years ago} \\ \text{③: the Scientific revolution, beginning a mere 500 years ago} \end{cases}$$

The above ① implies that

- we rule the world because we are the only animals that can create and believe in fictions like God, the state, money and human rights.

Did Plato know about the cognitive revolution?

////

Plato's say is as follows: No matter how much we argue, the issue of "relativism (rational sophistry) vs. absolutism (man of faith: Socrates)" can't be concluded. It is not a truth that men seek. There may not be the truth.

What people want is an "asserted fiction", not the truth.

And this is the philosophy. I think that's what Plato would have thought.

♠**Note 3.5.** S. Weinberg (1933 -), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said in his book [105] "To explain the word; The discovery of modern science" as follows:

(♯) [in Chapter 1] *There is an important feature of modern science that is almost completely missing in all the thinkers I have mentioned, from Thales to Plato: none of them attempted to verify or even (aside perhaps from Zeno) seriously to justify their speculations.*

Plato's philosophy is a representative of idealism (*cf.* for the definition of "idealism", see Definition 1.8), which is a completely different style of philosophy from (experiment-oriented) science. As noted above, Plato's philosophy is a philosophy that begins with "asserted fiction". Also, it is a time of F. Bacon (i.e., the father of British Empiricism) of the scientific revolution (the 17th century) that the importance of observation was, for the first time, emphasized. See Section 7.2

♠**Note 3.6.** In order to avoid eternal argument, Socrates invented "Socratic method", that is, the magic sentence "I know that I know nothing". Plato also invented the theory of Ideas.

For completeness, we add the following:

(F) I think that Plato did not believe in the existence of the world of Idea. If he believed in it, he was not a philosopher but a founder of religion. He also understood that the theory of Ideas is sophistry, and there is no truth in ethics.

Even so, there may be a reason to consider that

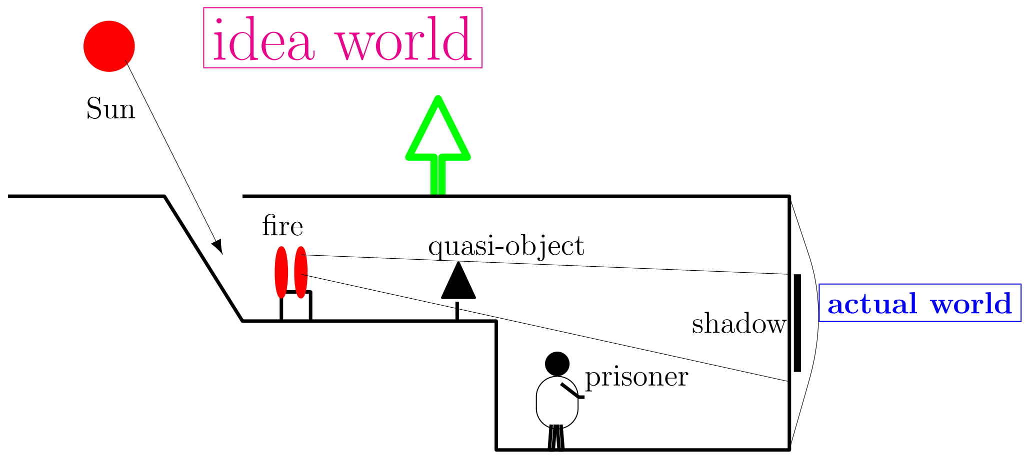
(G) something such as the sense of ethics of the human commonness is printed in the arrangement of a human DNA,

However, this idea may not be within philosophy.

3.2.2 Allegory of the cave

The allegory of the cave was presented by Plato in his work "Republic" in order to promote the understanding of "the theory of Ideas".

Allegory 3.3. [Allegory of the cave]



A group of prisoners are looking at the shadows on the back wall of a cave. The shadows just are reality since they have never experienced anything other than shadows. The shadows represent physical reality. One day, one of the prisoners gets free. Namely, he is the philosopher, the lover of wisdom. Let's assume he is Socrates. He turns around and the first thing he sees is objects of stone and wood made to resemble the shapes of real things such as a tree. Further along, generating the light that hits these objects that then produce shadows on the back wall of the cave, is a fire. Beyond the fire is the entrance/exit of the cave. The philosopher exits the cave and is temporarily blinded by the light. The first thing he sees is a real tree. Finally, the philosopher sees the sun which Plato called The Idea of the Good.

The freed prisoner (who is assumed to be Socrates) would think that the world outside the cave was superior to the world he experienced in the cave. And he would want to bring his fellow cave dwellers out of the cave and into the sunlight. The returning prisoner, whose eyes have become accustomed to the sunlight, would be blind when he re-enters the cave, just as he was when he was first exposed to the sun. The prisoners would infer from the returning man's blindness that the journey out of the cave had harmed him and that they should not undertake a similar journey. Hence, there is a possibility that the prisoners would therefore reach out and kill anyone who attempted to drag them out of the cave. In fact Socrates was killed.

Remark 3.4. Thus, Allegory of the cave says that three key-words in Plato philosophy:

$$[\text{A}](\textbf{mind}) \quad \underset{\textbf{(medium)}}{\longleftarrow [\text{B}] \longrightarrow} \quad [\text{C}](\textbf{matter})$$

correspond to as follows:

\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
Plato (cave)	actual world	? (See Review 3.10 later)	/ [Idea world]
quantum language	**measured value**	**observable**	**state** **[system]**

3.2.3 The theory of anamnesis

As a mediator between the real world and the idea world, Plato presents the concept of "anamnesis". Namely,

- We had seen the Idea before we were born. But we forget that when we are born. Therefore, to know an Idea is to recall the Idea. In other words, learning is nothing less than recalling (= anamnesis).

Remark 3.5. If we believe in the theory of anamnesis, three key-words in Plato philosophy:

$$[\mathrm{A}](\textbf{mind}) \quad \underset{\textbf{(medium)}}{\longleftarrow [\mathrm{B}] \longrightarrow} \quad [\mathrm{C}](\textbf{matter})$$

correspond to as follows:

\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
Plato (anamnesis)	actual world	anamnesis (See Review 3.10 later)	/ [Idea world]
quantum language	**measured value**	**observable**	**state** **[system]**

3.3 The allegory of the sun: Measurement theoretical aspect of Idea theory

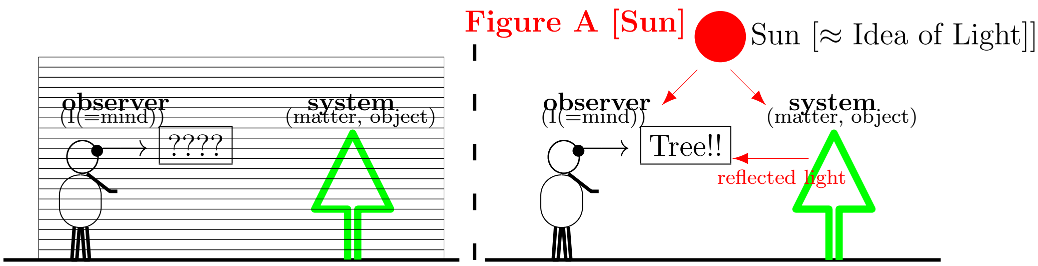
The analogy of the sun is found in "Republic", written by Plato. Upon being urged by Glaucon (Plato's elder brother) to define goodness, a cautious Socrates professes himself incapable of doing so. Instead he draws an analogy and offers to talk about **"the child of goodness"**. For the answer to "Why a child?", see Note 3.8.

3.3.1 The allegory of the sun

Note that the North Star can be also regarded as a measuring instrument for orientation. With this in mind, I hope you will read the following.

Allegory 3.6. [The Allegory of the Sun]: The Allegory of the Sun explains what the "Idea of Good" is all about. No matter how much you open your eyes, you cannot see anything "visible" in the "visible" world, such as a flower, a tree, or a dog, without the light of the sun.

Figure A [Sun]



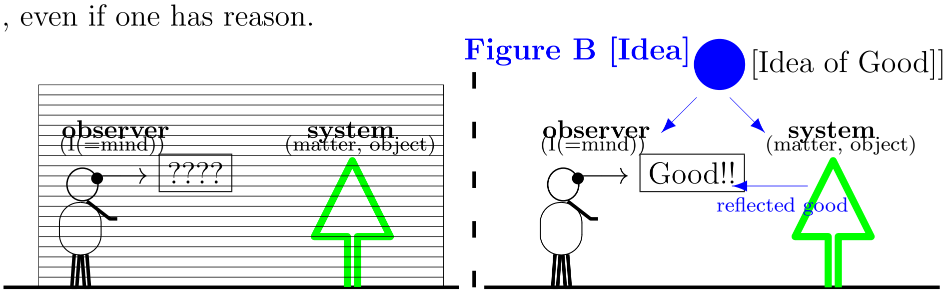
In the beginning, when it's dark as shown on the left figure, you can't see anything, even if your vision is normal.

However, by developing the skills to use a measuring instrument called the "Sun [= Idea of Light]," you can see that it is a "tree".

Thus, in this fable, the ternary relation ("the beholder," "the sun (the mediating thing)," "the thing to be seen") are clear and very easy to understand.

In the same way, things that exist in the invisible realm, such as virtue and courage, cannot be known unless one has the skill to use the measuring instrument called the "idea of the good", even if one has reason.

Figure B [Idea]



In other words, if you learn how to use a measuring instrument called the sun, you can make the object "visible". In the same way, the mastery of the measuring instrument, which is called the "idea of Good," makes clear virtue and courage.

Remark 3.7. The polar star can be regarded as a measuring instrument such as a kind of compass (*cf.* Note 1.4 (ii)). Thus, it is reasonable to regard the sum (Idea) as a measure of "Good". That is, I want to think:

$$\text{"Idea"} = \text{"instrument to make Idea world visible"}.$$

though this may not be standard.

Therefore, three key-words in Plato philosophy:

$$[\mathrm{A}](\textbf{mind}) \quad \longleftarrow \underset{\textbf{(medium)}}{[\mathrm{B}]} \longrightarrow \quad [\mathrm{C}](\textbf{matter})$$

correspond to as follows:

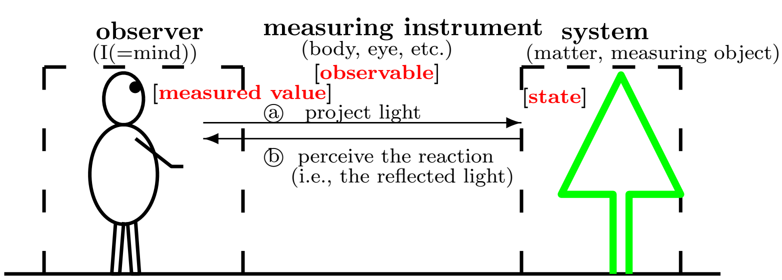
\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
Plato (Sun)	actual world	Idea (See Review 3.10 later)	/ [Idea world]
quantum language	**measured value**	**observable**	**state [system]**

It should be noted that this response is somewhat unreasonable. That is because Idea Theory was not proposed with the intention of a theory of measurement (or epistemology).

3.3.2* The measurement theoretical understanding of the allegory of the sun

Recall Figure 1.2 [Descartes figure] in Sec. 1.1.2, namely,

Figure 1.2 ; [Descartes Figure]: Image of "measurement(=ⓐ+ⓑ)" in dualism



Here, it should be noted that this is essentially the same as the figure [Allegory of the sun].

Summary 3.8. We see:

$\boxed{\text{Figure B [Idea of good]}}$ in Allegory 3.6 $\underset{\leftarrow\text{philosophy}}{\overset{\text{science}\rightarrow}{=\!=\!=}}$ $\boxed{\text{Figure A: [Sun]}}$ in Allegory 3.6 $\overset{\text{equivalent}}{=\!=\!=}$ $\boxed{\text{Descartes Figure}}$ in Figure 1.2

$\uparrow$ Define goodness

Plato philosophy (Theory 3.2)
$\boxed{\text{Theory of Ideas}}$

$\downarrow$ Allegory

$\boxed{\text{Allegory of the cave}}$
(Allegory 3.3)

$\downarrow$ + Anamnesis

$\boxed{\text{Anamnesis}}$
(Sec. 3.2.3)

$\uparrow$ Illustration

science
$\boxed{\text{Axiom 1 [measurement]}}$

philosophy | science

We consider that

- the understanding of the first line above, namely,

  $\boxed{\text{Figure B [Idea of good]}}$ in Allegory 3.6 $\underset{\leftarrow\text{philosophy}}{\overset{\text{science}\rightarrow}{=\!=\!=}}$ $\boxed{\text{Figure A: [Sun]}}$ in Allegory 3.6 $\overset{\text{equivalent}}{=\!=\!=}$ $\boxed{\text{Descartes Figure}}$ in Figure 1.2

  is equal to the measurement theoretical understanding of Allegory 3.6 [Allegory of the sun].

Thus, "Descartes figure" should have been called "Plato figure".

3.4 Plato: The fictional worldview (=Plato's way of telling philosophy)

3.4.1 The necessity of idealism and dualism

Let us review Plato's way of telling philosophy (=the fictional worldview, *cf.* Sec. 1.3.3.1).

(A): The fictional worldview (=Plato's way of telling philosophy)

Plato's way of telling philosophy (in the main current of western philosophy) is as follows.

(A) $\underset{\text{preface, introduction, (fictional)premise, expedient}}{\overset{\text{world is so}}{\boxed{\text{fictional worldview (literary truth, pseudo- truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{you should do so}}{\boxed{\text{ethics, morals}}}}$

Therefore,

(A_1) [world is so] is secondary,

(A_2) [you should do so] is main theme

In Plato philosophy, the theory of Ideas (=the fictional worldview) is only the fiction (= parable, fable). That is,

the theory of Ideas is only a prologue for Plato's ethics.

Plato's way of telling philosophy is common to all philosophies in the genealogy of the dualistic idealism as follows:

(B) Plato(the theory of Ideas)$\longrightarrow$Augustinus$\longrightarrow$Thomas Aquinas $\longrightarrow$Descartes
$\longrightarrow$Kant(epistemology)

If so, we may hesitate to reply "Yes" for the following question:

- Does the philosophy of worldview proposed by them merit serious and scientific discussion?

As mentioned throughout this paper, we consider that

- every worldview in the genealogy (B) of the dualistic idealism is an allegory as similar as the theory of Ideas.

In fact, epistemology (including Kant) is, from a scientific point of view, more an allegory than a brain science.

♠**Note 3.7.** Kant is highly regarded on today. As seen later (Chapter 10: Kant), the reason is that Kant understood Plato's intention perfectly.

- $\underset{\text{preface, introduction, (fictional)premise, expedient}}{\overset{\text{"Critique of Pure Reason (1781)"}}{\boxed{\text{fictional worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{"Critique of Practical Reason (1788)" "Critique of Judgment(1790)"}}{\boxed{\text{ethics, morals, aesthetics}}}}$

where it should be noted that Critique of Pure Reason is not scientific.

3.4.1.1 The necessity of the worldview

Even if the theory of Ideas is a fairy tale, Plato's idea has a point.

(C) Ethics·morals is dependent on the world (=environment around).

It is a matter of course that there is a difference between the ancient Japanese ethics and the modern American ethics. In this sense, strictly speaking, the following (=Plato's way of telling philosophy) is true:

(D) (= (F_2) in Sec. 3.2.1)

$$\underset{\substack{\text{introduction·preface·fiction}\\ \text{common sensory soil}}}{\overset{\text{world is so}}{\boxed{\text{fictional worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{we should live so}}{\boxed{\text{Ethics·morals}}}}$$

3.4.1.2 The necessity of idealism (= metaphysical world)

If the (D) is strictly put into practice, this is not philosophy but life consultation. For example,

- If you are really depressed, go to a psychiatrist and ask him to prescribe some medication. Philosophers are not to be trusted.

Thus, philosophers have to assume an unrealistic world (metaphysical world).

This is because if the settings can be verified by experiment, mistakes may be pointed out. Therefore,

(E) Metaphysical worldview (i.e., idealism) is desirable

This is the reason to adopt the idealism (i.e., metaphysical world) in Plato's way of telling philosophy

3.4.1.3 The necessity of dualism

Also,

(F) since the goal is ethics and morality, a world that reflects human beings in some way is preferable. In other words, the dualism of "things" and "people" is preferable.

For example,

- We don't usually take moral lessons from monism, such as Newtonian mechanics.

This is the reason to adopt "mind-matter dualism" in philosophy.

After all, we conclude that

(G) **in the philosophy of worldview, dualistic idealism is desirable.**

Remark 3.9. Recall that our purpose of this paper is to show that

$$\boxed{\text{Plato}} \xrightarrow[\text{progress}]{} \boxed{\text{Descartes}} \xrightarrow[\text{progress}]{} \boxed{\text{Locke}} \xrightarrow[\text{progress}]{} \boxed{\text{Kant}} \xrightarrow[\text{progress}]{} \boxed{\text{Quantum language}} \tag{3.1}$$

where $\xrightarrow[\text{progress}]{}$ means "getting closer to quantum language". However, some ask the following question:

- Is the series (3.1) inevitable?

Concerning this question, Allegory 3.6 [the Sun] is suggestive. Note that the following facts:

($\sharp_1$) Plato proposed Idea theory, which was an fairy tale as dualistic idealism.

($\sharp_2$) Plato attempted to explain Idea theory in a variety of ways, but the most scientific and rational explanation was Allegory 3.6 [the Sun]. (Of course, "scientific and rational" does not necessarily mean "good" since Idea theory is not scientific and rational.)

($\sharp_3$) Allegory 3.6 [the Sun] is similar to quantum language (= measurement theory).

Further note that

($\flat$) [History of Western philosophy]
= [dualistic idealism] + [the spirit of being scientific and rational]
= [Find the scientific destination of dualistic idealism]

If so (i.e. if ($\sharp_1$), ($\sharp_2$), ($\sharp_3$) and ($\flat$) are true), we can expect that the series (3.1) is inevitable.

KSTS/RR-20/001
November 15, 2020

3.5 Key words of dualism

3.5.1 Three key-words of dualism

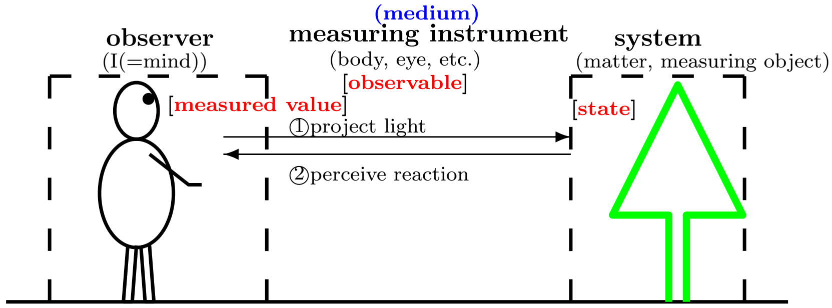
The key-words of dualism are simple. That is because

- Since dualism is a theory concerning measurement, [A](observer) and [C](matter) are needed. However, if the two are not related, this implies only that there are two monism. Therefore, there must be [B](medium(=device that mediate [A] and [C]))

Recall Figure 1.2 (in Section 1.1.2: The linguistic Copenhagen interpretation) below:

Figure 1.2 (in Section 1.1.2); [Descartes Figure]: Image of "measurement(=①+②)" in dualism

$$[\mathrm{A}](\textbf{mind}) \quad \longleftarrow \underset{(\textbf{medium})}{[\mathrm{B}]} \longrightarrow \quad [\mathrm{C}](\textbf{matter})$$



(A)		(B)		(C)
$\boxed{\text{measured value}}$	,	$\boxed{\text{observable (= measuring instrument)}}$	,	$\boxed{\text{state}}$.
(observer, brain, mind)		(thermometer, eye, ear, body, polar star)		(matter, measuring object)

If so,

- The structure of mind-matter dualism is as follows

$$[\mathrm{A}](\textbf{mind}) \quad \longleftarrow \underset{(\textbf{medium})}{[\mathrm{B}]} \longrightarrow \quad [\mathrm{C}](\textbf{matter})$$

  That is, it is composed of [A](mind),[B](medium),[C](matter).

In history, there are incomplete dualism that does not include three [A](mind), [B](medium), [C](matter).

- We consider that [B](medium) is the most important than the other two (*cf.* Linguistic Copenhagen interpretation (E_3) in Sec.1.1.2). Therefore, if the theory includes [B](medium), it is called "dualism".

3.5.2 Is Idea theory related to measurement (i.e., dualism) ?

Now we have a question:

- Where should these key-words (i.e., Idea world, actual world, Idea, anamnesis) be assigned next?

$$\underset{\text{(observer, brain, mind)}}{\overset{\text{(A)}}{\boxed{\text{measured value}}}}, \quad \underset{\text{(thermometer, eye, ear, body, polar star)}}{\overset{\text{(B)}}{\boxed{\text{observable (= measuring instrument)}}}}, \quad \underset{\text{(matter, measuring object)}}{\overset{\text{(C)}}{\boxed{\text{state}}}}.$$

This is answered in what follows.

Review 3.10. Recall that Remark 3.4 (Allegory of the cave), Remark 3.5 (Anamnesis), Remark 3.7 (Allegory of the cave) in the following table:

\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
①:Plato: cave Remark 3.4	actual world	/	/ [Idea world]
②:Plato: anamnesis Remark 3.5	actual world	anamnesis	/ [Idea world]
③:Plato: sun Remark 3.7	actual world	Idea	/ [Idea world]
quantum language	**measured value**	**observable**	**state [system]**

Plato's theory of ideas was not created for science, so what follows may be too forceful. However, In this text, I consider the following. Here,

(♯) **I don't know Plato's intention, but I want to choose Remark 3.7(Allegory of the sun)**, which is more scientific than the other two.

That is, I assert that

\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
③: Plato: sun Remark 3.7	actual world	Idea	/ [Idea world]
quantum language	**measured value**	**observable**	**state [system]**

The reason the I think that the above is somewhat persuasive is as follows.

- Since the Sophists said "Man is the measure of all things", Socrates-Plato must have thought the exact opposite claim, "Idea is the measure of all things".

  Namely,

  (♭) There exists the absolute standard (= idea) of "love", "beauty", "goodness", etc.

  Thus it is not unnatural to regard Idea as the meter standard, or the touchstone (i.e., a kind of measuring instrument).

♠**Note 3.8.** I am not confident in the correspondence ③ in Review 3.10 because the theory of ideas was not created with the intention of measurement. However, I don't care if the above claim is wrong. That is because our purpose of this paper is to show that

($\sharp$) Plato(the theory of Ideas) $\xrightarrow[\text{progress}]{}$ Descartes $\xrightarrow[\text{progress}]{}$ Kant $\xrightarrow[\text{progress}]{}$ Quantum language

if approaching quantum language is called "progress". It is easy to see that the above ($\sharp$) holds in all cases of ①, ②, and ③.

♠**Note 3.9.** However, I think that

the Allegory of the Sun is not a proper example for Idea theory,

since it is too scientific and thus too easy to understand. Probably, I think Plato gave too much scientific explanation in an attempt to make readers understand Idea theory. I think Plato was not satisfied with the parable in "the Allegory of the Sun", so he referred to "the Allegory of the Sun" as "the child of goodness".

The book : "History of western philosophy" due to B. Russell says that (*cf.* ref. [98]),

- *Aristotle's metaphysics, roughly speaking, may be described as Plato diluted by common sense. He is difficult because Plato and common sense do not mix easily.*

In the same sense, Idea theory and "the Allegory of the Sun" do not mix easily. Thus, this allegory may hinder our understanding of Idea theory.

However, it is interesting that the scientific explanation of Idea theory is similar to measurement theory. Thus, in this paper, I choose ③ as mentioned in Review 3.10.

♠**Note 3.10.** Literature has various genres. As an example, it is a love story, a detective story, SF (science fiction), poetry, nonfiction. In the same sense, **the philosophy of worldview is a kind of literature.**

Therefore, I think that the emphasis on the relationship to mathematics hinders our understanding of ideation theory.

The following is famous:

($\sharp$) It was written in the gate of the entrance of the school (Platonic Academy) which Plato established, **"The person who does not know the geometry should not pass through this gate"**

If this is to be taken seriously, Plato's researchers must be well trained in mathematics. But in reality, that has not happened. In science, it is more experimental verification than logic. Thus, the importance of logic is not so emphasized in science. That is, *the proof of the pudding is in the eating.* The field where importance of the logic is emphasized is the field where it is hard to do logical and quantitative arguments. For example, the importance of logic is emphasized in courts. There must be many unknowns in many cases. Still, the judge must judge the person. Therefore, the impression of "logically judged" is very important.

However, "mathematical, logical, reasonable" must be emphasized in philosophy. That is because the difference between philosophy and religion becomes obscure without the emphasis. The reason why the importance of logic is not emphasized in science is that I think it's because if we emphasize the importance of experimentation, it won't be confused with religion.

3.5.3* Plato's Idea theory ≈ Locke's secondary quality ≈Sausuure's linguistic theory ≈ Zadeh's fuzzy theory

If we assume that

(D) Plato's Idea theory, Locke's secondary quality, Sausuure's linguistic theory and Zadeh's fuzzy theory are one of aspects of quantum language

then, we can discuss as follows.

$$ \text{①:Idea} \xrightarrow[\text{cognitive}]{} \text{②: secondary qualities} \xrightarrow[\text{quantitative}]{} \text{③: observable (= measuring instrument)} $$

where,

① Idea
"the meter standard of beauty, the meter standard of goodness, $\cdots$"
$\Longrightarrow$
"device that make beauty visible", "device that make the good visible"

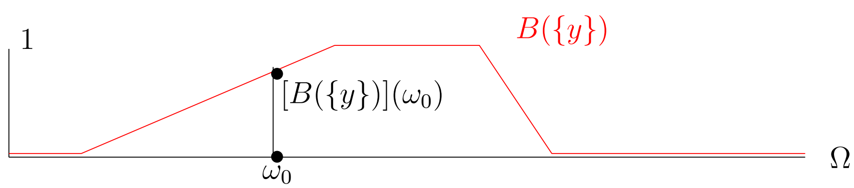
② secondary qualities(=sensations of inherent nature (=primary qualities))
"sweet, pungent", "hot, cold", "beautiful,ugly" $\cdots$

③ observable(=measuring instrument)
saccharimeter, thermometer, $\cdots$

In this real world, "Man is the measure of all things" may be almost true. On the other hand, in the Idea world, the absolute goodness, the absolute beauty. are always believed. That is,

- There exists the absolute standard of "love", "beauty", "goodness", etc.

Example 3.11. [The argument under the assumption that Idea theory is regarded as measurement theory]
Let Ω be a compact state space. Here, any tree can be represented by a certain state $\omega(\in \Omega)$. Thus, we have the basic structure $[C(\Omega) \subseteq L^\infty(\Omega,\nu) \subseteq B(L^2(\Omega,\nu))]$. Let $\mathsf{O}_B = (\{y,n\}, 2^{\{y,n\}}, B)$ be the continuous observable, which is assumed to be measurement instrument of beauty. Let T_0 be a tree with the state $\omega_0(\in \Omega)$. Put $\omega_0 = \widetilde{\omega}(T_0)$. Then, we have the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_B = (\{y,n\}, 2^{\{y,n\}}, B), S_{[\widetilde{\omega}(T_0)]})$, where "$y$" [resp. "$n$"] means "yes" [resp. "no"].



Axiom 1 (in Section 1.5.2) implies that

- the probability that a measured value y is obtained by the measurement $\mathbb{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}_B = (\{y,n\}, 2^{\{y,n\}}, B), S_{[\widetilde{\omega}(T_0)]})$ is given by $[B(\{y\})](\omega_0)$.

That is,

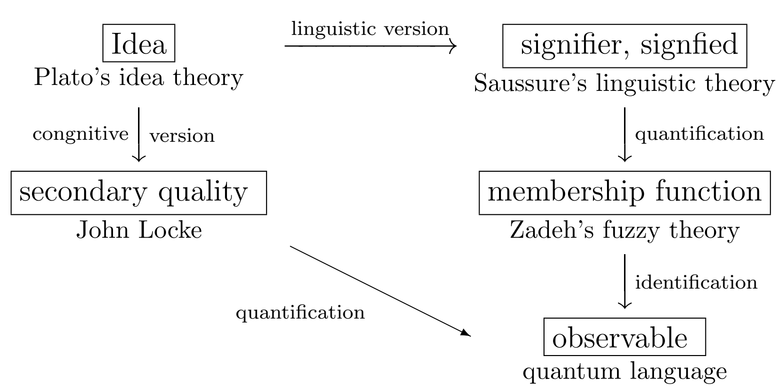
- the probability that the tree T_0 is beautiful is given by $[B(\{y\})](\omega_0)$.

Remark 3.12. [Plato's Idea theory $\approx$ Locke's secondary quality $\approx$Sausuure's linguistic theory $\approx$ Zadeh's fuzzy theory] Readers may find this example trivial. However, it should be noted that this example is essentially the same as Definition 11.16 (”signifier” and ”signified”). That is, we see, from the quantum linguistic point of view, the following three are similar:

$$
\begin{cases}
\text{Plato's Idea theory} \\
\text{Locke's secondary quality } (\textit{cf.} \text{ Sec. } 9.1) \\
\text{Saussure's linguistic theory } (\textit{cf.} \text{ Sec. } 11.5) \\
\text{Zadeh's Fuzzy theory } (\textit{cf.} \text{ refs. } [35, 36, 37, 38])
\end{cases}
$$

Or more precisely,

(D)



3.6 Summary: Plato's way of telling philosophy

3.6.1 Summary

(A):
Plato's way of telling philosophy (i.e., the fictional worldview)

In Plato's way of telling philosophy,

(A_1) a fictional worldview is characterized as the premise (or, introduction, preface, fiction) of the main theme (i.e., ethics, moral).

In other words, consider the following figure, i.e., Plato's way of telling philosophy:

world is so

(A_2) $\boxed{\text{fictional worldview (literary truth, pseudo- truth)}}$

preface, introduction, (fictional)premise, expedient, prelude

you should do so

$\xrightarrow{\text{therefore}}$ $\boxed{\text{practical logic, ethics, morals}}$

main subject

Literature has various genres. As an example, it is a love story, a detective story, SF (science fiction), poetry, nonfiction. In the same sense, the above fictional worldview (as the support of the main assertion [ethics. moral]) is a kind of literature.

(Notice)

(A_3) Some consider that the term: "$\xrightarrow{\text{therefore}}$" implies that the fictional worldview should be "logical". Here, it should be noted that the "logical" is similar to the "logical" of detective story.

If we think about it simply, **the "introduction (= fictional world view)" part is unnecessary.** In fact, Socrates, Confucius, etc. focused only on ethics and morality. However, the "introduction" part had a variety of benefits. In what follows, let us explain some benefits.

Remark 3.13. Whitehead(1861 - 1947) said:

(B) Western philosophy is characterized as a series of footnotes to Plato

Although we do not know his true intention, our understanding is as follows.

(C) The various fashions of the Buddhism, Confucianism and the Taoism Confucius jumbled up at the Orient. And those continued for 2500 years by a subtle strained relation. On the other hand, in the Western, Christianity was too strong. Thus, the main theme (i.e., ethics, morals) is mostly due to "Christianity + Socrates", and thus, the various fashions were not born.

However, according to Plato's way of telling philosophy, the introduction part (i.e., the fictional worldview) is changeable.

Therefore, the progress of Western philosophy (which was not realized in the East) can be realized as follows.

- Plato⟶Augustinus⟶Thomas Aquinas ⟶Descartes⟶Locke⟶...⟶Kant

That is,

Plato's way of telling philosophy could keep freshness.

If the example is say, we think that there has been an effect, such as **the model change of car**. The Plato's way of telling philosophy is almost always the main current of Western philosophy. This device (i.e., the model change) brought the prosperity of Western philosophy.

If we do not consider so, we cannot explain the fact that useless world-description (in western philosophy) lasted for 2500 years.

♠**Note 3.11.** We consider that

($\sharp$) Philosophy of ethics is common to mankind and is the world standard.

Or, we want to consider so. Otherwise, world peace cannot be achieved. There is a point to the theory that ethics is a rule that keeps society alive. Depending on what kind of society we envision, ethics will be somewhat different. However, we want to consider ($\sharp$). For example, "Don't lie" or "Don't kill people", etc., are common in the world. Also, the Golden Rule "Do unto others what you would have them do unto you" has been chanted by many philosophers and religious figures (Christ, Confucius , Muhammad, etc.).

In this sense, I think that ethics is logical.

However, worldview (= Plato's fictional worldview in the way he tells his philosophy) is not universal. In other words, it is one of the local philosophies that has evolved in its own way. We might say "Galapagos philosophy". For example, consider

$$\boxed{\text{Idea theory}} \xrightarrow[\text{cogito turn}]{} \boxed{\text{epistemology}} \xrightarrow[\text{linguistic turn}]{} \boxed{\text{analytic philosophy}}$$

If there was no epistemology, it is no wonder. In fact, there is no epistemology in the East or the United States. In this sense, the world description is not logical.

On the other hand,

- the realistic worldview is the world standard (moreover, the universe standard)
  Aristotle⟶Archimedes⟶Newton⟶ $\cdots$
- The logical worldview can't be called the world standard since it is quite influenced by Descartes=Kant philosophy.
  Aristotle ⟶ Frege ⟶ Wittgenstein⟶ $\cdots$
  (I think that philosophers need not be more logical than scientists. Thus, I can't understand why philosophers like logic. Kant philosophy was too literary, and it may be the reaction. I highly estimate Wittgenstein's "Tractatus Logico-Philosophicus (widely abbreviated and cited as TLP)", which is more poetic than logical.)
- the mechanical worldview is also the world standard:
  $$\underset{\text{(motion function method)}}{\text{Parmenides·Zeno}} \xrightarrow[\text{Tunnel of two thousand several hundred years}]{} \text{Fisher(statistics)}$$

And, it is a matter of course that the above two (the realistic worldview and the mechanical worldview) are useful. It is remarkable that the philosophy of worldview (=the fictional worldview) isn't useful at all but it continued for 2500 years. That is, we think that "Galapagos" $\Leftrightarrow$ "scientifically useless". This may be due to the shadow supporter (i.e., Christianity), i.e., as mentioned in the above remark, Christianity is too strong.

♠**Note 3.12.** Now we have the following classification of philosophers. (*cf.* Classification 1.11 [the classification of philosophers]): Recall the figure (Allegory of the cave) in Allegory 3.3, which is clearly related to measurement (*cf.* Definition 1.8 of "idealism"). Thus, Plato's Idea theory belongs to the fictional worldview (Western philosophy):

$$(\flat)\quad \begin{cases} (\flat_1)\text{: the realistic worldview (physics)} \\ \quad \text{Hērakleitos, Aristotle, Archimedes, Galileo, Newton, Einstein,} \cdots \\ \\ (\flat_{21})\text{: the fictional worldview (Western philosophy)} \\ \quad \text{Plato, Scholasticism, Descartes, Locke, Leibniz, Berkeley, Hume, Kant, Husserl} \\ \\ (\flat_{22})\text{: the logical worldview} \\ \quad \text{Boole, Frege, Peirce, Saussure, Russell, Wittgenstein, Hempel, Popper} \\ \\ (\flat_{23})\text{: the mechanical worldview (statistics, quantum language)} \\ \quad \text{Parmenides, Zeno, J. Bernoulli, statistics (e.g., Fisher), quantum language} \end{cases}$$

Spirit of Pythagoras is inherited, and Parmenides and Zeno have argued establishment of the worldview as science sincerely. However, Plato used the fictional worldview as a means of the protection of Socrates(ethic philosophy) and has dwarfed the worldview in non-scientific way. But, as Whitehead said "Plato's footnote", the fictional worldview, that is,

$$(\sharp_1)\quad \text{Plato}\rightarrow\text{Augustinus}\rightarrow\text{Thomas Aquinas}\longrightarrow\text{Descartes}\longrightarrow\text{Locke}\rightarrow...\rightarrow\text{Kant}\longrightarrow\text{Husserl}$$

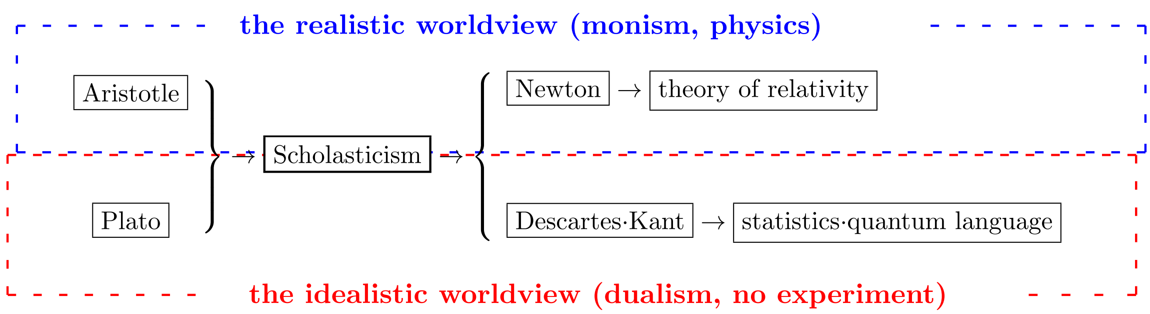
has continued to be supported over a long time of more than 2000 years. On the other hand, the scientific idealistic worldview was established by Fisher , etc. as follows:

$$(\sharp_2)\quad \text{Parmenides}\cdot\text{Zeno} \rightarrow \xrightarrow{\text{Tunnel of more than 1700 years}} \rightarrow \underset{\text{J.Bernoulli,Bayes,Laplace,etc.}}{\overset{\text{the classical mechanical worldview}}{\boxed{\text{Early modern period}}}}$$
$$\rightarrow \text{Fisher(statistics)}\longrightarrow\text{quantum language}$$

Chapter 4

The Big Three in Greek Philosophy (Aristotle)

Although Aristotle was a student of Plato, he proposed the realistic worldview, which was completely different from Plato's philosophy. He is called the father of all sciences ($\approx$ the father of the realistic worldview). It is no exaggeration to say that philosophy was started by these two men. That is, we see:



Therefore, it's best to assume that the two men have very different areas of expertise.

4.1 Aristotle (BC.384 - BC.322)

4.1.1 Realistic worldview vs. idealistic worldview

Aristotle (BC.384 - BC.322), the student of Plato, is called the father of all sciences ($\approx$ the father of the realistic worldview). He could not accept Plato's theory of Ideas(=[asserted fiction]). Namely,

- **Philosopher Plato preferred asserted fiction (without experiment) to truth (with experiment)**

On the other hand,

- **Scientist Aristotle preferred truth (with experiment) to asserted fiction (= without experiment)**

♠**Note 4.1.** S. Weinberg (1933 -), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said in his book [105] "To explain the word; The discovery of modern science" as follows:

(♯) [in Chapter 3] *I confess that I find Aristotle frequently tedious, in a way that Plato is not, but although often wrong Aristotle is not silly, in the way that Plato sometimes is.*

Plato was not aiming for science, that is, Plato's purpose is to support Socrates's ethical philosophy. Therefore, from the scientific point of view, some may feel Plato silly. Namely,

(1) a fiction that was asserted (by Plato) over 2000 years ago is somewhat silly, if not tedious.

On the other hand, Aristotle might be aim for science. Thus, from the modern point of view, some may feel Aristotle tedious. Namely,

(2) truths discovered (by Aristotle) over 2000 years ago are tedious and often wrong, if not silly

The above (1) and (2) are merely statements of the commonplace. And thus, he may not be saying anything negative. Science progresses, so you'll find the old science boring. But literature hasn't faded after 2,000 years. However, our purpose of this text is to show that

$$\boxed{\text{Plato}} \xrightarrow[\text{progress}]{} \boxed{\text{Descartes}} \xrightarrow[\text{progress}]{} \boxed{\text{Kant}} \xrightarrow[\text{progress}]{} \boxed{\text{Wittgenstein}} \xrightarrow[\text{progress}]{} \boxed{\text{Quantum language}}$$

4.1.2 Edios and Hyle

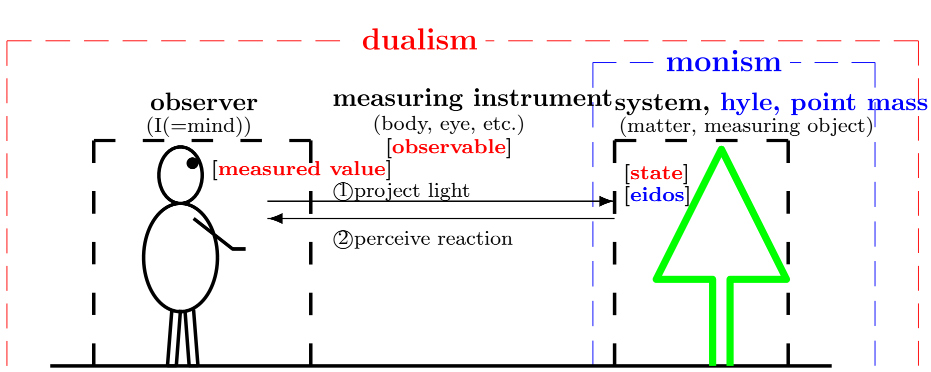
Aristotle proposed the concepts such as "eidos" and "hyle" as follows.

(A): Edios(Aristotle's Idea) and hyle

Aristotle said that

- **Edios** (= Aristotle's Idea = true form) is not in the heaven, but in **hyle** (= matter = particle).

Figure 1.2 (in Section 1.1.2); [Descartes Figure]: Image of "measurement(=①+②)" in dualism: **dualism** and **monism (= Aristotle's worldview)**



Assertion 4.1. (= Assertion 1.14)

[The key-words of the realistic worldview] The realistic worldview is monism, and its completed version is realized as Newtonian mechanics, whose key-words are "point mass" and "state". Thus, we see:

\	[A](= mind)	[B](Mediating of A and C)	[C](state) [matter]
Aristotle	———	———	eidos [hyle]
Newton	———	———	state [point mass]

That is, we consider the following progress:

$$\text{[eidos]} \xrightarrow[\text{progress}]{} \text{[state]} \qquad \text{[hyle]} \xrightarrow[\text{progress}]{} \text{[point mass]}$$

We can easily use Newtonian mechanics as follows.

- **point mass**(=particle with the mass m) with the **state**(=(position, momentum)= $(x,p) \in \mathbb{R}^2$(= state space))

  Thus, it is a matter of course that we conclude that

  - **Aristotle is the founder of the realistic worldview (= physics)**

[Note]: The above table should be compared to the following table in Review 3.10 [i.e., Plato's Idea theory]:

\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
③: Plato: Sun Remark 3.7	actual world	Idea	/ [Idea world]
quantum language	**measured value**	**observable**	**state [system]**

♠**Note 4.2.** The book : "History of western philosophy" due to B. Russell says that (*cf.* ref. [98]),

- *Aristotle's metaphysics, roughly speaking, may be described as Plato diluted by common sense. He is difficult because Plato and common sense do not mix easily.*

I think this representation is misleading since the two men have very different areas of expertise. Aristotle proposed a realist philosophy that is quite different from the Platonic philosophy of idealism. As the following diagram shows, Plato and Aristotle are water and oil. In fact, Scholasticism (= a compromise between Plato and Aristotle) was not successful because Plato and Aristotle do not mix.

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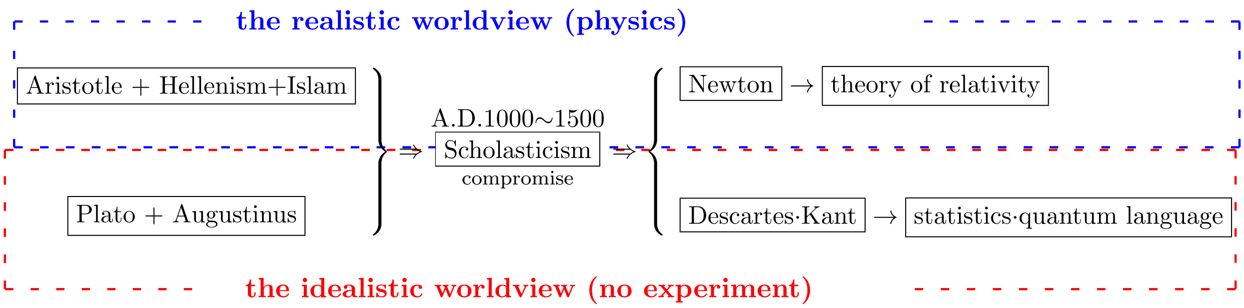


"Realistic worldview (= realism) or idealistic worldview (= metaphysics) ?" is the biggest dispute in the history of philosophy as shown in Assertion 1.12, that is,

realistic worldview vs. idealistic worldview (*cf.* Assertion 1.12)

dispute ＼ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	"Nominalismus"(Ockham)	"Realismus"(Anselmus)
ⓓ: space-time	Newton	Leibniz
ⓔ: quantum theory	Einstein	Bohr

ⓐ is my fiction, ⓒ is more of a confusion than a dispute. ⓓ is the Leibniz=Clarke correspondence (*cf.* Sec. 9.3.2), ⓔ is Bohr-Einstein debates. Quantum language is proposed as one of answers to Bohr-Einstein debates(*cf.* ref. [66]).

For further information, see my homepage

4.2 Why does the motion happen?

4.2.1 From purpose to causality: Modern science started from the discovery of "causality"

When a certain thing happens, the cause always exists. This is called *causality*(=*causal relation*). You should just recall the next proverb:

Smoke is not located on the place which does not have fire.

However the situation is not so simple as you think. Consider, for example,

- This morning I feel good.

  ($\sharp_1$) Is it because that I slept sound yesterday ?
 or

  ($\sharp_2$) Is it because I go to favorite golf from now on ?

You will find the difficulty in using the word "causality". In daily conversation, the word "causality" is used in many contexts, mixing up "a cause (past)", "a reason (implication)", and "the purpose and a motive (future)".

As mentioned in Sec. 2.3, the pioneers in the study of movement and change are Hērakleitos and Parmenides:

(A) $\begin{cases} \bullet \text{ Hērakleitos(BC.540 -BC.480): "Everything flows."} \\ \\ \bullet \text{ Parmenides (born around BC. 515): "There is no movement."} \\ \qquad\qquad \text{(Zeno's teacher)} \end{cases}$

I think the reader will have the following question.

- Why are their names still there, even though it was 2,500 years ago?

As I mentioned before, "motion and change" is the most important keyword in science (= "worldview"), that is, I consider:

(B) **[The beginning of World description]**

$$= \text{[The discovery of movement and change]} = \begin{cases} \text{Hērakleitos} \\ \\ \text{Parmenides} \end{cases}$$

This is why their names are still there.

However, Aristotle (BC384–BC322) pursued an even more fundamental problem:

(C) What is the essence of movement and change?

and concluded as follows.

(D):Purpose (Aristotle)

Aristotle asserted that **all the movements had the "purpose".**

- For example, a stone falls because it has the purpose to go downward, and smoke rises because it has the purpose to go upward.
  A heavy stone falls fast because it has a strong purpose of "falling fast".

4.2.1.1 From purpose to causality

Under the influence of Aristotle, "*Purpose*" had remained as a mainstream idea of "Movement" for a long period of 1500 years or more.

We were freed from the spell of "Purpose", only after Galileo, Bacon, Descartes, and Newton et al. discovered the essence of movement and change lies in "Causality".

Scientific revolution from "Purpose" to "Causality"

is the greatest paradigm shift in the history of science. It is not an exaggeration even if we call the shift "*birth of modern science*".

$$\text{(E)} \quad \underset{\text{(Hērakleitos, Parmenides, Zeno, Aristotle)}}{\overset{\text{the birth of worldview}}{\boxed{\text{Movement}}}} \xrightarrow[\text{Aristotle's spirit :(About 1500 years)}]{\text{"purpose" (and no experiment)}} \underset{\text{(Bacon, Galileo, Descartes, Newton)}}{\overset{\text{the birth of modern science}}{\boxed{\text{Causality (and experiment)}}}}$$

I cannot emphasize too much the importance of the discovery of the term: "causality". That is,

($\sharp$) Science is the discipline about phenomena that can be represented by the term "causality". (i.e., "No smoke without fire")

Thus, I consider that the discovery of "causality" is equal to that of science.

In the realistic worldview, Newtonian kinetic equation (i.e., the equation of the chain of causality) was final in a sense. However, in the idealistic worldview, the problem "What is causality?" is not solved yet. For the complete answer to the problem, we had wait for the appearance of quantum language (Axiom 2 (causal relation) in Sec.1.1.1, also, see ref. [66]).

Summary 4.2. [Solutions to the causality problem] For example, we see:

(F_1) The causality is represented by Newtonian kinetic equation in Newtonian mechanics

(F_2) The causality is represented by Maxwell's equations in electromagnetism

(F_3) The causality is represented by Schrödinger equation (or equivalently, Heisenberg's kinetic equation) in quantum mechanics

(F_4) The causality is represented by Axiom 2 (in Section 1.1) in quantum language

(Continued to Sec. 10.3: What is causality?).

♠**Note 4.3.** S. Weinberg (1933 -), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said in his book [105] ”To explain the word; The discovery of modern science” as follows:

- [in Chapter 3]: *We can agree with the classical scholar R. J. Hankinson that ”we must not lose sight of the fact that Aristotle was a man of his time and for that time he was extraordinarily perspicacious, acute, and advanced.” Nevertheless, there were principles running all through Aristotle’s thought that had to be unlearned in the discovery of modern science. For one thing, Aristotle’s work was suffused with teleology: things are what they are because of the purpose they serve.*

Recall the above (E). I think that only people after the scientific revolution (17th century) understand the meaning of ”science”.

Aristotle is also the father of biology. I think the reason Aristotle failed to discover causality is that biology was one of his major research topics. In biology, we tend to think in terms of purpose theory, and causality is very difficult to understand. It is in astronomy and physics that the causal relationship is very easy to see. In addition, causality is relatively easy to experimentally verify. Thus, the scientific revolution was born out of [(i) geodynamics, (ii) causality (Newtonian mechanics), and (iii) the importance of experimentation (i.e., British empiricism)]. That is,

$$\left.\begin{array}{l}\text{(i) Geocentrism}\\ \text{(ii) causality (Newtonian mechanics)}\\ \text{(iii) the importance of experimentation}\end{array}\right\} \Longrightarrow \text{ scientific revolution}$$

4.3 Practical logic

4.3.1 Aristotle's syllogism in ordinary language

If the ecology of various animals is observed, it will be clear that the base of language was due to intimidation, solidarity, reproduction. Language was one of the strongest arms for the survival and breeding. Such a time have continued for millions of years. Of course, the biggest events in the "history of language" happened one after the other. For example,

(A) "rhythm and song", "logical structure", "quantity concept", "grammar", "tense", "character", etc.

However, it was done gradually by many people, tens of thousands of years ago, and it is not possible to identify the names of the contributors. But the **surprise** that ordinary language had a logical structure is passed down as "Aristotle's syllogism," Namely,

①: Aristotle's syllogism in ordinary language

(B) Since Socrates is human being, and human being is mortal, it follows that Socrates is mortal.

Although this is quite famous, I have several questions concerning this as follows,

(C_1) Syllogism is essential for mathematical proofs. Therefore, it is natural to assume that Pythagoras already knew syllogism.

(C_2) Also, it is natural to consider that syllogism was frequently used in the debate between Socrates and sophists.

Thus I guess that

(D) The knowledge of syllogism of the time was summarized in Aristotle's book: "Organon", which was compiled by his followers about B.C. 40. And, syllogism was endorsed by Aristotle and remained authoritative for almost 2,000 years.

In fact, Immanuel Kant said that there was nothing else to invent after the work of Aristotle.

4.3.2 Can we trust syllogism ($A \to B$, $B \to C$, therefore, $A \to C$) ?

Again recall Sec. 2.4 (Zeno's paradoxes), in which we studied the fact:

(E) the logic in ordinary language cannot be trusted. That is, we must start from the worldview.

That is, our belief is the worldviewism (i.e., the worldview first) as follows:

(F): worldviewism (*cf.* Sec. 1.3.1)

The worldviewism has the following form:

(F) $\underset{\text{premise}}{\overset{\text{world is so}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{subject}}{\overset{\text{conclusion}}{\boxed{\text{discussions, calculation, practical logic, properties (motion, etc.)}}}}$

that is, the spirit such that "Start from worldview"

As mentioned before, Zeno's paradoxes suggest the necessity of the worldviewism. After the proclamation of the worldview (e.g., Newtonian mechanics, the theory of relativity, quantum language, etc.) , we have to discuss Zeno's paradoxes (*cf.* Sec. 1.3.1).

Of course, the worldviewism is indispensable to the arguments "syllogism" as well as "motion". That is,

(G_1) After a worldview (i.e., language system) is decided, logic is decided automatically and naturally. On the contrary, when a worldview (i.e., language system) is not decided, logic is not decided. That is,

the worldview takes precedence over syllogism (or logic).

In this sense, it is not exaggeration even if we say "language = logic". If so, we may nod that there is the meaning of both of "language" and "logic" for Greek "logos".

After all, I think (*cf.* Note 1.15) that

(G_2)
- Axiomatic logic is closely related to mathematics
- Practical logic is closely related to worldviews

Thus, I have an opinion that philosophers should be more interested in practical logic than in axiomatic logic.

♠**Note 4.4.** The above arguments may be obvious. For example, consider the worldview called Newtonian mechanics. It is a matter of course that

- After the declaration of the worldview (=the law of Newtonian mechanics), the language system called Newtonian mechanics is established. That is,

  there is no calculation without Newton's law

  Hence, under the language system, the reason (e.g., calculation, logic) is formed.

Therefore, it is obvious that the worldviews takes precedence over syllogism (or logic).

4.3.3* Syllogism holds in classical systems, but not in quantum systems

We have the following theorem:

Theorem 4.3. Consider quantum language as the worldview. Then,

(H_1) in classical cases, syllogism always holds

(H_2) in quantum cases, syllogism does not necessarily hold.

[Proof]:
For classical cases (H_1), see Sec. 4.3.5. Also, see refs. [35, 66])
For quantum cases (H_2), see Sec. 4.3.4. Also, see refs [47, 66]).

♠**Note 4.5.** Here, we have the following question:

($\sharp$) Why are "Zeno's paradoxes" and "Aristotle's syllogism" famous? Why are those discussed repeatedly in philosophy?

General philosophers might feel "something which doesn't fit nicely (i.e., the neglect of the worldviewism)" in "Zeno's paradoxes" and "Aristotle's syllogism". This feeling is transmitted more than 2,000 years. This is our answer to the above ($\sharp$).

♠**Note 4.6.** To compare Review 3.10 and Assertion 4.1, we can guess that Aristotle cannot understand Plato's theory of Ideas. That is, Aristotle tried to propose the other theory (i.e., the realistic worldview) than Plato's theory (i.e., the fictional worldview). And he discovered "eidos" and "hyle", which are the most basic concepts in mechanics. Thus, we conclude that Aristotle is the founder of the realistic worldview. As mentioned before, I am skeptical of the logical worldview, but the following diagram is usual in philosophers:

$$\boxed{\text{Arithtotle}} \longrightarrow \boxed{\text{Boole}} \longrightarrow \boxed{\text{Frege}} \longrightarrow \boxed{\text{Russell}} \longrightarrow \ldots$$

Aristotle's syllogism may not belong to symbolic logic, thus, I do not add Aristotle to ($\flat_{22}$).

$$(\flat)\begin{cases} (\flat_1)\text{: the realistic worldview (physics)} \\ \quad \text{Hērakleitos, Aristotle, Archimedes, Galileo, Newton, Einstein,} \cdots \\ \\ (\flat_{21})\text{: the fictional worldview (Western philosophy)} \\ \quad \text{Plato, Scholasticism, Descartes, Locke, Leibniz, Berkeley, Hume, Kant, Husserl} \\ \\ (\flat_{22})\text{: the logical worldview} \\ \quad \text{Boole, Frege, Peirce, Saussure, Russell, Wittgenstein, Hempel, Popper} \\ \\ (\flat_{23})\text{: the mechanical worldview (statistics, quantum language)} \\ \quad \text{Parmenides, Zeno, J. Bernoulli, statistics (e.g., Fisher), quantum language} \end{cases}$$

In science, quantitative discussion and computation become important. In this sense, we consider that physics (or, mechanics) is located in the center of science. On the other hand, the logical worldview is rather qualitative, and therefore, the logical worldview is rather influential in the field of humanities. For example, trials are an area where the importance of logic is most emphasized. Also, I think that philosophers are people who like logic most.

4.3.4 Syllogism does not always hold in quantum systems

Proof 4.4. The proof of Theorem 4.3 (H_2);

(H_2) : [Syllogism does not hold in quantum systems]:

i.e., the following does not always hold in quantum language:

- if $P_1 \longrightarrow P_2$ and $P_2 \longrightarrow P_3$, then it holds $P_1 \longrightarrow P_3$.

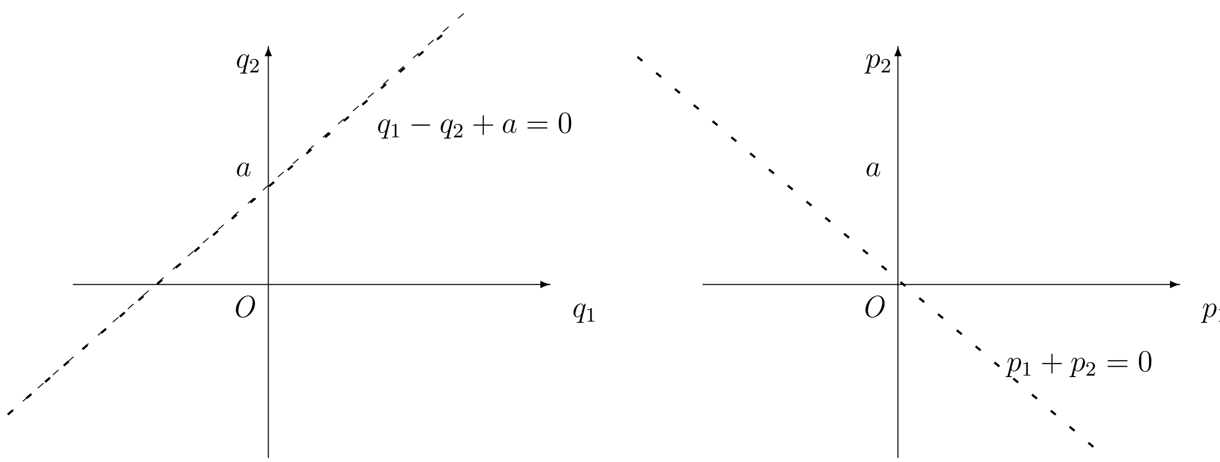
Let us prove it as follows. A quantum two particles system S is formulated in a tensor Hilbert space $H = H_1 \otimes H_1 = L^2(\mathbb{R}_{q_1}) \otimes L^2(\mathbb{R}_{q_2}) = L^2(\mathbb{R}^2_{(q_1,q_2)})$. The state u_0 ($\in H = H_1 \otimes H_1 = L^2(\mathbb{R}^2_{(q_1,q_2)})$) $\Big($ or precisely, $\rho_0 = |u_0\rangle\langle u_0|\Big)$of the system S is assumed to be

$$u_0(q_1, q_2) = \sqrt{\frac{1}{2\pi\epsilon\sigma}} e^{-\frac{1}{8\epsilon^2}(q_1-q_2+a)^2 - \frac{1}{8\sigma^2}(q_1+q_2)^2} \tag{4.1}$$

where $a \neq 0$, a positive number ϵ is sufficiently small, and a positive number σ is sufficiently large. Thus, we see that

$$|\widehat{u_0}(p_1, q_2)| = |\sqrt{\frac{1}{2\pi\epsilon\sigma}} e^{-\frac{1}{8\sigma^2}(p_1-p_2)^2 - \frac{1}{8\epsilon^2}(p_1+p_2)^2}| \tag{4.2}$$

where $\widehat{u_0}$ is the Fourier transform of u_0



For each $k = 1, 2,$ define the self-adjoint operators $Q_k : L^2(\mathbb{R}^2_{(q_1,q_2)}) \to L^2(\mathbb{R}^2_{(q_1,q_2)})$ and $P_k : L^2(\mathbb{R}^2_{(q_1,q_2)}) \to L^2(\mathbb{R}^2_{(q_1,q_2)})$ by

$$\begin{aligned} Q_1 &= q_1, \qquad P_1 = \frac{\hbar\partial}{i\partial q_1} \\ Q_2 &= q_2, \qquad P_2 = \frac{\hbar\partial}{i\partial q_2} \end{aligned} \tag{4.3}$$

($\sharp_1^0$) Let $\mathsf{O}_1 = (\mathbb{R}^3, \mathcal{B}_{\mathbb{R}^3}, F_1)$ be the observable representation of the self-adjoint operator $(Q_1 \otimes P_2) \times (I \otimes P_2)$. And consider the measurement $\mathsf{M}_{B(H)}(\mathsf{O}_1 = (\mathbb{R}^3, \mathcal{B}_{\mathbb{R}^3}, F_1), S_{[|u_0\rangle\langle u_0|]})$. Assume that the measured value $(q_1^0, p_2^0, p_2^0)(\in \mathbb{R}^3)$. That is,

$$\underset{\text{(the position of } A_1\text{, the momentum of } A_2)}{(q_1^0, p_2^0)} \Longrightarrow \underset{\text{the momentum of } A_2}{p_2^0}$$

($\sharp_2$) Let $\mathsf{O}_2 = (\mathbb{R}^2, \mathcal{B}_{\mathbb{R}^2}, F_2)$ be the observable representation of $(I \otimes P_2) \times (P_1 \otimes I)$. And consider the measurement $\mathsf{M}_{B(H)}(\mathsf{O}_2 = (\mathbb{R}^2, \mathcal{B}_{\mathbb{R}^2}, F_2), S_{[|u_0\rangle\langle u_0|]})$. Assume that the measured value $(p_2^0, -p_2^0)(\in \mathbb{R}^3)$. That is,

$$\underset{\text{the momentum of } A_2}{p_2^0} \Longrightarrow \underset{\text{the momentum of } A_1}{-p_2^0}$$

($\sharp_3$) Therefore, if syllogism holds, we may conclude that

$$\underset{\text{(the position of } A_1\text{, the momentum of } A_2)}{(q_1^0, p_2^0)} \quad \Longrightarrow \quad \underset{\text{the momentum of } A_1}{-p_2^0}$$

$$\Big(\text{ that is, the momentum of } A_1 \text{ is equal to } -p_2^0\Big)$$

But, the above argument (particularly, "syllogism") is not true. That is because

($\sharp_4$) $(Q_1 \otimes P_2) \times (I \otimes P_2)$ and $(I \otimes P_2) \times (P_1 \otimes I)$ (therefore, O_1 and O_2) do not commute, and thus, the simultaneous observable does not exist.
Thus, we can not test the ($\sharp_3$) experimentally.

4.3.5 Why does logic arise in classical QL?

The close relationship between measurement and logic was first discussed in ref. [35].

- Ishikawa,S. *Fuzzy inferences by algebraic method,* Fuzzy Sets and Systems 87, 181–200 (1997) doi:10.1016/S0165-0114(96)00035-8
  Or, see ref. [71] (In preparation)

The argument in this section are regarded as a slight variation of the argument in ref. [35].

4.3.5.1 The proof of Theorem 4.3 (H$_1$)

Recall Theorem 4.3 (H$_1$), that is,

(H$_1$) : [Syllogism holds in classical systems], i.e., [Classical logic (i.e., $\wedge$, $\vee$, $\neg$) holds in classical systems]:

Or more precisely,

[In a class of classical binary projective measurements, measurement has properties like propositional logic]:

In this section we will devote ourselves to the above proof as follows.

Proof 4.5. The proof of Theorem 4.3 (H$_1$): Consider a classical basic structure:

$$[C_0(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu)]$$

Here, assume that Ω is a locally compact space with a Borel measure ν on Ω such that $\nu(D) > 0$ (for any open set D ($\subseteq \Omega$)). Consider many tomatoes, that is, roughly speaking, consider T as the set of all tomatoes. Assume that any tomato $t(\in T)$ is represented by a state ω, which is an element of the state space Ω. Thus, we have the map $\widehat{\omega} : T \to \Omega$. That is, the quantitative property of a tomato t is represented by $\omega(t)$. For example, it suffices to consider Ω as $\mathbb{R}^N$ (= N-dimensional real space), where N is sufficiently large natural number. That is,

$$\Omega \ni \omega = \Big(\omega^{(1)}(=\text{weight}), \omega^{(2)}(=\text{diameter}), \omega^{(3)}(=\text{diameter}), \omega^{(4)}(=\text{color value}),$$
$$\omega^{(5)}(=\text{calorie}), \omega^{(6)}(=\text{sugar content}), ..., \omega^{(N)}(= ...)\Big) \in \mathbb{R}^N$$

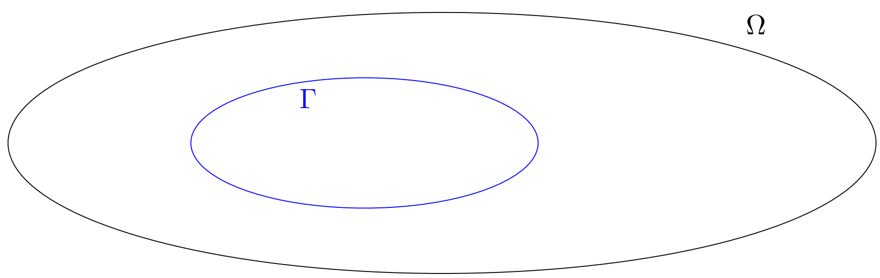
Consider a binary projective observable (i.e., $\{1,0\}$-valued projection observable, or $\{x_1, x_0\}$-valued projection observable) $\mathsf{O} \equiv (X, 2^X, F)$ in $L^\infty(\Omega, \nu)$, where $X = \{1, 0\}$ (or, $X = \{x_1, x_0\}$) and $F(\Xi) = [F(\Xi)]^2$ $(\forall \Xi \in \mathcal{F})$ (*cf,* Example 1.19).

Further, recalling Assumption 1.16, we, for the sake of simplicity, assume that $\{\omega \in \Omega \mid [F(\{1\})](\omega) = 1, a.e.\}(\equiv \Gamma)$ is an open set such that

$$\Gamma = [\overline{\Gamma}]^\circ, \qquad \nu(\Omega \setminus (\Gamma \cup [\Gamma^c]^\circ)) = 0$$

where D^c is the complement of D, i.e., $\Omega \setminus D$, $\overline{D}$ ="the closure of D", D° ="the interior of D)". Note that we can assume that $\Gamma^c = [\Gamma^c]^\circ$ in the sense of "almost everywhere concerning ν". The $\{1,0\}$-valued projective observable $\mathsf{O} \equiv (X, 2^X, F)$ in $L^\infty(\Omega, \nu)$ is also denoted by

$$\mathsf{O}^\Gamma \equiv (X, 2^X, F^\Gamma) \tag{4.4}$$



♠**Note 4.7.** (i): Someone might say that the term "the set of all tomatoes" is ambiguous. However, for the sake of convenience, here we use the term "the set of all tomatoes". In Sec. 11.8 [Hempel's raven problem], we will discuss it precisely.

(ii): If we want to both tomato's world Ω_1 and apple's world, it suffices to start from the product space $\Omega_1 \times \Omega_2$. Thus, we consider the world also is represented by a large state space $\widehat{\Omega}$.

4.3.5.2 Classical logic (i.e., not, and, or, implication)

Put $X = \{1, 0\}$. Put Γ =RD, or Γ =SW in the formula (4.4). Consider a binary projective observables $\mathsf{O}^{\rm RD} \equiv (X(=\{1,0\}), 2^X, F^{\rm RD})$ and $\mathsf{O}^{\rm SW} \equiv (X, 2^X, F^{\rm SW})$ in $L^\infty(\Omega, \nu)$. Consider a measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$. That is, we consider that the following three are equivalent (i.e., Axiom 1 (measurement) says that $(\mathrm{I}_1) \Leftrightarrow (\mathrm{I}_2)$. Also, (I_3) is the expression of (I_1) in ordinary language):

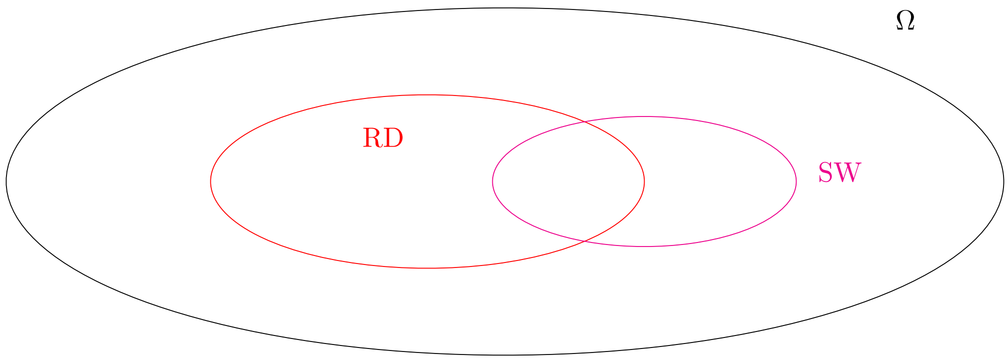
- (I_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$.
  (strictly speaking, the probability that a measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$ is equal to 1.)
- (I_2) $\widehat{\omega}(t) \in \mathrm{RD}(\equiv \{\omega \in \Omega \mid [F^{\rm RD}(\{1\})](\omega) = 1\})$
- (I_3) A tomato t is "red".

Similarly, consider a measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})$. That is, we consider that the following three are equivalent:

- (J_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]})$.

(J$_2$) $\widehat{\omega}(t) \in \mathrm{SW}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{SW}}(\{1\})](\omega) = 1\})$

(I$_3$) A tomato t is "sweet".



[Not]
It is clear that the following four are equivalent:

(K$_0$) A measured value 0 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{SW}}, S_{[\widehat{\omega}(t)]})$.

(K$_1$) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\Theta_\neg(\mathsf{O}^{\mathrm{SW}}), S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\neg\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{SW}}, S_{[\widehat{\omega}(t)]})$), where $\Theta_\neg : \{1,0\} \to \{1,0\}$ is defined by $\Theta_\neg(1) = 0$, $\Theta_\neg(0) = 1$ (*cf.* Definition 1.23).

(K$_2$) $\widehat{\omega}(t) \in [\overline{\mathrm{RD}}]^c(\equiv \{\omega \in \Omega \mid [F^{\mathrm{SW}}(\{0\})](\omega) = 1\})$

(K$_3$) A tomato t is not "red".

[And]
We see that the following four are equivalent:

(L$_0$) A measured value $(1,1)$ is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{SW}} \times \mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]})$.

(L$_1$) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\Theta_\wedge(\mathsf{O}^{\mathrm{SW}} \times \mathsf{O}^{\mathrm{RD}}), S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{SW}}, S_{[\widehat{\omega}(t)]}) \bigwedge \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]})$), where $\Theta_\wedge : \{1,0\}^2 \to \{1,0\}$ is defined by $\Theta_\wedge(1,1) = 1$, $\Theta_\wedge(1,0) = \Theta_\wedge(0,1) = \Theta_\wedge(0,0) = 0$.

(L$_2$) $\widehat{\omega}(t) \in \mathrm{RD}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{SW}}(\{1\})](\omega) = 1\}) \bigcap \mathrm{SW}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{RD}}(\{1\})](\omega) = 1\})$

(L$_3$) A tomato t is "red" and "sweet"

[Or]
We see that the following four are equivalent:

(M$_0$) A measured value (x_1, x_2) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{SW}} \times \mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(1,0),(0,1)\}$

(M$_1$) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\Theta_\vee(\mathsf{O}^{\mathrm{SW}} \times \mathsf{O}^{\mathrm{RD}}), S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{SW}}, S_{[\widehat{\omega}(t)]}) \bigvee \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\mathrm{RD}}, S_{[\widehat{\omega}(t)]})$), where $\Theta_\vee : \{1,0\}^2 \to \{1,0\}$ is defined by $\Theta_\vee(1,1) = \Theta_\vee(1,0) = \Theta_\vee(0,1) = 1$, $\Theta_\vee(0,0) = 0$.

(M$_2$) $\widehat{\omega}(t) \in \mathrm{RD}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{SW}}(\{1\})](\omega) = 1\}) \bigcup \mathrm{SW}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{RD}}(\{1\})](\omega) = 1\})$

(M$_3$) A tomato t is "red" or "sweet"

[Implication]
We see that the following four are equivalent:

(N_0) A measured value (x_1, x_2) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(0,1),(0.0)\}$

(N_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\Theta_{\rightarrow}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}), S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RP}, S_{[\widehat{\omega}(t)]})$), where $\Theta_{\rightarrow} : \{1,0\}^2 \to \{1,0\}$ is defined by $\Theta_{\rightarrow}(1,1) = \Theta_{\rightarrow}(0,1) = \Theta_{\rightarrow}(0,0) = 1$, $\Theta_{\rightarrow}(1,0) = 0$.

(N_2) $\widehat{\omega}(t) \in {\rm SW}^c(\equiv \{\omega \in \Omega \mid [F^{\rm SW}(\{1\})](\omega) = 0\}) \vee {\rm RD}(\equiv \{\omega \in \Omega \mid [F^{\rm RD}(\{1\})](\omega) = 1\})$

(N_3) A tomato t is not "sweet", or it is "red"

Remark 4.6. When $\omega_1 \neq \omega_2$, it should be noted that the symbol "$\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\omega_1]}) \bigwedge \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\omega_2]})$" is not yet defined. This should be defined by the parallel $\mathsf{M}_{L^\infty(\Omega,\nu)\otimes L^\infty(\Omega,\nu)}(\Theta_\wedge(\mathsf{O}^{\rm SW} \bigotimes \mathsf{O}^{\rm RD}), S_{[(\omega_1,\omega_2)]})$. More generally, the symbol "$\bigwedge_{\lambda\in\Lambda} \mathsf{M}_{L^\infty(\Omega_\lambda,\nu_\lambda)}(\mathsf{O}^{\Gamma_\lambda},\ S_{[\omega_\lambda]})$" is defined by

$$\mathsf{M}_{\bigotimes_{\lambda\in\Lambda} L^\infty(\Omega_\lambda,\nu_\lambda)}(\bigotimes_{\lambda\in\Lambda} \mathsf{O}^{\Gamma_\lambda}, S_{[(\omega_\lambda)_{\lambda\in\Lambda}]})$$

It might have been better to use parallel measurements (and not simultaneous measurements) all the time to avoid confusion.

Remark 4.7. Note that propositional logic (i.e., $\neg$, $\wedge$, $\vee$, $\rightarrow$) and predict logic (i.e., $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\forall$, $\exists$) are essentially the same since $P_1 \wedge P_2 \wedge P_3 \wedge ... = (\forall n)[P_n]$ and $P_1 \vee P_2 \vee P_3 \vee ... = (\exists n)[P_n]$. Thus, this paper distinguishes propositional logic and predict logic.

4.3.5.3 Syllogism

Further, consider a measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RP}, S_{[\widehat{\omega}(t)]})$. That is, we consider that the following three are equivalent:

(O_1) A measured value 1 is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RP}, S_{[\widehat{\omega}(t)]})$.

(O_2) $\widehat{\omega}(t) \in {\rm RP}(\equiv \{\omega \in \Omega \mid [F^{\rm RP}(\{1\})](\omega) = 1\})$

(O_3) A tomato t is "ripe".

Theorem 4.8. [Syllogism]:
Let t be a tomato, and let $\widehat{\omega}(t)(\in \Omega)$ be the state of t. Assume the followings:

(P_0) A measured value (x_1, x_2) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RP}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(0,1),(0.0)\}$

which is equivalent to

(P_1) $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RP}, S_{[\widehat{\omega}(t)]})$

(P_2) $\widehat{\omega}(t) \notin {\rm SW}(\equiv \{\omega \in \Omega \mid [F^{\rm SW}(\{1\})](\omega) = 1\}) \vee \widehat{\omega}(t) \in {\rm RP}(\equiv \{\omega \in \Omega \mid [F^{\rm RP}(\{1\})](\omega) = 1\})$

(P_3) A tomato t is not "sweet", or it is "ripe".

and

(P'_0) A measured value (x_2, x_3) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RP} \times \mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(0,1),(0.0)\}$

which is equivalent to

(p'_1) $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RP}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$

which is equivalent to

(P'_2) $\widehat{\omega}(t) \notin {\rm RP}(\equiv \{\omega \in \Omega \mid [F^{\rm RP}(\{1\})](\omega) = 1\}) \vee \widehat{\omega}(t) \in {\rm RD}(\equiv \{\omega \in \Omega \mid [F^{\rm RD}(\{1\})](\omega) = 1\})$

(P'_3) A tomato t is not "ripe", or it is "red".

Then the following holds:

(Q_0) A measured value (x_1, x_3) obtained by the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$ belongs to $\{(1,1),(0,1),(0.0)\}$

(Q_1) $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm RD}, S_{[\widehat{\omega}(t)]})$

(Q_2) $\widehat{\omega}(t) \notin {\rm SW}(\equiv \{\omega \in \Omega \mid [F^{\rm RD}(\{1\})](\omega) = 1\}) \vee \widehat{\omega}(t) \in {\rm RD}(\equiv \{\omega \in \Omega \mid [F^{\rm RP}(\{1\})](\omega) = 1\})$

(Q_3) A tomato t is not "sweet", or it is "red".

[Proof]: Recalling the linguistic Copenhagen interpretation "Only one measurement is permitted" (in Sec. 1.1.2), we have enough to see the simultaneous observable $\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RP} \times \mathsf{O}^{\rm RD}$, which uniquely exists (*cf.* Proposition 4 (i)). Thus, we have the measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD} \times \mathsf{O}^{\rm RP}, S_{[\widehat{\omega}(t)]})$. Let (x_1, x_2, x_3) be the measured value. We easily see that (x_1, x_3) belongs to $\{(1,1),(0,1),(0.0)\}$. Thus, (Q) holds. □

4.3.5.4 Elementary binary projective observables

Consider the state space Ω, which is finite (or, countable) with a metric d (i.e., $d(\omega_1,\omega_2) = 1 \ (\omega_1 \neq \omega_2), = 0 \ (\omega_1 = \omega_2)$. Further, assume that the Borel measure ν is defined by the point measure, i.e., $\nu(\{\omega\}) = 1 \ (\forall \omega \in \Omega)$.

Definition 4.9. Let λ be any element of Ω. Putting $\Gamma = \{\lambda\}$ in the formula (4.4), define the elementary binary projective observable $\mathsf{O}^{\{\lambda\}} = (X(=\{1,0\}), 2^X, F^{\{\lambda\}})$ in $L^\infty(\Omega,\nu)$ such that

$$[F^{\{\lambda\}}(\{1\})](\omega) = \begin{cases} 1 & (\text{if } \omega = \lambda) \\ 0 & (\text{if } \omega \neq \lambda) \end{cases}, \qquad [F^{\{\lambda\}}(\{0\})](\omega) = 1 - [F_{\{\lambda\}}(\{1\})](\omega)$$
$$(\forall \omega \in \Omega)$$

The measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\{\lambda\}}, S_{[\omega]})$ $(\lambda, \omega \in \Omega)$ is called an *elementary measurement.*

////

It is clear that it holds that

(R$_1$) A measured value 1 is obtained by the elementary measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\{\lambda\}}, S_{[\omega]})$ $\Longleftrightarrow \lambda = \omega$

(R$_2$) A measured value 1 is obtained by the elementary measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\{\lambda\}}, S_{[\omega]})$ $\Longleftrightarrow \lambda \neq \omega$

Theorem 4.10. Let Γ be a subset of Ω. And let $\omega \in \Omega$. Then we see that

$$\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\Gamma}, S_{[\omega]}) = \bigvee_{\lambda\in\Gamma} \mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^{\{\lambda\}}, S_{[\omega]})$$

♠**Note 4.8.** Many readers may not consider the above theorem to be particularly important. I have the same view. However, the theorem was prepared in preparation for Sec. 11.6.1. Of course, the spirit of expressing complex observables in simple observables is quite important. In quantum language, this spirit is realized by von Neumann's spectral decomposition theorem (and Holevo's theorem), that is, "Any observable can be composed of binary projective observables", *cf.* [29, 103].

Chapter 5

Around Alexandria; Hellenistic period

Wisdom of pyramid building for thousands of years was accumulated by Egypt (Alexandria). Bright people studied in Egypt from each place of the Mediterranean Sea coast to learn it. For example,

Euclid$\cdots$geometry
Archimedes$\cdots$buoyancy, lever
Ptolemaeus$\cdots$Geocentrism
Aristarchus$\cdots$Heliocentrism
Eratosthenes$\cdots$the measurement of the earth

5.1 Around Alexandria; Hellenistic period

Influenced by the tradition of pyramid construction engineering, the studies of the Alexandrian school are solid and scientific. Put differently, it could be said that there was no philosophical appeal that transcended mathematical logic. The Alexandrians must have known of Plato's work. However, they had little influence from Plato. I guess that

- they didn't think that Plato's philosophy would survive more than 2,000 years later

Hellenistic period is located as follows:

$$\begin{tikzcd}
\overset{\text{poem}}{\boxed{\text{The arche is } \bigcirc\ \bigcirc}} \arrow[r] & \boxed{\text{Aristotle}} \arrow[r] & \overset{\text{science}}{\boxed{\text{Hellenistic period}}} & & \\
\underset{\text{v.s. Sophists}}{\boxed{\text{Socrates}}} \arrow[r, "\text{student}"] & \boxed{\text{Plato}} \arrow[u, "\text{student}"'] \arrow[r] & \cdots \arrow[rr] & & \underset{\text{Middle Ages}}{\boxed{\text{Scholasticism}}}
\end{tikzcd}$$

Many Greek philosophers were not willing to put their theories to work. But, in Hellenistic period, Practical research was respected.

In Section 5.3, I will explain Heliocentrism of Aristarchus a little in detail as the preparation of Chap. 6, in which we say that

- "Geocentrism vs. Heliocentrism" is a metaphysical problem that cannot be put on black-and-white in the experiment. That is,

  ($\sharp$) "Geocentrism vs. Heliocentrism" is a philosophical problem, and not the problem of truth or falsehood.

5.2 Euclid(BC.330 - BC.275)

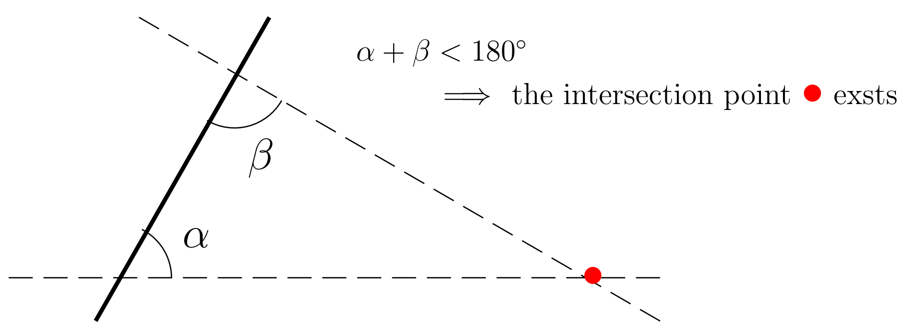
5.2.1 Euclid geometry - Parallel postulate

Three great pyramids in the Egyptian Giza desert (deceased person is Khufu, Khafre, Menkaure) erecting time of is the 2500 BC. Since then more than 2,000 years later, Euclid (BC.330 - BC.275) was born. Euclid is referred to as the "father of geometry" who was active in Alexandria (the mouth of the Nile). His book "Elements" is one of the most influential works in the history of mathematics. It has been estimated to be second only to the Bible in the number of editions published since the first printing in 1482 AD. When I think from now,

(A) Euclid advocated geometric axiomatization and considered the parallel postulate, and was the mathematician who intuited that the concept of "self-evident" isn't self-evident.

Here, the parallel postulate is as follows:

- If a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.



In spite of close attention of Euclid, the next **wrong belief** has been formed by "Element".

(B) **It is the best method to start from a self-evident thing.**

Much philosophers (Descartes and Spinoza, etc.) have fallen into this wrong belief.

It is well known by now that Descartes' cogito proposition "I think, therefore I am" is a proposition that is far from self-evident. That is, there is nothing self-evident.

♠**Note 5.1.** In "Elements", geometry is not only written but also algebra. For example, it is shown that prime numbers are infinite. The proof is as follows.

(♯) Assume that the set of prime numbers is finite, that is, $\{2, 3, 5, 7, ..., n\}$. Put

$$N = (2 \times 3 \times 5 \times 7 \times ... \times n) + 1$$

Then, N is a prime number or it can be divided by the larger prime number than n. In each case, it contradicts the assumption that n is the largest prime number. □

5.2.2 non-Euclidean revolution

Discovery of non-Euclid geometry (due to Gauss(1777 - 1855), etc.) defeated the wrong belief (B) and asserted

(C) **Start from "productive" than "self-evident"!**

that is, "all is well that ends well".

In this paper, the (C) is called the non-Euclidean revolution, that is,

(D) non-Euclidean revolution $\Big[$(B): self-evident $\xrightarrow[\text{non-Euclidean revolution}]{}$ (C): productive$\Big]$

It can't be said that the non-Euclidean revolution is still generally also recognized sufficiently in today. There is no successful theory which starts from "self-evident things". For example, Newtonian mechanics, the theory of relativity, quantum mechanics, etc. do not start from "self-evident things". Paradoxically saying, we see that

(E) The question: "What is 'self-evidence'?" is not self-evident.

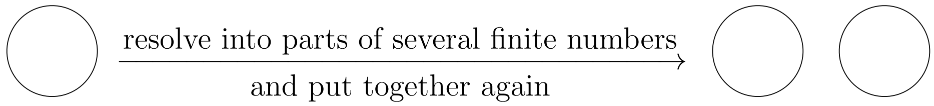
Axiom of choice of mathematics is not self-evident, where axiom of choice is as follows.

- Given any set X of pairwise disjoint non-empty sets, there exists at least one set C that contains exactly one element in common with each of the sets in X.
  (For example, consider a set $X = \{\{a, b\}, \{c, d, e\}, \{g\}, \{h, i, j, k\}\}$. Then, we can construct a set $C = \{a, c, g, j\}$)

This is not self-evident (i.e., trivial). For instance, Banach-Tarski theorem says that

(F) If we adopt axiom of choice, we have to admit the following

- A ball B is resolved into parts of several finite numbers, and we assume that it's put together again. Then, we can get the same two balls which are also the same as the ball B.



Then, we want to doubt axiom of choice, but a description of the mathematics largely decreases when I do not accept axiom of choice. Hence, we usually accept axiom of choice.

Remark 5.1. I have an opinion that the main streem of Western philosophy is

$$\left.\begin{array}{c} \text{Plato} \\ \boxed{\text{dualism}} \\ \\ \text{Euclid} \\ \boxed{\text{axiomatc approach}} \end{array}\right\} \Longrightarrow \begin{array}{c} \text{Find the axiomatic approach to dualism} \\ \boxed{\text{Descartes-Kant philosophy}} \end{array} \xrightarrow[\text{not yet completed}]{} ?$$

If it were not for Euclid's "Elements" and the Bible, Western philosophy would be just like Eastern philosophy.

♠**Note 5.2.** There was also tradition of pyramid construction, and Egypt was an advanced country of mathematics. Pythagoras and Archimedes also learned geometry in Egypt. Then Alexandria was an academic city as there was Alexandria library having 700,000 collection of books. After Euclid, we know that

- Eratosthenes (BC.275 - BC.194) : He was determined to 46250km the whole circumference of the earth. Cf. Sec. 5.5.
- Cleopatra(BC.69 - BC.30): The most beautiful woman in human history.
- Ptolemaeus (AD.83 - 168): Geocentrism

5.3 Aristarchus (BC.310 - BC.230)

5.3.1 the diameter of the moon : the diameter of the sun

Aristarchus (BC.310 - BC.230) was an ancient Greek astronomer and mathematician who presented Heliocentrism. He calculated as follows.

Proposition 5.2.

(A_1) the diameter of the moon : the diameter of the earth ≈1:3 (Recent result says that 1 : 3.669) , where $a : b = c : d$ means $a/b = c/d$.

(A_2) the diameter of the moon : the diameter of the sun≈1:19

(A_3) Thus, the diameter of the earth : the diameter of the sun ≈1:6.333 (Recent result says that 1 : 109)

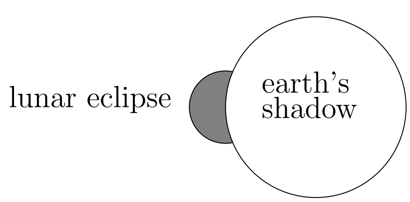
(A_4) Since each volume is proportional to [diameter]^3, the sun is much larger than the earth.

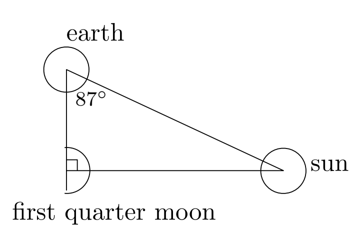
The answer to (A_1) Look at the lower left figure (lunar eclipse). Since the sun is very far, it suffices to consider that

$$\text{the diameter of the earth} \approx \text{the diameter of the earth's shadow}$$

Hence, measuring by eye, we see (A_1).
The answer to (A_2): Look at the lower left figure (the first quarter moon). Note that $\cos 87° \approx 1/19$. And using the fact that The sun and the moon are seen as the same size, we can calculate:

$$\frac{\text{the diameter of the moon}}{\text{the diameter of the sun}} = \frac{\text{the distance between the moon and the earth}}{\text{the distance between the sun and the earth}} = \cos 87° \approx \frac{1}{19}$$





5.3.2 Ancient Heliocentrism

Aristarchus considered as follows:

(B_1) The sun is overwhelmingly larger than the Earth. If so, it is wrong that the big sun goes around the small earth. It is sure that the small earth goes around the big sun.

That is,

(B_2) Aristarchus proposed Heliocentrism

His argument is almost complete since the difference between "the volume" and "the mass" is trivial.

Next problem is as follows.

(C) measuring the diameter of the earth

This was solved by Eratosthenes (*cf.* Sec.5.5).

5.4 Archimedes (BC.287 - BC.212)

Archimedes was born in Syracuse on the island of Sicily in the Mediterranean. Archimedes studied in Alexandria that was a center of the study and engaged in the study of "Elements" with pupils of Euclid afterwards. He returned to Syracuse later and spent life in Syracuse.

5.4.1 Buoyancy (Archimedes' principle)

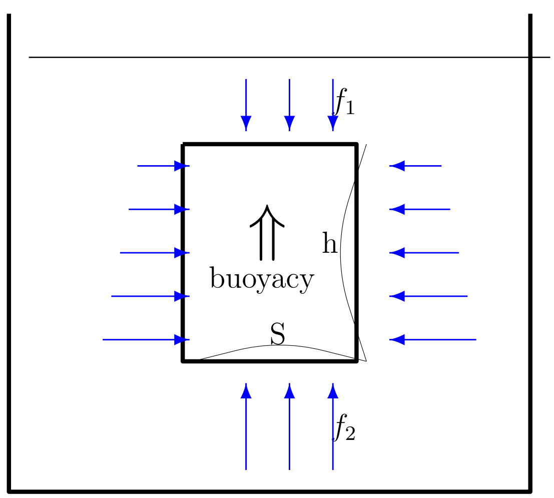
Archimedes' principle on buoyancy is as follows.

(A) Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object. If some want to avoid the term "force", then

$$\begin{aligned}&\big[\text{the weight of the matter in water}\big]\\=&\big[\text{the weight of the matter}\big]-[\text{buoyacy}]/g\end{aligned}\tag{5.1}$$

where (g: gravitational constant, the shape of this matter is assumed a cone.)

$$\begin{aligned}[\text{buoyacy}]=&\ \big[\text{Sum of the water pressure from the bottom of the object}\big]\\&-\big[\text{Sum of the water pressure from the top of the object}\big]\\=&\ (f_1+f_2)S=(|f_2|-|f_1|)S=hSg=Vg\quad(\text{the density of water is }\rho(=1).)\end{aligned}$$



S: bottom area, h: height
$V=hS$; volume
f_k: water pressure, $k=1,2$

♠**Note 5.3.** A famous anecdote of the golden crown is the delicate anecdote that there is not connected with Archimedes' principle. In like there is a relationship, I try to write this in what follows.

- The King of Syracuse asked Archimedes "Can you check whether silver is not mixed by the crown without breaking the crown". Archimedes notices next answer ($\sharp$) during bathing: while shouting with joy too much "Heureka!"(="I have found it!"), was running around the streets naked without even wearing clothes.

He found:

($\sharp$) Preparing the gold bullion of the weight same as the crown, compare the weight the gold bullion and the weight is the crown in water. Then, we can, by the (5.1), compare the volume of the gold bullion and the volume of the crown.

♠**Note 5.4.** For each great discovery, an anecdote (or, a catch copy, stage effect) is left as follows.

($\sharp_1$) Archimedes$\cdots\cdots$golden crown, heureka! (*cf.* Sec.5.4)

($\sharp_2$) Galileo$\cdots\cdots$Leaning Tower of Pisa, "And Yet It Moves" (*cf.* Sec.7.3.4)

($\sharp_3$) Newton$\cdots\cdots$ ①:Newton's apple, "Geocentrism vs. Heliocentrism"(*cf.* Note 7.9)

($\sharp_4$) Descartes$\cdots\cdots$ ①:fly on the ceiling (*cf.* Note 8.2), ②: I think, therefore I am, (*cf.* Sec.8.2)

($\sharp_5$) Kant$\cdots\cdots$clock (*cf.* Note 10.2), dogmatic slumber (*cf.* Note 10.7)

($\sharp_6$) Wttgenstein$\cdots\cdots$primary school teacher, Guardian: Russell (*cf.* Sec.11.6.1)

($\sharp_7$) Einstein$\cdots\cdots$Elevator

($\sharp_8$) quantum mechanics$\cdots\cdots$Heisenberg's uncertainty principle (*cf.* Note 4.1 of ref. [66])

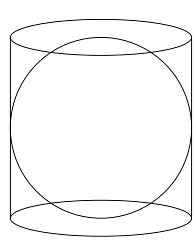
Here, the ($\sharp_8$) is my opinion.

5.4.2 The tomb of Archimedes

Consider the ball B of radius r. Archimedes showed the followings:

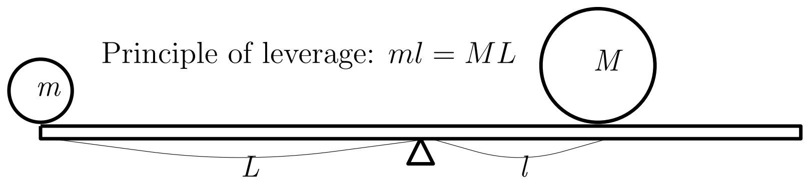
$$\text{The volume of the ball } B = \frac{4\pi r^3}{3}, \qquad \text{The surface area of the ball } B = 4\pi r^2$$

If you are a genius, you may find the proof by seeing the lower illustration('the cylinder which is circumscribed to a ball' called "the tomb of Archimedes"). If you are not genius, you can calculate it by using the differential and integral calculus.



5.4.3 Principle of leverage

Archimedes found" principle of a lever" and did more various invention with a lever.



Archimedes said "Give me a lever long enough and a fulcrum on which to place it, and I shall move the world". In spite that he referred Aristarchus' Heliocentrism in his book: "The Sand Reckoner", he supported Aristotle's Geocentric model. However, Archimedes, found " principle of a lever", have to restate Aristarchus' Heliocentrism((B_1) in Sec.5.3) as

(B) Because the sun is so much bigger than the earth. The center of gravity of the combined Earth and Sun is extremely close to the Sun. Hence, **the Sun and the Earth revolve around the gravity of both the earth and the Sun.**

If Archimedes said so, science history would be history which is completely different from now.

♠**Note 5.5.** Archimedes' arguments were so clear that even elementary school students could understand them, and he did not say ambiguous and unintelligible things like philosophy (Plato, etc.). This clarity is a factor in Archimedes' popularity. As I have mentioned before, his words pierce our hearts,

- Heureka!"(="I have found it!")
- Give me a lever long enough and a fulcrum on which to place it, and I shall move the world

and so on. And the "last words" was

- Do not disturb my circles!

The city of Syracuse, where Archimedes lived, was a battleground between Carthage (Hannibal the General) and Rome. The Roman army knew that Archimedes was a famous scientist, so they instructed him not to do any harm. However, when Archimedes was thinking about writing a figure on the sand, he was almost taken away by the Roman soldiers, who refused to do so, saying "Do not disturb my circles! And thus he was killed. It can be said that he was the "greatest star of the ancient scientists" until the end of his life.

♠**Note 5.6.** Note that

- Archimedes did not speak ambiguous things like Plato's philosophy.

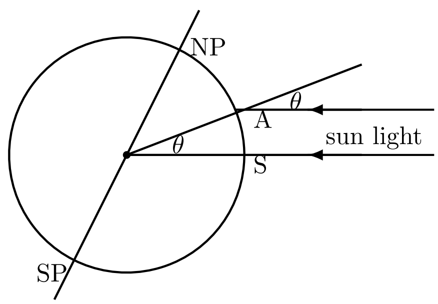
Therefore the work of Archimedes is quantitative, clear and easy to understand. Since political power could interpret the vague philosophy conveniently, philosophy could influence to maintain harmony with religion or politics. In fact, philosophy survived in the middle ages as a maid of theology. On the other hand, Archimedes' work was almost forgotten.

5.5 Eratosthenes (BC.275 - BC.194)

5.5.1 The biggest ancient observer

Pythagoras believed that the earth must be a beautiful shape and believed that the earth was a sphere. Aristotle deduced that the lunar eclipse was the shadow of the Earth and believed that the Earth was a sphere. When you look at the ocean in the distance, it looks like an arc, so there were probably people who believed that the earth was a sphere since long ago. However, if we were to mention the two certain discoverers, it would be the scientist Eratosthenes (BC.275 - BC.194) and the explorer Magellan (AD.1480 - AD.1521).

Eratosthenes measured the whole circumference of the earth as follows.



NP:North pole, SP:South pole, A:Alexandria, S:Syene(=Aswan)

- Syene is on the tropic of cancer, thus, the sun is seen in right above at noon on the summer solstice.
- Aswan is located just south of Alexandria. The distance $=AS$=925km.

Hence,

$$\text{the whole circumference of the earth} = 2 \times 3.14 \times [\text{the radius of the Earth}] = 360AS/\theta$$
$$=360 \times 925/7.2 = 46250km$$

As the recent result:40009km, it may be surprising.

♠**Note 5.7.** Since Aristarchus discovered

$$\big[\text{the diameter of the moon}\,\big] : \big[\text{the diameter of the earth}\,\big] : \big[\text{the diameter of the sun}\,\big] = 1 : 3 : 19$$

then, by Eratosthenes's result, we know that

$$\big[\text{the diameter of the moon}\,\big], \big[\text{the diameter of the earth}\,\big], \big[\text{the diameter of the sun}\,\big].$$

5.6 Claudius Ptolemaeus (AD.83 - AD.168)

5.6.1 The ancient scientific collected studies

Ptolemaic Dynasty is ruined by the death of Cleopatra, Rome became the heyday of the Five Good Emperors era. At this time, Ptolemaeus (AD.83 - 168) played an active part in Alexandria. In his book "Almagest", he adopted Aristotle's Geocentrism (i.e., the sun goes around the earth). Ptolemaeus explained the retrogression seen at a planet in Mars such as Mars revolves around the earth while drawing a small circle as "epicycle". Ptolemaeus compiled the latest theory in those days and concluded the Geocentrism (= Ptolemaic system) under the enormous measured data.

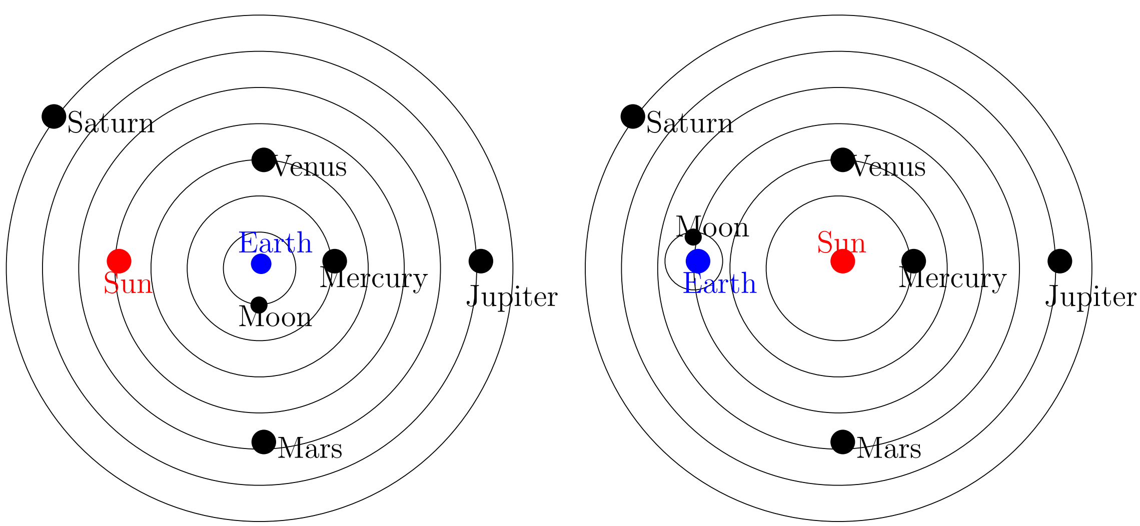
(A) Ptolemaeus followed Aristotle, Archimedes, etc.

And it is sure

(B) **Ptolemaeus is a top-notch researchers.**

He was the scientist who gave the most importance to observation among the ancient scientists.

Although, approximately 1500 years later (at Galileo's trial (1633)), his Ptolemaic system was replaced by the Copernican system, he was surely one of scientists who thought observation and experiment as important most.



Geocentrism (=Ptolemaic system) **Heliocentrism (=Copernican system)**

♠**Note 5.8.** S. Weinberg (1933 -), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said of his book [105] "To explain the word; The discovery of modern science" as follows:

- [in Chapter 8]: *In one respect the work on this theory described in the Almagest is strikingly modern in its methods. Mathematical models are proposed for planetary motions containing various free numerical parameters, which are then found by constraining the predictions of the models to agree with observation.*

Decades ago, when I saw the planetarium when I was a kid, a commentator explained that "Ptolemaeus asserted a foolish Geocentrism, and this was corrected by Copernicus." However, when I went to the planetarium a few months ago, a commentator praised Ptolemaeus, saying, "His epicycle model is groundbreaking at the time." It was great to hear a similar opinion to Weinberg's at a planetarium in the Far East island nation.

♠**Note 5.9.** Recall the following figure:

$$\begin{tikzcd} \overset{\text{poem}}{\boxed{\text{The arche is } \bigcirc\bigcirc}} \arrow[r] & \boxed{\text{Aristotle}} \arrow[r] & \overset{\text{science}}{\boxed{\text{Hellenistic period}}} & \\ \underset{\text{v.s. Sophists}}{\boxed{\text{Socrates}}} \arrow[r, "\text{student}"] & \boxed{\text{Plato}} \arrow[u, "\text{student}"'] \arrow[r] & \cdots \arrow[r] & \underset{\text{Middle Ages}}{\boxed{\text{Scholasticism}}} \end{tikzcd}$$

That is, Aristotle bridged the gap between this pseudoscience ($\approx$ the arche is $\bigcirc\bigcirc$) and the base for the foundation that would become science. Archimedes is the discoverer of "principle of buoyancy", which belongs the realistic worldview. Ptolemaic system is based on the realistic motion function method. Thus, we can get as follows (*cf.* Classification 1.11 [the classification of philosophers]).

$$(\flat)\ \left\{ \begin{array}{l} (\flat_1)\text{: the realistic worldview (physics)} \\ \quad \text{Hērakleitos, Aristotle, Aristarchus, Archimedes, Eratosthenes, Ptolemaeus,} \\ \quad \text{Galileo, Newton, Einstein}, \cdots \\ \quad \text{(Although mathematics is not a worldview, Pythagoras, Eudoxus, Euclid)} \\ \\ (\flat_{21})\text{: the fictional worldview (Western philosophy)} \\ \quad \text{Plato, Scholasticism, Descartes, Locke, Leibniz, Berkeley, Hume, Kant, Husserl} \\ \\ (\flat_{22})\text{: the logical worldview} \\ \quad \text{Boole, Frege, Peirce, Saussure, Russell, Wittgenstein, Hempel, Popper} \\ \\ (\flat_{23})\text{: the mechanical worldview (statistics, quantum language)} \\ \quad \text{Parmenides, Zeno, J. Bernoulli, statistics (e.g., Fisher), quantum language} \end{array} \right.$$

In the above, the following two are greatest (*cf.* Note 2.6):

$$(\mathrm{C}_1)\ \text{[astronomy]:}\quad \underset{\text{BC582-496}}{\overset{\text{the earth is round}}{\boxed{\text{Pythagoras}}}} \longrightarrow \underset{\text{BC400-347}}{\overset{\text{geocentrism 1}}{\boxed{\text{Eudoxus}}}} \longrightarrow \underset{\text{BC384-322}}{\overset{\text{geocentrism 2}}{\boxed{\text{Aristotle}}}} \longrightarrow \underset{\text{AD83-168}}{\overset{\text{geocentrism 3}}{\boxed{\text{Ptolemaeus}}}}$$

$$(\mathrm{C}_2)\quad [\text{math}]: \quad \underset{\text{BC582-496}}{\overset{\text{Pythagorean theorem}}{\boxed{\text{Pythagoras}}}} \longrightarrow \underset{\text{BC400-347}}{\overset{\text{method ofexhaustion}}{\boxed{\text{Eudoxus}}}} \longrightarrow \underset{\text{BC330-275}}{\overset{\text{Elements}}{\boxed{\text{Euclid}}}} \longrightarrow \underset{\text{BC287-212}}{\overset{\text{quadrature}}{\boxed{\text{Archimedes}}}}$$

Chapter 6

The Middle Ages

The Middle Ages may be characterized as "the time of the thought stop for about 1500 years" Thus, it is called

"Philosophy is a maidservant of theology"

In this chapter, we discuss:

($\sharp_1$) Augustinus(354 - 430): Christianity became the state religion of the Roman Empire. Subjective time theory

($\sharp_2$) Anselmus(1033 - 1110): the father of Scholasticism, Arguments for the existence of God, "Realismus" in Problem of universals

($\sharp_3$) Thomas Aquinas(1225 - 1274): Completion of the scholasticism (Summa Theologica): Compromise between Plato philosophy and Aristotle philosophy

($\sharp_4$) Ockham(1285 - 1347): Occam's razor, "Nominalismus" in Problem of universals

6.1 Augustinus(AD. 354 - AD.430)

6.1.1 Philosophy is a maidservant of theology

One of the largest events in the Western history is

AD.380: Christianity became the state religion of the Roman Empire

A wonder of Western philosophy is:

(A) Western philosophy met with a dying crisis many times. Each time, Western philosophy was rescued by a hand of someone's help (such as a hand of help of a god).

Augustinus (AD. 354 - 430) is one who extended a helping hand to dying Plato philosophy. Catholic father Augustinus used Plato philosophy to reinforce a theoretical backbone of Christianity.

For this,

(B) It's desirable that God (in Christianity) and Idea (in Plato philosophy) have the similar nature.

The opinion of the sophist ("Man is the measure of all things") is contrary to the opinion of Socrates-Plato, and also to the opinion of Christianity ("God is the measure of all things"). Thus, Socrates-Plato and Christianity have an affinity. In other words, there are reasons to think as follows.

$$\begin{aligned}\textbf{Plato's Idea} &= \text{(A device that make the absolute goodness visible)}\\ &\approx \text{(A device that make the intellect of "God" visible)} = \textbf{church}\end{aligned}$$

Augustinus might think so.

Of course, they cannot accept Aristotle's Idea (i.e., the Idea that came down to earth) . Because it would be more valuable if the Idea were in heaven.

It could be said that "Christianity was Idea theory arranged for the people (Nietzsche)," and conversely, it could be said that "Idea theory was celebrated as Christianity for the intellectuals."

The Plato philosophy got the strongest supporter (i.e., Christianity).

(C) Philosophy won a help from Christianity. But this implied "Philosophy is a maidservant of theology". And philosophy fell into a thought stop, but, at least, Philosophy survived.

All proceeded as Augustinus' plan.

♠**Note 6.1.** If we emulate Review 3.10 and forcefully apply the three key words of dualism to Augustine, we get the following

dualism \ three key-words	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
③: Plato: sun Remark 3.7	actual world	Idea	/ [Idea world]
Augustinus; Christianity	earthly city	church	/ [the city of God]
quantum language	**measured value**	**observable**	**state [system]**

If so, we have to acknowledge the following,

- $\boxed{\text{Plato}} \xrightarrow[\text{regress}]{} \boxed{\text{Augustinus}} \xrightarrow[\text{progress}]{} \boxed{\text{Descartes}} \xrightarrow[\text{progress}]{} \cdot \xrightarrow[\text{progress}]{} \boxed{\text{Quantum languge}}$

but we do not stick to this since Christianity is not related to a measurement theory.

It is unclear whether the Christian fathers (such as Augustine) took Plato's philosophy seriously or not. However, the famous philosopher Anselmus barely appeared 600 years after Augustine, so I don't think they were serious.

♠**Note 6.2.** Readers may have the following question:

($\sharp_1$) Why is paranormal theory always the mainstream of Western philosophy?

My opinion is as follows. Recall the Platonic method of telling philosophy:

$$(\sharp_2) \quad \underset{\text{introduction·preface·fiction}}{\overset{\text{world is so}}{\boxed{\text{fictional worldview (literary truth, pseudo- truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{Live so}}{\boxed{\text{Ethics·morals}}}}$$

Therefore,

- [world is so] is secondary, subsidiary,
- [you should do so] is main theme

Since the ethics and morality of the main subject is fully backed by Christianity, the "description of the world" in the introduction part can be any fairy tale or fiction. However, as I explained in Plato's section,

($\sharp_3$)) The description of the world in philosophy requires a "dualistic idealism".

Also, note that dualistic idealism is paranormal. This is the answer to the question ($\sharp_1$).

However, assume that there is a scientific theory in dualistic idealism. Then, we can expect that

($\sharp_4$)) while fancy theories wander around in dualistic idealism, they gradually evolve and converge into a solid theory.

It is the purpose of this paper to show this. Specifically, we show that

- $\boxed{\text{Plato}} \to \boxed{\text{Descartes}} \to \boxed{\text{Rocke}} \to \boxed{\text{Kant}} \to \boxed{\text{Wittgenstein}} \to \underset{\text{scientific theory}}{\boxed{\text{Quantum languge}}}$

6.1.2* "Confessions" by Augustinus: Only the present exists

This section is written in the following reference:

- [68]:Ishikawa, S: Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language Open Journal of philosophy, Vol. 8, No.5 , 466-480, 2018, DOI: 10.4236/ojpp.2018.85032 (https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862)

  [Revised version] (https://philpapers.org/rec/ISHLCB)
  (http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf)

We want to know:

① How should we live?

② How is the world made?

Augustine (354-430), the greatest Catholic priest, used Plato's philosophy as the "God's intellect = Idea" and armed Christianity. And everything proceeded according to Augustine's plan and intention. What we ordinary people want to know most is, how should we live? Christian Fathers, as God's spokespersons, preached this to the people as the teachings of Christ. Therefore, Christian fathers, like God, had to be able to answer any questions immediately. Among them, the question that puzzled them were, "how is the world made?", "if this world was made by God, what was it like before God made it?" etc.

Bible says:

(D_1) This world was made by God.

If so, people may have a question:

(D_2) How about before God made it?

However, if we believe in (D_1), then we consider that

(D_3) Time was also made at the same time as the world.

Therefore,

(D_4) The sentence "before God made it" is nonsense.

If we are told by fathers of Christianity so, we think that

(D_5) Well, I didn't read the Bible very well myself. I had a boring question ((D_2)

It should be noted that people want such a short story, and not scientific arguments. That is, note that it is not an understanding of the world for the sake of truth, but an understanding of the world for the sake of deepening our faith.

Augustinus asserted the following in his book "Confessions".

(E):Augustinus' theory of time as a short story

(E) **Only present exists, and neither future nor past exist.**

In fact,

(F) the future is in "prediction", the past is in "memory". There is what we can realize "only now".

This is the beginning of the subjective time (which may be a main theme in philosophy). Although this "time" cannot be used in science, this time can be used in the Platonic method of telling philosophy as follows.

$$\underset{\text{introduction·preface·fiction}}{\overset{\text{Only present exists}}{\boxed{\text{fictional worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{Live joyfully now!}}{\boxed{\text{Ethics·morals}}}}$$

It is well known that St. Augustinus said that

- *the past does not exist because of its being already gone, that the future does not exist because of its not coming yet, and that the present really exists.*

Here, consider

(G) "Only the present exists"

Note that this proposition (G) is related to "tense". Thus, the linguistic Copenhagen interpretation (E_2) in Sec. 1.1.2 says that this (K) is not a statement in quantum language. Thus, the (G) is not scientific, that is, there is no experiment to verify the (G).

Now,

- Augustinus' tense (past, present, future) is a kind of sermon. But it may be interesting in comparison with the linguistic Copenhagen interpretation (*cf.* (E_2) in Sec.1.1.2), i.e.,

There is no tense in science.

Thus, it is prohibited that Augustinus' tense (i.e., the subjective time) is discussed in science. However, we can appreciate literary pleasure from the philosophical discussions.

6.1.3 "Subjective time" is a magic word which excites our delusion

Augustine's problems such as "subjective time," "tense," and "observer's time" did not enter into the realm of science, but they continued to attract the interest of philosophers. For example, Bergson, a philosopher of "subjective time", tried to challenge Einstein of "theory of relativity" to an argument. But he was rejected by Einstein, saying "I don't know the time of philosophers". Even now, some are still misled by this "observer's time". In quantum mechanics, for example, observer's time is often assumed. For example, some researchers may accept "So-called Copenhagen interpretation" such as

- at the moment when an observer measures it, a wave function collapses.

In order to explain "At the moment when observer measured it", von Neumann made a non-scientific word "abstract ego", and said

- "At the moment when observer measured it" is "at the moment when a signal reach abstract ego"

which is of course prohibited by the linguistic Copenhagen interpretation (*cf.* (E_2) in Sec.1.1.2 earlier). For the quantum linguistic understanding of "wave function collapse", see:

[59] S. Ishikawa, *Linguistic interpretation of quantum mechanics; Projection Postulate,* JQIS, Vol. 5, No.4 , 150-155, 2015, DOI: 10.4236/jqis.2015.54017
(`http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=62464`)

♠**Note 6.3.** "What is the subjective time?" This is a problem of brain science. It is sure that cats and dogs etc. have clock gene or biological clock, thus they surely feel the subjective time. This is a scientific problem. Also, measuring the time with a clock is also a measurement. However, when you measure time with your brain clock, it is not a measurement. A measurement that only you can make is not a measurement. In science, "I", "now" and "here" are forbidden. Thus, 'Now I am here' is not a scientific proposition (i.e., a proposition in quantum language). Enjoyment of wordplay is an important part of a successful philosophy, e.g., "I know I know nothing", "Only present exists", "I think, therefore I am", etc. However, it should be noted that these are not statements in quantum language, i.e., these violate the linguistic Copenhagen interpretation.

♠**Note 6.4.** For completeness, let us rewrite as follows.

① How shoul we live? ② How is the world made?

Here,

- "①: the problems of life" and "②: the problem of world" are different things

In spite of the difference, we prefer to Platonic method of telling philosophy:

- the "logic" which is dressed so that ① may be derived from ②.

In this sense, the ② is a reason added later. We might be, by common sense, convinced that "the worldview was to describe the world plainly and with no fiction". However, Plato and Augustinus consider that

- the worldview is to create the world that it is convenient for faith or doctrine.

This is a replacement of the problem. However, this succeeds in science as well as philosophy. As seen later (Kant's Copernican revolution, Wittgenstein' words "The limits of my language mean the limits of my world", and finally, quantum language),

- the worldview is not to to describe the world plainly and with no fiction, but to create the world that it is convenient for faith or doctrine.

That is, "More abstract painting than realistic painting". Concretely saying, for example,

- When there is a kind of the paint only in "red" and "green", We draw as much as possible it seems realistic picture in this two colors

This is not only the philosophical case but also the scientific case (i.e., quantum language). Quantum language is prohibited from using anything other than two axioms (Axiom 1 and Axiom 2 in Sec.1.1), which is the same as the paint example above.

Thus I think that

- It is not an exaggeration to say that, since Plato, the Copernican revolution has been the norm of philosophy.

6.2 Scholasticism –Graft Bamboo (=Aristotle) to a tree (=Plato) –

6.2.1 Aristotle's philosophy spread to the Islamic world

Plato's philosophy survived with the backing of Christianity (Augustine). The philosophy of Aristotle spread to Islam. I don't know the details of the reason why,

(A) Plato's philosophy must have spread to the Christian world and Aristotle's philosophy to the Muslim world.

Probably, there were various conflicts in Christianity, and the winners stayed in Rome and supported Plato's philosophy. The losers were driven to the Muslim world, and Aristotle's philosophy must have spread to the Muslim world by such a process.

Eastern Islam was centered in Baghdad which was famous on the Arabian Nights. Western Islamic culture developed around Cordoba in the Andalusian region of southern Spain and became the largest city in the world in the 10th century. At that time, the Islamic world learned a lot of wisdom from the books of ancient Greeks and Romans and developed its own thought and technology. The Islamic culture was at the forefront of the world under Aristotle's followership.

♠**Note 6.5.** In this paper we adopt the story such as (A). Actually, it may not be such a simple story.

6.2.2 Crusade expedition and Inflow of Islamic culture

In the era of crusade expedition (1096 - 1270), the Western countries were in a downturn. Such public opinion had been drifting.

- The achievements of the Crusades do not rise by Plato's way. Thus, let's study Aristotle at the tip of the Islamic culture!

I think it is true that

- in every age and every place in the world, the human resources required are the humanities in peacetime, and the sciences in wartime.

As a byproduct of the pilgrimage to the Holy Land of Jerusalem and the crusade to recapture it, interaction with Islamic culture was facilitated. Aristotle's philosophy flowed into the West, merged with Plato's philosophy, and settled in as Schola's philosophy.

That is, Scholasticism was born. As the typical persons of Scholasticism, we list up as follows.

(B_1) Anselmus (1033 - 1109) "The father of Scholasticism", "Realismus"

(B_2) Thomas Aquinas (1225 - 1274) "Summa Theologica", Greatest man in Scholasticism

(B_3) Ockham (1285 - 1347) "Ockham's razor", "Nominalismus"

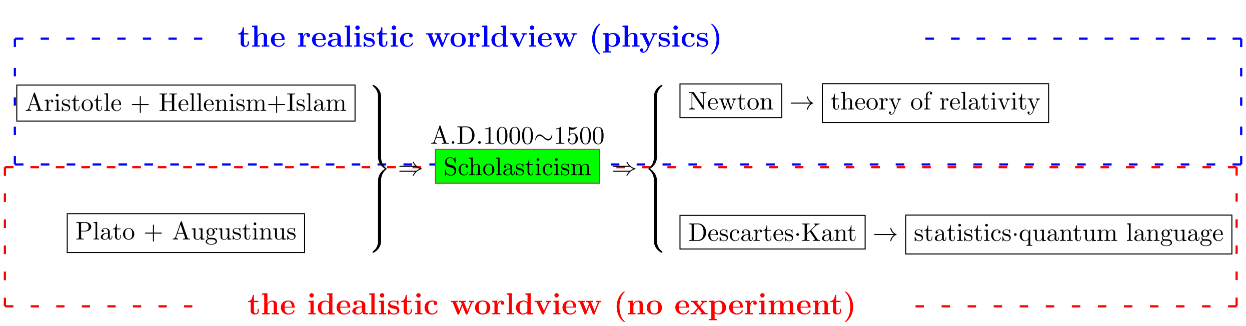
After all,

- in the beginning, Plato's dualistic idealism was the most popular , but gradually Aristotle's influence increased. It has become so "science-like" that it has abandoned dualistic idealism. And It has become like a product of the fusion of Platonic and Aristotelian philosophies.

Of course, it is impossible to succeed this "fusion". That is because Plato philosophy and Aristotle philosophy are "oil (idealistic dualism) and water (realistic monism)", and these are different categories (*cf.* Assertion 1.4 [the history of worldview], Assertion 1.14). However, in this paper, we prepare the story such as

- in the process of fusion of Plato philosophy and Aristotle philosophy, disadvantages of the theory of Ideas became clear, which led to Descartes.

Figure 6.1 (Scholasticism is a compromise between Plato and Aristotle)**.**



Also, by-product of crusade expedition, we have to note

- "Positional notation (= the discovery of zero)" of the origin in India

which will be mentioned in what follows.

6.3 The discovery of zero

6.3.1 Positional notation (= the discovery of zero): Arabic numerals

As mentioned in the previous section,

(A) Plato was introduced to the Christian world and Aristotle to the Muslim world.

Although many people must have suffered miserably during the Crusade expedition, the merits for Christian culture is that Aristotle philosophy and the positional notation flowed into Europe from Islam.

♠**Note 6.6.** Which was influential, Aristotle philosophy or the positional notation? If this question is the same as "Which was indispensable for the proposal of Newtonian mechanics?", we may choose the positional notation, because Newton was a calculation maniac.

The positional notation is how to write numbers to learn in an elementary school. For example,

$$+5040302, \quad -15, \quad +39.045, \quad -81.5, \quad +3.1415$$
$$-1000, \quad +0.009876, \quad +0.3333\cdots, \quad 0,$$

and so on. That is, By 13 symbols "0, 1, 2, 3, 4, 5, 6, 7, 8, 9, $+$, $-$, . (radix point)", we can express all real numbers by the positional notation.

Hence, we may say

- the discovery of the positional notation (= Arabic numerals)
  = the discovery of all real numbers.

(the radix point was discovered in Europa of 16 century AD.)

Of course, the discovery of zero is

(B) the discovery of how to use zero called the positional notation

6.3.2 Arabic numerals and Roman numeral

Roman numerals are often used on the clock face such as

$$1{=}\text{I},\ 2{=}\text{II},\ 3{=}\text{III},\ 4{=}\text{IV},\ 5{=}\text{V},...,10{=}\text{X},\ 11{=}\text{XI},$$

However, it is too hard to represent large numbers such as

$$495 = \text{CDXCV},\ 1888 = \text{MDCCCLXXXVIII},\ 3999 = \text{MMMCMXCIX}$$

6.3.3 The explosion of mathematics

European mathematics originally had the high potential of Euclidean geometry. With the introduction of the positional notation (the discovery of zeros), computation became easier. The word "ALGEBRA (algebra)" is originally an Arabic word. The formula for the solution of the quadratic equation : $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

was also understood by the Arabic mathematician al-Khwarizmi (about 790 - about 850)

Mathematician Gauss(1777 - 1855) said

(C) "If genius Archimedes invented the positional notation, I am certain that the mathematics must have progressed drastically."

The positional notation triggered the "math explosion" such that

(D) Solution of algebraic equations, complex numbers, the function concept, betting of problem (probability), analytic geometry (Descartes coordinates), calculus, differential equations, linear algebra, number theory, etc.

♠**Note 6.7.** There may be several opinions about the three big discoveries of mathematics. We think as follows.

① the discovery of the plane (geometry)

② the discovery of zero (positional notation)

③ the discovery of sets

Of course, it is needless to say that the biggest discovery is "⓪:the discovery of natural numbers".

6.4 The proof of the existence of God

6.4.1 Anselmus (1033 - 1109)

Anselmus is the founder of Scholastic philosophy, famous for his "Proof of the Existence of God". His proof is nothing more than a play on words that we use on a daily basis. The premise is not clearly stated (i.e., it is not proven under any one worldview). In particular, the meaning of "existence" is not clear. Therefore, there is no need to read the proofs seriously.

Proof 6.2. Anselmus: the proof of God's Existence

①: God is a being than which none greater can be imagined . That is, the greatest possible being that can be imagined.

②: It is obvious that God exists as an idea in the mind.

③: A being that exists as an idea in the mind and in reality is greater than a being that exists only as an idea in the mind.

④: Thus, if God exists only as an idea in the mind, then we can imagine something that is greater than God. But we cannot imagine something that is greater than God.

⑤: Therefore, God exists in reality.

$\square$

The above proof is not worth validating since it is not discussed under a certain worldview. This is a play on words.

♠**Note 6.8.** (i): I don't understand the above proof. However, I associate the above argument with "a set of all sets", which is related to Russell's paradox. Thus, the above may be a kind of self-referential word play.
(ii): If the meaning of "existence" is defined in the sense of Definition 1.7, the existence of "God" is obvious. Because "God" is the most important key-word in the Bible.

6.4.2 Review: the worldviewism

Let us review the worldviewism.

(A): Worldviewism (*cf.* Sec.1.3.1)

The worldviewism is as follows.

$$\text{(A)} \qquad \underset{\text{premise}}{\overset{\text{world is so}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{subject}}{\overset{\text{conclusion}}{\boxed{\text{discussions, calculation, logic, properties}}}}$$

That is,

(B) The worldviewism is the spirit "Start from the worldview" or "Start from the firm premise!".

Thus, from our standing-point (i.e., worldviewism), Proof 6.2 is not trusted.

So far, we have seen many paradoxes caused by arguments that ignore worldviewism. For example,

(C) **The list of our answers for philosophical unsolved problems** (*cf.* (D) in Sec. 1.2.2)

- What is probability (or, measurement, causality) ? *cf.* Sec. 1.1.1)
- Zeno paradox (Flying arrow), (*cf.* Sec. 2.4.2)
- Zeno paradox (Achilles and a tortoise), (*cf.* Sec. 2.4.3)
- the measurement theoretical understanding of Plato's allegory of the sum , (*cf.* Sec. 3.3.2)
- Plato's Idea theory≈Zadeh's fuzzy theory≈Sausuure's linguistic theory (*cf.* Sec. 3.5.3)
- Syllogism holds in classical systems, but not in quantum systems (*cf.* Sec. 4.3.2)
- Only the present exists (*cf.* Sec. 6.1.2)

6.4.3 The inflow of Aristotle philosophy

Although we cannot understand Proof 6.2, we think that Anselmus did not say much more than the following:

(D_1) "Aristotle philosophy flowed in via Islam, and I underwent the influence".

That is,

$$\begin{array}{ccccccc}
\text{Aristotle} & \longrightarrow & \text{syllogism} & \longrightarrow & \text{logic} & \longrightarrow & \text{proof} \\
 & & & & & & \text{via Islam} \big\downarrow \text{inflow} \\
\text{Plato} & \longrightarrow & \text{Augustinus} & \longrightarrow & \text{God} & \longrightarrow & \text{The existence of God}
\end{array}$$

In other words, a priest who had studied Aristotle's syllogism felt like he had "proved the existence of God" by using the syllogism.

We can see that "Proof of the Existence of God" is a product of Schola's philosophy (a reckless attempt to fuse and compromise Plato and Aristotle).

If one were to choose Aristotle's "syllogism" over Christ's "Bible, we think that

(D_2) the expiry date of Plato's myth "God's intellect = Idea" by Augustine has expired, and Aristotle's influence has increased.

Anselmus' argument itself is nonsense. However, to exaggerate, the Schola philosophy, which is based on Anselmus, is thought as

(E) It is a revolution in Christianity that allows not only "faith" but also "reason".

Of course, even if the theme of thinking is limited, there would be no objection if it is "proof of God's existence".

The reason for Anselmus' fame is the above (E), namely,

(F) It is the discovery of the magic word "proof of the existence of God" to be freed from the mind control of "stop thinking".

After all, we see,

(G) the history of Schola's philosophy is that the "eyes to see reality" of Aristotle's style gradually matured from the blind state of faith alone, and Descartes received the baton and opened the curtain of the modern era.

Nevertheless, great philosophers in Europe have repeatedly challenged the "proof of God's existence", but I (especially non-Christian Japanese) would have no right to speak to the significance and motivation of this challenge.

♠**Note 6.9.** All scientists are interested to "God". "What is God ?(= How about neuronal circuit concerning belief?)" and "What is subjective time? (= How about biological clock?) are one of the most interesting problems in brain science.

6.5 Scholasticism; Problem of universals

6.5.1* What is the problem of universals?

The following is a review. Augustine (354-430), the greatest Catholic priest, adopted Plato's Idea theory to reinforce Christianity. Fortunately, the fundamental keywords (=existence) of both God (Christianity) and Idea (Plato's philosophy) are quite similar, since both of them have the mood of heavenly existence, so Augustine's plan succeeded and the honeymoon era between Christianity and Plato's philosophy lasted for more than 500 years.

There, Aristotle's philosophy flowed in via Islam. Plato's philosophy, which is a dualistic and idealistic description of the world, and Aristotle's philosophy, which is a monistic and realistic description of the world, are like water and oil (or, grafting a bamboo to a tree), and thus , naturally it was confusing. As shown in the previous section, the first confusion was the "proof of God's existence" of Anselmus. The second confusion is the "Problem of universals" in this section.

The problem of universals is the biggest dispute in Scholasticism. This problem is as follows.

(A): What is "Problem of universals"?

Problem 6.3. "Problem of universals" is as follows.

(A_1) It is certain that Mr. Smith, Mr. White, Mr. Brown, etc. exist. Then, we have the following problem:

Do "American", "Japanese", "honesty", "intelligence", etc. exist?

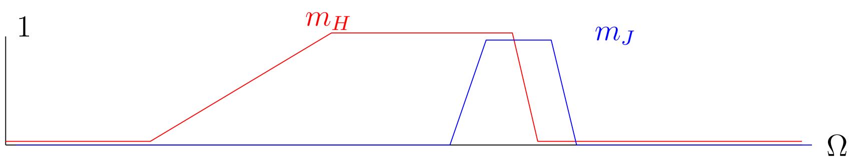
If "Yes", then, "Realismus". If "No", then, "Nominalismus".

Most people may have the following question:

- Why did great Fathers argue eagerly in a problem like such word game?

Thus, our present problem is **"What is 'Problem of universals'?"**. In what follows, this will be answered from the quantum mechanical worldview ?

Let Ω be a compact space, in which every human's state is assumed to be represented. Let $m_H : \Omega \to [0,1]$ [resp. $m_J : \Omega \to [0,1]$] be the membership function of "honesty" [resp. "Japanese"] (see the picture below).



Also, it is assumed that a membership function $m : \Omega \to [0,1]$ induces the observable $\mathsf{O}_m = (\{y,n\}, 2^{\{y,n\}}, F_m)$ in $C(\Omega)$, where $F_m(\{y\})](\omega) = m(\omega)(\forall \omega \in \Omega)$.

Recall Example 3.11 (and Remark 3.12), that is,

(A$_2$)

$$
\begin{tikzcd}
\underset{\text{Plato's idea theory}}{\boxed{\text{Idea}}} \arrow[rr, "\text{linguistic version}"] \arrow[d, "\text{congnitive}"', "\text{version}"] & & \underset{\text{Saussure's linguistic theory}}{\boxed{\text{signifier, signfied}}} \arrow[d, "\text{quantification}"] \\
\underset{\text{John Locke}}{\boxed{\text{secondary quality}}} \arrow[drr, "\text{quantification}"'] & & \underset{\text{Zadeh's fuzzy theory}}{\boxed{\text{membership function}}} \arrow[d, "\text{identification}"] \\
& & \underset{\text{quantum language}}{\boxed{\text{observable}}}
\end{tikzcd}
$$

That is,

(A$_3$) "Idea of honesty"(= "Degree of honesty")

$$\xrightarrow[\text{quantification}]{} \text{"Menbership function of honesty } m_H : \Omega \to [0,1]\text{"}$$

$$\xrightarrow[\text{probabilistic interpretation}]{} \text{"the observable concerning honesty} \mathsf{O}_{m_H} = (\{y,n\}, 2^{\{y,n\}}.F_{m_H})\text{"}$$

Also, note that

\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
Plato idealism (no experiment)	actual world	Idea	/ [Idea world]
Aristotle realism (experiment)	——	——	eidos [hyule]
quantum language	**measured value**	**observable**	**state** **[system]**

Thus, we see:

Answer 6.4. (A_3) says that

(B_1) "Honesty" is an observable

Also, recall Definition 1.7, that is,
(i): in quantum language (i.e., idealistic dualism), "observable", "measured value" and "state" exist
(ii): in Aristotle standing point (i.e., realistic monism), "state" exists.
Therefore, we say that

(B_2) Plato school agree to "Realismus": that is, "honesty" exists.
e.g., Anselmus (1033 - 1109)

(B_3) Aristotle school agree to "Nominalismus": that is, "honesty" does not exist.
e.g., William of Ockham (1285 - 1347)

After all, we think

- This is the problem such that "(since Augustinus) Plato school ("Realismus") vs. (via Islam)Aristotle school ("Nominalismus"). The power in the church gradually shifted to the Aristotle group. In this sense, it may be called "confusion" than "dispute".

As seen in the following table, the problem "realistic worldview(monism) vs. idealistic worldview(dualism)" is the biggest dispute in philosophy and science. "Nominalismus" (Ockham) in Problem of universals is a little irrational since religion is not realistic.

Table 1.1 : realistic worldview vs. idealistic worldview

dispute ＼ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	"Nominalismus"(Ockham)	"Realismun"(Anselmus)
ⓓ: space-time	Newton	Leibniz
ⓔ: quantum theory	Einstein	Bohr

ⓐ is my fiction, ⓒ is more of a confusion than a dispute. ⓓ is the Leibniz=Clarke correspondence (*cf.* Sec. 9.3.2), ⓔ is Bohr-Einstein debates. Quantum language is proposed as one of answers to Bohr-Einstein debates(*cf.* ref. [66]).

6.6 Thomas Aquinas (1225 - 1274); Scholasticism

6.6.1 Grafting Bamboo (Aristotlelian science) to a Tree (Plato's literature)

The Catholic priest: Thomas Aquinas (1225 - 1274) wrote "Summa Theologica" as the summing-up of Scholasticism. He was the most important at the intermediate time of Scholasticism (or, Problem of universals), that is,

$$\underset{(\text{"Realismus"})}{\overset{(1033-1109)}{\boxed{\text{Anselmus}}}} \longrightarrow \underset{(\text{compromise or fusion})}{\overset{(1225-1274)}{\boxed{\text{Thomas Aquinas}}}} \longrightarrow \underset{(\text{"Nominalismus"})}{\overset{(1285-1347)}{\boxed{\text{Ockham}}}} \tag{6.1}$$

His proposal is the compromise of "Realismus" (due to Plato) and Ockham (due to Aristotle), thus, his theory has three key-words:

(A_1) :[universalia ante res] as Plato's Idea (i.e., measuring instrument in quantum language)

(A_2) :[universalia in rebus] as Aristotle's eidos (i.e., state in quantum language)

(A_3) :[universalia post rem] as actual world (i.e., measured value in quantum language).

Since Plato philosophy and Aristotle philosophy are "oil and water", and these are different categories (*cf.* Assertion 1.4[the history of worldview]), it is a matter of course that Aquinas' idea is irrational. However, as seen in Table 5.2, we say:

(B) in the process of fusion of Plato philosophy and Aristotle philosophy, deficiencies in the theory of Ideas is turned to reveal, this led to Descartes.

that is,

Table 5.2 : Key-words in each worldview

Plato idealism (no experiment)	actual world	Idea	/ [Idea world]
Aristotle realism (experiment)	——	——	eidos [hyle]
Thomas Aquinas	universale post rem	universale ante rem	[universale in re] /
Descartes	I, mind, brain	body	/ [matter]
quantum language	**measured value**	**measuring instrument**	**state** **[system]**

♠**Note 6.10.** As mentioned in Sec.3.5, I am not confident in the above table, because the theory of ideas and Scholasticism were not created with the intention of measurement. However, I will proceed with this policy of discussion in this paper.

Review 6.5. In the above table, the meaning of the correspondence of key-words is as follow.

$$
\underset{\quad}{\overset{\text{(actual world,Idea world)}}{\boxed{\text{Plato}}}} + \underset{\text{(eidos,hyle)}}{\boxed{\text{Aristotle}}} \xrightarrow[\text{Impossible attempt}]{\text{compromise}} \underset{\text{(in re, /) or (/, in re)}}{\overset{\text{(post rem,ante rem)}}{\boxed{\text{Thomas Aquinas}}}} \xrightarrow[\text{Descartes' genius}]{} \underset{\text{(/,matter)}}{\overset{\text{(mind,body)}}{\boxed{\text{Descartes}}}}
$$

$$
\xrightarrow[\text{progress}]{} \underset{\text{(state,system)}}{\overset{\text{(measured value,measuring instrument)}}{\boxed{\text{quantum language}}}}
$$

Also, the formula 6.1(=the history of Scholasticism) is rewritten as follows.

$$
\underset{\text{("Realismus")}}{\overset{\text{Plato}}{\boxed{\text{Augustinus}}}} \longrightarrow \underset{\text{("Realismus")}}{\overset{\text{Plato (+ Aristotle)}}{\boxed{\text{Anselmus}}}} \longrightarrow \underset{\text{(compromise or fusion)}}{\overset{\text{Plato + Aristotle}}{\boxed{\text{Thomas Aquinas}}}} \xrightarrow[\text{overstep}]{} \underset{\text{("Nominalismus")}}{\overset{\text{Aristotle}}{\boxed{\text{Ockham}}}} \tag{6.2}
$$

Thus, I guess that

(C) Ockham wanted to shave Plato's Idea theory with a razor. (*cf.* (C_3) in Sec. 6.7).

♠**Note 6.11.** It's said that the problem of universals is incomprehensible. This is due to the fact:

- The fusion of Plato and Aristotle is an unreasonable trial,

That is,

($\sharp_1$) Aristotle (as well as Newton) do not fit in Christianity.

($\sharp_2$) Although the key-words of Thomas Aquinas philosophy and those of Descartes philosophy are similar (i.e., those have three key-words as seen in Table 5.2), this may be accidental. (*cf.* Review6.5).

($\sharp_3$) The formula (6.2) implies the confusion in Scholasticism. Therefore, the "$\longrightarrow$ in formula (6.2)" does not mean "progress".

♠**Note 6.12.** A seen in the above, the problem of universals is in confusion. However, we think that Scholasticism belongs to the linguistic worldview. Thus, we have (*cf.* Classification 1.11 [the classification of philosophers]).

$(\flat)$

$(\flat_1)$: the realistic worldview (physics)
Hērakleitos, Aristotle, Aristarchus, Archimedes, Eratosthenes, Ptolemaeus, Galileo, Newton, Einstein, $\cdots$
(Although mathematics is not a worldview, Pythagoras, Eudoxus, Euclid)

$(\flat_{21})$: the fictional worldview (Western philosophy)
Plato, Scholasticism, Descartes, Locke, Leibniz, Berkeley, Hume, Kant, Husserl

$(\flat_{22})$: the logical worldview
Boole, Frege, Peirce, Saussure, Russell, Wittgenstein, Hempel, Popper

$(\flat_{23})$: the mechanical worldview (statistics, quantum language)
Parmenides, Zeno, J. Bernoulli, statistics (e.g., Fisher), quantum language

6.7 Ockham's razor

William of Ockham (1285 - 1347), a Scholastic philosopher or theologian born in Ockham in England, is known as an advocate of Ockham's razor(=the law of parsimony) in philosophy and science.

(A):Ockham's razor(=the law of parsimony)

Ockham's razor is as follows:

(A) Shave unnecessary assumptions with a razor!

However, this may be a self-evident truth.

For example,

(B) Assume that you were a student of Plato and Plato asked you

- "The sun goes around the earth? or the earth goes around the sun?"

Then, which did you answer to Plato?

Probably, you, by Ockham's razor, answer that the sun goes around the earth. In fact Aristotle did so. Ockham's razor is dependent on the environment around. Thus I have a question:

(C) Is there a case as which Ockham's razor is functioning effectively (besides the mathematical theorems) ?

We see that

(D_1) Relying on Occam's razor, Ptolemy claimed the theory of celestial motion from the observational data of the time.

And

(D_2) relying on Occam's razor, Galileo claimed for a geodynamic theory from the observational data of the time.

Occam's Razor is an explanation added later and thus, powerless. But then again...

(E) Is there a case (other than mathematics) in which "Occam's Razor" is working effectively?

This makes me skeptical of Occam's razor.

Even in the Middle Ages, it is difficult to imagine that people would have been impressed by such a statement if it had been made without any background.

The question is, "In what kind of background and context did Ockham say it? I think so.

The author is a layman and does not know much about it. Occam, however, is the leading edge of the Aristotelian sect.

Therefore, I guess that

(F) What Ockham wanted to shave with a razor is the theory of Ideas

(*cf.* (C) in Sec. 6.6). It makes sense, then. However, it would be too short-sighted to think that it must be better to eliminate such hypotheses as the existence of "something ideal in the heavens.

Plato must have thought that the idea theory was a fairy tale. It would be immature and foolish to cut off the fairy tale.

Chapter 7

Early modern – Scientific revolution: From Geocentrism to Heliocentrism

We assume that the three greatest paradigm shifts are as follows

($\sharp_1$) Aristotelian worldview (purpose) $\longrightarrow$ Newtonian worldview (causal relation)

($\sharp_2$) Ptolemaic system: Geocentrism $\longrightarrow$ Copernican system: Heliocentrism

($\sharp_3$) Christianity: Adam and Eve $\longrightarrow$ Darwin: evolution theory

In this chapter, we are concerned with ($\sharp_1$) and ($\sharp_2$), and conclude that

- ($\sharp_2$) is a metaphysical dispute, which cannot be made clear by experiments. And it was clarified by ($\sharp_1$). In this sense, ($\sharp_2$) is included in ($\sharp_1$).

7.1 Paradigm shift

Eastern Roman Empire was made to be ruined by Ottoman Turkey in 1453.

- 1453:The Eastern Roman Empire extinction (Constantinople surrender)

The influence on Christ cultural area of this great event is immeasurable.

Traffic of "Silk Road" became inconvenient. And thus,

Age of Discovery had begun

Also, engineers, artists, cultural people, etc. (of Eastern Roman Empire) had flowed into Western Europe as refugees. And hence,

Renaissance rose suddenly.

The timeline is as follows.

Table 7.1. Scientific revolution; Chronological table

Before Galileo: The era of observation and experiment

- 1450: Gutenberg's printing press — inventor
- 1453:The Eastern Roman Empire extinction (Constantinople surrender)
- 1492: Columbus, discovery of the American Continental — navigator
- 1498: Vasco da Gama, discovery of the sea route to India — navigator
- 1500s: Leonardo da Vinci, "Mona Lisa's smile" — artist
- 1510: Copernicus, Heliocentrism — scientist
- 1510: Raffaello, "The School of Athens", Admiration to ancient Greece — artist
- 1517: Luther, Protestant Reformation — religionist
- 1519 - 20: Magellan, the first circumnavigation of the Earth
- 1540s: Michelangelo, "The Last Judgment" — artist
- 1600: G. Bruno, who supported heliocentrism along with Galileo, was burned at the stake by the Vatican — philosopher
- 1609∼1619: Kepler's laws of planetary motion — scientist
- 1610: Galileo, A telescope was made and moons of Jupiter were found — scientist
- 1620: F. Bacon, "knowledge is power",The father of British Empiricism — illuminator
- 1633 Galileo's trial "And yet it moves" — scientist

After Galileo: The era of thought

- 1637: "Discourse on the Method", Rene Descartes (1596 - 1650), the father of modern philosophy, Cogito proposition — philosopher
- 1670: Pascal, "Pensèes" — enlightener
- 1685 - 1750: Bach — artist
- **1687: Newton, "Principia"** — scientist
- 1688: Glorious Revolution
- 1690: John Locke, the father of British Empiricism, "An Essay Concerning Human Understanding", tabula-rasa, the secondary quality — philosopher
- around 1700: Jakob Bernoulli, the law of large numbers — mathematician
- 1703: Leibniz, "New Essays on Human Understanding" — philosopher
- 1715 - 16: Leibniz-Clarke correspondence (*cf.* Sec. 9.3.2)
- 1739: Hume, "A Treatise of Human Nature" — philosopher
- 1781: Kant, "Critique of Pure Reason" — philosopher

♠**Note 7.1.** The law of large numbers, discovered by J. Bernoulli(1654 - 1705), is as follows.

($\sharp$) If a fair coin (one with probability of heads equal to 1/2) is flipped a large number of times, the proportion of heads will tend to get closer to 1/2 as the number of tosses increases.

I think that Bernoulli's achievement equals Galileo's achievement. That is,

$$\begin{cases} \text{Scientific pioneer in the realistic worldview} \cdots \text{Galileo} \\ \\ \text{Scientific pioneer in the idealistic worldview} \cdots \text{J. Bernoulli} \end{cases}$$

It is difficult to identify the founder of the probability theory to one person. But, I think that J. Bernoulli is one of the most important founders (e.g., P.S. Laplace (1749-1827), A. Kolmogorov (1903-1987), etc.).

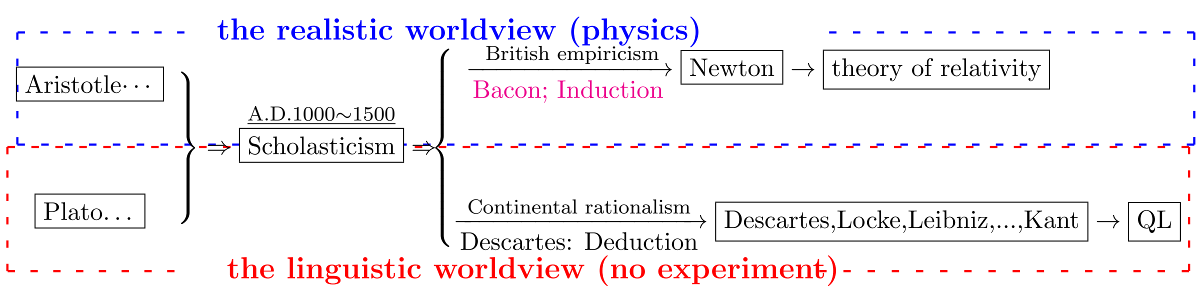
7.2 Francis Bacon (1561 - 1626): The father of empiricism, Inductive reasoning

7.2.1 How to create science: The exclusion of idols (=prejudice, preconception)

F. Bacon has been called the father of empiricism. He was the greatest enlightener of "scientific revolution" as follows.

- $\underset{\text{(Hērakleitos, Parmenides, Zeno, Aristotle)}}{\overset{\text{the birth of worldview}}{\boxed{\text{Movement}}}} \xrightarrow[\text{Aristotle's spirit :(About 1500 years)}]{\text{"purpose" (and no experiment)}} \underset{\text{(Galileo, Bacon, Descartes, Newton)}}{\overset{\text{the birth of modern science}}{\boxed{\text{Causality (and experiment)}}}}$

Or,



In 1620, he proposed "how to create science" (called inductive reasoning, or induction principle) in his book "Novum organum".

(A):Induction principle (by bad idols), how to create science

His proposal is as follows.

(A) ①:Exclusion of bad idols
$\longrightarrow$②:data, collection by observation and experiment$\longrightarrow$③:scientific theory, principle

Let us explain this in what follows.

① : Firstly, we have to exclude bad idols (=prejudice, preconception) Here, idols is as follows.
Idols of the Tribe: prejudice due to sense organs
Idols of the Cave: prejudice due to custom, the education
Idols of the Market: prejudice due to language
Idols of the Theatre: prejudice due to thought, theory

② : Next, we have to collect data by observation, experiments,

③ : Lastly, find the essence from the data, and build science theory.

Here, "②+③" is called "induction".

Recall that Greek philosophy starts from "principle", e.g., "the arche (= the first principle of all things) is ◯◯", or, Euclid advocated geometric axiomatization, and a lot of theorems were derived from Euclid axioms, that is,

$$(\mathrm{B}_1)\ \text{Mathematics:} \qquad \text{Euclid axioms} \xrightarrow{\text{deduction}} \text{theorems}$$

On the other hand, Bacon emphasized the importance of observation (or experiments), that is,

$$(\mathrm{B}_2)\ \text{Science:} \qquad \text{data, collections} \xrightarrow{\text{induction}} \text{principle}$$

which is the scientific method proposed by Bacon (who was called the father of empiricism).

7.2.1.1 Isaac Newton (the exclusion of bad idols) (1642-1727)

Newton said:

"I frame no hypotheses"

And he practiced Bacon's induction principle, and proposed Newtonian mechanics as follows.

(C): ①exclude bad idols (i.e., Aristotle's purpose, Geocentrism) $\longrightarrow$ ②Data collection (due to Tycho Brahe, Kepler, Galileo) $\longrightarrow$ ③Science theory (Newtonian mechanics)

♠**Note 7.2.** This may be say in a philosophy side. We must add the next section (good idols).

7.2.1.2 Isaac Newton (good idols) (1642-1727)

Bacon's induction principle is not simple. there is another way (by good idols) such as

(D): Induction principle (by good idols)

Induction principle (by good idols) is as follows.

(D) ①believe good idols ⟶ ②Data collection ⟶ ③Science theory

Newton said:

"I frame no hypotheses"

And he practiced Bacon's induction principle (good idols), and proposed Newtonian mechanics as follows.

(E): ①believe good idols (i.e., Causal relation) ⟶ ②Data collection (due to Tycho Brahe, Kepler, Galileo) ⟶ ③Science theory (Newtonian mechanics)

♠**Note 7.3.** Although ironically,

($\sharp$) Bacon, who proposed the exclusion of idols, was also one of discoverers of "good idols" called "causal relation".

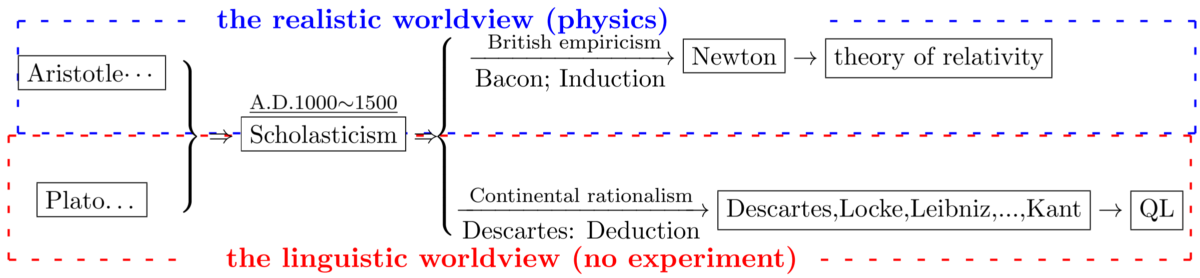
If so, what Bacon wanted to say really may be

$$\Big[\text{ bad idols }\Big] = \Big[\text{ dogmatism in Scholasticism (or, Aristotle's purpose)}\Big]$$

♠**Note 7.4.** S. Weinberg (1933 -), a physicist at the University of Texas, Austin, won a Nobel Prize in 1979 for work that became a cornerstone of particle physics, said in his book [105] "To explain the word; The discovery of modern science" as follows:

($\sharp_1$) [in Chapter 13]: *They (i.e., Bacon and Descartes) are, in my opinion, the two individuals whose importance in the scientific revolution is most* **overrated**. *.... Scientists in the seventeenth and eighteenth centuries would invoke Bacon as a counterweight to Plato and Aristotle, ... It is not clear to me that anyone's scientific work was actually changed for the better by Bacon's writing. Galileo did not need Bacon to tell him to do experiments, and neither I think did Boyle or Newton.*

From the pure scientific point of view, I almost agree to the above opinion. However, it should be noted that the scientific revolution was not only achieved by scientists, but by the collective power of philosophers, enlighteners, astronomers, mathematicians, adventurers, artists, educators, politicians, religionists, and the general public. The difficult and important thing is to get public support for the scientific revolution. I think Bacon did this work well. He enlightened the method to create science (i.e., the importance of causality and experiment) to the general public, not the scientists. As I will show in the next chapter, I consider as follows:



And, the part:

$(\sharp_1)$ " $\xrightarrow[\text{Descartes}]{\text{Continental rationalism}}$ $\boxed{\text{Descartes,Locke,Leibniz,...,Kant}}$ "

is usually rewritten by the following $(\sharp_2)$, which is called "the flower of modern philosophy"

$$
(\sharp_2) \quad \begin{array}{c}\text{Deduction}\\ \boxed{\text{Descartes}}\\ \text{[The father of modern philosophy]}\end{array} \longrightarrow \left\{ \begin{array}{c}\text{Induction (due to Bacon)}\\ \boxed{\text{British Empiricism}}\\ \text{[Locke, Berkeley, Hume]}\\ \\ \text{Deduction (due to Descartes)}\\ \boxed{\text{Continental Rationalism}}\\ \text{[Leibniz]}\end{array} \right\} \longrightarrow \begin{array}{c}\boxed{\text{Kant philosophy}}\\ \text{[summing-up]}\end{array}
$$

(Note that, in spite of "British Empiricism", Locke, Berkeley, Hume were not concerned with experiment. Thus, I assume that they did not belong to the realistic worldview.)

7.3 From Geocentrism to Heliocentrism

7.3.1 What is "Geocentrism vs. Heliocentrism"?

As mentioned in Chapter 5,

- Heliocentrism due to Aristarchus (BC.310 - BC.230) is based on the arguments:

  The big sun cannot go around the small earth.

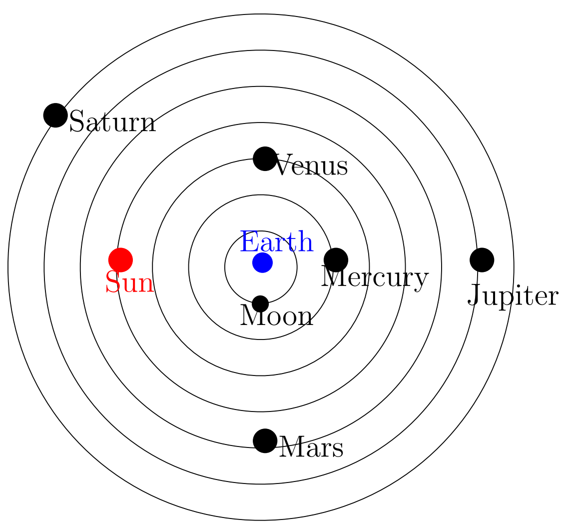
  I think his Heliocentrism to have reached the scientific level. (*cf.* Sec. 5.3).

- Geocentrism due to Ptolemaeus (AD.83 - AD.168) can explain the motion on planets by epicycle theory. Thus, I also think his Geocentrism to have reached the scientific level at the time. (*cf.* Sec. 5.6).

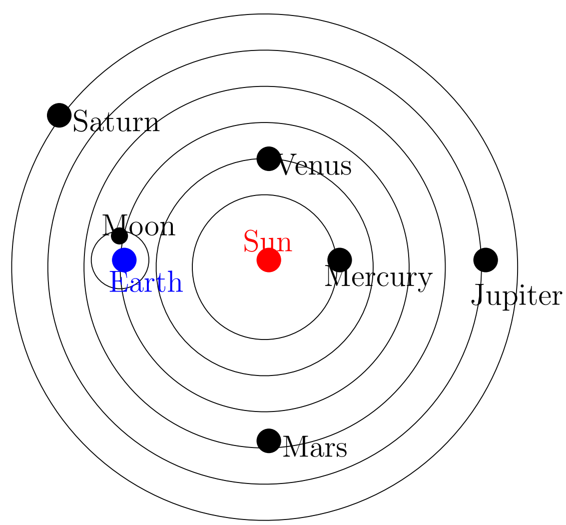
However, Heliocentrism due to Copernicus (1473 - 1543) might be controversial whether it had reached the scientific level. For example, there is an opinion that

- At the time, Europe is in the cold period, the masses were hungry for "the sun". The public was hungry for the sun central principle. Therefore, there is a foundation that allows the germination of Heliocentrism.

In spite of the above, I want to assert that Copernicus is a great scientists. The reason I think so is written in the following Note 7.5.



Geocentrism (=Ptolemaic system)



Heliocentrism (=Copernican system)

♠**Note 7.5.** S. Weinberg (1933 -), An American physicist, won a Nobel Prize of physics in 1979, said in his book [105] "To explain the word; The discovery of modern science" as follows:

(♯) [Chapter 11] *Copernicus could not claim in the Commentariolus that his scheme fitted observation better than that of Ptolemy. For one thing, it didn't. Indeed, it couldn't, since for the most part Copernicus based his theory on data he inferred from Ptolemy's Almagest, rather than on his own observations. Instead of appealing to new* **observations***, Copernicus pointed out a number of his theory's* **aesthetic** *advantages.*

It was very interesting to me that Copernicus, the flag bearer of the scientific revolution, valued beauty over observation. In fact, my conclusion in section 7.4 of this paper, "Heliocentrism vs. Geocentrism," is also "Heliocentrism was chosen because it is more beautiful." I've grown to love Copernicus. Thus I would like to regard him as the great scientist.

In this paper, I discuss the next.

(A): What is "Heliocentrism vs. Geocentrism"?

Now,

(A) Note that motion is relative. Thus, if the earth is assumed to be at center, the sun goes around the earth (i.e., Geocentrism). Also, if the sun is assumed to be at center, the earth goes around the sum (i.e., Heliocentrism). Hence,

The difference between Heliocentrism and Geocentrism may be only a difference of how to take the coordinate system.

7.3.2 Somehow "from Geocentrism to Heliocentrism"

In what follows, I will arrange the history of "Heliocentrism vs. Geocentrism".

- 1510: Copernicus, Heliocentrism in the Commentariolus
- 1600: G. Bruno, who supported heliocentrism along with Galileo, was burned at the stake by the Vatican

Analyzing the enormous data obtained by Tycho Brahe's steady astronomical observation, Kepler found the following laws:

- Kepler's laws of planetary motion:
  1609:

  ($\sharp_1$) The first law of elliptical orbits,

  ($\sharp_2$) The second law of areal velocity,

  1619:

($\sharp_3$) The third law of Periods:

And

1610: Galileo found the moons of the Jupiter by his telescope of the self-made

And further,

1633: Galileo said "And yet it moves" in the Trial of Galileo

♠**Note 7.6.** In the next year of 1642 when Galileo died, Isaac Newton was born in the British country.

1687: Newton "Principia"

In this way, we think:

(B) Somehow the air "to Heliocentrism" has been formed.

Still, I am worried about this problem (A)"What is 'Geocentrism vs. Heliocentrism'?"

That is,

(C_1) Did Both Galileo and the church understand the essence of 'Geocentrism vs. Heliocentrism'?

which is equivalent to

(C_2) In order to win the definitive victory, what should they (Galileo or the church) have done?

In order to answer to this question, we first have to clarify the meaning of "Heliocentrism vs. Geocentrism".

7.3.3 "Geocentrism vs. Heliocentrism" is the problem of the world-view

As mentioned in the previous section, how to decide "Geocentrism vs. Heliocentrism" is somewhat difficult. That is because

(D) Thus, if the earth is assumed to be at center, the sun goes around the earth (i.e., Heliocentrism). Also, if the sun is assumed to be at center, the earth goes around the sum (i.e., Geocentrism). Hence, The difference between Heliocentrism and Geocentrism is only a difference of how to take the coordinate system.

In the same sense, we say that

(E) **No matter how much there are exact observation data, we cannot decide "Geocentrism or Heliocentrism"**

In the famous trial of Galileo, he said

"And Yet It Moves"

However, I wonder if Galileo knew the (E)?

♠**Note 7.7.** No matter how much there are exact observation data, we cannot decide "Geocentrism vs. Heliocentrism" we have to need the worldview. Namely,

($\sharp_1$) it is a matter of course that there is no science without measurement

However, we believe that

($\sharp_2$) there is no science without worldview

Thus, as seen later, we cannot decide "Geocentrism vs. Heliocentrism" without worldview

7.3.4 The Galileo legend; Leaning Tower of Pisa, Trial of Galileo

The worldview of Aristotle has kept its position for 1500 years such as (*cf.* Sec. 4.2.1):

$$\underset{\text{(Hērakleitos, Parmenides, Zeno, Aristotle)}}{\overset{\text{the birth of worldview}}{\boxed{\text{Movement}}}} \xrightarrow[\text{Aristotle's spirit :(About 1500 years)}]{\text{"purpose" (and no experiment)}} \underset{\text{(Bacon, Galileo, Descartes, Newton)}}{\overset{\text{the birth of modern science}}{\boxed{\text{Causality (and experiment)}}}}$$

Thus, this worldview is not a so bad worldview.

But, this worldview was a little inconvenient to organize the data, obtained by technological innovation (e.g., telescope, navigation, etc.). The history of the increase of the inconvenience is as follow:

$$\big[\text{Copernicus}\big] \Rightarrow \big[\text{Kepler}\big] \Rightarrow \big[\text{Galileo}\big]$$

However, these are not sufficient to decide "Geocentrism vs. Heliocentrism". It is a matter of course that there were excellent persons in the church. And they might think:

- if they insisted that motion is relative, they did not lose the dispute, at least, they could make "Geocentrism vs. Heliocentrism" endless dispute.

Galileo legend

Galileo (1564 - 1642) was an active leader of the overthrow of the worldview of Aristotle, and his targets were the following (F_1) and (F_2):

(F_1) Ptolemaic Geocentrism

(F_2) Aristotelian purpose such as "Heavy objects fall faster"

Concerning the two, We have two episodes called "Galileo legend" as follows.

For (F_1), "And Yet it moves" in trial of Galileo

For (F_2), Leaning Tower of Pisa

Thus, Galileo thought it was a matter of measurement data, but the church thought it was a matter of worldview. Therefore,

- at the time (1633) of the trial of Galileo, the Church was convinced that it would not lose its argument with Galileo and, at worst, could bring it into an endless dispute.

An endless dispute implies the win of the church. The church is not so stupid.

No way, the church did not think that Newton would appear

No one would have predicted the appearance of Newton.

After all, Galileo was the active leader of the overthrow of the worldview of Aristotle, but he could not propose the new worldview. In that sense, the Galileo legend is just the beginning of Newton's appearance

7.4 Principia; Newtonian worldview

7.4.1 Principia (1687)

"PhilosophiæNaturalis Principia Mathematica" (in short, "Principia") , written by Newton (1687), is the most famous and important book in science. Three laws of Kepler were derived from three laws of dynamics and the law of universal gravitation. Principia was written based on elementary geometry and not the differential and integral calculus. Why did Newton (= advocate of differential and integral calculus) not write Principia based on differential and integral calculus? Although there may be several opinions for this question, The work (based on differential and integral calculus) was succeeded by Leibniz, J. Bernoulli, Euler, d'Alembert, Lagrange and Laplace, etc. and was completed.

7.4.2* What is "Geocentrism vs. Heliocentrism"? After all, the worldviewism

The following biggest paradigm shift in the history of science is as follows.

(A) Motion [Motion function method: (Parmenides, Zeno, Aristotle)]

$\xrightarrow[\text{paradigm shift}]{}$

Causal relation [Kinetic differential equation method: (Newton)]

That is, we see:

(B): Paradigm shift [Newtonian mechanical worldview]

Now,

(B) $\underset{\text{(purpose)}}{\boxed{\text{Aristotle's worldview}}} \xrightarrow[\text{paradigm shift}]{} \underset{\text{(causal relation)}}{\boxed{\text{Newtonian worldview}}}$

Here, Newtonian worldview means Newtonian mechanics, that is,

Newton's kinetic equation (i.e., the chain of causal relations)
\+ the law of universal gravitation

Recall that the main theme of this paper is the worldviewism(*cf.* Sec. 1.3.1). As mentioned frequently up to this point,

(C_1) The argument in ordinary language (or, in the motion function method (*cf.* Sec.2.3.3)) is fuzzy, and thus, "Geocentrism vs. Heliocentrism" cannot be decided. Thus, we need a new worldview.

In Principia, Newton proposed Newtonian mechanics (i.e., Newtonian worldview) and showed that

(C_2) **When the motion of the sun and the planets is studied, the calculation becomes easy under the assumption that the planets go around the sun.**

Therefore, even the definitions "center" and "go around" depend on the worldview. After all, we conclude that

(D) "Geocentrism or Heliocentrism" is not an issue that can be settled, no matter how accurate the observations are. That is, "Geocentrism vs. Heliocentrism" is not the problem of measurements, but the problem of the worldview.

♠**Note 7.8.** Next is said to be the three major discoveries of modern science

($\sharp_1$) Aristotelian worldview (purpose) $\longrightarrow$ Newtonian worldview (causal relation)

($\sharp_2$) Ptolemaic system: Geocentrism $\longrightarrow$ Copernican system: Heliocentrism

($\sharp_3$) Christianity: Adam and Eve $\longrightarrow$ Darwin: evolution theory

However, it should be noted that ($\sharp_2$) is a consequence of ($\sharp_1$). There may be a reason to consider that ($\sharp_2$) is a great episode of the birth of ($\sharp_1$).

♠**Note 7.9.** Here, we have (*cf.* Classification 1.11 [the classification of philosophers]).

($\flat$)

($\flat_1$): the realistic worldview (physics)
Hērakleitos, Aristotle, Aristarchus, Archimedes, Eratosthenes, Ptolemaeus, Galileo, Newton, Einstein, $\cdots$
(Although mathematics is not a worldview, Pythagoras, Eudoxus, Euclid)

($\flat_{21}$): the fictional worldview (Western philosophy)
Plato, Scholasticism, Descartes, Locke, Leibniz, Berkeley, Hume, Kant, Husserl

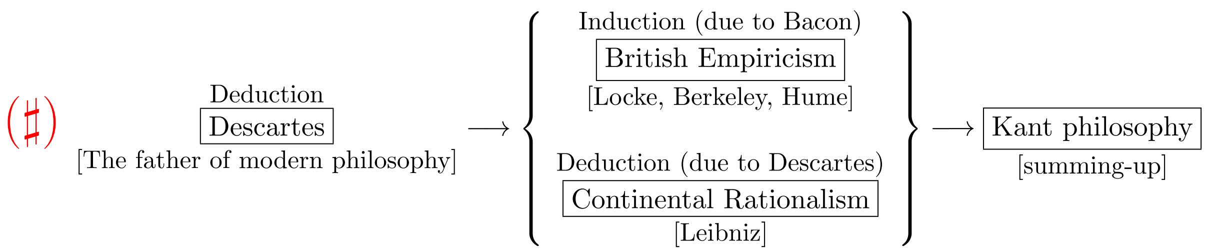
($\flat_{22}$): the logical worldview
Boole, Frege, Peirce, Saussure, Russell, Wittgenstein, Hempel, Popper

($\flat_{23}$): the mechanical worldview (statistics, quantum language)
Parmenides, Zeno, J. Bernoulli, statistics (e.g., Fisher), quantum language

Chapter 8

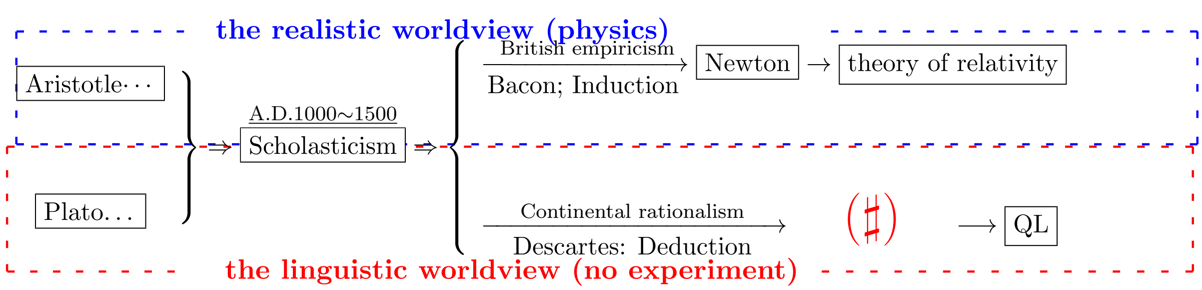
Modern philosophy(Descartes)

The following is called the flower of modern philosophy:



(Note that in spite of "British Empiricism", Locke, Berkeley, Hume were not concerned with experiment. Thus, I assume that they did not belong to the realistic worldview.)

Here, the (♯) is located as follows:



It is unthinkable now, but I think it was a time when philosophers were respected and believed that "philosophy is the king of science". Even so,

- Why were top elites in those days absorbed in the **useless** philosophy?

As mentioned before, I think that this is due to Platonic method of telling philosophy. Is it a desire to replicate the success of Euclidean geometry (i.e., logic about figure) in philosophy? Namely, I think they wanted to pursue a "science of dualism" (*cf.* Remark 3.9, Remark 5.1).

In this chapter, we discuss Descartes, British empiricism, Continental rationalism.

8.1 The theory of Ideas has expired

Let us review Platonic method of telling philosophy:

$$\underset{\text{introduction·preface·preliminary fiction·prologue}}{\overset{\text{world is so}}{\boxed{\text{fictional worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{Live so}}{\boxed{\text{Ethics·morals}}}}$$

Then, ethics, morals is main, and the fictional worldview is only preface. Although it is desirable that the worldview in preface is related to a dualistic idealism (*cf.* Sec.3.4.1), but it didn't matter even if it was merely a fable. In fact, the theory of Ideas is just a fable. In an extreme case, using the psychological tricks such as

(A) "Intellectual's remark can be trusted",
"As for the beautiful woman, a heart is fair",
"We can trust the assertion of Kant who was too serious and stiff.", etc.

In the preface (=view of the world), it is enough to win the trust of the reader. A scientist is not expected to be a person of character, but a moralist must be a person of character. Therefore, Kant had to be as earnest and honest as a clock.

The main current of western philosophy adhered to Platonic method of telling philosophy. In Plato philosophy, the worldview (=the theory of Ideas) is completely an allegory, and the main subject is due to Socrates' ethics. In cases of Augustinus and Scholasticism, the main subject is of course Christianity. Hence, the worldview in the preface is not so important. For example, "only the present exists" [resp. "barren discussion: Existence of God, Plato or Aristotle?"] is the intellectual act of killing time in Augustinus philosophy [resp. Scholasticism].

However, after the age of geographical discovery and the Renaissance, the momentum for the renewal of fairy tales had increased.

And Descartes thought that

The expiry date of the theory of Ideas (≈ a kind of fable) was expired.

Descartes, using self-referential cogito proposition "I think, therefore I am", proposed the worldview (i.e., Descartes philosophy (= mind-matter dualism)). Roughly speaking,

$$\underset{\text{(Plato)}}{\boxed{\text{the theory of Ideas}}} \xrightarrow[\text{cogito turn}]{} \underset{\text{(Descartes)}}{\boxed{\text{mind-matter dualism}}} \tag{8.1}$$

The philosophy of worldview is only an anecdote, however, it should be effective for general people (and thus, quantitative arguments are not desirable). After all, following Socrates' self-referential statement "I know that I know nothing", Descartes also wanted to use self-referential trick "I think, therefore I am".

♠**Note 8.1.** There is a good reason for the birth of Newtonian mechanics, for example, the Age of Discovery, positional notation (= the discovery of zero), etc. Newtonian mechanics was powerful, for example, and had the power to refute the theory of celestial motion. On the other hand, there may not be a firm reason for the birth of Descartes philosophy as the continuation of Platonic method of telling philosophy. My opinion is as follows.

($\sharp_1$) Under the Christian strong influence, it could not be free to discuss the ethics. And thus, western philosophy devoted itself to the preface (i.e., worldview) than the main subject (i.e., ethics).

Platonic method of telling philosophy might be a clever strategy for western philosophy to coexist with Christianity. The following question is significant:

($\sharp_2$) **Is the cogito turn (8.1) progress, or a change in trend?**

As mentioned in Preface, to answer this is one of the main purposes of this paper. In Chapter 11, we will conclude that it is "progress". in spite that the change from Plato philosophy (i.e., Idea theory) to Descartes philosophy (i.e., mind-matter dualism) is not inevitable.

8.2 ”I think, therefore I am” is meaningless from the scientific point of view

Descartes(1596-1650) was a French philosopher, mathematician, and scientist. He is also widely regarded as one of the founders of modern philosophy.

♠**Note 8.2.** One of Descartes's most enduring legacies was his development of Cartesian or analytic geometry, which uses algebra to describe geometry. There is even a plausible legend that Descartes, who was sleeping in his bed, came up with the system in order to track the position of a fly moving around the ceiling.

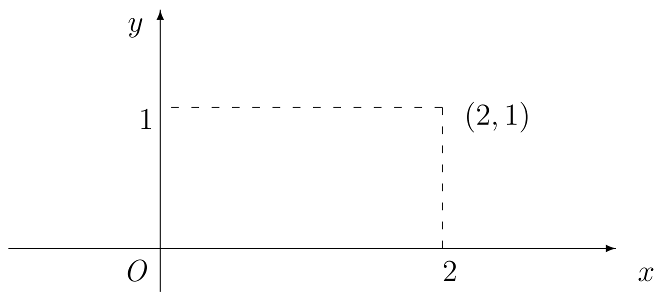


Figure 8.1: [Cartesian coordinates]

8.2.1 Discourse on the Method (1637)

Let us study the most famous book in philosophy: ”Discourse on the Method (1637)” by René Descartes, which is the abbreviation of ”The method of rightly conducting one's reason and of seeking truth in the sciences”. Probably after Descartes read Bacon's ”Novum organum”, he decided ”Start from the unquestionable truth”. And he found the most famous philosophical proposition (= cogito proposition):

”I think, therefore I am”

That is, Descartes think:

I think that 'I think, therefore I am'

In spite that this is a most doubtful proposition (and thus, it is not in quantum language, as seen in Note 1.7 or, Proposition 8.2), Descartes believed that it was the unquestionable truth.

And further, bearing in mind Euclid's success in geometry, Descartes was convinced that

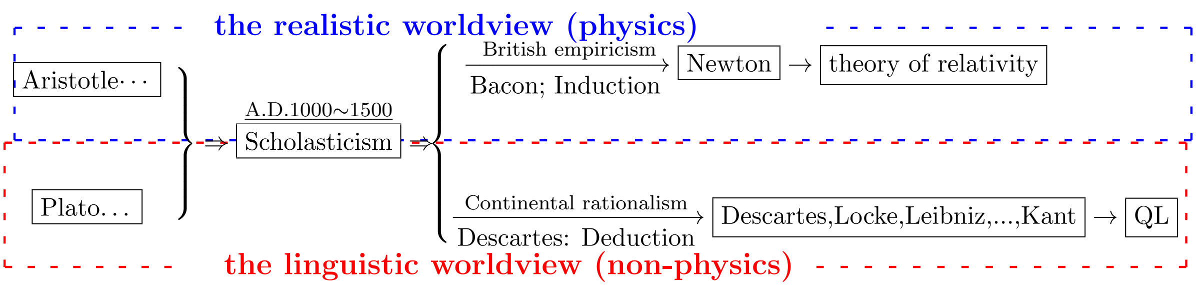
- Every statements derived from the cogito proposition are absolutely trusted

This is just "The method of rightly conducting one's reason and of seeking truth in the sciences". Therefore, after all, Bacon and Descartes are in opposite positions. That is,

(A_1) Bacon emphasized the importance of experiments, and so asserted the inductive method. He was oriented to physics (and further, British empiricism)

(A_2) Descartes believed "good sense", and so asserted the deductive method. He was oriented to metaphysics (and further, Continental rationalism)

In this paper, the following figure is actually drawn so far repeatedly.



However, the part:

($\sharp_1$) " $\xrightarrow[\text{Descartes}]{\text{Continental rationalism}}$ $\boxed{\text{Descartes,Locke,Leibniz,...,Kant}}$"

is usually rewritten by the following ($\sharp_2$), which is called "the flower of modern philosophy"

$$
(\sharp_2) \quad \begin{matrix} \text{Deduction} \\ \boxed{\text{Descartes}} \\ \text{[The father of modern philosophy]} \end{matrix} \longrightarrow \left\{ \begin{matrix} \text{Induction (due to Bacon)} \\ \boxed{\text{British Empiricism}} \\ \text{[Locke, Berkeley, Hume]} \\ \\ \text{Deduction (due to Descartes)} \\ \boxed{\text{Continental Rationalism}} \\ \text{[Leibniz]} \end{matrix} \right\} \longrightarrow \begin{matrix} \boxed{\text{Kant philosophy}} \\ \text{[summing-up]} \end{matrix}
$$

(Note that in spite of "British Empiricism", Locke, Berkeley, Hume were not concerned with experiment. Thus, I assume that they did not belong to the realistic worldview.)

Descartes asserted:

Proclaim 8.1. The first principle (= cogito proposition) in philosophy
Now,

(A) Descartes doubted everything. And he arrived in the cogito proposition which has no doubted room. That is, he arrived in

(B) I think, therefore I am.

And, he proclaimed that the cogito proposition (B) is the first principle in philosophy.

8.2.2* Two (scientific or non-scientific) interpretations of "I think, therefore I am"

What is described in this section has been mentioned many times in the references [40]∼[68].

Now we will show two (scientific or non-scientific) interpretations of the cogito proposition: "I think, therefore I am".

Proposition 8.2. **[(i): Scientific interpretation of the cogito proposition]:**
Let start from the scientific interpretation of the cogito proposition.
For the sake of convenience, put "I"="Tom". Thus,

- "I think, therefore I am"= "Tom thinks, therefore Tom is"

If this is a scientific proposition, it must be experimentally verifiable. However, it is easy. That is because it is usual that

(C_1) a doctor says "Tom's brainwaves are normal, so he's alive."

which is of course a statement in quantum language. However, the interpretation (C_1) is a trivial one, and this would not be Descartes' intention.
[(ii): Non-scientific interpretation (i.e., Descartes' intention) of the cogito proposition]:
Similarly, put "I"="Tom". Put

($\flat$) "I think, therefore I am"= "Tom thinks, therefore Tom is"

Thus, we have the situation:

(C_2) Tom thinks "Tom thinks, therefore Tom is"

which means that

$$\text{"observer = Tom"}, \qquad \text{"matter = Tom"},$$

That is, this violates the linguistic Copenhagen interpretation. Therefore, the (C_2) is not in quantum language, that is, it is just a play on words.
Thus, the ($\flat$) is a non-scientific proposition (i.e., an experimentally unverifiable proposition), but this play on words had captured the interest of philosophy enthusiasts. That is,

This play on words ushered in the modern era.

It is obvious that the above (ii) is the recommended answer.

8.3 Descartes' strategy

8.3.1 From cogito to dualistic idealism

The most important key-word in Descartes' philosophy is "I". Descartes thought that

> Nobody pays attention even if Descartes appeals for the importance of "I" aloud.

Thus,

(A) Descartes used the advertising slogan (i.e., the cogito proposition): "I think, therefore I am"

The cogito proposition is nonsensical, but it is very impressive. Thus, this proposition could be a perfect advertising slogan.

$$\boxed{\text{cogito proposition}} \xrightarrow[\text{derivation}]{\text{non-scientific}} \boxed{\text{the existence of "I"}} \xrightarrow[\text{Def. 1.7}]{\text{"exist"=keyword}} \boxed{\text{"I" as keyword}}$$

Thus, by the cogito proposition, what Descartes wanted to say was

(B) "I" is the most important key-word in Descartes philosophy.

His strategy succeeded wonderfully. If "I" is accepted, the existence of "matter" (which is perceived by "I") is accepted. And further, the medium of "I" and "matter" is automatically accepted as "body (= sensory organ)".

Therefore, the key-words of Descartes philosophy (= mind-matter dualism) is

(C) $\boxed{\text{"I"(="mind", "brain")}}$, $\boxed{\text{"body"(="sensory organ")}}$, $\boxed{\text{"matter"}}$

For completeness, it should be noted that this is not a consequence of the cogito proposition. That is the cogito proposition is the reason added afterwards. The cogito proposition is nothing more than a catch copy.

Principle 8.3. The principle of Descartes' philosophy is

(D) **Think of everything in terms of mind-matter dualism and idealism.**

That is, Descartes proposed the fictional worldview the called mind-matter dualism and idealism which starts from the three key-words

$$\underset{\boxed{\text{"I"(=mind, brain, observer)}}}{\text{[A](mind)}}, \quad \longleftarrow \underset{\boxed{\text{"body"}}}{\overset{\text{(medium)}}{\text{[B]}}} \longrightarrow , \quad \underset{\boxed{\text{"matter"}}}{\text{[C](matter)}}$$

(=sensory organ, measuring instrument)

For the definitions of "idealism" and "idealism", see Definition 1.8, i.e., idealism = the spirit such that thinking takes precedence over experimentation (since experiments on the mind are almost impossible). Also, dualism=the worldview concerning measurement.

That is, we see

$$(\sharp) \qquad \underset{\text{introduction·preface·fiction}}{\overset{\text{"I think, therefore I am" (= mind-matter dualistic idealism)}}{\boxed{\text{fictional worldview (literary truth, pseudo- truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{Live so}}{\boxed{\text{Ethics·morals}}}}$$

Although Descartes did not emphasize measurement (≈ cognition, epistemology), the importance of cognition was emphasized by his successors (Locke, Hume, etc.).

Also, ethics, moral, etc. are not emphasized in ($\sharp$), but Descartes asserted "The existence of God is derived".

And, Descartes proposed the following two problems;

Problem 8.4. [mind-body problem, qualia problem]

(D_1) **mind-body problem:**
How are "body" and "mind" connected?

(D_2) **subjectivity problem (= qualia problem):**
Is the world I perceive the same as the world you perceive?

For the answer of (D_1) in the quantum mechanical worldview, see Sec. 11.9
For the answer of (D_2) in the quantum mechanical worldview, see Sec. 9.5.1

Descartes might think as follows.

(F) The theory of Ideas has expired expiration date. Thus, in order to refresh philosophy, a new model-change (or, a new wrapping paper) is needed such as Descartes' problem (D). Even if this is a non-sense problem, this model-change won't fade for about 100 years.

In fact, the cogito turn:

$$\underset{\text{Plato}}{\boxed{\text{Theory of Ideas}}} \xrightarrow{\text{Cogito Turn}} \underset{\text{Descartes}}{\boxed{\text{Mind-matter dualism}}}$$

is the biggest model-change in the history of philosophy.

♠**Note 8.3.** Descartes' book: "The method of rightly conducting one's reason and of seeking truth in the sciences" begins with the non-scientific proposition "I think, therefore I am", which is not experimentally verifiable (*cf.* Proposition 8.2.). Some may find this ironic. However, I'd like to think that Descartes had it all figured out. That is because

- Euclid's "Elements" begins with seven axioms, which cannot be proved.

Thus, I guess that,

- As Euclid aimed at a science of figures, Descartes aimed at a science of dualism.

If so, I can expect that Western philosophy may have a destination.

♠**Note 8.4.** As mentioned in ref. [66], I rewrite as follows. It is not true to consider that every phenomenon can be described in terms of quantum language. Although readers may think that the following can be described in measurement theory, but we believe that it is impossible. For example, the followings cannot be written by quantum language:

$$\begin{cases} \text{①} : \text{ tense—past, present, future —} & \text{②} : \text{ Heidegger's saying "In-der-Welt-sein"} \\ \text{③} : \text{the measurement of a measurement,} & \text{④} : \text{Bergson's subjective time} \\ \text{⑤} : \text{ observer's space-time,} & \\ \text{⑥} : \text{Only the present exists (due to Augustinus(354-430))} & \end{cases}$$

If we want to understand the above words, we have to propose the other scientific languages (except quantum language). We have to recall Wittgenstein's sayings

The limits of my language mean the limits of my world

I consider that ①–⑥ are related to self-reference in the wide sense.

8.4 The correspondence of key-words in Descartes philosophy and quantum language

The key-words of Descartes philosophy (= mind-matter dualism) is

"I"(="brain", "mind"), "body"(="sensory organ"), "matter"

However, we cannot expect the substantial result even if we consider Descartes' problem. However, the above three key-words are essentially important in the relation with the quantum language: Now let us explain this.

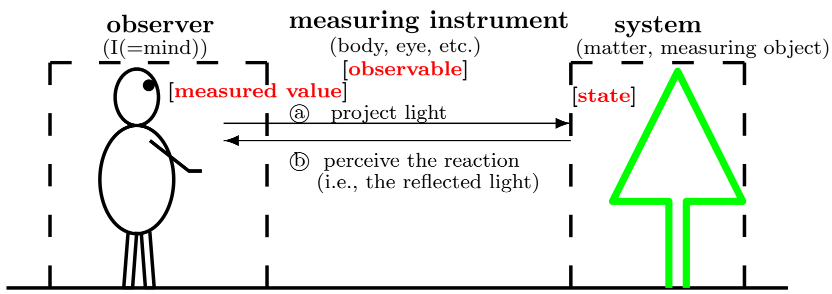


Figure 8.1: [Descartes Figure]: Image of "measurement(=①+②)" in dualism

For example, consider:

- Examine whether the hot or cold water in the bath and put your hands in the bathtub.

In this case, "hand" is regarded as "measuring instrument". In the same sense, "eye" is also regarded as "measuring instrument". Conversely, Glasses, microscope, telescope, etc. is a kind of body (= sensory organ). If so, we want to conclude that

body (particularly, sensory organ) $\doteqdot$ measuring instrument

In the above Descartes figure, slightly incomprehensible one may be

"I"(="brain", "mind") $\doteqdot$ measured value

However, it suffices to consider "there is no measured value without brain". For example when a needle of a voltmeter just moved, it is only a physical phenomenon. Nevertheless a movement of this needle is read, and it's sensed by a brain. Then, it for the first time becomes "measured value".

The reason that Descartes philosophy is useless is as follows.

(A) In spite that three key-words "mind", "body", "matter" are gathered, Descartes philosophy has no computable structure. This is only the fictional worldview, and not the mechanical worldview.

Table 8.1: Key-words in each worldview (*cf.* Assertion 1.14)

mind-matter dualism	[A](= mind)	[B](between A and B)	[C](= matter)
Plato	actual world	Idea	/ [Idea world]
Descartes	I, mind, brain	body	/ [matter]
quantum language	**measured value**	**measuring instrument (= observable)**	**state [system]**

That is, using the following change:

$$[\mathrm{I}] \Longrightarrow [\text{measured value}],\ [\text{body}] \Longrightarrow [\text{measuring instrument}],\ [\text{matter}] \Longrightarrow [\text{system}]$$

we get the computable worldview (i.e., Axioms 1 and 2 in Sec. 1.1.1), i.e., quantum language.

If so, the following problem is essential.

Problem 8.5. Descartes' model-change:

$$\text{the theory of Ideas} \xrightarrow[\text{model-change}]{} \text{Descartes philosophy}$$

can be regarded as a progress? That is, it is sure that Descartes' model-change is supported by many people. Almost people certainly believe that science makes progress, that is, science development is not fashionable change. However, there may be a lot of opinions about philosophy. We completely agree that it is fun to think of dualistic idealism (= dualistic metaphysical world). However, we have the question:

- Did western philosophy make essential progress?

which is essentially the same as

- Does dualistic idealism (= dualistic metaphysical world) deserve to study as science?

Brief explanation of Problem 8.5:
Although this will be answered throughout this paper, I add a simple explanation as follows. Our answer to Problem 8.5 is "essential progress". As the reason mentioned in Assertion 1.14 of Sec.1.4, the following key-words converge to "observable" such as

$$\underset{\text{(Plato)}}{\boxed{\text{Idea}}} \to \underset{\text{(Descartes)}}{\boxed{\text{body}}} \to \underset{\text{(Locke)}}{\boxed{\text{secondary qualities}}} \to \cdots \to \underset{\text{(quantum language)}}{\boxed{\text{observable(=measuring instrument)}}}$$

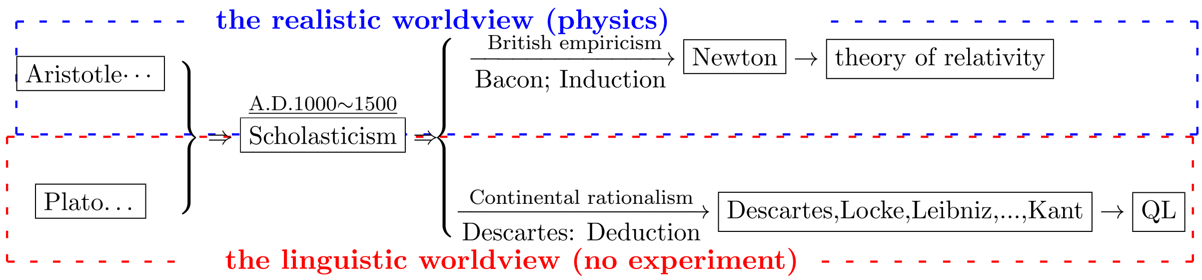
It should be noted that there is no settlement (i.e., "essential progress" or "fashionable change"?) without ultimate goal (= quantum language). Therefore, this will be summarized in the final chapter (=Postscript) . □

KSTS/RR-20/001
November 15, 2020

Chapter 9

Modern philosophy(Locke, Leibnitz, Berkeley, Hume)

The fact may be represented by the figure.



However, the part:

$$(\sharp_1) \quad \text{"} \xrightarrow[\text{Descartes}]{\text{Continental rationalism}} \boxed{\text{Descartes,Locke,Leibniz,...,Kant}} \text{"}$$

is usually rewritten by the following ($\sharp_2$), which is called "the flower of modern philosophy"

$$(\sharp_2) \quad \begin{matrix} \text{Deduction} \\ \boxed{\text{Descartes}} \\ \text{[The father of modern philosophy]} \end{matrix} \longrightarrow \left\{ \begin{matrix} \text{Induction (due to Bacon)} \\ \boxed{\text{British Empiricism}} \\ \text{[Locke, Berkeley, Hume]} \\ \\ \text{Deduction (due to Descartes)} \\ \boxed{\text{Continental Rationalism}} \\ \text{[Leibniz]} \end{matrix} \right\} \longrightarrow \begin{matrix} \boxed{\text{Kant philosophy}} \\ \text{[summing-up]} \end{matrix}$$

(Note that, in spite of "British Empiricism", Locke, Berkeley, Hume were not concerned with experiment. Thus, I assume that they did not belong to realistic worldview.)

It's unbelievable now, but it was a time when philosophy was believed to be the "king of the academy," and a time when philosophy was respected.

Even so,

- Why were top elites in those days absorbed in the **useless** philosophy?

As I have said many times, I think that this is due to Euclid's success and Platonic method of telling philosophy.

.

9.1 Locke (1632 - 1704): The father of British Empiricism

9.1.1 "An Essay Concerning Human Understanding" by Locke (1689)

There may be a reason to consider that

(A) The role of Descartes was the elimination of such "spiritual power" and "supernatural being", and to prepare the social environment of the appearance of Newton. That is, Descartes was only the opening performer. In this sense, "I think, therefore I am" (the existence of "I") was only the side show of the opening performer.

And so,

(B) The role of Descartes, as the opening performer, had been finished by the appearance of Newton.

Even if there was such history, it wasn't strange.

However, strangely, there were people who took "the existence of I" or "Descartes Figure 1.2" seriously. For example, John Locke (1632 - 1704) thought as follows.

(C) :"An Essay Concerning Human Understanding" by Locke (1689)

Locke is the successor of Descartes philosophy. He philosophically (i.e., without experiments) discussed the Descartes figure (i.e., the relation among "I"(="brain", "mind"), "body"(="sensory organ"), "matter"). He is called "The father of British Empiricism ($\approx$ epistemology)".
Since Isaac Newton published his "Principia Mathematica" (1686), John Locke might think in his "Essay Concerning Human Understanding" (1689) as follows.

(D) In the field of "matter" of Descartes figure, activity of Newton is remarkable. However, concerning the relation among "I"(="brain", "mind"), "body"(="sensory organ"), "matter", he wanted to reach the summit.

If so,

(E) It was too early more than 300 years to study "epistemology" in science in earnest.

Thus, it is impossible to expect the result.

However,

(F) If we think that the work of philosophers is "model-change", then the achievements of Locke is enormous.

♠**Note 9.1.** If the above (D) is true, Locke may have been trying to create another kind of physics that is different from Newtonian mechanics. If so, his theory should belong to dualistic realism. However, Locke and others (Descartes, Leibniz, Hume, Kant, etc.) were not concerned with experimental verification (*cf.* for our definition of "realism", see Definition 1.7). Thus, in this paper, we think that Descartes-Kant philosophy belongs to dualistic idealism. This was confused in my previous paper [61].

9.1.2 "Tabula rasa", primary quality and secondary quality

9.1.2.1 "tabula rasa"

Tabula rasa is a Latin phrase often translated as "blank paper" in English, that is,

(G) The "brain circuit" is a blank paper state at the start, but we look and hear in various ways, then "concept (= complex brain circuit)" is made.

Present-day brain science may say:

"It's equal to say nothing by such general opinion."

however, at any rate, the (G) is the starting point of British Empiricism.

♠**Note 9.2.** (i): Maybe Locke wasn't claiming "tabula rasa" in the scientific sense. I think he was following Bacon's empiricism and insisting on a "tabula rasa".

(ii): As mentioned later (*cf.* Note 10.3, Summary 11.31), considering "language" and not "cognition", then, in several languages (ordinary language, mathematics, Newtonian mechanics, programing language, etc.), we say tat

($\sharp$) "ordinary language" is like tabula rasa

When a baby was born, a baby doesn't know ordinary language at all (i.e., a baby is with tabula rasa state). The baby is acquiring ordinary language by trial and error.

9.1.2.2 Primary quality and secondary quality

According to Locke,

(H_1) **primary quality** (i.e., inherent nature (=primary quality)) $\cdots$ weight, temperature, length, etc.

(H_2) **secondary quality** (i.e., sensations of inherent nature)$\cdots$ sweet, red, hot, salty, etc.

That is,

(I) :Locke's worldview

The world is composed of two (i.e., "matter" and "mind (= observer)". "Matter" has inherent nature (= primary quality), "observer" has body (="sensory organ"). Through the sensory organ, secondary quality (sweet, red, hot, salty, etc.) is felt by our brain.

In terms of quantum language, we say:

primary quality$\Longrightarrow$state,

secondary quality$\Longrightarrow$observable (= measuring instrument)

as seen in the table below.

Table 9.1: The key-words of worldviews (*cf.* Assertion 1.14)

mind-matter dualism	[A](= mind)	[B](between A and B)	[C](= matter)
Plato	actual world	Idea	/ [idea world]
Descartes	I, mind, brain	body	/ [matter]
Locke	mind	secondary quality	primary quality [matter]
quantum language	**measured value**	**observable**	**state [system]**

Here,

(J) Locke represents the most important concept in dualistic idealism as the term "secondary quantity". The terms such as Idea, body, etc. may be not comprehensive. However,

- "secondary quantity" is a word we can understand. Hence, Locke's achievement should be honored.

Again, note that "secondary quantity" is a word that forms the foundation of dualism.

If so, we may affirmatively answer Problem 8.5, i.e.,

Can the direction:"Descartes $\xrightarrow[\text{model-change}]{}$ Locke" be regarded as progress?

That is, we may assert that

$$\underset{\text{Plato}}{\overset{\text{(Idea)}}{\boxed{\text{Plato}}}} \xrightarrow[\text{model-change}]{\text{progress}} \overset{\text{(body)}}{\boxed{\text{Descartes}}} \xrightarrow[\text{model-change}]{\text{progress}} \overset{\text{(secondary quality)}}{\boxed{\text{Locke}}} \xrightarrow[\text{model-change}]{\text{progress}} \overset{\text{(measuring instrument)}}{\boxed{\text{quantum language}}}$$

if "to make progress" is defined by "to come near quantum language" (*cf.* Assertion 1.14). It should be noted that this result cannot be confirmed without an understanding of quantum language. Therefore, this will be summarized in the final chapter (=Postscript)

♠**Note 9.3.** (i): The polar star can be regarded as a measuring instrument such as a kind of compass.
(ii): By the way, Merleau-Ponty (1908 - 1961) might think in the following manner.

($\sharp$) I shake hands with my right hand and the left hand. In this case, if I regard the right hand as the measuring instrument, I feel the existence of my left hand. On the contrary, if I regard the left hand as the measuring instrument, I feel the existence of my right hand.

It may be interesting, however, I do no know whether such thing is worth arguing.

9.2 Dramatic presentation of "British Empiricism vs. Continental Rationalism"

Modern philosophy became popular through the following rival relation:

British Empiricism vs. Continental Rationalism

That is,

$$(A)\qquad \underset{\text{[The father of modern philosophy]}}{\overset{\text{Deduction}}{\boxed{\text{Descartes}}}} \longrightarrow \left\{ \begin{array}{c} \text{Induction (due to Bacon)} \\ \boxed{\text{British Empiricism}} \\ \text{[Locke, Berkeley, Hume]} \\ \\ \text{Deduction (due to Descartes)} \\ \boxed{\text{Continental Rationalism}} \\ \text{[Leibniz]} \end{array} \right\} \longrightarrow \underset{\text{[summing-up]}}{\boxed{\text{Kant philosophy}}}$$

(Note that in spite of "British Empiricism", Locke, Berkeley, Hume were not concerned with experiment. Thus, I assume that they did not belong to physics, but idealism.)

Let us explain the above:

Modern philosophy

British Empiricism["tabula rasa"(= blank paper)]
"An Essay concerning Human Understanding" (by Locke, 1690) says that

(B) He eliminated the possibility of innate knowledge before experience. Human being is born as the blank state ("tabula rasa") . (Locke, Berkeley, Hume, $\cdots$)

Continental Rationalism[nativism]
"New Essays on Human Understanding" (by Leibniz, 1703) says that

(C) nativism (= Anti-"tabula rasa"). the human mind as it is at birth, with ideas or thoughts in it.

(Leibniz,$\cdots$)

That is, from

"An Essay concerning Human Understanding" vs. "New Essays on Human Understanding"

the rival relation:

British Empiricism["tabula rasa"] vs. Continental Rationalism[nativism]

began. After nearly 100 years of twists and turns,

Appearance of Kant (Critique of Pure Reason: 1781)

And

(D) Kant has integrated "tabula rasa vs. nativism"

It is well known such above story that regards Kant as a hero.

9.2.1 Would Leibniz be serious for this argument (i.e., nativism) ?

Gottfried Wilhelm Leibniz (1646-1716) was one of the great thinkers of the seventeenth and eighteenth centuries and is known as the last "universal genius". He made deep and important contributions to the fields of metaphysics, epistemology, logic, philosophy of religion, as well as mathematics, physics, geology, jurisprudence, and history.

Everyone may have the following question:

(E) Why did such a genius participate in a nonsense argument (i.e., "nativism vs. tabula rasa") ?

It is clear that Locke's theory is too extreme, and thus, it is a matter of course that Leibniz did not completely agree with "tabula rasa". However,

(F_1) However, the story that Leibniz disputed Locke in "New Essays on Human Understanding" is too exaggerated.

The argument about "nativism vs. tabula rasa" is non-sense in the following sense:

(F_2) Even if future brain science will make a decision favorable to one of them (i.e., "nativism vs. tabula rasa"), it is independent of Leibniz's (or, Locke's) evaluation. That is because "Continental Rationalism vs. British Empiricism" is regarded as a pre-science problem.

What is the most important is to form the ground on which the dualistic idealism (or the problem concerning brain) can be argued scientifically. Without the ground, it is useless even if they said something

For example, from the scientific point of view, atomism due to Democritus (BC.460-BC.370) is non-sense. It is poem. .

For the argument about "Continental Rationalism vs. British Empiricism" from the linguistic point of view, see Summary 11.31 in Chap. 11.

Ordinary people might have wanted to find the big name of "Genius Leibniz" in the debates of "British Empiricism vs. Continental Rationalism". Maybe they just enjoyed a play of the name as "the dawn of modern times" as an entertainment.

♠**Note 9.4.** As mentioned later (*cf.* Note 10.3, Summary 11.31), considering "language" and not "cognition", then, we say that

($\sharp_1$) mathematics is nativism

That is because mathematics is based on set theory. That is, any mathematical theorem can be derived from Zermelo=Fraenkel Axioms.
Also, recall Note 9.2, in which we say tat

($\sharp_2$) "ordinary language" is like tabula rasa

That is because "ordinary language" is not based on some axioms.

9.3* Leibniz-Clarke Correspondence: What is space-time?

This section is published in the following:

- ref. [68]: S. Ishikawa; *Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language*
  Open Journal of philosophy, Vol. 8, No.5 , 466-480, 2018, DOI: 10.4236/ojpp.2018.85032
  (`https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862`)

- ref. [69]; S. Ishikawa; *Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language*; [Revised version] ; Keio Research report; 2018; KSTS/RR-18/001, 1-15 (`https://philpapers.org/rec/ISHLCB`)
  (`http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf`)

The problems ("What is space?" and "What is time?") are the most important in modern science as well as the traditional philosophies. In this section, we give the quantum linguistic answer to these problems. As seen later, our answer is similar to Leibniz's relationalism concerning space-time. In this sense, we consider that Leibniz is one of the discoverers of the linguistic Copenhagen interpretation

9.3.1 "What is space?" and "What is time?"

9.3.1.1 Space in quantum language (How to describe "space" in quantum language)

In what follows, let us explain "space" in measurement theory (= quantum language).

For example, consider the simplest case, that is,

(A) "space"$=\mathbb{R}_q$(one dimensional space)

Since classical system and quantum system must be considered, we see

$$
\text{(B)} \quad \begin{cases} (\mathrm{B}_1)\text{: a classical particle in the one dimensional space } \mathbb{R}_q \\ \\ (\mathrm{B}_2)\text{: a quantum particle in the one dimensional space } \mathbb{R}_q \end{cases}
$$

In the classical case, we start from the following state:

$$
(q,p) = (\text{"position"}, \text{"momentum"}) \in \mathbb{R}_q \times \mathbb{R}_p
$$

Thus, we have the classical basic structure:

(C$_1$) $[C_0(\mathbb{R}_q \times \mathbb{R}_p) \subseteq L^\infty(\mathbb{R}_q \times \mathbb{R}_p) \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p)]$

Also, concerning quantum system, we have the quantum basic structure:

(C$_2$) $[\mathcal{C}(L^2(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q)]$

Summing up, we have the basic structure

$$
\text{(C)} \;\; [\mathcal{A} \subseteq \overline{\mathcal{A}} \subseteq B(H)] \begin{cases} \text{(C}_1\text{): classical } [C_0(\mathbb{R}_q \times \mathbb{R}_p) \subseteq L^\infty(\mathbb{R}_q \times \mathbb{R}_p) \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p)] \\ \\ \text{(C}_2\text{): quantum } [\mathcal{C}(L^2(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q)] \end{cases}
$$

Since we always start from a basic structure in quantum language, we consider that

$$
\begin{aligned} & \text{How to describe "space" in quantum language} \\ \Leftrightarrow \; & \text{How to describe [(A):space] by [(C):basic structure]} \end{aligned} \tag{9.1}
$$

This is done in the following steps.

Assertion 9.1. [The linguistic Copenhagen interpretation concerning "space"]
How to describe "space" in quantum language

(D$_1$) Begin with the basic structure:

$$[\mathcal{A} \subseteq \overline{\mathcal{A}} \subseteq B(H)]$$

(D$_2$) Next, consider a certain commutative C^*-algebra $\mathcal{A}_0 (= C_0(\Omega))$ such that

$$\mathcal{A}_0 \subseteq \overline{\mathcal{A}}$$

(D$_3$) Lastly, the spectrum Ω ($\approx \mathfrak{S}^p(\mathcal{A}_*)$) is used to represent "space".

Therefore, in quantum language, we see

- space is a kind of state of a "thing".

For example,

(E$_1$) in the classical case (C$_1$):

$$[C_0(\mathbb{R}_q \times \mathbb{R}_p) \subseteq L^\infty(\mathbb{R}_q \times \mathbb{R}_p) \subseteq B(L^2(\mathbb{R}_q \times \mathbb{R}_p))]$$

we have the commutative $C_0(\mathbb{R}_q)$ such that

$$C_0(\mathbb{R}_q) \subseteq L^\infty(\mathbb{R}_q \times \mathbb{R}_p)$$

And thus, we get the space $\mathbb{R}_q$ as mentioned in (A)

(E_2) in the quantum case (C_2):

$$[\mathcal{C}(L^2(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q)) \subseteq B(L^2(\mathbb{R}_q))]$$

we have the commutative $C_0(\mathbb{R}_q)$ such that

$$C_0(\mathbb{R}_q) \subseteq B(L^2(\mathbb{R}_q))$$

And thus, we get the space $\mathbb{R}_q$ as mentioned in (A)

9.3.1.2 Time in quantum language (How to describe "time" in quantum language)

In what follows, let us explain "time" in measurement theory (= quantum language). This is easily done in the following steps.

Assertion 9.2. [The linguistic Copenhagen interpretation concerning "time"]
How to describe "time" in quantum language

(F_1) Let T be a tree in Axiom 2 in Sec. 1.1. (Don't mind the finiteness or infinity of T) For each $t \in T$, consider the basic structure:

$$[\mathcal{A}_t \subseteq \overline{\mathcal{A}}_t \subseteq B(H_t)]$$

(F_2) Next, consider a certain linear subtree $T'(\subseteq T)$, which can be used to represent "time".

Therefore, in quantum language, we see

- time is an order of occurring in succession which changes one after another.

9.3.2 Leibniz-Clarke Correspondence

The above argument urges us to recall Leibniz-Clarke Correspondence (1715–1716: *cf.* [1]), which is important to know both Leibniz's and Clarke's (=Newton's) ideas concerning space and time.

(G) [The realistic space-time]
Newton's absolutism says that the space-time should be regarded as a receptacle of a "thing." Therefore, even if "thing" does not exits, the space-time exists.

On the other hand,

(H) [The metaphysical space-time]

Leibniz's relationalism says that

(H_1) Space is a kind of state of "thing".

(H_2) Time is an order of occurring in succession which changes one after another.

Therefore, I regard this correspondence as

$$\underset{\text{(realistic view)}}{\boxed{\text{Newton } (\approx \text{Clarke})}} \underset{\text{v.s.}}{\longleftrightarrow} \underset{\text{(linguistic view)}}{\boxed{\text{Leibniz}}}$$

which should be compared to

$$\underset{\text{(realistic view)}}{\boxed{\text{Einstein}}} \underset{\text{v.s.}}{\longleftrightarrow} \underset{\text{(linguistic view)}}{\boxed{\text{Bohr}}}$$

(also, recall Note 4.3).

Again, we emphasize that Leibniz's relationalism in Leibniz-Clarke correspondence is clarified in quantum language, and it should be regarded as one of the most important parts of the linguistic Copenhagen interpretation of quantum mechanics.

Many scientists may think that

> Newton's assertion is understandable, in fact, his idea was inherited by Einstein. On the other, Leibniz's assertion is incomprehensible and literary. Thus, his idea is not related to science.

However, recall the classification of the world-description (Classification 1.9):

$$\begin{cases} ① : \underset{\text{(realistic world view)}}{\text{Newton, Clarke}} \cdots \overset{\text{(space-time in physics)}}{\underset{\text{"What is space-time?"}}{\boxed{\text{realistic space-time}}}} \quad \text{(successors: Einstein, etc.)} \\ \\ ② : \underset{\text{(linguistic world view)}}{\text{Leibniz}} \cdots \overset{\text{(space-time in measurement theory)}}{\underset{\text{"How should space-time be represented?"}}{\boxed{\text{linguistic space-time}}}} \quad \text{(i.e., spectrum, tree)} \end{cases}$$

in which Newton and Leibniz respectively devotes himself to ① and ②. Although Leibniz's assertion is not clear, we believe that

- Leibniz found the importance of "linguistic space and time" in science,

Also, it should be noted that

($\sharp_1$) Newton proposed the scientific language called Newtonian mechanics,
on the other hand,
Leibniz could not propose a scientific language

After all, we conclude that

($\sharp_2$) **the cause of philosophers' failure is not to propose a language**.

Talking cynically, we say that

($\sharp_3$) Philosophers continued investigating "linguistic interpretation" (="how to use Axioms 1 and 2") without language (i.e., Axiom 1(measurement:§2.7) and Axiom 2(causality:§10.3)).

Table 1.1 : realistic worldview vs. idealistic worldview

dispute ＼ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	"Nominalismus"(Ockham)	"Realismus"(Anselmus)
ⓓ: space-time	Newton	Leibniz
ⓔ: quantum theory	Einstein	Bohr

ⓐ is my fiction, ⓒ is more of a confusion than a dispute. ⓓ is the Leibniz=Clarke correspondence (*cf.* Sec. 9.3.2), ⓔ is Bohr-Einstein debates. Quantum language is proposed as one of answers to Bohr-Einstein debates(*cf.* ref. [66]).

9.4 Subjective idealism: Berkeley, ”To be is to be perceived”

9.4.1 Priest: Berkeley

Berkeley (1685 -1753) is famous as follows.

(A_1) Berkeley is a priest, and he interpreted Locke's primary quality as the state of things that come from a supernatural power such as a god. Thus his philosophy is called subjective idealism.

Table 9.2: The key-words of worldviews (*cf.* Assertion 1.14)

mind-matter dualism	[A](= mind)	[B](between A and B)	[C](= matter)
Plato	actual world	Idea	/ [idea world]
Descartes	I, mind, brain	body	/ [matter]
Locke	mind	secondary quality	primary quality [matter]
Berkeley	mind	secondary quality	/ [God]
quantum language	**measured value**	**observable**	**state [system]**

Thus I don't think that

$$\boxed{\text{Locke}} \xrightarrow[\text{progress}]{} \boxed{\text{Berkeley}}$$

though there may be people who want to believe that.

However, he is an important figure in the following sense.

(A_2) Berkeley indicated that the mathematical definition of $\lim \frac{0}{0}$ is not complete

(A_3) He said ”To be is to be perceived”, which represented the essential spirit of dualism. Also, ”If a tree falls in a forest and no one is around to hear it, does it make a sound?” is said to be due to Berkeley.

Summing up, Berkeley was always the standpoint of anti-Newton (= anti-realism).

- If we think that modern philosophy (from Descartes to Kant) has significance as a buffer zone of Christianity with Newtonian mechanics, we can conclude that Berkeley is honest.

♠**Note 9.5.** The mathematical definition of $\lim \frac{0}{0}$ (i.e., (ϵ, δ)-definition of limit) was more important than Newton thought, and it was discovered one hundred and tens of years later (by Cauchy (1789-1857), Weierstrass (1815-1897), etc.). When I think from now on, there was a possibility that Leibniz discovered it, but he was busy in the other things.

9.4.2 (A_3): To be is to be perceived

Consider the following saying:

(B_1) There is no science without measurement
($\approx$ [To be is to be perceived])

Everyone may believe that this saying (B_1) is absolutely true. In fact, the importance of "measurement" is emphasized as follows (*cf.* Sec. 1.1).

$$\underset{\text{measurement theory}}{\boxed{\text{(=quantum language)}}} = \overset{\text{[Axiom 1]}}{\boxed{\text{measurement}}} + \overset{\text{[Axiom 2]}}{\boxed{\text{causal relation}}} + \underset{\text{[the manual to use Axioms 1 and 2]}}{\boxed{\text{linguistic Copenhagen interpretation}}} \tag{9.2}$$

But, it is Genius Newton (and Einstein) that neglect this absolute truth (H_1). In fact, Newtonian mechanics is formulated as follows.

$$\boxed{\text{Newtonian mechanics}} = \boxed{\text{No measurement}} + \overset{\text{[Newtonian kinetic equation]}}{\boxed{\text{causal relation}}} \tag{9.3}$$

Here, note that Newton removed "measurement" from (9.2) in spite of the maxim that there is no science without measurement.

The insightfulness of Newton is surprising. A genius isn't confused by "the absolute maxim (B_1)".

The following is my fiction:

(B_2) "Exclusion of measurement" is the conclusion reached by the deep consideration of Newton. However, Berkeley, an excellent controversialist of anti-Newton, considered that the exclusion was a weak point of Newtonian mechanics. And he said

To be is to be perceived

This is the golden rule of anti-Newtonianism (i.e., anti-physicalism, idealism). The opposing structure: [Newton vs. Berkeley] continues to [Einstein vs. Bohr] as mentioned in next section.

9.4.3 "Einstein-Tagore Meeting" and "Bohr-Einstein debates"

Concerning "realistic worldview vs. idealistic worldview", Einstein-Tagore (poet, thinker in India) meeting in 1930 is famous, in which they asserted as

- Tagore: "Truth is always limited by human perception."
- Einstein: "Truth is independent of our consciousness, For instance, if nobody is in this house, yet that table remains where it is[1]."

In the above, Tagore's assertion is similar to Berkeley's "To be is to be perceived", which belongs to the situation of dualistic idealism(=idealistic worldview).

On the other hand, Einstein's saying:

(C_1) if nobody is in this house, yet that table remains where it is (= Does the moon disappear when I'm not looking at it?)

is the same as

(C_2) Truth is independent of us (= realistic worldview)

Thus, Einstein and Newton are similar, in the sense that

Truth is independent of human being (i.e., physics holds without measurement)

Therefore, it should be noted that (9.3) is significant.

In this paper, we are not concerned with Bohr-Einstein debates in quantum mechanics (in order to solve this problem, I proposed quantum language), (*cf.* ref. [66])). However, Bohr-Einstein debates is similar to the above. Thus, summing up, we see:

realistic worldview vs. idealistic worldview (*cf.* Table 1.1 in Assertion 1.12)

Realistic worldview [monism, realism, no measurement]	Idealistic worldview [dualism, idealism, measurement]
Newton	Berkeley
Newton (and Clarke)	Leibniz
Einstein	Tagore
Einstein	Bohr

Now, concerning Bohr-Einstein debates, The impression that Einstein lost now has been left, but the author does not think so (*cf.* ref. [66]).

[1]Einstein often said this kind of statement at various places, for example, "Does the moon disappear when I'm not looking at it?"

♠**Note 9.6.** Omitting "Newton vs. Berkeley" and "Einstein vs. Tagore" in the above table, I repeatedly mention the following table (*cf.* Assertion 1.12):

Table 1.1 : realistic worldview vs. idealistic worldview

dispute ╲ [R] vs. [L]	Realistic worldview (monism, realism, no measurement)	Idealistic worldview (dualism, idealism, measurement)
ⓐ: motion	Hērakleitos	Parmenides
ⓑ:Ancient Greece	Aristotle	Plato
ⓒ: Problem of universals	"Nominalismus"(Ockham)	"Realismus"(Anselmus)
ⓓ: space-time	Newton	Leibniz
ⓔ: quantum theory	Einstein	Bohr

ⓐ is my fiction, ⓒ is more of a confusion than a dispute. ⓓ is the Leibniz=Clarke correspondence (*cf.* Sec. 9.3.2), ⓔ is Bohr-Einstein debates. Quantum language is proposed as one of answers to Bohr-Einstein debates(*cf.* ref. [66]).

♠**Note 9.7.** In Japan, I learned the dualistic proposition: "If a tree falls in a forest and no one is around to hear it, does it make a sound?" in a Zen dialogue (i.e., a question-and-answer exchange between Zen priests and their followers). Zen is one school of Buddhism. In modern Japan, most people may think that Zen monologue is a kind of wordplay.

9.4.4 Bohr-Einstein debates: Do the laws of physics require measurement?

For Bohr-Einstein debates, I discussed in ref. [66]. However, I would like to write something in this section. Almost people agree to the following maxim:

- There is no science without measurement

However, genius Newton neglected this maxim, and proposed Newtonian mechanics as follow: Newtonian mechanics and quantum mechanics are formulated as follows:

$$(\sharp_1)\quad \boxed{\text{Newtoinan mechanics}} = \text{Nothing} + \underset{\text{(Newtonian equation)}}{\boxed{\text{Causality}}}$$

On the other hand, quantum mechanics is formulated as follows.

$$(\sharp_2)\quad \boxed{\text{quantum mechanics}} = \underset{\text{(Born's quantum measurement)}}{\boxed{\text{Measurement}}} + \underset{\text{(Heisenberg (and Schrödinger) equation)}}{\boxed{\text{Causality}}}$$

N. Bohr, the leader of the Copenhagen school, agreed to the $(\sharp_2)$, on the other hand, A. Einstein asserted that measurement is not needed for physics since he believed in

($\sharp_3$) *The moon is there whether one looks at it or not.*

in Einstein and Tagore's conversation.

So far, many experimental results support Bohr. However, if Einstein says the following ($\sharp_4$), everyone has no choice but to shut up.

($\sharp_4$) Then, did the laws of physics not work before the birth of humankind?

Thus, I think that Bohr-Einstein debates is not settled yet.

9.5 Qualia problem and Brain in a vat argument

This section was written with reference to the following.

- [68]:Ishikawa, S: Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language Open Journal of philosophy, Vol. 8, No.5 , 466-480, 2018, DOI: 10.4236/ojpp.2018.85032 (https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862)

  [Revised version] (https://philpapers.org/rec/ISHLCB) (http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf)

9.5.1* The problem of qualia

Jack and Betty were looking at the pink flowers. Betty had the following question (i.e., qualia problem) .

($\sharp$) Is the "pink" that I felt the same as the "pink" that Jack felt?

This question can be answered in the following way.
[(A): Scientific answer (To measure is to believe)]:
With Jack and Betty as test subjects, you (i.e., scientist) can perform various tests (colorblindness test, EEG measurement, electroretinogram, etc.). And if no difference is found in any of the tests, we can conclude that each "pink" felt by Jack and Betty is the same.
[(B): Non-scientific argument (To think is not to measure)]:
However, Betty may say:

($\flat_1$) I like the "pink" so much that I has pink walls in my room too. Jack loves blue and always wears blue. Therefore, I find "pink" to be more beautiful than Jack.

However, the scientist may say that

($\flat_2$) According to the linguistic Copenhagen interpretation (*cf.* (E_1) in Sec. 1.1.2), you (= Betty) are not qualified to be an observer. Even if you are qualified to be an observer, you should perform the same tests as in [(A):Scientific answer] for Jack and yourself as test subjects, that is, various tests (colorblindness test, EEG measurement, electroretinogram, etc.). However, if so, it is the same as the situation [(A):Scientific answer].

Then, Betty says:

($\flat_3$) OK. I (=Betty) can understand. In short, the measurements I can only make against myself are not scientific measurements. Or, a measurement that only I can make is not a measurement. Is it OK?

Then, the scientist says to Betty:

($\flat_4$) That's right.

□

♠**Note 9.8.** In the book "The astonishing hypothesis" (by F. Click (the most noted for being a co-discoverer of the structure of the DNA molecule in 1953 with James Watson)), Dr. Click said that

($\sharp_1$) *You, your joys and your sorrows, your memories and your ambitions, your sense of personal identity and free will, are in fact no more than the behavior of a vast assembly of nerve cells and their associated molecules.*

That is, he believed the monistic realism such as

($\sharp_2$) the movement of the human spirit is also a kind of physical phenomenon.

I agree to his opinion. And I believe that with the development of brain science, even consciousness can be measured in the future. However, no matter how much brain science develops, I believe that there is no solution to the qualia problem other than the one described above.

9.5.2* Brain in a vat argument

Let us introduce a famous problem "Brain in a vat" due to H. Putnam (*cf.* ref. [92]) as follows:

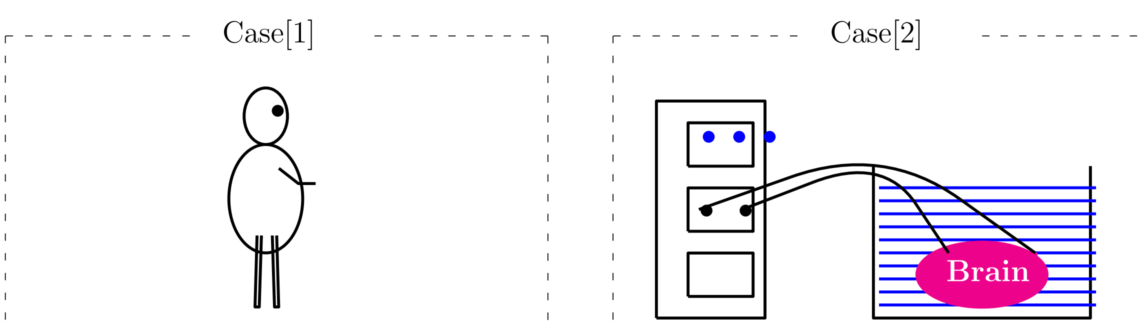


Figure 8.1: [Brain in a vat]

There is a possibility of the following.

(C_1) a mad scientist has removed your brain, and placed it into a vat of liquid to keep it alive and active. The scientist has also connected your brain to a powerful computer, which sends neurological signals to the brain in the way the brain normally receives them. Thus, the computer is able to send your the data to you brain to fool you into believing that you are still walking around a forest.

Then, you may say;

(C_2) "Am I a brain in a vat?" Or, "Can I check whether I am a brain vat or not?"

And you think:

(C_3) "I cannot decide if I am a brain vat or not".
That is, "I cannot decide if I am in Case[1] or Case[2]?

Therefore, since (C_3) is true, you may say:

(C_4) "I cannot know if I have a limb or not."

Then, we have the following problem:

Problem 9.3. [The problem concerning "Brain in a vat"]

- Is the (C_4) true?

[Answer]:
The (C_3) clearly is true since (C_1) is assumed. However, (C_4) is not true. That is because

($\sharp_1$) if you are in Case[1], you find that you have a limb.

($\sharp_2$) if you are in Case[2], you also find that you have a limb (under the assumption (C_1)).

Thus, (H_4) is wrong.

[Alternative explanation]:
For completeness, let's rephrase the same thing as follows. You ask someone "Do I have a limb?" If they reply, "Of course you have," you can be sure that you have a limb. In short,

- you only have to believe in the measurement results.

Or,

- To be is to be perceived.

This is just the linguistic Copenhagen interpretation (*cf.* Sec. 9.4.3: Einstein-Tagore meeting). □

♠**Note 9.9.** Recall the worldviewism (in Sec 1.3.1), which says

($\sharp$) Without the principle (i.e., the worldview), we can't say anything

Under the worldview called the quantum mechanical worldview (i.e., quantum language), we have seen the followings.

- What is probability (or, measurement, causality) ? *cf.* Sec. 1.1.1)
- Zeno paradox (Flying arrow), (*cf.* Sec. 2.4.2)
- Zeno paradox (Achilles and a tortoise), (*cf.* Sec. 2.4.3)
- the measurement theoretical understanding of Plato's allegory of the sum , (*cf.* Sec. 3.3.2)
- Plato's Idea theory≈Zadeh's fuzzy theory≈Sausuure's linguistic theory (*cf.* Sec. 3.5.3)
- Syllogism holds in classical systems, but not in quantum systems (*cf.* Sec. 4.3.2)
- Only the present exists (*cf.* Sec. 6.1.2)
- What is the problem of universals? (*cf.* Sec. 6.5.1)
- What is Geocentrism vs. Heliocentrism? After all, the worldviewism (*cf.* Sec. 7.4.2)
- Two (scientific or non-scientific) interpretations of I think, therefore I am.(*cf.* Sec. 8.2.2)
- The problem of qualia (*cf.* Sec. 9.5.1)
- Brain in a vat argument (*cf.* Sec. 9.5.2)

9.6 Hume; skeptic who didn't measure, "A Treatise of Human Nature"

9.6.1 The review of Descartes

Let us review Descartes philosophy.

(A_1) Descartes found the indisputable truth, i.e., cogito proposition "I think, therefore I am". Therefore, everything derived from cogito proposition can be trusted. That is, he started from "the existence of I".

The purity of Descartes is mind-blowing, but in the first place "I think, therefore I am" and "the existence of I" is suspicious (*cf.* Note 1.7, or Proposition 8.2 in Sec.8.2). Hence, the following is also suspicious:

(A_2) "The existence of I" is certain. **Therefore**, the matters that I perceive exist. And further, Descartes introduced "body (= sensor organ)" which mediates between "I" and "matter". After all, he reached and discussed "mind-matter dualism" (= Descartes problem 8.4), that is, "the problem of mind-matter dualism" and "mind-body problem".

Although Descartes problem 8.4 is, from the scientific point of view, a barren discussion, Descartes philosophy was supported a lot of people. Since the philosophy of worldview is a kind of fashion or "model-change", to be supported by many people is the most important.

(A_3) If Descartes and Locke asserted that

- there is a possibility that mind-matter dualism (with keywords "matter", "I (= mind, brain)", "body(=secondary quality)", "matter") succeeds.

then, I think that they are, from the quantum linguistic point of view, true.

If so, we may affirmatively answer Problem 8.5, i.e.,

$$\text{Can the direction:"Descartes} \xrightarrow[\text{model-change}]{} \text{Locke" be regarded as progress?}$$

That is, we may assert that

$$\text{Plato} \xrightarrow[\text{model-change}]{\text{progress}} \text{Descartes} \xrightarrow[\text{model-change}]{\text{progress}} \text{Locke}$$

9.6.2 Hume's straying[Less than brain science]; Hume's wordplay

Descartes philosophy is a philosophy which has the risk of entering the science. In fact, Hume approached the zone of science.

In "A Treatise of Human Nature" (1739), Hume pointed out the leap in logic of **"Therefore"** in the above (A_2). As Hume says, it is sure that "the existence of matter" cannot be derived from "the existence of I"[2]. Also, it is not guaranteed that "matter I perceive" is equal to "true matter". Thus, the existence of "matter" is doubtful. However, it is sure that I feel so. Hume states that

(B) "a bundle of perceptions" (= brain circuit) exists

That is,

(C) "matter" and "causal relation" are a kind of bundle of perceptions

I think that the above "$(A_1) \to (K) \to (L)$" is self-referential in the wide sense. That is,

- "$(A_1) \to (K) \to (L)$" is a kind of psychological illusion as having been able to understand all events by the word "bundle of perceptions".

To enjoy such convinced form may also be a pleasure of philosophy. Thus,

(D) It is said Hume's philosophy is the goal of British Empiricism

Hume took the faultfinding of Descartes, and Hume has entered into wrong direction "brain science". The research of "the bundle of perceptions" belongs to brain science.

(E) If Hume was a scientist, he was too early for 300 years

The cause of victory of Galileo was a "telescope". Hume studied "brain science" without measuring instrument in spite that Hume thought that he himself is a philosopher and not scientist. Hence, I think that

$$\boxed{\text{Descartes}} \xrightarrow[\text{progress}]{} \boxed{\text{Locke}} \xrightarrow{\text{regress}} \boxed{\text{Hume}}$$

However, Hume was revived by Kant (*cf.* next Chapter 10: Kant: Copernican revolution). Then, I want to say that

$$\boxed{\text{Descartes}} \xrightarrow[\text{progress}]{} \boxed{\text{Locke}} \xrightarrow[\text{progress}]{} \boxed{\text{Hume}} \xrightarrow[\text{progress}]{} \boxed{\text{Kant}}$$

[2]This kind of logic is a typical self-reference (*cf.* Note 1.7, or Proposition 8.2 in Sec.8.2). Thus, Hume's logic (or generally, philosophical logic) in ordinary language cannot be trusted. That is, it is only a wordplay.

9.6.3 Hume; The causality problem

It is a matter of course that the representation of "causal relation" is the most important theme in worldview. In Newtonian mechanics, the causality is represented by Newtonian kinetic equation. In Descartes=Kant philosophy, the representation of "causal relation" is as follows.

($\sharp_1$) **[Cognitive causality]:** David Hume, Immanuel Kant, etc. thought as follows. :

> We cannot say that "Causality" actually exists in the world, or that it does not exist in the world. And when we think that "something" in the world is "causality", we should just believe that the it has "causality".

Most readers may regard this as "a kind of rhetoric", however, several readers may be convinced in "Now that you say that, it may be so." Surely, since you are looking through the prejudice "causality", you may look such. This is Kant's famous "Copernican revolution" (i.e., "Kant was awakened from his dogmatic slumber by Hume's idea and came up with the Copernican revolution", this will be discussed in Sec. 10.3 [What is causality?]), that is,

"recognition constitutes the world."

which is considered that the recognition circuit of causality is installed in the brain, and when it is stimulated by "something" and reacts, "there is causal relationship." Probably, many readers doubt about the substantial influence which this ($\sharp$) had on the science after it. However, in this book, I adopted the friendly story to the utmost to Kant.

This will be discussed again in the next chapter (*cf.* Section 10.3 What is causality?).

♠**Note 9.10.** In the book "The astonishing hypothesis" (by F. Click (the most noted for being a co-discoverer of the structure of the DNA molecule in 1953 with James Watson)), Dr. Click said that

($\sharp_1$) *You, your joys and your sorrows, your memories and your ambitions, your sense of personal identity and free will, are in fact no more than the behavior of a vast assembly of nerve cells and their associated molecules.*

That is, he believed the monistic realism such as

($\sharp_2$) the movement of the human spirit is also a kind of physical phenomenon.

Also, since the title of his book is "The astonishing hypothesis", Click must have felt that

($\sharp_3$) Descartes philosophy is based on dualistic realism.

However, the definition of "realism" is different from our definition (Definition 1.8), where we consider that realism is a worldview based on experimental verification. It is clear that Descartes, Locke, Hume were not concerned with about experimental validation. That is, they are not scientists but philosophers. Therefore, it may be "astonishing" by common sense, but it's not "astonishing" by our definition. In Chapter 11, we will show that the monistic realism and the dualistic idealism (i.e., quantum language) are compatible.

9.7 Hume's problem of induction

♠**Note 9.11.** Philosophy has the habit of enjoying problems that are impossible to solve or beyond the frame of science. For example, some may be interested in the justifications of the followings:

($\sharp_1$) Newtonian mechanics has been right to this day. So, will the Newtonian mechanics be right tomorrow?

($\sharp_2$) Quantum language have been very useful to this day. So, will quantum language be useful tomorrow?

The justification problem of these may be called Hume's problem of induction, though David Hume suspected the justification of induction. That is, Hume's problem of induction is out of worldviewism. That is because, even if a great worldview is proposed, everybody can say :

($\sharp_3$) Will the worldview be right tomorrow?

Therefore, thinking about these issues is a time-sucking pleasure, and while it may not be advisable, what we should do is finding a problem that is similar to the above problems and can be solved in the framework of quantum language.

9.7.1* The solution of Hume's problem of induction

This section was written with reference to the following.

- [70]:Ishikawa, S: Philosophy of science for scientists; The probabilistic interpretation of science Journal of quantum information science, Vol. 9, No.3 , 140-154, DOI: 10.4236/jqis.2019.93007 (https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

For example, consider the following inferences (= inductive inferences):

(A$_1$) Until now, the sun has always risen in the east. So, tomorrow the sun will rise in the east again.

(A$_2$) When a coin is thrown ten times, we get (H,H,T,H,H,H,H,H,H,T) (where H="head", T="tail"). Thus, we infer that "H" will be obtained with probability 8/10 by the next coin-tossing

Here, our present problem is

(B) Can such induction (= inductive inference) as above be justified?

Recall that our spirit in this paper is "No scientific arguments without worldview", or, "true justification" = "justification under a certain worldview".

♠**Note 9.12.** In Note 7.1, we say that Bernoulli's achievement (i.e., the discovery of the law of large numbers) equals Galileo's achievement. That is,

$$\begin{cases} \text{Scientific pioneer in the realistic worldview} \cdots \text{Galileo} \\ \\ \text{Scientific pioneer in the idealistic worldview} \cdots \text{J. Bernoulli} \end{cases}$$

The reason that we consider so is that Galileo=Newton mechanics and Bernoulli's law of large numbers have the power to predict the future. Further we think that there are essentially only two of these theories that have the power to predict the future. Therefore, we are convinced that Hume's problem of induction and Bernoulli's law of large numbers are closely related as mentioned below.

In this section we show that the justification is easily solved in our quantum mechanical worldview. If we expect a scientific answer to Hume's problem, we must start with the scientific definition of "the uniformity principle of nature", i.e., the following Definition 9.4 [The uniformity principle of nature]. Some may feel that the uniformity principle of nature (i.e., the condition in Definition 9.4) is quite different from what Hume thought. However, we think that it is impossible to propose the different quantitative definition of the uniformity principle of nature that leads to a result like Theorem 9.6 [Inductive reasoning] (i.e., *If similar measurements are performed, the similar measured values are obtained*).

Definition 9.4. [The uniformity principle of nature] For simplicity, consider a classical basic structure $[C(\Omega) \subseteq L^\infty(\Omega,\nu) \subseteq B(L^2(\Omega,\nu))]$ such that Ω is compact and $\nu(\Omega) = 1$. A family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ is said to satisfy the *uniformity principle of nature* (*concerning* μ), if there exists a probability space $(X, \mathcal{F}, \mu)$ such that

$$[F_i(\Xi)](\omega_i) = \mu(\Xi) \qquad \forall \Xi \in \mathcal{F}, \forall i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$$

Remark 9.5. [No scientific arguments without worldview] The uniformity principle of nature is not a principle (= axiom) of a worldview, and thus it should be defined as an assumption under a certain world description. The argument without world description leads contraction as follows.

(C_1) It worked well, assuming the uniformity principle of nature up to now.

(C_2) So it will continue to work.

(C_3) Thus, the uniformity principle of nature can be justified.

This is not true, since it is a cyclic argument. That is, the above is a wordplay.

Under the above definition, we assert the following theorem (essentially the same as the law of large numbers), which should be regarded as the fundamental theorem in philosophy of science.

Theorem 9.6. [Inductive reasoning, the quantum linguistic solution of Hume's problem of induction]. Let $[C(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu))]$ be a basic structure such that Ω is compact and $\nu(\Omega) = 1$. Assume that a family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ satisfies the uniformity principle of nature (concerning μ). Let $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., x_N) \in \times_{i=-n}^N X$ be a measured value by the parallel measurement $\bigotimes_{i=-n}^N \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}_i := (X, \mathcal{F}, F_i),\ S_{[\omega_i]})$. Then, we see that

$$
\begin{aligned}
&\frac{\sharp\{k \mid x_k \in \Xi, k = -n, -n+1, ..., -1, 0\}}{n} \approx \mu(\Xi)(= [F_i(\Xi)](\omega_i)) \\
&\big(\Xi \in \mathcal{F}, i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\big)
\end{aligned}
\tag{9.4}
$$

where n is sufficiently large. Here $\sharp[\Theta]$ is the number of elements in a set Θ.

Proof. Let $\Xi_i \in \mathcal{F}$ $(i = -n, -n+1, ..., -1, 0, 1, ..., N)$. Axiom 1 [measurement] (in Section 1.1) says that the probability that a measured value $(x_{-n}, x_{-n+1}, ...,\ x_{-1}, x_0, x_1, ..., x_N)$ obtained by the parallel measurement $\bigotimes_{i=-n}^N \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O}_i := (X, \mathcal{F}, F_i),\ S_{[\omega_i]})$ belongs to $\times_{i=-n}^N \Xi_i$ is given by $\times_{i=-n}^N [F_i(\Xi_i)](\omega_i) = \times_{i=-n}^N \mu(\Xi_i)$. Thus, the sequence $\{x_i\}_{i=-n}^N$ can be regarded as independent random variables with the identical distribution μ. Hence, using the law of large numbers, we can immediately get the formula (9.4). Also, this theorem is a direct consequence of the law of large numbers for parallel measurements (*cf.* refs. [38], or § 4.2 in ref.[66]). □

Remark 9.7. (i): Recall that the law of large numbers (which is almost equivalent to Theorem 9.6) says that

"frequency probability" = "the probability in Axiom 1 [measurement] (in Section 1.1)"

(*cf.* ref. [38]), though the probability in Axiom 1 [measurement] (in Section 1.1) has the several aspects. Also, note that the law of large numbers in statistics (*cf.* ref. [76]) has already been accepted as the fundamental theorem in science. Therefore, even if Theorem 9.6 ([Inductive reasoning]+(9.4)) is called the fundamental theorem in philosophy of science, we don't think it's exaggerated. We believe that our proposal (i.e., Theorem 9.6) is completely true in our worldview. Thus, we think that the solution of Hume's problem of induction was practically already found as the law of large numbers. In the framework of our worldview, we are convinced

that the above is the definitive solution to Hume's problem. However, there may be another idea if some start from another worldview. Hence, as described at the end of this paper, we hope that many philosophers propose various mathematical foundations of scientific philosophy, in which Hume's problem of induction are discussed from the various viewpoints.
(ii): In Definition 9.4 [The uniformity principle of nature] and Theorem 9.6 [Inductive reasoning], we consider the family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$. This may be too general. Usually, it suffices to consider that $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, \mathcal{F}, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$, i.e., $F = F_i$ $(-n \le \forall i \le N)$.
(iii): It may be understandable to consider two measurements: $\bigotimes_{i=-n}^{0} \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, \mathcal{F}, F_i)$, $S_{[\omega_i]})$ and $\bigotimes_{i=1}^{N} \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, \mathcal{F}, F_i)$, $S_{[\omega_i]})$. The reason that we do not consider two measurements is due to the linguistic Copenhagen interpretation (G_1), i.e., *only one measurement is permitted.*

Example 9.8. [Coin tossing]. Let us discuss the unfair coin tossing as the most understandable example of Theorem 9.6 [Inductive reasoning]. Consider a basic structure $[C(\Omega) \subseteq L^\infty(\Omega, \nu) \subseteq B(L^2(\Omega, \nu))]$. Let $\{\omega_i\}_{i=-n}^{N}$ be a sequence in Ω, where ω_i is the state of i-th coin tossing $(i = -n, -n+1, ..., 0, 1, 2, 3, ..., N)$. Let $\mathsf{O} = (X, 2^X, F)$ be an observable in $L^\infty(\Omega, \nu)$ such that

$$\begin{aligned} & X = \{H, T\}, (\text{where } H\text{: head, } U\text{: tail}) \ , \\ & [F(\{H\})](\omega_i) = \mu(\{H\}) = 2/3, \quad [F(\{T\})](\omega_i) = \mu(\{T\}) = 1/3 \\ & \qquad (\forall i = -n, -n+1, ..., -1, 0, 1, 2, ..., N) \end{aligned} \tag{9.5}$$

That is, a family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O} := (X, 2^X, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ satisfies the uniformity principle of nature (concerning μ). Let $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., N) \in \times_{i=-n}^{N} X$ be a measured value obtained by the parallel measurement $\bigotimes_{i=-n}^{N} \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, 2^X, F), S_{[\omega_i]})$, i.e., infinite coin throws. Here, Theorem 9.6 [Inductive reasoning] say that it is natural to assume that, for sufficiently large n,

$$\begin{aligned} & (x_{-n}, x_{-n+1}, ..., x_{-1}, x_0) = (\underbrace{\text{T H H T H H H T T ..... T H H}}_{n+1}) \\ & (\text{where the number of } H\text{s} \approx 2n/3,\ T\text{s} \approx n/3) \end{aligned} \tag{9.6}$$

Then we can believe that we see that $x_i = H$ with probability 2/3 [resp. $x_i = T$ with probability 1/3] for each $i = 1, 2, ..., N$. It should be noted that even without knowing (9.5), we can conclude that if we know (9.6).

Remark 9.9. It should be noted that the above example shows that Theorem 9.6 [Inductive reasoning] (or equivalently, the law of large numbers), like Newton's kinetic equation, has the power to predict the future. This is the reason that Hume's problem of induction keeps attracting much researcher's interest for a long time. If the justification of Hume's problem of induction is solved, it should be the most fundamental theory in science. Thus, we are convinced that our assertion (i.e., the law of large numbers $\approx$ the justification of Hume's problem of induction) is true.

9.7.2* Grue paradox cannot be represented in quantum language

This section was written with reference to the following.

- [70]:Ishikawa, S: Philosophy of science for scientists; The probabilistic interpretation of science Journal of quantum information science, Vol. 9, No.3 , 140-154,
DOI: 10.4236/jqis.2019.93007
(https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

If our understanding of inductive reasoning (mentioned in the above) is true, we can solve the grue paradox (*cf.* ref. [21]). Let us mention it as follows.
Consider a basic structure $[C(\Omega) \subseteq L^\infty(\Omega,\nu) \subseteq B(L^2(\Omega,\nu))]$. Let Ω_g, Ω_b be the open subsets of the state space Ω such that $\Omega_g \cap \Omega_b = \emptyset$. And put $\Omega_o = \Omega \setminus (\Omega_g \cup \Omega_b)$. Let $\mathsf{O} = (X \equiv \{g, b, o\}, 2^X, F)$ be the observable in $L^\infty(\Omega,\nu)$ such that

$$[F(\{g\})](\omega) = 1 \ \ (\omega \in \Omega_g), \quad = 0 \ \ (\omega \in \Omega \setminus \Omega_g) \quad [F(\{b\})](\omega) = 1 \ \ (\omega \in \Omega_b), \quad = 0 \ \ (\omega \in \Omega \setminus \Omega_b)$$
$$[F(\{o\})](\omega) = 1 - [F(\{g\})](\omega) - [F(\{b\})](\omega) \ \ (\omega \in \Omega) \tag{9.7}$$

where "g", "b", "o" respectively means "green", "blue", "others".
Let $\{e_{-n}, e_{-n+1}, ..., e_{-1}, e_0, e_1, e_2, ..., e_N\}$ be the set of (green) emeralds. And assume that $\omega_i (\in \Omega_g^\circ)$ is the state of emerald e_i $(i = -n, -n+1, ..., -1, 0, 1, 2, ..., N)$.
A family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, 2^X, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ clearly satisfies the uniformity principle of nature, that is, there exists an probability space $(X, 2^X, \mu)$ such that

$$[F(\Xi)](\omega_i) = \mu(\Xi) \quad \forall \Xi \in 2^X, \forall i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$$

where $\mu(\{g\}) = 1$, $\mu(\{b, o\}) = 0$.
Let $(x_{-n}, x_{-n+1}, ..., x_{-1}, x_0, x_1, ..., x_N) \in \times_{i=-n}^N X$ be a measured value obtained by the parallel measurement $\bigotimes_{i=-n}^N \mathsf{M}_{L^\infty(\Omega,\nu)}$ $(\mathsf{O} := (X, 2^X, F), S_{[\omega_i]})$. We see, of course, that $x_i = g$ $(i =$

$-n, -n+1, .., -1, 0$). And thus, we can believe, by Theorem 9.6 [Inductive reasoning], that $x_1 = x_2 = ... = x_N = g$. For the sake of completeness, note that we can predict $x_1 = x_2 = ... = x_N = g$ only by the data $x_{-n} = x_{-n+1} = ... = x_0 = g$. This is usual arguments concerning Theorem 9.6 [Inductive reasoning].

On the other hand, Goodman's grue paradox is as follows (*cf.* ref. [21]).

(D$_1$) Define that Y has a grue property iff Y is green at time i such that $i \le 0$ and Y is blue at time i such that $0 < i$. Suppose that we have examined the emeralds at $-n, -n+1, ... -1, 0$, and found them to all be green (and hence also grue). Then, "so-called inductive reasoning" says that emeralds at $1, 2, ..., N$ have the grue property (and hence blue) as well as green. Thus, a contradiction is gotten.

However, we think that this (D$_1$) cannot be described in quantum language. If we try to describe the (D$_1$), we may consider as follows.

(D$_2$) Let $\{e_{-n}, e_{-n+1}, ..., e_{-1}, e_0, e_1, e_2, ..., e_N\}$ be the set of emeralds. Let $\omega_i (\in \Omega_g^\circ)$ be the state of emerald e_i ($i = -n, -n+1, ..., -1, 0$), and let $\omega_i (\in \Omega_b^\circ)$ be the state of emerald e_i ($i = 1, 2, ..., N$). However, it should be noted that a family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, 2^X, F), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ does not satisfy the uniformity principle of nature. That is because

$$[F(\{g\})](\omega_i) = 1 \ \ (i = -n, -n+1, ..., 0), \qquad [F(\{g\})](\omega_i) = 0 \ \ (i = 1, 2, ..., N)$$

Hence Theorem 9.6 [Inductive reasoning] cannot be applied.

Or,

(D$_3$) Let $\{e_{-n}, e_{-n+1}, ..., e_{-1}, e_0, e_1, e_2, ..., e_N\}$ be the set of emeralds. And let $\omega_i (\in \Omega_g^\circ)$ is the state of emerald e_i such that $\omega = \omega_i$ ($i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$). Let $\mathsf{O}_i = (X, 2^X, F_i)$ be the observable ($i = -n, -n+1, ..., -1, 0, 1, 2, ..., N$) such that O_i is the same as $\mathsf{O}(= (X \equiv \{g, b, o\}, 2^X, F))$ in (9.7) (if $i = -n, -n+1, ..., -1, 0$), and $\mathsf{O}_i = (X, 2^X, F_i)$ (if $0, 1, 2, ..., N$) is defined by $F_i(\{g\}) = F(\{b\})$, $F_i(\{b\}) = F(\{g\})$, $F_i(\{o\}) = F(\{o\})$. However, in this case, it should be noted that a family of measurements $\{\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}_i := (X, 2^X, F_i), S_{[\omega_i]}) \mid i = -n, -n+1, ..., -1, 0, 1, 2, ..., N\}$ does not satisfy the uniformity principle of nature. That is because

$$[F_i(\{g\})](\omega_i) = [F(\{g\})](\omega_i) = 1 \qquad (i = -n, -n+1, ..., 0),$$

$$[F_i(\{g\})](\omega_i) = [F(\{b\})](\omega_i) = 0 \qquad (i = 1, 2, ..., N)$$

Hence Theorem 9.6 [Inductive reasoning] cannot be applied.

Therefore Goodman's grue paradox (D_1) cannot be described in quantum language.

Remark 9.10. We believe that there is no scientific argument without scientific worldview. Thus, we can immediately conclude that Goodman's discussion (D_1) is doubtful since his argument is not based on any scientific worldview. In this sense, the above arguments (D_2) and (D_3) may not be needed. That is, the confusion of grue paradox is due to lack of the understanding of Hume's problem of induction in the linguistic quantum mechanical worldview, and not lack of the term "grue" is non-projectible (*cf.* ref. [21]). Thus, we think that to solve Goodman's grue paradox is to answer the following:

(E) Propose a worldview! And further formulate Hume's induction as the fundamental theorem in the worldview! In this formulation, confirm that Goodman's paradox is eliminated naturally.

What I did is this.

Remark 9.11. Readers think that the Grue paradox is too unnatural. I agree. That is, the Grue paradox is like playing football and then before you know it, you're playing rugby. If the rules change in the middle of a game, there is no sport. I think that this is due to that N. Goodman (1906-1998) belonged to the school of analytic philosophy. Analytic philosophy does not emphasize much of the worldviewism (it cf. Sec. 1.3.1) such that

$$(\sharp_3) \qquad \underset{\text{premise}}{\overset{\text{world is so}}{\boxed{\text{worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{subject}}{\overset{\text{conclusion}}{\boxed{\text{discussions, calculation (= practical logic), properties}}}}$$

Thus, the Grue paradox is a paradox resulting from the failure to follow world descriptivism. I think that the worldviewism should be emphasized even in analytic philosophy. Thus, I think that the Grue paradox is unproductive for us, if the weaknesses of analytic philosophy are not pointed out. Also, see Wittgenstein's paradox in Sec. 11.6.3.

♠**Note 9.13.** Here, we have (*cf.* Classification 1.11 [the classification of philosophers]).

$$
(\flat) \begin{cases}
(\flat_1)\text{: the realistic worldview (physics)} \\
\quad \text{Hērakleitos, Aristotle, Aristarchus, Archimedes, Eratosthenes, Ptolemaeus,} \\
\quad \text{Galileo, Newton, Einstein, } \cdots \\
\quad \text{(Although mathematics is not a worldview, Pythagoras, Eudoxus, Euclid)} \\
\\
(\flat_{21})\text{: the fictional worldview (Western philosophy)} \\
\quad \text{Plato, Scholasticism, Descartes, Locke, Leibniz, Berkeley, Hume, Kant, Husserl} \\
\\
(\flat_{22})\text{: the logical worldview} \\
\quad \text{Boole, Frege, Peirce, Saussure, Russell, Wittgenstein, Hempel, Popper} \\
\\
(\flat_{23})\text{: the mechanical worldview (statistics, quantum language)} \\
\quad \text{Parmenides, Zeno, J. Bernoulli, statistics (e.g., Fisher), quantum language}
\end{cases}
$$

Chapter 10

Kant: Copernican revolution

The following is usually called the flower of modern philosophy (i.e., epistemology, or dualistic idealism):

$$\boxed{\text{Descartes}} \longrightarrow \left\{ \begin{array}{c} \text{[tabula rasa, experience]} \\ \boxed{\text{British Empiricism}} \\ \text{[Locke, Berkeley, Hume]} \\ \\ \text{[a priori, nativism]} \\ \boxed{\text{Continental Rationalism}} \\ \text{[Leibniz]} \end{array} \right\} \xrightarrow[\text{Copernican revolution}]{\qquad\qquad\qquad} \underset{\text{[summing-up]}}{\boxed{\text{Kant philosophy}}}$$

Kant completely followed Platonic method of telling philosophy (i.e., the fictional worldview) as follows.

$$(\sharp) \qquad \underset{\text{preface, introduction, (fictional)premise, expedient}}{\overset{\text{world is so}}{\boxed{\text{fictional worldview (literary truth, pseudo- truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{you should do so}}{\boxed{\text{ethics, morals}}}}$$

That is, Kant executed the following:

$(\sharp_1)$ $\big[$world is so$\big]$ is secondary,
"Critique of Pure Reason (1781)":

$(\sharp_2)$ $\big[$you should do so$\big]$ is main theme
"Critique of Practical Reason (1788)", "Critique of Judgment(1790)"

Hence, it is generally believed that Kant's philosophy has been a great success. That is because many people believe that

- Kant's attempt $(\sharp_1)$ at a metaphysical basis for the natural sciences is nothing less than a metaphysical basis for science in general, including Newtonian mechanics. Thus $(\sharp_2)$ is reliable.

Thus, Kant is the perfecter of Platonic method of telling philosophy. However, from the scientific point of view, the $(\sharp_1)$ is nothing but poetry.

But, a miracle happens. The $(\sharp_1)$ is very similar to quantum language (*cf.* Chapter 11).

10.1 ”Surely You’re Joking, Mr. Kant!”: Antinomy

10.1.1 Three Critiques

Immanuel Kant (1724 - 1804) is one of the most influential philosophers in the history of Western philosophy. His main work is ”Critique of Pure Reason (1781)”, ”Critique of Practical Reason (1788)”, ”Critique of Judgment(1790)”, whose theme is respectively ”truth” (i.e., ”pseudo-truth” in the sense of this paper), ”virtue”, ”beauty”. That is, he followed Platonic method of telling philosophy as follows.

$$
\text{(A)} \qquad \underset{\text{preface, introduction, (fictional)premise, expedient}}{\overset{\text{”Critique of Pure Reason (1781)”}}{\boxed{\text{fictional worldview (literary truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\substack{\text{”Critique of Practical Reason (1788)”} \\ \text{”Critique of Judgment(1790)”}}}{\boxed{\text{ethics, morals,aesthetics}}}}
$$

Of course, the interest of this paper is concentrated to the worldview (i.e., ”Critique of Pure Reason”). Have said many times in this book, philosophy of the worldview is only a ”preface”, thus, it might be immature. However,

(B) ”Critique of Pure Reason” might be perfect as a preface.

♠**Note 10.1.** As mentioned frequently up to this point, the fictional worldview is really ”asserted fiction”, however, we must pretend not to accept the fictional worldview as ”non-logical”, or we must accept it as ”logical in a wide sense”. That is because it must be prohibited that the difference between philosophy and religion becomes fuzzy. Therefore, we must use the terms such as ”logic”, ”reason”, etc. in the fictional worldview, for example, ”Critique of Pure Reason”, ”Tractatus Logico philosophicus (=TLP)”, etc. I think that Platonic method of telling philosophy (i.e., The fictional worldview) is a survival strategy for Western philosophy to co-exist with Christianity.

♠**Note 10.2.** It is often held that Kant lived a very strict and disciplined life, leading to an oft-repeated story that neighbors would set their clocks by his daily walks. In fact, he may have been a gregarious man. However, the image of Kant as strict and honest promotes an understanding of Kant’s philosophy. For the philosopher, his image itself is a part of his work.

10.1.2 Antinomy

Kant asserted that

(C) There is a proposition P such that ”P is true” and ”P is not true”

And he called such a proposition antinomy.

Antinomy 10.1. [Four Antinomies]
Kant assert that he finds the following four antinomies:

(D_1) The world has a beginning in time, and is also limited as regards space.

(D_2) Every composite substance in the world is made up of simple parts, and nothing anywhere exists save the simple or what is composed of the simple.

(D_3) Causality in accordance with laws of nature is not the only causality from which the appearances of the world can one and all be derived. To explain these appearances it is necessary to assume that there is also another causality, that of Spontaneity.

(D_4) There belongs to the world, either as its part or as its cause, a being that is absolutely necessary.

[**Notice**] Propositions (D_1)-(D_4) are only word play since the "logic" in ordinary language cannot be trusted, that is, it is not discussed under a certain worldview. Thus, **the following proof is not worth reading.**

Proof of (D_1)

Although each proposition (i.e., (D_1)-(D_4)), for example, according to Kant, let us show that the proposition (D_1) concerning time is antinomy as follows.

1. If the world has no beginning, then for any time t an infinite series of successive states of things has been synthesized by t.
2. An infinite series cannot be completed through successive synthesis.
3. The world has a beginning (is limited in time).

Therefore, Kant concludes that

- The proposition (D_1) concerning time is antinomy □

Some might read the above and say,

"Surely You're Joking, Mr. Kant!"

With this antinomy, Kant may have wanted to show the "limits of reason".

However, I would like to think that Kant's intention is worldviewism. That is, what Kant had to say in these antinomies is this:

- Without the principle (i.e., the worldview), we can't say anything

- A worldview with antinomies should not be adopted

If so, I agree with Kant. As emphasized throughout this paper, we must again emphasize the importance of worldviewism "Begin with the worldview!" (*cf.* (B) in Sec.1.3.1).

Remark 10.2. Discussions without a worldview are pointless. For example, we already know the followings.

($\sharp_1$) Zeno's paradoxes can be clarified in the motion function method (*cf.* Sec.2.4)

($\sharp_2$) Syllogism does not hold in quantum mechanics (*cf.* Sec.4.3),

($\sharp_3$) The statement "Only 'now' exists" violates the linguistic Copenhagen interpretation. (*cf.* Sec.6.1),

($\sharp_4$) The reason that Anselmus' "Arguments for the existence of God" is not clear is due to the fact that it is not being discussed under a certain worldview. (*cf.* Sec.6.4),

($\sharp_5$) The difference between Geocentrism and Heliocentrism was revealed only under Newtonian mechanics (*cf.* Chap.7),

($\sharp_6$) The qualia problem is solved in quantum language (*cf.* Sec.9.5.1)

($\sharp_7$) "Brain in a vat" is solved in quantum language (*cf.* Sec.9.5.2)

($\sharp_8$) The problem: "What is space-time?" can be answered from the linguistic point of view. (*cf.* Sec 9.2 [Leibniz-Clarke correspondence)

($\sharp_9$) Hume's problem of induction and Grue Paradox can be solved in quantum language. (*cf.* Sec.9.7)

($\sharp_{10}$) Kant's Antinomy arises from an argument without a worldview (in this section).

10.2 Kant's epistemology

10.2.1 Kant's compromise

Now, it is usually said that

(A) Kant's "Critique of Pure Reason" is a kind of compromise between Continental Rationalism and British Empiricism

That is,

$$
\underset{\text{[The father of modern philosophy]}}{\boxed{\text{Descartes}}} \longrightarrow \left\{ \begin{array}{c} \text{[tabula rasa, experience]} \\ \boxed{\text{British Empiricism}} \\ \text{[Locke, Berkeley, Hume]} \\ \\ \text{[a priori, nativism]} \\ \boxed{\text{Continental Rationalism}} \\ \text{[Leibniz]} \end{array} \right\} \longrightarrow \begin{array}{c} \text{["Critique of Pure Reason"]} \\ \boxed{\text{Kant philosophy}} \\ \text{[summing-up, compromise]} \end{array}
$$

The meaning of "compromise" is as follows.

A priori concept is, for example,

(B) sensibility (= space-time perception) and understanding (=thought)

which is within Continental Rationalism and not "tabula rasa". But,

(C) Cognition, judgment is going to be formed gradually through the experience

which is similar to British Empiricism.

♠**Note 10.3.** (= Summary 11.31) Consider "language" and not "cognition". That is, consider the following linguistic turn (*cf.* Chap.11):

$$
\begin{array}{c} \text{(cognition)} \\ \boxed{\text{mind-matter dualism(=epistemology)}} \\ \text{(Descartes}\cdot\text{Kant)} \end{array} \xrightarrow[\text{linguistic turn}]{} \begin{array}{c} \text{(language)} \\ \boxed{\text{linguistic philosophy}} \\ \text{(Wittgenstein)} \end{array}
$$

Then, in several languages (ordinary language, mathematics, Newtonian mechanics, programing language, etc.), we say tat

($\sharp_1$) "ordinary language" is like tabula rasa (i.e.,British Empiricism)(*cf.* Note 9.2)

($\sharp_2$) Mathematics is nativism like Continental Rationalism

($\sharp_3$) **quantum language is like Kant's compromise**

This will be again discussed in Summary 11.31 in Chap.11.

10.2.2 Thing-in-itself, Copernican revolution; from copy theory to constitution theory

Kant thought that

(D) We can understand the "world" only through the human perception. Also, cats can understand the "world" only through the cat perception. Thus,

There is "cat's world" for cats. and further, there is "butterfly's world" for butterflies.

If there are aliens whose cognition ability is finer than ours, their world is different ours. Although the difference of the worlds is made by that of the cognition ability, it is sure there exists something, which is called "thing-in-itself" by Kant.

That is, Kant thought as follows.

- we do not perceive the world as copy, but we perceive the world as it is constituted by cognition ability.

That is, Kant proposed so called Copernican revolution such that

from "copy theory" to "constitution theory"

namely,

(E_1) It's not "the world first, recognition later", but "recognition first, the world later".

or

(E_2) Recognition is not about painting a photorealistic picture, but it is similar to painting an abstract picture.

Also, we see the following table:
Seeing the above table, some may think:

(E_3) Is it true?

$$\boxed{\text{Locke}} \xrightarrow[\text{progress}]{} \boxed{\text{Kant}}$$

This will be answered in Problem 10.6 later.

♠**Note 10.4.** Many scientists may think of Kant's discovery (= the Copernican revolution) as a trifle, and may wonder, "Why did modest Kant give it such an exaggerated name?". Thus, it is natural to consider:

Table 10.1: The key-words of worldviews (*cf.* Assertion 1.14)

mind-matter dualism	[A](= mind)	[B](between A and B)	[C](= matter)
Plato	actual world	Idea	/ [idea world]
Descartes	I, mind, brain	body	/ [matter]
Locke	mind	secondary quality	primary quality [matter]
Kant	phenomenon	recognition	/ [thing-in-itself]
quantum language	**measured value**	**observable**	**state [system]**

- Kant believed the Copernican revolution as the greatest discovery in the history of philosophy.

I agree with him. Namely, I consider that

$$
(\mathrm{F}_1) \quad \underset{\text{(Descartes, Locke, Leibniz, Hume)}}{\overset{\text{idealism as immature realism}}{\boxed{\text{fake idealism (in pre-science)}}}} \xrightarrow[\text{Copernican revolution}]{} \underset{\text{(Kant)}}{\overset{\text{transendenatal idealism}}{\boxed{\text{true idealism (in philosophy)}}}}
$$

That is, I believe that

(F_2) "the Copernican revolution" = "the discovery of philosophy"

For the further arguments, see Problem 10.6 later.

♠**Note 10.5.** As said in the linguistic Copenhagen interpretation (E_3) in Sec. 1.1.2, "measuring instrument" is superior to "matter (= thing-in itself)". Recall Berkeley's saying:

- To be is to be perceived.

which is similar to "Recognition is previous, the world is later". However, it was Kant who fully understood (F).

10.2.3 "Critique of Pure Reason (1781)"

This section is the preparation of next chapter 11.

Explanation 10.3. (The preparation of Explanation 11.30 in Chap.11)
"Critique of Pure Reason (1781)" : Extracted from Microsoft Encarta (DVD version, Japanese (2009)

The Critique of Pure Reason made it clear that the [cognitive] ability of human beings is not merely to passively [copy] things in the world, but rather to actively work on the world and create the objects of their recognition on their own (British Empiricism) . The creation does not mean the creation from nothing. The world is already there in some way, and information from this world, which can be given to us through our senses, is necessary for [recognition] to be established. However, this information, as it is, is chaotic and confusing. The [cognitive] ability of human beings must give order to the information of these confused senses through a certain form (i.e., a priori comprehensive judgment and recognition : Continental rationalism) that is inherent to them, and only then can they bring together the objects of unified recognition (i.e., posteriori recognition). According to Kant, the form of [cognition] is a priori comprehensive judgment and recognition. That is,

(i) [Form of sensitivity(intuition)(Space-time $(=\mathbb{R} \times \mathbb{R}^3)$)]

(ii) [Form of understanding(thinking)] (e.g., the notion of quantity, such as single or many, and the notion of relations, such as causality)

If so, then the propositions "all things are in time and space" and "everything follows a causal relation" apply unconditionally to all objects of experience, even though they cannot be proved empirically. This is because the object can be constituted only in the form of space, time, and causal relations. This is similar to the case, for example, where the statement "The world is green" is considered to be a correct statement for all human beings if they all wear green sunglasses.
(MSN; Encarta encyclopedia. 2009 DVD Japanese version(translated by the author)).

That is, Kant proposed:

(G_1) the Reconciliation between Rationalism and Empiricism:

$$\left\{ \begin{array}{c} \text{[tabula rasa, experience]} \\ \boxed{\text{British Empiricism}} \\ \text{[Locke, Berkeley, Hume]} \\ \\ \text{[a priori, nativism]} \\ \boxed{\text{Continental Rationalism}} \\ \text{[Leibniz]} \end{array} \right\} \longrightarrow \begin{array}{c} \text{["Critique of Pure Reason"]} \\ \boxed{\text{Kant philosophy}} \\ \text{[summing-up, compromise]} \end{array}$$

(G_2) the Copernican revolution:

$$\underset{\text{(the world is previous, recognition is later)}}{\boxed{\text{Copy theory}}} \xrightarrow[\text{Copernican revolution}]{} \underset{\text{(recognition is previous, the world is later)}}{\boxed{\text{Constitution theory}}}$$

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10.3 * What is causality?

As mention in Sec 4.2, Aristotle considered the cause of the movement to be the "purpose" of the movement. Although this was what should be praised, it was not able to be said that "the purpose was to the point." For human beings to discover that the essence of movement and change is "causal relationship", we had to wait for the appearance of Galileo, Bacon, Descartes, Newton, etc.

Revolution to "Causality" from "Purpose"

is the greatest paradigm shift in the history of science. It is not an overstatement even if we call it "**birth of modern science**".

$$
\underset{\text{(Hērakleitos, Parmenides, Zeno)}}{\overset{\text{the birth of worldview}}{\boxed{\text{Movement}}}}
\xrightarrow[\text{Aristotle :(About 1500 years)}]{\text{"purpose"}}
\underset{\text{(Galileo, Bacon, Descartes, Newton)}}{\overset{\text{the birth of modern science}}{\boxed{\text{Causality}}}}
$$

♠**Note 10.6.** I cannot emphasize too much the importance of the discovery of the term: "causality". That is,

(♯) Science is the discipline about phenomena can be represented by the term "causality".

Thus, I consider that the discovery of "causality" is equal to that of science.

10.3.1 Four answers to "what is causality?"

As mentioned above, about "what is an essence of movement and change?", it was once settled with the word "causality." However, not all were solved now. We do not yet understand "causality" fully. In fact,

Problem 10.4. Problem:

"What is causality?"

is the most important outstanding problems in modern science.

Answer this problem!

There may be some readers who are surprised with saying like this, although it is the outstanding problems in the present. Below, I arrange the history of the answer to this problem.

(A) **[Realistic causality]:** Newton advocated the realistic describing method of Newtonian mechanics as a final settlement of accounts of ideas, such as Galileo, Bacon, and Descartes, and he thought as follows. :

> "Causality" actually exists in the world. Newtonian equation described faithfully this "causality". That is, Newtonian equation is the equation of a causal chain.

This realistic causality may be a very natural idea, and you may think that you cannot think in addition to this. In fact, probably, we may say that the current of the realistic causal relationship which continues like

"Newtonian mechanics⟶ Electricity and magnetism⟶ Theory of relativity⟶ $\cdots$ "

is the mainstream of science.

However, there are also other ideas, i.e., three "non-realistic causalities" as follows.

(B) **[Cognitive causality]:** David Hume, Immanuel Kant, etc. thought as follows. :

> We can not say that "Causality" actually exists in the world, or that it does not exist in the world. And when we think that "something" in the world is "causality", we should just believe that it has "causality".

Most readers may regard this as "a kind of rhetoric", however, some readers may believe it. It may look like that, because you are looking through the prejudice of "causality." This is Kant's famous "Copernican revolution" (i.e., "Kant was awakened from his dogmatic slumber by Hume's idea and came up with the Copernican revolution"), that is,

"recognition constitutes the world."

which is considered that the recognition circuit of causality is installed in the brain, and when it is stimulated by "something" and reacts, "there is causal relationship."

♠**Note 10.7.** About his discovery of "the Copernican revolution", Kant says in his book "Prolegomena" (1783):

(♯) *I freely admit that it was the remembrance of David Hume which, many years ago, first interrupted my dogmatic slumber and gave my investigations in the field of speculative philosophy a completely different direction.*

Readers may ask, "Why did honest Kant made such an exaggerated description?" It is a matter of course that Kant had great confidence such that it was the most greatest discovery in the history of philosophy. I agree to his opinion. For additional explanation about this, see Problem 10.6 later. Also, see Section 11.5.2. □

(C) **[Mathematical causality(Dynamical system theory)]:** Since dynamical system theory has developed as the mathematical technique in engineering, they have not investigated "What is causality?" thoroughly. However,

> In dynamical system theory, we start from the state equation (i.e., simultaneous ordinary differential equation of the first order) such that
>
> $$\begin{cases} \frac{d\omega_1}{dt}(t) = v_1(\omega_1(t), \omega_2(t), \ldots, \omega_n(t), t) \\ \frac{d\omega_2}{dt}(t) = v_2(\omega_1(t), \omega_2(t), \ldots, \omega_n(t), t) \\ \cdots\cdots \\ \frac{d\omega_n}{dt}(t) = v_n(\omega_1(t), \omega_2(t), \ldots, \omega_n(t), t) \end{cases} \tag{10.1}$$
>
> and, we think that
>
> ($\sharp$) the phenomenon described by the state equation has "causality."

This is the spirit of dynamical system theory (= statistics). Although this is proposed under the confusion of mathematics and worldview, it is quite useful. In this sense, I think that (C) should be evaluated more.

(D) **[Linguistic causal relationship (MeasurementTheory)]:** The causal relationship of measurement theory is decided by the Axiom 2 (causality; Sec. 1.1) of this chapter. If I say in detail,:

> Although measurement theory consists of the two Axioms 1 and 2, it is the Axiom 2 that is concerned with causal relationship. When describing a certain phenomenon in quantum language (i.e., a language called measurement theory) and using Axiom 2 (causality; Sec. 1.1) , we think that the phenomenon has causality.

Summary 10.5. The above is summarized as follows.

(A) World is first
(B) Recognition is first
(C) Mathematics(buried into ordinary language) is first
(D) Language (= quantum language) is first

Now, in measurement theory, we assert the next as said repeatedly:

Quantum language is a basic language which describes various sciences.

Supposing this is recognized, we can assert the next. Namely,

In science, causality is just as mentioned in the above (D).

This is my answer to "What is causality ?".

♠**Note 10.8.** Consider the following problems:

($\sharp_1$) What is time (space, causality, probability, etc.) ?

There are two ways to answer.

$$(\sharp_2) \quad \text{The answer of "What is XX ?"} \begin{cases} \text{(a): To show the definition of XX} \\ \\ \text{(b): To show how to use the term "XX"} \end{cases}$$

In this note, the answer to the question ($\sharp_1$) is presented from the linguistic point of view (b).

10.4 Summar; Descartes=Kant philosophy

10.4.1 Before Kant

Every thing started from Descartes figure(=Figure 1.2=Figure 8.1), i.e., "mind", "body", "matter". For example, in "An Essay Concerning Human Understanding (1689)", Locke might think as follows.

(A) In the field of "matter" of Descartes figure, activity of Newton is remarkable. However, concerning the relation among "I"(="brain", "mind"), "body"(="sensory organ"), "matter", he wanted to reach the summit.

Leibniz (in "The human being intelligence new discussion"1703) which advocated an objection in Locke can also be conscious of Newton. After all, Unproductive confrontation structure "Locke vs. Descartes·Leibniz" began. That is,

$$\underset{\text{[Locke, Berkeley, Hume]}}{\overset{\text{[tabula rasa, experience]}}{\boxed{\text{British Empiricism}}}} \quad \text{vs.} \quad \underset{\text{[Descartes, Leibniz]}}{\overset{\text{[a priori, nativism]}}{\boxed{\text{Continental Rationalism}}}}$$

And further, through Berkeley, Hume, modern philosophy had been flowering as follows.

$$\underset{\text{[The father of modern philosophy]}}{\overset{\text{["Discourse on the Method"]}}{\boxed{\text{Descartes}}}} \longrightarrow \left\{ \begin{array}{c} \underset{\text{[Locke, Berkeley, Hume]}}{\overset{\text{[tabula rasa, experience]}}{\boxed{\text{British Empiricism}}}} \\ \\ \underset{\text{[Leibniz]}}{\overset{\text{[a priori, nativism]}}{\boxed{\text{Continental Rationalism}}}} \end{array} \right\} \longrightarrow \underset{\text{[summing-up]}}{\overset{\text{["Critique of Pure Reason"]}}{\boxed{\text{Kant philosophy}}}}$$

According to Platonic method of telling philosophy (*cf.* (G) in Sec. 1.3), we may expect the following:

(B): The fictional worldview (western philosophy)

The fictional worldview (=How to tell philosophy due to Plato) is as follows.

$$\text{(B)} \qquad \underset{\text{preface, introduction, (fictional)premise, expedient}}{\overset{\text{world is so}}{\boxed{\text{fictional worldview}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\text{you should do so}}{\boxed{\text{ethics, morals}}}}$$

Therefore,

(B_1) [world is so] is secondary,

(B_2) [you should do so] is main theme

In spite of the above (B_1), we think that

(C) In modern philosophy, philosophers might be too eager to the preface (i.e., fictional world-view).

This might be due to the fact that rivalry to Newtonian mechanics was too strong. Or, Christianity might hope that modern philosophy played a role of rivalry to Newtonian mechanics.

10.4.2 Is Kant a progress from Descartes ?

10.4.2.1 The inevitability of Kant's appearance

We think that

> when it comes to the 1770s, the expiration date of epistemology was running out.

Therefore, many people might want to say

(D) Newtonian mechanics moved the world. Does the world move by epistemology? It was too early for 300 years? After all, is the "epistemology" important or not ?

Thus, many people thought:

(E_1) **We are tired of the epistemology. Someone please finish the epistemology nicely.**

This is the atmosphere of 1770's, in which Kant appeared. Thus, Kant was accepted easily. The philosophy enthusiasts of the time might think:

(E_2) Kant's book is difficult to read, more than 600 pages long. Kant's consideration of "Understanding", "reason" and "sensibility" is not even in the realm of scientific hypothesis. It is more of a literary art than a science. Therefore, it is unlikely that scientists will still be paying attention to it in 300 years' time. However, we would be grateful if this brought an end to epistemology. Dr. Kant's brain is as accurate as a clock, let us believe him. Since it's a compromise of British empiricism and Continental Rationalism, none will complain about it.

Thus, Kant theory was the conclusion declaration named "unification".

(F_1) Greatness of Kant is to have prevented that epistemology faces the direction of the brain science by the showy name called "Copernican revolution".

That is, Kant understand that, even if the epistemology is clarified by the brain science, this is non-sense from the philosophical point of view.

We think that

(F_2) **using the term ”Copernican revolution”, Kant prevented that epistemology enters into the zone of brain science.**

Naming of ”Copernican revolution” does not mean that self-congratulation of Kant. I'd like to believe that strong intention of Kant which says ”Epistemology is not experimental science, but philosophy” (or, ”philosophy should not aim at experimental sciences”), is included in the term: ”Copernican revolution”.

Thus, Kant follows Platonic method of telling philosophy such that

$$\text{(G)} \qquad \underset{\text{preface, introduction, (fictional)premise, expedient}}{\overset{\text{”Critique of Pure Reason (1781)”}}{\boxed{\text{fictional worldview (literary truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\substack{\text{”Critique of Practical Reason (1788)”} \\ \text{”Critique of Judgment(1790)”}}}{\boxed{\text{ethics, morals, aesthetics}}}}$$

This was a great success. Today, no scientist is interested in ”Critique of Pure Reason (1781)”. However, ”Critique of Practical Reason (1788)” is a must for any ethicist. Thus, I think that the (G) is the most typical example of Platonic method of telling philosophy.
This implies the end of Grand Narratives (i.e., epistemology), and the start of ”non-physical law (i.e., the philosophy of proverb)” such as

(H_1) Bentham(1789): ”the greatest happiness of the greatest number”

(H_2) Hegel (1770 - 1831) : ”thesis-antithesis-synthesis”

(H_3) Darwin(1809 - 1882) :”the survival of the fittest”

(H_4) Nietzsche(1844 - 1900) : ”God is dead”

It is natural, since the job of the philosopher is to make non-physical laws (= golden sayings, proverbs, copies) that are supported by many philosophy enthusiasts.

10.4.2.2 Is Kant a progress from Descartes ?

I think that

(I) Nevertheless, many readers must think that the Copernican revolution is far too overrated.

In the next chapter, I will show that this is a fair estimation. Here, I will say somewhat more about this issue as follows.

Problem 10.6. It is sure that both Descartes worldview and Kant worldview are useless. Thus, the following problem may not be easy.

(J) is the following reform progress?

$$\underset{\text{(Descartes, Locke, Leibniz, Hume)}}{\overset{\text{mind-matter dualistic idealism}}{\boxed{\text{Descartes' worldview (literary truth)}}}} \xrightarrow[\text{progress}]{} \underset{\text{(Kant)}}{\overset{\text{mind-matter dualistic idealism}}{\boxed{\text{Kant worldview (literary truth)}}}} \quad ?$$

Namely, what parts of Descartes' worldview have advanced?

Brief Answer: The difference between the two are as follows.

($\sharp_1$) "Idealism" in Descartes' worldview means "theory without experimental verification"

($\sharp_2$) "Idealism" in Kant's worldview means "transcendental idealism", that is, "idealism made by the Copernican revolution".

And, we can show that "transcendental idealism" is similar to the idealism of quantum language. Thus, we can conclude that the (J) is yes. The precise answer will be presented in Answer DDD11.12Answer of Chapter 11.

♠**Note 10.9.** Physics and science make up a theory while making modifications by the result of the experiment. Thus, physics and science can expect sound development. On the other hand, the philosophy of worldview is metaphysics, which cannot be determined by experiments. Thus, the question "Did the philosophy of worldview make a progress?" is not easy to answer. That is because, if we consider that

- the western philosophy was able to keep freshness for a long time by renewing a preface part (i.e., the worldview), much like a car model change.

then, we must conclude that the philosophy of worldview does not make a progress. However, in this paper, we assert that

($\sharp_1$) the philosophy of worldview has been making a progress. And moreover, it finally converges to quantum language.

More precisely, we assert that

($\sharp_2$) If "to make progress" is defined by "to come near quantum language" (*cf.* Assertion 1.5), then the philosophy of worldview has been making a progress.

10.4.3 After Kant: Hegel and Husserl

Recall that our purpose of this paper is to show that

$$\boxed{\text{Plato}} \xrightarrow[\text{progress}]{} \boxed{\text{Descartes}} \xrightarrow[\text{progress}]{} \boxed{\text{Kant}} \xrightarrow[\text{progress}]{} \boxed{\text{Wittgenstein}} \xrightarrow[\text{progress}]{} \boxed{\text{Quantum language}} \tag{10.2}$$

where $\xrightarrow[\text{progress}]{}$ means "getting closer to quantum language".

If so, readers may think, "Why aren't the names of great philosophers like Hegel and Husserl mentioned?"

10.4.3.1 The reason that Hegel is omitted in (10.2)

Hegel (1770-1831) was a German philosopher and an important figure of German idealism. He studied Kant's work:

$$\text{(K)} \qquad \underset{\text{preface, introduction, (fictional)premise, expedient}}{\overset{\text{"Critique of Pure Reason (1781)"}}{\boxed{\text{fictional worldview (literary truth)}}}} \xrightarrow{\text{therefore}} \underset{\text{main subject}}{\overset{\substack{\text{"Critique of Practical Reason (1788)"} \\ \text{"Critique of Judgment(1790)"}}}{\boxed{\text{ethics, morals, aesthetics}}}}$$

And he might think that Kant's explanation of the above "$\boxed{\text{the truth}} \xrightarrow{\text{therefore}} \boxed{\text{the good}}$" is not sufficient, that is, the relation between the truth (Critique of Pure Reason) and the good (Critique of Practical Reason) is not clear. And thus, he wrote "The Phenomenology of Spirit" (1807), which is his most widely discussed philosophical work. Compared to Kant's book "Critique of Practical Reason", this work may be popular with general philosophy enthusiasts. However, as I have noted many times in this paper, I think that there is little relationship between truth and ethics. Also, note that our interest is concentrated to the worldview. Hence, Hegel is farther from quantum language than Kant. Thus, we omit "Hegel" in (10.2).

10.4.3.2 The reason that Husserl is omitted in (10.2)

Edmund Husserl (1859–1938) was the principal founder of phenomenology and thus one of the most influential German philosophers of the 20th century. He wrote "Ideen" (1913), in which he tried to reinforce Kant's work (Critique of Pure Reason) from a psychological perspective and to establish "the foundation for various disciplines". In fact Ideen is one of the most influential books in 20th century. However, as I have noted many times in this paper, I think that the truth in philosophy is rather literary. And thus, his phenomenology is rather literary and not scientific. It was supposed to be a big theory (such as "the foundation for various disciplines"), but most scientists today are not interested in it. Also, note that our true interest is concentrated to the (scientific) worldview. Hence, Husserl is farther from quantum language than Kant. Thus, we omit "Husserl" in (10.2).

10.4.3.3 It's not that I have a low opinion of Hegel and Husserl.

It's not that I have a low opinion of Hegel and Husserl. Rather, I don't have the philosophical background to be able to evaluate their accomplishments. Thus, I think that

$$
\begin{tikzcd}
 & & & \boxed{\text{Hegel}} & \\
\boxed{\text{Plato}} \arrow[r, "\text{progress(QL)}"'] & \boxed{\text{Descartes}} \arrow[r, "\text{progress(QL)}"'] & \boxed{\text{Kant}} \arrow[ur, "\text{progress}'"] \arrow[r, "\text{progress(QL)}"'] \arrow[dr, "\text{progress}''"'] & \boxed{\text{Wittgenstein}} \arrow[r, "\text{progress(QL)}"'] & \boxed{\text{QL}} \\
 & & & \boxed{\text{Husserl}} &
\end{tikzcd}
$$

where [progress(QL)] is defined by "to come near quantum language" (*cf.* Assertion 1.5). Also, the above [progress′] and [progress″] is respectively different from [progress(QL)]. Of course, the definition of "progress" is not always determined uniquely. For example, Husserl may say that

$$
\begin{tikzcd}
 & & & \boxed{\text{Hegel}} & \\
\boxed{\text{Plato}} \arrow[r, "\text{progress}''"'] & \boxed{\text{Descartes}} \arrow[r, "\text{progress}''"'] & \boxed{\text{Kant}} \arrow[ur, "\text{progress}'"] \arrow[r, "\text{progress(QL)}"'] \arrow[dr, "\text{progress}''"'] & \boxed{\text{Wittgenstein}} \arrow[r, "\text{progress(QL)}"'] & \boxed{\text{QL}} \\
 & & & \boxed{\text{Husserl}} &
\end{tikzcd}
$$

where [progress″] is defined by "to come near Husserl". However, it should be noted that

- Of the worldviews mentioned above, quantum language is the only one that is scientifically useful.

♠**Note 10.10.** Here, we have (*cf.* Classification 1.11 [the classification of philosophers]).

$$
(\flat) \begin{cases}
(\flat_1)\text{: the realistic worldview (physics)} \\
\quad \text{Hērakleitos, Aristotle, Aristarchus, Archimedes, Eratosthenes, Ptolemaeus,} \\
\quad \text{Galileo, Newton, Einstein,} \cdots \\
\quad \text{(Although mathematics is not a worldview, Pythagoras, Eudoxus, Euclid)} \\
\\
(\flat_{21})\text{: the fictional worldview (Western philosophy)} \\
\quad \text{Plato, Scholasticism, Descartes, Locke, Leibniz, Berkeley, Hume, Kant, Husserl} \\
\\
(\flat_{22})\text{: the logical worldview} \\
\quad \text{Boole, Frege, Peirce, Saussure, Russell, Wittgenstein, Hempel, Popper} \\
\\
(\flat_{23})\text{: the mechanical worldview (statistics, quantum language)} \\
\quad \text{Parmenides, Zeno, J. Bernoulli, statistics (e.g., Fisher), quantum language}
\end{cases}
$$

Chapter 11

Linguistic philosophy and quantum language

The purpose of this Chapter is to show the following

($\sharp_1$)

$$\boxed{\text{Descartes}} \xrightarrow[\text{progress ①}]{\text{Answer 11.12}} \boxed{\text{Kant}} \xrightarrow[\text{progress ①}]{} \underset{\substack{\text{Saussure (Remark 11.13)} \\ \text{Wittgenstein (Sec. 11.6.3)}}}{\boxed{\text{linguistic phil.}}} \xrightarrow[\text{progress ②}]{} \boxed{\text{Quantum language}}$$

where $\xrightarrow[\text{progress ①}]{}$ means "getting closer to quantum language".

Assume that "linguistic philosophy" is classified as follows. (($\flat_{22}^1$),($\flat_{22}^2$) are mathematics and ($\flat_{22}^5$) is linguistics)

$$(\flat_{22}) \begin{cases} (\flat_{22}^1) \quad \text{foundation of math.} \quad \text{(Cantor, Gödel)}, \quad (\flat_{22}^2) \quad \text{math.} \quad \text{(Bourbaki)}, \\ (\flat_{22}^3) \quad \text{analytic philosophy, symbolic logic} \quad \text{(Frege, Russell, Wittgenstein,...)} \\ (\flat_{22}^4) \quad \text{analystic, scientific philosophy} \quad \text{(Hempel, Popper, ...)} \\ (\flat_{22}^5) \quad \text{linguistics, ordinary language} \quad \text{(Saussure, Wittgenstein (except logic))} \end{cases}$$

If so, the above ($\sharp_1$) can be rewritten more precisely as follow:

$$(\sharp_2) \quad \boxed{\text{Descartes=Kant}} \xrightarrow[\text{progress ①}]{} \underset{\substack{(\flat_{22}^5)\text{:Saussure (Remark 11.13)} \\ (\flat_{22}^3,\flat_{22}^5)\text{: Wittgenstein (Sec. 11.6.3)} \\ (\flat_{22}^4)\text{:Hempel (Remark 11.42)}}}{\boxed{\text{Analytic phil. } (\flat_{22}^3,\flat_{22}^4,\flat_{22}^5)}} \xrightarrow[\text{progress ②}]{} \boxed{\text{Quantum language}}$$

Also, for the "$\xrightarrow[\text{progress ②}]{}$", see List 11.26.

11.1 Linguistic philosophy, Analytic philosophy

11.1.1 Analytic philosophy; A philosophy without a worldview

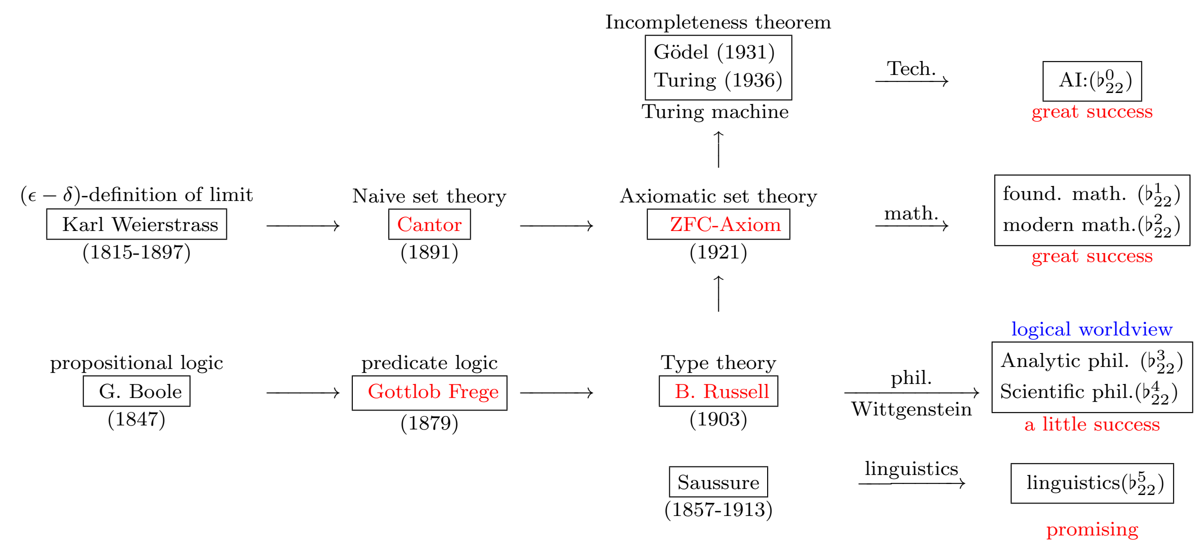
The expiration date of epistemology had expired in Kant's time. Philosophers had to find their next subject. Some philosophers (Frege, Russel, etc.) have tried to create philosophy (i.e., analytic philosophy) from scratch, abandoning the great tradition of the dualistic idealism (from Plato to Kant). I respect their pioneering spirit, but whether or not they succeed is another matter. That is, we have the following question:

- Is analytic philosophy a true philosophy?

I think no one has answered this question yet.

Roughly speaking, we have the following diagram:

Diagram 11.1. [A brief modern history around analytic philosophy]



Recall Classification 1.9 [the classification of worldviews]

($\flat_1$) : the realistic worldview (monism) : Aristotle, Archimedes, Galileo, Newton, Einstein

($\flat_2$) : the idealistic worldview (dualism)
- ($\flat_{21}$): the fictional worldview ($\approx$ western philosophy)
  Plato, Descartes, Locke, Kant
- ($\flat_{22}$): the logical worldview ($\approx$ analytic, scientific phil.)
  Frege, Saussure, Russell, Wittgenstein...
- ($\flat_{23}$): the mechanical worldview
  statistics, quantum language

($\flat_3$)Others (Thinking Tip, etc.) : Darwin's theory of evolution, Hegel's dialectic, etc.

Further, the above "the logical worldview ($\flat_{22}$)" is classified as follows.

Classification 11.2. [The location of the logical worldview] Assume that the above is classified as follows.

$$
(\flat_{22}) \begin{cases}
(\mathrm{A}_0)\text{: Artificial intelligence, "Logic circuit may be more powerful than any worldview"} \\
\qquad (\flat_{22}^0) \quad \text{Computer; (Gödel, Turing, von Neumann, etc.)} \\
(\mathrm{A}_1)\text{: mathematics: "Describe math. by logic (and set theory)!"} \\
\qquad \begin{cases} (\flat_{22}^1) \quad \text{foundation of math. (Boole, Frege, Cantor, Russell, Gödel,...),} \\ (\flat_{22}^2) \quad \text{math. (Bourbaki),} \end{cases} \\
(\mathrm{A}_2)\text{:}\textbf{the logical worldview} \text{ (or precisely, the naive logical worldview) :} \\
\quad \text{"Describe everything by logically!"} \\
\qquad \begin{cases} (\flat_{22}^3) \quad \text{analytic philosophy, symbolic logic (Frege, Russell, Wittgenstein,...)} \\ (\flat_{22}^4) \quad \text{scientific philosophy (Hempel, Popper, ...)} \end{cases} \\
(\mathrm{A}_3)\text{: linguistics: "Describe everything reasonably!"} \\
\qquad (\flat_{22}^5) \quad \text{linguistics, ordinary language (Saussure, Wittgenstein)}
\end{cases}
$$

In this chaper, we devote (A_2) and (A_3) since (A_0) and (A_1) are mathematics and technology.

♠**Note 11.1.** In ancient Greece, mathematics was a field of philosophy. Philosophers still tend to confuse philosophical and mathematical problems. For example, as mentioned in Section 2.4, Zeno's paradox is the problem of a worldview. However, there are still many philosophers who think that Zenon's paradox (Achilles and a tortoise) has been solved by considering it as a geometric series problem. I think that analytic philosophy is also confused between mathematics and philosophy. As mentioned in Diagram 11.1, it is Wittgenstein that made analytical philosophy independent of mathematics. That is,

$$
(\sharp_1) \qquad \underset{\text{Frege, Russell}}{\boxed{\text{Symbolic logic, Set theory}}} \xrightarrow[\text{Wittgenstein}]{\text{Philosophy}} \boxed{\text{Analytic philosophy}}
$$

It was enthusiastically supported by many amateur philosophy enthusiasts. However, I think that, from the theoretical point of view, the following problem is not yet settled.

- Was Wittgenstein's attempt successful?

As mentioned in this chapter, my opinion is "There is no direct relationship between logic and the world in analytic philosophy" and thus, "The ($\sharp_1$) is unpromising as a worldview". There are a lot of negative opinions for analytic philosophy (*cf.* [81]). My position has always been to consider quantum language:

$$
(\sharp_2) \qquad \boxed{\text{Quantum mechanics}} \xrightarrow{\text{lingiustic turn}} \boxed{\text{Quantum language}}
$$

And, in this section, I will say that the ($\sharp_2$) is significantly more productive than the ($\sharp_1$). And thus, I conclude that

- $\boxed{\text{Analytic philosophy}} \xrightarrow[\text{progress}]{} \boxed{\text{Quantum language}}$

////

♠**Note 11.2.** I am not proficient in the use of the terms "analytic philosophy", "scientific philosophy (=philosophy of science)", or "linguistic philosophy (=philosophy of language)". We might be forgiven for thinking that these three are the same. In this paper we consider the following. But there is no strict distinction among the three.

- scientific philosophy ······ the main theme is science
- "analytic philosophy"≈"linguistic philosophy". The linguistic philosophy is broader, if I may say so.

In short, I don't know much about these things as philosophy. Most people agree that (A_0), (A_1) and (A_3) do not belong to philosophy. Even if someone argues that $(\flat_{22})$ has nothing to do with philosophy, I may not dispute it. That is because analytic philosophy today names a style of doing philosophy (i.e., like (A_3): "Describe everything reasonably!" or "Aim for argumentative clarity and precision!"), not a philosophical program or a set of substantive views.

11.1.2 (A_1), (A_2) and (A_3) are related?

I predict the next progress:

$$(A_0) \quad \boxed{\text{logical worldview}} \xrightarrow[\text{progress}]{} \overset{\text{(quantum mechanical worldview)}}{\boxed{\text{quantum language}}} \xrightarrow[\text{progress}]{} \boxed{\text{AI}}$$

Therefore, (A_0) will be the most important after about 2050 AD. However, I omit it, since I don't know much about technology.

Let us (A_1), (A_2) and (A_3) explain more precisely.

11.1.2.1 (A_1): mathematics: "Describe math. by logic (and set theory)!" in Classification 11.2

Above, the part involving Cantor, Frege, Russell, Zermelo, Fraenkel and so on is the biggest event in the history of mathematics, because they built mathematics with axiomatic set theory. That is,

$$(A_1) \ : \ (\flat_{22}^1) \text{ and } (\flat_{22}^2): \quad \underset{\text{Cantor, Frege, Russell, Zermelo=Fraenkel and so on}}{\overset{\text{Describe mathematics by logic, set theory!}}{\boxed{\text{axiomatic logic, set theory}}}}$$

$$\xrightarrow{\text{therefore}} \boxed{\text{Math. (Found. math. } (\flat_{22}^1) \text{ and modern math. } (\flat_{22}^2))}$$

This was one of the most successful fields of the 20th century. However, it should be noted that the above is mathematics and not philosophy (i.e., worldview).

11.1.2.2 (A_2):the logical worldview: "Describe phenomena with logic!" in Classification 11.2

The above success of (A_1) led many philosophers to expect that science could be described by symbolic logic and set theory as well. This is called the logical worldview (or, the set theoretical worldview) in this text. That is,

(A_2) : ($\flat_{22}^3$) and ($\flat_{22}^4$): analytic philosophy

$$
\underset{\text{the logical (set theorical) worldview}}{\overset{\text{"Describe phenomena with logic (and set theory)!"}}{\boxed{\text{symbolic logic, set theory}}}} \xrightarrow{\text{therefore}} \boxed{\text{the world will be clear}}
$$

if the world is restricted to science, it is called "scientific philosophy ($\flat_{22}^4$)".

Many philosophers might consider that

- (A_1) is the greatest, and therefore, (A_2) must be promising too.

This optimistic outlook, I believe, prompted the spreading of analytic philosophy. However, I think that the above (A_2) did not achieve great success. There are many reasons for this failure, but I mention just one:

(B_1) the method of regarding a field of mathematics (e.g., logic, set theory) as a worldview is too straightforward.
(Also, see Remark 11.3 below.)

In fact, philosophers in scientific philosophy ($\flat_{22}^4$) found the attempt (A_2) difficult to succeed. This will be discussed in Section 11.7 (Flagpole problem) and Section 11.8 (Hempel's raven problem). Thus, I conclude that

(B_2) analytic philosophy is a philosophy without a worldview (or precisely, without a good worldview).

This means that the boundary line between analytic philosophy (A_2) and linguistics (A_3) is not clear. Therefore, philosophers in analytic philosophy ($\flat_{22}^3$) were also interested in linguistics ($\flat_{22}^5$).

Remark 11.3. (i): There may be a variety of opinions (A_2). What they (e.g., Frege, Cantor, Russel, Zermelo, Frankel, etc.) revealed is in (A_1) as follows.

(C) mathematics (or, symbolic logic) is not directly related to the world.

KSTS/RR-20/001
November 15, 2020

Thus, we need "worldview", which is a bridge between "logic (mathematics)" and "the world". That is,

$$(\sharp_1) \qquad \boxed{\text{mathematics(logic)}} \longleftarrow \boxed{\text{worldview}} \longrightarrow \boxed{\text{world}}$$

Therefore, any worldview should contain non-mathematical words as the key-words (as well as mathematical terms). For example, everyone knows that

$$(\sharp_2) \qquad \underset{\text{differential equation}}{\boxed{\text{mathematics}}} \longleftarrow \underset{\text{(space-time, force, mass, velocity, ...)}}{\boxed{\text{Newtonian mechanics}}} \longrightarrow \boxed{\text{world}}$$

Also, "the logical worldview" may be represented as loows.

$$(\sharp_3) \qquad \boxed{\text{logic, axiomatic set theory)}} \longleftarrow \underset{\text{(set, a collection of things)}}{\boxed{\text{logic, naive set theory}}} \longrightarrow \boxed{\text{world}}$$

(Also, again see Note 2.8: pure Pythagoreanism.)
In the above, don't get nervous about the difference between elementary set theory (=naive set theory= Cantor's set theory, *cf.* [25]) and mathematical set theory (=axiomatic set theory). Here, in elementary set theory, "set" is defined by "a collection of things". This is the same as the use of "set" in everyday language. Thus, using naive set theory, the above ($\sharp_3$) can be rewiritten by

$$(\sharp_4) \qquad \boxed{\text{logic, naive set theory}} \longleftarrow \boxed{\text{nothing}} \longrightarrow \boxed{\text{world}}$$

Since even most mathematicians are not familiar with mathematical set theory but elementary set theory, it is more natural to think of ($\sharp_4$) than ($\sharp_3$). Therefore, there is a reason to consider that the logical worldview (A_2) is not a worldview, at least is is not a worldview in the true sense.
(ii): Analytic philosophy can only be successful if any of modal logic, temporal logic, intuitionist logic, fuzzy logic, quantum logic, etc. are successful in terms of application. But I think this possibility is hopeless. There are various fields in mathematics, but I don't think there is a necessity to start from logic. For example, there is no reason to adhere to symbolic logic when expressing "time" mathematically.

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11.1.2.3 (A_3): linguistics: "Consider everything logically!" in Classification 11.2

It is a matter of course that linguistics is very important. And no one can disagree with the following spirit.

(A_3) Let's use the word correctly! Speak carefully! Consider everything logically! Try to be reasonable!

This is not a philosophical assertion; it may be a moral slogan. This spirit might be enlightened by Wittgenstein's saying:

(D) philosophical problems arise from insufficient attention to the variety of natural language use.

The (A_3) is having many meanings. However, the influence of Wittgenstein on the active exchange between linguistics and philosophy was immeasurable. With the development of automatic translation technology and other technologies in (A_0), the future of linguistics is promising.

However, it should be noted that neither linguistics nor mathematics is philosophy (i.e., worldview).

Remark 11.4. We have two types of logic (i.e., symbolic logic and practical logic) (*cf.* Note 1.15), i.e.,

(E) $\begin{cases} \text{symbolic logic (= axiomatic logic =mathematical logic) (} cf. \text{ Postulate 11.5)} \\ \text{practical logic (= inference)} \end{cases}$

However, in the logical worldview, we tend to confuse the two, or regard practical logic as symbolic logic. The most important "logic" in philosophy is practical logic. Recall the worldviewism in Sec. 1.3.1 as follows.

($\widehat{A}$): The worldviewism in Sec. 1.3.1

The worldviewism has the following form:

($\widehat{A}_1$) $\overset{\text{world is so}}{\underset{\text{premise}}{\boxed{\text{worldview}}}}$

$\xrightarrow{\text{therefore}} \overset{\text{conclusion}}{\underset{\text{subject}}{\boxed{\text{practical logic } (\approx \text{ inference, how to think, calculation), discussions, etc.}}}}$

That is,

($\widehat{A}_2$) **Since the world is like this, think like this!**

That is, the worldviewism is the spirit such that "Start from worldview". I think that

($\widehat{A}_3$) It's not an exaggeration to say **"philosophy = worldviewism"**.

Thus we have three kinds of "logic" as follows:

KSTS/RR-20/001
November 15, 2020

($\sharp_1$) "logic" in (A_1) = "logic" in (A_2) = symbolic logic

($\sharp_2$) "logical" in (A_3)≈"reasonable"≈"careful"

($\sharp_3$) practical logic ($\widehat{A}$)

In analytic philosophy, the importance of "logic" (i.e., "logic" in ($\sharp_1$)) is often emphasized. For example, they may sometimes say that

($\sharp_4$) *Doing philosophy without knowing logic is like doing French literature without knowing French.*

However, it should be noted that this "quote" is said under the conflation of symbolic logic and practical logic.

In order to master analytical philosophy (i.e., the logical worldview), I think it's enough to know the logic and set theory taught in the first year of university in the math department. It doesn't matter if the philosopher cannot solve problem concerning the definition of "$\lim_{n\to\infty} a_n = a$", which may be taught in the second year of university (*cf.* Exercise 11.8 (iii))). Predicate logic, etc., is not available to most physicists either. Only mathematicians should be able to use it. As mentioned in Remark 11.3 (ii), I think all attempts to use logic at the graduate level have failed.

It is not the study of logic that is important, but the following.

- anaytic philosophy has been studied and supported under the confusion between symbolic logic and practical logic.

In this sense, analytic philosophy is not a good theory. But still, I can say the following.

$$\boxed{\text{Kant}} \xrightarrow[\text{progress}]{\text{Answer 11.12}} \underset{\text{(the logical worldview)}}{\boxed{\text{linguistic, analytic philosophy}}} \Big(\xrightarrow[\text{progress}]{} \underset{\text{(the quantum mechanical worldview)}}{\boxed{\text{quantum language}}} \Big)$$

This will be discussed in this chapter.

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11.2 George Boole (1815-1864) and Gottlob Frege (1848-1925)

George Boole was a mathematician, philosopher and logician in England. Particularly, it's said that the propositional logic (i.e., Boolean algebra) was proposed by him. Also, Gottlob Frege was a German philosopher, logician, and mathematician. He is called the father of analytic philosophy, concentrating on the philosophy of language, logic, and mathematics. Particularly, it's said that the predicate logic was proposed by him.

Postulate 11.5. [Symbolic logic (i.e., Propositional logic and predicate logic)]
For any proposition P, the truth function $\phi(P)$ is determined such that

$$\phi(P) = \begin{cases} 1 & (\text{if } P \text{ is true}) \\ 0 & (\text{if } P \text{ is wrong (i.e., not true)}) \end{cases}$$

[Propositional logic] : G. Boole

(A$_1$) Assume that P_1, P_2 are propositions. Then, $P_1 \wedge P_2$, $P_1 \vee P_2$, $\neg P_1$, $P_1 \to P_2$ are propositions.
And it holds that $\phi(P_1 \wedge P_2) = \min\{\phi(P_1), \phi(P_2)\}$, $\phi(P_1 \vee P_2) = \max\{\phi(P_1), \phi(P_2)\}$, $\phi(\neg P) = 1 - \phi(P)$.

where $\wedge$, $\vee$, $\neg$, $\to$ respectively means "and", "or", "not", "implies". Note that $P_1 \to P_2$ is defined by $\neg P_1 \vee P_2$". Also, De Morgan's laws say that $P_1 \vee P_2 = \neg(\neg P_1 \wedge \neg P_2)$ holds.

Also, assume that P_θ ($\theta \in \Theta \equiv \{1, 2, ..., N\}$), then it clearly holds
(i): $P_1 \wedge P_2 \wedge ... \wedge P_n$ (denoted by $\bigwedge_{\theta \in \Theta} P_\theta$) or $\forall \theta (\in \Theta)[P_\theta]$) is a proposition
(ii): $P_1 \vee P_2 \vee ... \vee P_n$ (denoted by $\bigvee_{\theta \in \Theta} P_\theta$) or $\exists \theta (\in \Theta)[P_\theta]$) is a proposition.

[Predicate logic]: G. Frege

(A$_2$) The above finite set $\Theta (\equiv \{1, 2, ..., N\})$ can be extended to any infinite set Θ.

♠**Note 11.3.** Some may think that the predicate logic (i.e., infinite operations) is a slight generalization of the propositional logic (i.e., finite operations). However, the predicate logic is powerfully expressive. In fact, Zermelo-Fraenkel Axiom asserts that all mathematical propositions can be described by only "predicate logic" and "$\in$". That is, Postulate 11.5 completely works for all mathematical propositions.

Remark 11.6. As mentioned in Note 11.3, Postulate 11.5 completely works for all mathematical propositions. However, Postulate 11.5 does not necessarily work for non-mathematical propositions. Most propositions we use in our daily lives do not have a definite truth function.

For example,

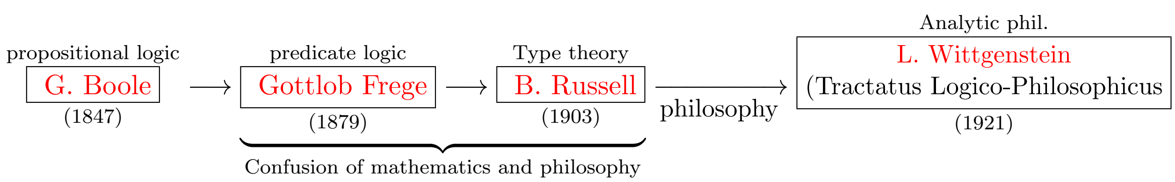
- if P_1="American are cheerful", P_2="Tyrannosaurs are violent", P_3="every raven is black", it is difficult to determine the truth function $\phi(P_i)$ $(i = 1, 2, 3)$.

As seen in Sec 11.8 later, even the definition of "P_3 is true" is not easy. Thus, I doubt if this P is even a proposition. Strictly speaking, I believe that

(♯) Postulate 11.5 can only be applied to mathematical propositions. That is, predicate logic is the language of mathematics.

In short, the definition of "proposition" is not clear in non-mathematical fields. Hence, there is some difficulty in using Postulate 11.5 in philosophy. Therefore, I am skeptical of the strict use of Postulate 11.5 in philosophy.

Remark 11.7. The works of Boole, Frege and Russell belong to mathematics. It was Wittgenstein who arranged their work as philosophy such that Amateur philosophy enthusiasts supported



Russell may have written the plot of the drama above, but Wittgenstein was a great actor who played his role perfectly. And thus, after Wittgenstein, analytic philosophy has become a major trend in Western philosophy. Wittgenstein tried to do something that was theoretically impossible to do, so the assessment of him is mixed. In this text, I value him highly. Philosophy is not a matter of logic, but of asserting in strong terms. In this sense, Wittgenstein understood philosophy better than Frege and Russell did.

I think that Frege and Russell have been admired under the confusion of mathematics and philosophy. Of course, there are other opinions. It can be also said that Frege and Russell, who used the confusion of mathematics and philosophy to spread analytic philosophy, were more strategic. On the other hand, Wittgenstein was too simple, in a good way. This will be again discussed in Sec. 11.6.

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Exercise 11.8. (i): The proof of syllogism: $(p \to q) \wedge (q \to r) \to (p \to r)$

Truth Table

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$(p \to q) \wedge (q \to r) \to (p \to r)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	0	1	0	1	1	0	1
1	0	0	0	1	0	0	1
0	1	1	1	1	1	1	1
0	1	0	1	0	1	0	1
0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1

Thus, syllogism: $(p \to q) \wedge (q \to r) \to (p \to r)$ is always true.

(ii):$((p \to q) \wedge p) \to q$ is always true, but $((p \to q) \wedge q) \to p$ is not always true.

Proof of (ii)

p	q	$p \to q$	$(p \to q) \wedge p$	$(p \to q) \wedge q$	$((p \to q) \wedge p) \to q$	$((p \to q) \wedge q) \to p$
1	1	1	1	1	1	1
1	0	0	0	0	1	1
0	1	1	0	1	1	0
0	0	1	0	0	1	1

(iii):[The elementary problem concerning predicate logic]

"$\lim_{n \to \infty} a_n = a$ (i.e., "A real-valued sequence $\{a_n\}_{n=1}^{\infty}$ converges to a") is defined by

$$\forall \epsilon (> 0) \exists N (\text{natural number}) [|a_n - a| < \epsilon (\forall n > N)]$$

This was due to the great mathematicians Cauchy and Weierstrass etc. As mentioned in Note 9.5, Even though Newton did not know the elementary use of such predicate logic, he was able to make physics. I don't know any physicist saying:

- *Doing physics without knowing logic is like doing French literature without knowing French.*

I think the only correct one is

- *Doing mathematics without knowing logic is like doing French literature without knowing French.*

$\square$

♠**Note 11.4.** I am not an expert in history of mathematics or history of philosophy, so I am not familiar with G. Frege. However, in this text, I follow the opinions of the philosophers in analytic philosophy, and regard him as a genius since Aristotle.

11.3 Peirce (1839-1914)

This section was written with reference to the following.

- [70]:Ishikawa, S: Philosophy of science for scientists; The probabilistic interpretation of science Journal of quantum information science, Vol. 9, No.3 , 140-154, DOI: 10.4236/jqis.2019.93007 (https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

11.3.1 * What is Peirce's abduction?

Charles Sanders Peirce (1839-1914) was an American philosopher, logician, mathematician, and scientist who is sometimes known as "the father of pragmatism". This paper will not touch on pragmatism, but will speak of his "abduction" (also called abductive reasoning, abductive inference, or retroduction).

We have two types of logic (i.e., symbolic logic and practical logic) (*cf.* Note 1.15), i.e.,

- $\begin{cases} \text{symbolic logic (= axiomatic logic =mathematical logic)} \\ \text{practical logic (= inference)} \end{cases}$

However, in the logical worldview, we tend to confuse the two, or regard practical logic as symbolic logic.

For example, some philosophers have tried to construct worldviews using some "symbolic logic" (e.g., modal logic, temporal logic). These may be successful to some extent, but they are unlikely to become mainstream analytical philosophy.

I think that Peirce's inference can be also regarded as one of the confusions of the logical worldview.

Let us explain it as follows.

Let P="It rains", Q="the lawn in the garden gets wet." Assume that $P \to Q$ is true. Since $[[P \longrightarrow Q] \wedge P] \longrightarrow Q]$ is true (see Exercise 11.8), it may be natural to consider that

($\sharp_1$) It rained last night, so the lawn is wet this morning.

This may be correct, but what we like to think is next.

($\sharp_2$) the lawn is wet this morning, so it must have rained last night.

that is, $[[P \longrightarrow Q] \wedge Q] \longrightarrow P]$, which does not always hold in the sense of Boole's logic as seen in Exercise 11.8. Thus, Peirce wanted to propose the new symbolic logic in which $[[P \longrightarrow Q] \wedge Q] \longrightarrow P]$ is true. (He might be inspired by the success of non-Euclidean geometry.)

His idea was not successful as mathematical logic, it (= the abduction) was popularized as a "thinking tip". I think

- "If he had formulated his ideas as practical logic, he would have succeeded".

This will be discussed in Sec. 11.3.1.1 (the logical worldview) and Sec. 11.3.1.2 (the quantum mechanical worldview). And I will show that Peirce's abduction has been supported under the confusion between symbolic logic and practical logic.

11.3.1.1 Deduction, abduction and induction in "logic" (i.e., in the logical worldview)

According to Peirce, three kinds of inferences (i.e., deduction, abduction, induction) are important. Let us explain deduction, abduction and induction as follows.

The reader will be convinced that Pearce believed that "the language of science is logic."

[Deduction]

A typical example of deduction is as follows: $\Big($ In the following, (A_1') and $(\widehat{\mathrm{A}}_1')$ are often omitted. $\Big)$

(A_1) All the beans in this bag B_1 are white: [bag $\mathrm{B}_1 \longrightarrow$"$w$"($\approx$ white)]

(A_1') All the beans in that bag B_2 are white or black fifty-fifty (or generally, the ratio of white beans to black beans is $p/(1-p)$ where $0 < p < 1$): [bag $\mathrm{B}_2 \longrightarrow$"$w$"($\approx$ white) or "b"($\approx$ black)]

(A_2) This bean is from this bag B_1: [bag B_1]

(A_3) Therefore, this bean is white: ["w"($\approx$ white)]

It is, of course, obvious and ordinary.

♠**Note 11.5.** It is clear that the following is a tautology:

$(\sharp_1)$ $\qquad \big[[B_1 \longrightarrow w] \wedge [B_2 \longrightarrow [w \vee b]] \wedge B_1\big] \longrightarrow [w]$

Thus, the above conclusion (A_3) can be understood as a consequence of this tautology $(\sharp_1)$. However, this has not solved everything. We have the following problem:

$(\sharp_2)$ Is there a more natural solution than the above solution (due to the logical worldview)?

This new solution will be introduced in the following section.

[Abduction]

On the other hand, C.S, Peirce (*cf.* ref. [88]) proposed abduction. The example of abduction is as follows:

($\widehat{\mathrm{A}}_1$) All the beans in this bag B_1 are white: [bag $\mathrm{B}_1 \longrightarrow$"$w$"($\approx$ white)]

($\widehat{\mathrm{A}}'_1$) All the beans in that bag B_2 are white and black fifty-fifty (or generally, the ratio of white beans to black beans is $p/(1-p)$): [bag $\mathrm{B}_2 \longrightarrow$"$w$"($\approx$ white) or "b"($\approx$ black)]

($\widehat{\mathrm{A}}_2$) This bean (from B_1 or B_2) is white: ["w"($\approx$ white)]

($\widehat{\mathrm{A}}_3$) Therefore, this bean is from this bag B_1 : [bag B_1]

♠**Note 11.6.** It is clear that the following is not a tautology:

$$(\sharp_3) \qquad \big[[B_1 \longrightarrow w] \wedge [B_2 \longrightarrow [w \vee b]] \wedge w\big] \longrightarrow [B_1]$$

Thus, the above conclusion ($\widehat{\mathrm{A}}_3$) can not be understood since this ($\sharp_3$) is not a tautology. However, the above conclusion ($\widehat{\mathrm{A}}_3$) has a point. For simplicity, assume that $0 \le p \ll 1$ (i.e., most of the beans in the B_2 are black).

($\sharp_4$) After ($\widehat{\mathrm{A}}_2$), assume that this white bean is from the bag B_2.

If so, this is a very rare event that has happened since most of the beans in the B_2 are black. The assumption ($\sharp_4$) is unreasonable. That is, it is reasonable to consider that this white bean is from the bag B_1, namely, ($\widehat{\mathrm{A}}_3$).

Now from the theoretical point of view, we have two ways to think the above problem.

$$(\sharp_5) \quad \begin{cases} \text{①:To invent another symbolic logic in which } (\sharp_3) \text{ is derived} \\ \\ \text{②:To propose a different worldview than the logical worldview.} \\ \quad \text{And to derive } (\widehat{\mathrm{A}}_3) \text{ in the new worldview.} \end{cases}$$

I think that ① is impossible. ② will be introduced in the following section.

[Induction]

Further, induction (inductive reasoning) is as follows.

($\widetilde{\mathrm{A}}_1$) $1000p$ white beans and $1000(1-p)$ black beans are mixed well in this bag B_3 (here, $0 < p < 1$). Assume that we do not know the value p ($0 < p < 1$).

($\widetilde{\mathrm{A}}_2$) When we took 20 beans out of this bag B_3, every bean was white.

($\widetilde{\mathrm{A}}_3$) Therefore, the bean picked out from this bag B_3 next can be presumed to be white.

This will be again discussed in the following section.

11.3.1.2 Deduction, abduction and induction in quantum language (i.e., the quantum mechanical world view)

In our worldview (i.e., the quantum mechanical worldview $\approx$ the quantum linguistic worldview), the relation among deduction, abduction and abduction is characterized as follows.
First, we will show that the abduction $[(\widehat{\mathrm{A}}_1)$-$(\widehat{\mathrm{A}}_3)]$ can be justified in quantum language. Consider the state space $\Theta = \{\theta_1, \theta_2\}$ with the discrete topology, and the classical basic structure $[C(\Theta) \subseteq L^\infty(\Theta, \nu) \subseteq B(L^2(\Theta, \nu))]$, where $\nu(\{\theta_1\}) = \nu(\{\theta_2\}) = 1/2$. Assume that

$$\theta_1 \approx \text{the state of the bag B}_1, \quad \theta_2 \approx \text{the state of the bag B}_2,$$

Assume that 1000 white beans belong to bag B_1, and further, $1000p$ white beans and $1000(1-p)$ black beans belong to the bag B_2 (where $0 < p < 1$). Thus we have the observable $\mathsf{O} = (\{w, b\}, 2^{\{w,b\}}, F)$ in $L^\infty(\Theta, \nu)$ such that

$$\begin{aligned} &[F(\{w\})](\theta_1) = 1 \qquad && [F(\{b\})](\theta_1) = 0 \\ &[F(\{w\})](\theta_2) = p \qquad && [F(\{b\})](\theta_2) = 1 - p \qquad (0 < p < 1) \end{aligned}$$

where "w" and "b" means "white" and "black" respectively.
Thus, we have the measurement $\mathsf{M}_{L^\infty(\Theta,\nu)}(\mathsf{O} := (\{w, b\}, 2^{\{w,b\}}, F), S_{[\theta_i]})$, $i = 1, 2$. For example, Axiom 1 [measurement] (in Section 1.1) says that

(B_1) [measurement]: The probability that the measured value w is obtained by $\mathsf{M}_{L^\infty(\Theta,\nu)}(\mathsf{O} := (\{w, b\}, 2^{\{w,b\}}, F), S_{[\theta_1]})$ is equal to 1

This is the same as the deduction (i.e., (A_1)–(A_3)).
Next, under the circumstance that bags B_1 and B_2 cannot be distinguished, we consider the following inference problem:

($\widehat{\mathrm{B}}_2$) [inference problem]: When the measured value w is obtained by the measurement $\mathsf{M}_{L^\infty(\Theta,\nu)}(\mathsf{O} := (\{w, b\}, 2^{\{w,b\}}, F), S_{[*]})$, which do you infer, $[*] = \theta_1$ or $[*] = \theta_2$?
Fisher's maximum likelihood method Theorem 1.27 [Fisher's maximum likelihood method] says that $[*] = \theta_1$, since

$$\max\{F(\{w\})](\theta_1), F(\{w\})](\theta_2)\} = \max\{1, p\} = 1 = [F(\{w\})](\theta_1)$$

This implies $(\widehat{\mathrm{A}}_3)$.

Therefore, the above $(\widehat{\mathrm{B}}_2)$ is the quantum linguistic representation of abduction (i.e., $(\widehat{\mathrm{A}}_1)$–$(\widehat{\mathrm{A}}_3)$). For the sake of completeness, note that (B_1) and $(\widehat{\mathrm{B}}_2)$ are in reverse problem (*cf.* Remark 1.28). That is, we have the following correspondence:

$$
\begin{array}{ccc}
\begin{array}{c}(\mathrm{B}_1)\text{: measurement} \\ \text{Axiom 1 [measurement] (in Section 1.1)}\end{array} & \xrightarrow[\text{simplified form}]{} & \begin{array}{c}[(\mathrm{A}_1),(\mathrm{A}_1'),(\mathrm{A}_2) \longrightarrow (\mathrm{A}_3)] \\ \text{deduction}\end{array} \\
\Big\downarrow \text{reverse} & & \Big\downarrow \text{reverse} \\
\begin{array}{c}(\widehat{\mathrm{B}}_2)\text{: inference} \\ \text{(Fisher's maximum likelihood method)}\end{array} & \xrightarrow[\text{simplified form}]{} & \begin{array}{c}[(\widehat{\mathrm{A}}_1),(\widehat{\mathrm{A}}_1'),(\widehat{\mathrm{A}}_2) \longrightarrow (\widehat{\mathrm{A}}_3)] \\ \text{abduction}\end{array}
\end{array}
\tag{11.1}
$$

Thus, the scientific meaning of abduction can be completely clarified in the translation from logic to quantum language.

Lastly we should mention that

$(\widetilde{\mathrm{B}}_3)$ the above $(\widetilde{\mathrm{A}}_1)$-$(\widetilde{\mathrm{A}}_3)$ (i.e., inductive reasoning) are already discussed in quantum language (*cf.* Section 9.7: Hume's problem of induction).

Remark 11.9. Recall the worldviewism in Sec. 1.3.1 such as

($\sharp$): The worldviewism

The worldviewism has the following form:

$$(\sharp_1)\quad \underbrace{\overbrace{\boxed{\text{worldview}}}^{\text{world is so}}}_{\text{premise}} \xrightarrow{\text{therefore}} \underbrace{\overbrace{\boxed{\text{practical logic } (\approx \text{ inference, how to think, calculation), etc.}}}^{\text{think like this! (conclusion)}}}_{\text{subject}}$$

That is,

($\sharp_2$) **Since the world is like this, think like this!**

Note that the formula (11.1 is the typical example of the worldviewism ($\sharp_1$). That is,

[The practical logic is produced by a worldview]

$\boxed{\textbf{the quantum mechanical worldview}}$ (quantum language)	$\xrightarrow[\text{therefore}]{}$	$\boxed{\textbf{practical logic}}$
(B_1): measurement Axiom 1 [measurement] (in Section 1.1)	$\xrightarrow[\text{therefore}]{}$	$[(A_1),(A_1'),(A_2) \longrightarrow (A_3)]$ deduction
$\downarrow$ reverse		$\downarrow$ reverse
($\widehat{B}_2$): inference (Fisher's maximum likelihood method)	$\xrightarrow[\text{therefore}]{}$	$[(\widehat{A}_1),(\widehat{A}_1'),(\widehat{A}_2) \longrightarrow (\widehat{A}_3)]$ abduction

The above says that

- a lot of information is lost from the phenomenon in order to express it in logical form if it is possible.

In this sense, I think that

- logic is useful for making broad arguments.

It's common knowledge in science that, for us humans, we can't have a rigorous discussion without doing the calculation, not the logic. Of course, it should be note that the method of calculation is based on a worldview.

I'm not completely denying Peirce's abduction. In fact, I think that Detective Sherlock Holmes was a great master of Peirce's abduction.

////

♠**Note 11.7.** Philosophers may like logic (i.e., the logical worldview), thus, they may want to think that deduction, abduction and induction are kinds of logic. However, I think that the argument in Sec. 11.3.1.1 is not natural. On the other hand, we think that our quantum mechanical formulation is reasonable as follows.

$$(C_1) \qquad \underset{\text{(possible but unnatural)}}{\text{deduction}} \quad \cdots\cdots \quad \underset{\text{(possible and natural)}}{\text{measurement}}$$

$$(C_2) \qquad \underset{\text{(theoretically impossible)}}{\text{abduction}} \quad \cdots\cdots \quad \underset{\text{(possible and natural)}}{\text{inference (Fisher's maximum likelihood method)}}$$

$$(C_3) \qquad \underset{\text{(theoretically impossible)}}{\text{induction}} \quad \cdots\cdots \quad \underset{\text{(possible and natural)}}{\text{the law of large numbers}}$$

Thus, I think that Peirce's argument, from the theoretical point of view, highlighted the weaknesses of the logical worldview.

11.4 Bertrand Russell: five-minute hypothesis

Bertrand Russell (1872-1970) is a great British logician.

The naive set theory (i.e., Cantor's set theory) involves contradictions. For example, Russell showed Russell's paradox such that

- if it is assumed that $\{U \mid U \notin U\}$ is a set, then it leads contradiction.

To avoid such a paradox, Alfred North Whitehead and Bertrand Russell proposed "type theory" (i.e., a kind of axiomatic set theory), which was published as the Principia Mathematica on the foundations of mathematics in 1910–1913. Also, the axiomatic system of set theory was developed by Zermelo and others. Most modern mathematicians study mathematics developed under a system of axioms called ZFC (1921), which consists of eight axioms by Zermelo and Fraenkel plus an axiom called the Axiom of Choice.

Also, Russell was a great intellectual and one of the founders of analytic philosophy along with his predecessor Gottlob Frege (and student Ludwig Wittgenstein). From the analytic philosophical point of view, his greatest achievement may have been the production of the "genius Wittgenstein".

I know almost nothing about Russell's philosophy by skipping over his book [96]. Thus, I can understand the mathematician Russell, but not the philosopher Russell. Therefore, I can't speak for Russell's philosophy

I will mention "Five-minute hypothesis" in Sec. 11.4.1 and "McTaggart's paradox" in Sec. 11.4.2. Also, see ref. [99] for the delicate relationship between Russell and McTaggart.

11.4.1 * Five-minute hypothesis

This section was written with reference to the following.

- [68]:Ishikawa, S: Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language Open Journal of philosophy, Vol. 8, No.5 , 466-480, 2018, DOI: 10.4236/ojpp.2018.85032 (`https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862`)

  [Revised version] (`https://philpapers.org/rec/ISHLCB`) (`http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf`)

As mentioned frequently in this text, I think that mathematical logic is one of fields of mathematics and thus, Russell's study of logic has little to do with traditional Western philosophy (i.e., Descartes-Kant philosophy). Thus, in this text I am not interested in Russell's logical work.

The five-minute hypothesis is a skeptical hypothesis put forth by the philosopher Bertrand Russell. However, as seen later, I do not think that this hypothesis is not related to skepticism though my understanding to skepticism may be insufficient. The five-minute hypothesis, proposed by B. Russell (*cf.* ref. [97]), is as follows.

(A_1) The universe was created five minutes ago. Or equivalently, the universe was created ten years ago.

Now we show that this (A_1) is not the statement in quantum language as follows (i.e., The first answer (i) and the second answer (ii))

Answer:
Note that this hypothesis (A_1) is related to "tense". Thus, the linguistic Copenhagen interpretation (E_2) in Sec. 1.1.2 says that this (A_1) is not a statement in quantum language. Thus, the (A_1) is not scientific, that is, there is no experiment to verify the statement (A_1).

Some may want to relate this hypothesis to skepticism (*cf.* ref. [97]), However we do not think that this direction is productive.

Remark 11.10. Also, the above (A_1) should be compared to the following (A_2)

(A_2) The universe was created in A.D. 2010. (Or equivalently, now is A.D. 2020, and the universe was created ten years ago.)

This (A_2) can be denied by experiment, that is, it is different from the fact. Thus, this is a proposition in quantum language.

11.4.2 * McTaggart's paradox

This section was written with reference to the following.

- [68]:Ishikawa, S: Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language Open Journal of philosophy, Vol. 8, No.5 , 466-480, 2018, DOI: 10.4236/ojpp.2018.85032 (`https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862`)

[Revised version] (https://philpapers.org/rec/ISHLCB)
(http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf)

J.M.E. McTaggart (1866–1921) was an English philosopher. He was a member of the Cambridge Apostles, along with B. Russell (1877-1970).

In ref. [82], McTaggart asserted ”the Unreality of Time” as follows.

The sketch of McTaggart's proof

(B_1) Assume that there are two kinds of times. i.e., ”observer's time (A-series)” and ”objective time (B-series)”. (Note that this assumption is against the linguistic Copenhagen interpretation (E_2) in Sec. 1.1.2.)

(B_2) $\cdots\cdots$

(B_3) After all, the contradiction is obtained

Therefore, by the reduction to the absurd (i.e., the proof by contradiction), we get;

(B_4) A-series does not exist (in science).

About this proof, there are various opinions also among philosophers. Although I cannot understand the above part (B_2) (since the properties of A-series are not clear), I agree to him if his assertion is (B_4) (*cf.* ref. [43]). That is, I agree that McTaggart noticed first that observer's time is not scientific.

Recall the linguistic Copenhagen interpretation (E_2) in Sec. 1.1.2:

- (i.e.,(E_2) in Sec. 1.1.2): While ”matter” is in the space-time, the observer is not.

Thus, I agree to the opinion that McTaggart is one of discoverers of the linguistic Copenhagen interpretation. I think, from the quantum linguistic point of view, that he should be estimated more highly.

11.5 Saussure: Copernican revolution in language

Ferdinand de Saussure (1857-1913) was a Swiss linguist. He is widely considered one of the founders of 20th-century linguistics.

11.5.1 Saussure's linguistics: What comes first, things or words?

Let's think a little more about the implications of Saussureian linguistics. We tend to think that there are things at the beginning, and that we give each thing a name, just like we put a label on it. However, that is not the case. Rather, Saussureian linguistics says that we understand the order of things by the act of giving them names. I think that his theory is almost the linguistic version of the Copernican revolution of Kant. That is,

$$\underset{\text{copy theory}}{\boxed{\text{things} \longrightarrow \text{words}}} \xrightarrow[\text{Copernican revolution in language}]{\text{Saussure's discovery}} \overset{\text{constructionism}}{\underset{\text{Saussure's linguistics}}{\boxed{\text{words} \longrightarrow \text{things}}}} \tag{11.2}$$

For example,

(A_1) I, who live in Japan, an island nation, know the names of many fish, but rarely distinguish between "cow, bull, ox, calf," etc.

(A_2) Also, in Japan, the rainbow has seven colors (i.e., red, orange, yellow, green, blue, indigo, violet), and I was surprised when I first heard that there is a country where the rainbow does not have seven colors.

(A_3) Also, I think that there is a possibility that quantum language changes our scientific worldview.

In the above, I'd like to assume that (A_1)$\sim$(A_3) are about the same.

11.5.2 Several Copernican revolutions

Saussure's linguistics bridges Kant and quantum language as follows.

Assertion 11.11. Summing up, we can write as follows.

$$
\begin{tikzcd}
\underset{\text{Descartes=Locke}}{\overset{\text{copy theory}}{\boxed{\text{the world} \longrightarrow \text{cognition}}}} \arrow[r, "\text{Copernican revolution in cognition}", "\textcircled{1}"'] \arrow[d] & \underset{\text{Kant}}{\overset{\text{constructionism}}{\boxed{\text{cognition} \longrightarrow \text{the world}}}} \arrow[d, "\textcircled{2}"', "\text{linguistic turn}\ {\scriptstyle(\text{Wittgenstein})}"] \\
\overset{\text{copy theory}}{\boxed{\text{things} \longrightarrow \text{words}}} \arrow[r, "\text{Copernican revolution in language}"] \arrow[d] & \underset{\text{Saussure's linguistics}}{\overset{\text{constructionism}}{\boxed{\text{words} \longrightarrow \text{things}}}} \arrow[d, "\textcircled{3}"', "\text{mechanical turn}"] \\
\underset{\text{quantum mechanics (=QM)}}{\overset{\text{copy theory}}{\boxed{\text{phenomena} \longrightarrow \text{QM}}}} \arrow[r, "\text{Copernican revolution in mechanics}"] & \underset{\text{quantum language (=QL)}}{\overset{\text{constructionism}}{\boxed{\text{QL} \longrightarrow \text{phenomena}}}}
\end{tikzcd}
\tag{11.3}
$$

Therefore, I think that

- **"The Copernican revolution (due to Kant)" is the greatest discovery in the history of philosophy.**

  That is,

  $$\textbf{the Copernican revolution} \quad = \quad \textbf{the discovery of "true idealism"}$$

  where we think that "true idealism"="transcendental idealism".

♠**Note 11.8.** As mentioned in Note 10.7, Kant named his discovery the Copernican revolution. Readers may ask, "Why did honest Kant made such an exaggerated description?" It is a matter of course that Kant had great confidence such that it was the most greatest discovery in the history of philosophy. I believe that Kant said "the Copernican revolution" because Kant himself thought it was the greatest discovery in the history of philosophy. This is by no means Kant's big talk. I think that his confidence is realized in Assertion 11.11.

Answer 11.12. [The answer of Problem 10.6: Is "From Descartes to Kant" progress?]
Now we can answer Problem 10.6, i.e.,

- $$\underset{\text{(Descartes, Locke, Leibniz, Hume)}}{\boxed{\text{Descartes philosophy}}} \xrightarrow[\text{progress}]{} \underset{\text{(Kant)}}{\boxed{\text{Kant philosophy}}} \quad ?$$

The above formula (11.3) in Assertion 11.11 says that

$$\boxed{\text{Descartes}} \xrightarrow[\textcircled{1}\text{ in (11.3)}]{} \boxed{\text{Kant}} \xrightarrow[\textcircled{2}\text{ in (11.3)}]{} \boxed{\text{Saussure}} \xrightarrow[\textcircled{3}\text{ in (11.3)}]{} \boxed{\text{QL}}$$

Therefore, we can be convinced that

$$\underset{\text{(Descartes, Locke, Leibniz, Hume)}}{\boxed{\text{Descartes philosophy}}} \xrightarrow[\text{progress}]{\text{Copernican revolution}} \underset{\text{(Kant philosophy)}}{\boxed{\text{Kant}}}$$

$$\xrightarrow[\text{progress}]{\text{Linguistic turn}} \underset{\text{(Saussure linguistics)}}{\boxed{\text{linguistics}}} \xrightarrow[\text{progress}]{\text{mechanical turn}} \boxed{\text{QL}} \tag{11.4}$$

where $\xrightarrow[\text{progress}]{}$ means "getting closer to quantum language". $\square$

Remark 11.13. I will restate what I wrote at the beginning of this chapter as follows.

$$(\flat_{22}) \begin{cases} (\flat_{22}^1) & \text{foundation of math.} \quad \text{(Boole, Frege, Russel, Cantor, Zermelo, Fraenkel, Gödel),...,} \\ (\flat_{22}^2) & \text{math.} \quad \text{(Bourbaki),} \\ (\flat_{22}^3) & \text{analytic philosophy, symbolic logic} \quad \text{(Frege, Russell, Wittgenstein,...)} \\ (\flat_{22}^4) & \text{scientific philosophy} \quad \text{(Hempel, Popper, ...)} \\ (\flat_{22}^5) & \text{linguistics, ordinary language} \quad \text{(Saussure, Wittgenstein (except logic))} \end{cases}$$

If so, the above formula (11.4) can be rewritten more precisely as follow:

$$(\sharp_2) \quad \boxed{\text{Descartes=Kant}} \xrightarrow[\text{progress}]{} \boxed{\text{linguistic phil. } (\flat_{22}^3), (\flat_{22}^4), (\flat_{22}^5)} \to \boxed{\text{Quantum language}}$$

ⓐ: $(\flat_{22}^5)$:Saussure (Remark 11.12)
ⓑ: $(\flat_{22}^5)$: Wittgenstein (Sec. 11.6.3)
ⓒ: $(\flat_{22}^4)$:Hempel (Remark 11.42)

ⓑ and ⓒ will be discussed later.

11.5.3 * The quantitative expressions of "signifier" and "signified"

Now let us explain the terms "signifier" and "signified", which were introduced by Saussure.

Definition 11.14. ["signifier" and "signified" in ordinary language]
For example, we explain "signifier" and "signified" concerning "dog"

(B_1) The "dog" in front of you is itself a physical being, isn't it? The image of the dog you have in mind, the barking, or the image of the dog in your head, is "signified".

(B_2) When this becomes a word (letter/sound) such as "dog", it is called "signifie".

Although Saussure's proposal (i.e., the above definition) is very significant, this violates the worldviewism (*cf.* Sec. 1.3.1). As emphasized frequently throughout the paper, I consider

(C) Without a worldview, we can't say anything solid

In fact, Saussure's idea does not play an important role in science.
Thus, I propose Definition 11.14, in which Saussure's idea is realized in quantum language as follows.

Definition 11.15. [Membership function (= Fuzzy set, *cf.* [35, 36, 37, 38, 109])] Let Ω be a state space. A continuous function $m : \Omega \to [0,1]$ (i.e., the closed interval in the real $\mathbb{R}$) is called a membership function. Assume that the state (i.e., quantitative property) of any animal can be expressed by a point in the state space Ω. Define the membership functions $m_D : \Omega \to [0,1]$ of dogs as follows. Suppose that there are 100 zoologists and the following question is made them.

(D) Is this animal with the sate ω_1 ($\in \Omega$) a dog or not?

The answer is as follows.

$$\text{(E)} \left\{ \begin{array}{l} \text{70 zoologists say that this bird is a dog.} \\ \text{30 zoologists say that this animal is not a dog.} \end{array} \right.$$

Then the value of $m_D(\omega_1)$ is defined by 0.7. For many animals with the state ω_i ($i = 2, 3, ...N$), repeating the experiment in the same way, the value of $m_D(\omega_i)$ ($i = 2, 3, ...N$) is determined. And the membership function $m_D : \Omega \to [0,1]$ of dogs is defined by the interpolation method (which may be rather subjective). Put $\Omega_D = \{\omega \in \Omega \mid m_D(\omega) = 1\}$, which is called the dog state class.
Also, for the probabilistic interpretation of a membership function, see Interpretation 11.39.

With the above preparation, we get the following definitions.

Definition 11.16. ["signifier" and "signified" in quantum language]

(F_1) "signifier"——"dog"

(F_2) "signified"——$m_D : \Omega \to [0,1]$ (or roughly, Ω_D)

The above will play an essential role in Hempel's raven prpblem in Sec. 11.8.

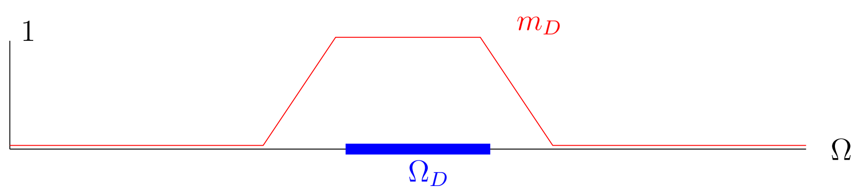
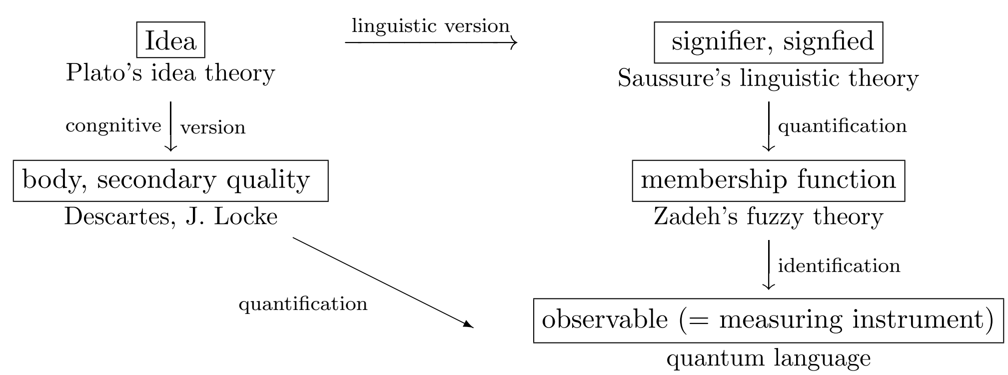


Figure 11.1: $[m : \Omega \to [0,1]$, and $\Omega_D]$

♠**Note 11.9.** (i); The above definition is essential to the solution of Hempel raven problem. See Section 11.8

(ii): From the quantum linguistic point of view, As mentioned in Remark 3.12, Saussure's theory is located in the following.

- 

Some readers may disagree with me about Plato's Idea, as I've made a rather dogmatic assumption about it. However, I like the diagram above because it makes me feel like I understand the history of Western philosophy as the history of dualistic idealism.

11.6 Linguistic turn: Wittgenstein

Ludwig Wittgenstein (1889 - 1951), who was the student of B. Russel, was the most famous philosopher (in linguistic philosophy) in the 20th century.

His position is as follows.

Simbolic logic, Set theory

ⓐ : Gottlob Frege (predicate logic) ⟶ B. Russell (Type theory)

ⓑ : Saussure: natural language (Linguistics)

$\xrightarrow{\text{philosophy}}$

L. Wittgenstein
ⓐ:Tractatus Logico-Philosophicus (=TLP) (1921)
ⓑ:Philosophical Investigations (1953)
the logical worldview

That is,

(A) Frege and Russell asserted that symbolic logic is the foundation of mathematics. Also, they wanted to assert that sybbolic logic is the foundation of science.

(B) Saussure discovered the Copernican revolution in linguistics

(C) Wittgenstein asserted that "logic" is the foundation of science (*cf.* Sec. 11.5.2).

However, I think that Wittgenstein's attempt was a social success, but it may not have been a success in theory. That is because it is ambiguous whether his "logic" is symbolic logic or practical logic. This will be discussed in what follows.

11.6.1 * My scientific understanding of "Tractatus Logico-Philosophicus (=TLP)"

In the preface of his book "Tractatus Logico-Philosophicus", L. Wittgenstein said that

(D) *This book will perhaps only be understood by those who have themselves already thought the thoughts which are expressed in it — or similar thoughts.*

This is a very significant sentence for me. That is because the (D) implies that, I can understand his theory if what I have said in Sec 4.3.5 is "similar thoughts".

♠**Note 11.10.** "Tractatus Logico-Philosophicus (= TLP)" is a curious book, one of the leading philosophy books of the 20th century. It's a book that encourages people to "be logical," yet it's not written logically. Furthermore, probably not a single reader has read this book logically. No reader has understood this book, but it is a philosophical book that everyone is convinced that it must have some great things to say. Recently, some people have even declared this theory to be nonsensical (*cf.* refs. [12, 78]). However, this happens frequently in the history of philosophy (*cf.* Note 3.10). It is not surprising, for example, that both "Discourse on the Method" and "Critique of Pure Reason" are the same kind of thing. But something surprising happens in TLP. In fact, TLP can be read really logically, as I will write below, as I will write below. In this sense TLP is groundbreaking.

As mentioned in Remark 11.25 later, Wittgenstein's picture theory is only a theory of Venn diagram in a logical space (= set), which is the most elemental part in naive set theory. However, since my purpose is to compare his theory and quantum language, I will explain his theory by comparing it to quantum language.

Now let us review Sec.4.3.5: "**[In a class of classical binary projection measurements, measurement has properties like propositional logic]**", using the word "proposition" (without using the word "measurement").

Table 11.17. [Logic in quantum language (Sec. 4.3.5) vs. logic in Wittgenstein's picture theory (this section)]

Table: [Proposition is a kind of measurement]

Logic in quantum language (Sec. 4.3.5)	Logic in Wittgenstein's theory (Sec. 11.6.1)
Axiom 1 in Section 1.1 the linguistic Copenhagen interpretation	Definition 11.19 (Naive set theory?)
system, particle, object, the world, tomato	object, thing, the world, tomato
state space (state)	logical space (case, fact,, atomic fact)
measured value $\{1,0\}$	truth value $\{\mathrm{T},\mathrm{F}\}$
binary projection measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma \equiv (X(=\{1,0\}),2^X,F^\Gamma),S_{[\widehat{\omega}(t)]})$	proposition $\mathsf{P}_\Omega(\Gamma,S_{[\widehat{\omega}(t)]})$
Theorem 4.8 (Syllogism in measurements)	Theorem 11.21 (Syllogism in propositions)
elemetary measurement	elementary proposition
Theorem 4.10 Elementary measurements are not fundamental	Theorem 11.24 Elementary propositions are fundamental

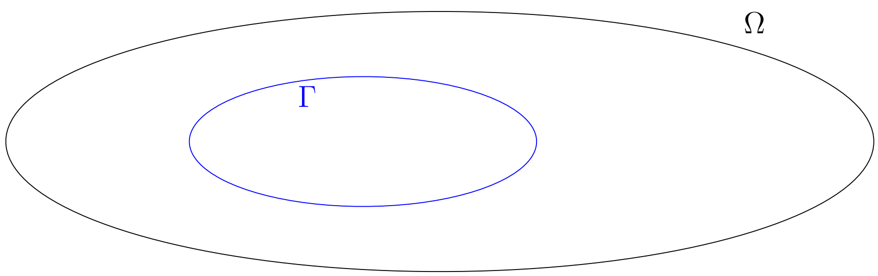
I encourage you to read the following, referring to the above table. If you understand Sec. 4.3.5, you should be able to understand this section in a few minutes. The terms and concepts are not ambiguous in Wittgentein's theory. Using the above table, we can translate the language of Wittgenstein's theory into quantum language.

////

Seeing Table 11.17, assume that $(\Omega,\nu),\Gamma,\widehat{\omega}(t),\mathrm{SW},\mathrm{RP},\mathrm{RD}$, ..are the same as in Sec. 4.3.5. That is, Ω is a locally compact space with a Borel measure ν on Ω such that $\nu(D)>0$ (for any open set $D(\neq\emptyset)$ $(\subseteq\Omega)$). Here, note that we can assume that Ω is compact without loss of generality (*cf.* Stone-Čech compactification; ref. [15]). And, $\Gamma (\subseteq \Omega)$ is an open set such that

$$\Gamma = [\overline{\Gamma}]^\circ, \qquad \nu(\Omega \setminus (\Gamma \cup [\Gamma^c]^\circ)) = 0$$

where D^c is the complement of D, i.e., $\Omega \setminus D$, $\overline{D}$ ="the closure of D", D° ="the interior of D)". where D^c is the complement of D, i.e., $\Omega \setminus D$, $\overline{D}$ ="the closure of D", D° ="the interior of D)". Note that we can assume that $\Gamma^c = [\Gamma^c]^\circ = [\overline{\Gamma}]^c$ in the sense of "almost everywhere concerning ν".



Consider many tomatoes, that is, roughly speaking, consider T as the set of all tomatoes. Assume that any tomato $t (\in T)$ is represented by a state ω, which is an element of the logical space Ω ($\Leftrightarrow$state space Ω in Sec. 4.3.5). Thus, we have the injective map $\widehat{\omega} : T \to \Omega$. That is, the quantitative property of a tomato t is represented by $\omega(t)$. For example, it suffices to consider Ω as $\mathbb{R}^N$ (= N-dimensional real space), where N is sufficiently large natural number (or, $N = \infty$). That is,

$$\begin{aligned}\Omega \ni \omega = \Big(& \omega^{(1)}(= \text{weight}), \omega^{(2)}(= \text{diameter}), \omega^{(3)}(= \text{diameter}), \omega^{(4)}(= \text{color value}), \\ & \omega^{(5)}(= \text{calorie}), \omega^{(6)}(= \text{sugar content}), ..., \omega^{(N)}(= ...) \Big) \in \mathbb{R}^N\end{aligned}$$

♠**Note 11.11.** (=Note 4.7); Someone might say that the term "the set of all tomatoes" is ambiguous. However, for the sake of convenience, here we use the term "the set of all tomatoes". In Sec. 11.8 [Hempel's raven problem], we will discuss it precisely.

Remark 11.18. In Sec. 4.3.5, the binary projection measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma, S_{[\widehat{\omega}(t)]})$ follows Axiom 1 (and the linguistic Copenhagen interpretation). On the other hand, the proposition $\mathsf{P}_\Omega(\Gamma, S_{[\omega]})$ is not related to Axiom 1. Thus, we need the definition below.

////

Definition 11.19. A pair (Γ, ω) $(\Gamma \subseteq \Omega, \omega \in \Omega)$ is called a proposition (i.e., "a system with a state ω has *Gamma*-property"), which is denoted by $\mathsf{P}_\Omega(\Gamma, S_{[\omega]})$ (*cf.* a binary projection measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma, S_{[\omega]})$) in Sec. 4.3.5).

Define as follows.

(E$_1$) a truth value of the proposition $\mathsf{P}_\Omega(\Gamma, S_{[\omega]}) = \begin{cases} \text{T (true)} & (\text{ if } \omega \in \Gamma) \\ \text{F (false)} & (\text{ if } \omega \notin \Gamma) \end{cases}$

That is, truth values $\{\mathrm{T}, \mathrm{F}\}(= X)$ correspond to measured values $\{1, 0\}(= X)$ (or, $X = \{x_1, x_0\}$).

If the true value of $\mathsf{P}_\Omega(\Gamma, S_{[\widehat{\omega}(t)]})$ is equal to "T", we may say that

(E$_2$) a tomato t has Γ-property.

////

11.6.1.1 Classical logic (i.e., not, and, or, implication)

Consider a proposition $\mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})$. Definition 11.18 says that the following three are equivalent (i.e., (I$_1$) $\Leftrightarrow$ (I$_2$). Also, (I$_3$) is the expression of (I$_1$) in ordinary language):

(I$_1$) A truth value of the proposition $\mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})$ is equal to "T".

(I$_2$) $\widehat{\omega}(t) \in \mathrm{RD}$

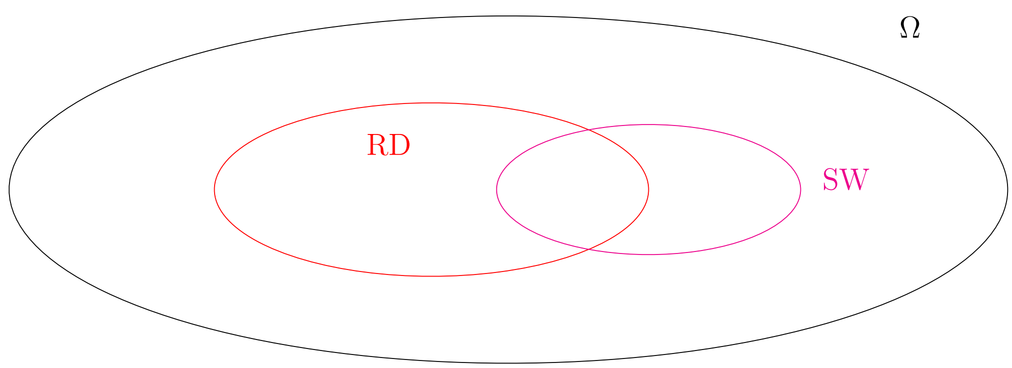
(I$_3$) A tomato t is "red".

Similarly, consider a proposition $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})$. Thus, we see that the following three are equivalent:

(J$_1$) A truth value of the proposition $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})$ is equal to "T".

(J$_2$) $\widehat{\omega}(t) \in \mathrm{SW}$

(J$_3$) A tomato t is "sweet".



[Not]

It is clear that the following four are equivalent:

(K_1) A truth value of the proposition $\mathsf{P}_\Omega(\mathrm{SW}^c, S_{[\widehat{\omega}(t)]})$ is equal to "T"
(Here, $\mathsf{P}_\Omega(\mathrm{SW}^c, S_{[\widehat{\omega}(t)]})$ is also denoted by $\neg\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]})$),

(K_2) $\widehat{\omega}(t) \in [\mathrm{RD}]^c$

(K_3) A tomato t is not "red".

[And]

We see that the following three are equivalent:

(L_1) A truth value of the proposition $\mathsf{P}_\Omega(\mathrm{SW} \bigcap \mathrm{RD}), S_{[\widehat{\omega}(t)]})$ is equal to "T" (which is also denoted by $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]}) \bigwedge \mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})$)

(L_2) $\widehat{\omega}(t) \in \mathrm{RD} \bigcap \mathrm{SW}$

(L_3) A tomato t is "red" and "sweet"

[Or]

We see that the following three are equivalent:

(M_1) A truth value of the proposition $\mathsf{P}_\Omega(\Theta_\vee(\mathrm{SW} \bigcup \mathrm{RD}), S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]}) \bigvee \mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})$) is equal to "T"

(M_2) $\widehat{\omega}(t) \in \mathrm{RD} \bigcup \mathrm{SW}$

(M_3) A tomato t is "red" or "sweet"

[Implication]

We see that the following three are equivalent:

(N_1) A truth value of the proposition $\mathsf{P}_\Omega(\mathrm{SW}^c \bigcup \mathrm{RD}, S_{[\widehat{\omega}(t)]})$ (which is also denoted by $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{P}_\Omega(\mathrm{RP}, S_{[\widehat{\omega}(t)]})$) is equal to "T",

(N_2) $\widehat{\omega}(t) \in \mathrm{SW}^c \bigcup \mathrm{RD}$

(N_3) a tomato t is not "sweet", or it is "red"

Remark 11.20. It is clear that logical operation (Boolean algebra; $[\neg, \wedge, \vee, \rightarrow]$) holds in the above.

11.6.1.2 Syllogism

Further, consider a proposition $\mathsf{P}_\Omega(\mathsf{O}^{\mathrm{RP}}, S_{[\widehat{\omega}(t)]})$. That is, we consider that the following three are equivalent:

(O_1) A truth value of the proposition $\mathsf{P}_\Omega(\mathsf{O}^{\mathrm{RP}}, S_{[\widehat{\omega}(t)]})$ is equal to "T".

(O_2) $\widehat{\omega}(t) \in \mathrm{RP}(\equiv \{\omega \in \Omega \mid [F^{\mathrm{RP}}(\{\mathrm{T}\})](\omega) = 1\})$

(O_3) A tomato t is "ripe".

Theorem 11.21. [Syllogism]:
Let t be a tomato, and let $\widehat{\omega}(t)(\in \Omega)$ be the state of t. Assume the followings:

(P_1) $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{P}_\Omega(\mathrm{RP}, S_{[\widehat{\omega}(t)]})$

which is equivalent to

(P_2) $[\widehat{\omega}(t) \notin \mathrm{SW}] \vee [\widehat{\omega}(t) \in \mathrm{RP}]$

(P_3) A tomato t is not "sweet", or it is "ripe".

And further, assume

(P'_1) $\mathsf{P}_\Omega(\mathrm{RP}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{P}_\Omega(\mathrm{RD}, S_{[\widehat{\omega}(t)]})$

which is equivalent to

(P'_2) $[\widehat{\omega}(t) \notin \mathrm{RP}] \vee [\widehat{\omega}(t) \in \mathrm{RD}]$

(P'_3) A tomato t is not "ripe", or it is "red".

Then the following holds:

(Q_1) $\mathsf{P}_\Omega(\mathrm{SW}, S_{[\widehat{\omega}(t)]}) \longrightarrow \mathsf{P}_\Omega(\mathrm{RP}, S_{[\widehat{\omega}(t)]})$

(Q_2) $[\widehat{\omega}(t) \notin \mathrm{SW}] \vee [\widehat{\omega}(t) \in \mathrm{RD}]$

(Q_3) A tomato t is not "sweet", or it is "red".

[Proof using Definition 11.19]

A simple calcultion shows that

$$
\begin{aligned}
&[\mathrm{SW}^c \cup \mathrm{RP}] \cap [\mathrm{PR}^c \cup \mathrm{RD}] = [\mathrm{SW}^c \cap \mathrm{PR}^c] \cup [\mathrm{SW}^c \cap \mathrm{RD}] \cup [\mathrm{RP} \cap \mathrm{PR}^c] \cup [\mathrm{RP} \cap \mathrm{RD}] \\
=&[\mathrm{SW}^c \cap \mathrm{PR}^c] \cup [\mathrm{SW}^c \cap \mathrm{RD}] \cup [\mathrm{RP} \cap \mathrm{RD}] \subseteq \mathrm{SW}^c \cup \mathrm{RD}
\end{aligned}
$$

Hence, recalling (P_2) and (P'_2), we immediately see (Q_2). and thus, (Q_1), (Q_3).

□

♠**Note 11.12.** Note that the proof of this theorem (due to Definition 11.19 is simple compared to Theorem 4.8 (using Axiom 1 and the linguistic Copenhagen interpretation in Sec. 1.1.2). That is, in the proof of Theorem 11.21, we do not need to check the existence and uniqueness of the simultaneous observable $\mathsf{O}^{\rm SW} \times \mathsf{O}^{\rm RD} \times \mathsf{O}^{\rm RP}$,

The following exercise promote the reader's understanding of "Proposition".

Exercise 11.22. Let $\Gamma \subseteq \Omega$ and $\Lambda \subseteq \Omega$. Then, we have the following question.

- Is the statement "$\Lambda \subseteq \Gamma$" a proposition?

[Answer]: Inspired by Remark 4.6 (due to the linguistic Copenhagen interpretation: "only one measurement is permitted"), we get the following discussion:

$$
\begin{aligned}
&\Lambda \subseteq \Gamma \\
\Longleftrightarrow & (\forall \lambda \in \Lambda)[\lambda \in \Gamma] \\
\Longleftrightarrow & (\omega_\lambda)_{\lambda \in \Lambda} \in \bigtimes_{\lambda \in \Lambda} \Gamma_\lambda \qquad (\text{where } \omega_\lambda := \lambda \ (\forall \lambda \in \Lambda)), \Gamma_\lambda := \Gamma \ (\forall \lambda \in \Lambda)) \\
\Longleftrightarrow & \mathsf{P}_{\Omega^\Lambda}[\bigtimes_{\lambda \in \Lambda} \Gamma_\lambda, S_{[(\omega_\lambda)_{\lambda \in \Lambda}]}] \qquad (\text{where } \Omega^\Lambda \text{ is the } \Lambda\text{-dimensional produc space})
\end{aligned}
$$

Thus, Definiytion 11.19 says that the statement "$\Lambda \subseteq \Gamma$" is a proposition. Also, note that this problem is closely related to Hempel's raven problem (*cf.* Sec. 11.8). □

11.6.1.3 Elementary proposition

Consider the state space Ω, which is finite (or, countable) with a metric d (i.e., $d(\omega_1, \omega_2) = 1 \ (\omega_1 \neq \omega_2), = 0 \ \ (\omega_1 = \omega_2)$. Further, assume that the Borel measure ν is defined by the point measure, i.e., $\nu(\{\omega\}) = 1 \ \ (\forall \omega \in \Omega)$.

Definition 11.23. [elementary proposition] Let λ be any element of Ω. Putting $\Gamma = \{\lambda\}$, define the proposition $\mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]})$, which is called an elementary proposition ($\Leftrightarrow$ elementary measurement in Sec. 4.3.5).

////

It is clear that it holds that

(R$_1$) A truth value of the elementary proposition $\mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]})$ is equal to "T"
$\Longleftrightarrow \lambda = \omega$

(R_2) A truth value of the elementary proposition $\mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]})$ is equal to "F"
$\Longleftrightarrow \lambda \neq \omega$

Theorem 11.24. Let Γ be a subset of Ω. And let $\omega \in \Omega$. Then we see that

$$\mathsf{P}_\Omega(\Gamma, S_{[\omega]}) = \bigvee_{\lambda\in\Gamma}\mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]})$$

That is, every proposition can be represented by the sum of elementary propositions. This is not trivial since Exercise 11.22 is not trivial.

[Proof].
We see that

$$\begin{aligned}
&\text{The true value of } \mathsf{P}_\Omega(\Gamma, S_{[\omega]}) \text{ is equal to "T"}\\
\Longleftrightarrow &\omega \in \Gamma\\
\Longleftrightarrow &\exists\lambda(\in \Gamma)[\omega \in \{\lambda\}]\\
\Longleftrightarrow &\text{The true value of } \bigvee_{\lambda\in\Gamma}\mathsf{P}_\Omega(\{\lambda\}, S_{[\omega]}) \text{ is equal to "T"}
\end{aligned}$$

□

♠**Note 11.13.** The above is the main assertion in Wittgenstein's theory. However many readers may not consider the above theorem to be particularly important. I have the same view. In fact, this theorem holds only in special cases such as Ω has a discrete topology. Of course, the spirit of expressing complex observables in simple observables is quite important. In quantum language, this spirit is realized by von Neumann's spectral decomposition theorem (and Holevo's theorem), that is, "Any observable can be composed of binary projective observables", *cf.* [29, 103].

Remark 11.25. In this section, I was devoted to understanding Wittgenstein's theory in comparison to quantum language. However, it is more easy to understand Wittgenstein's theory in comparison to naive set theory as follows.

(S_1) a set (logical space) Ω is not "a collection of things"" but "a collection of facts (states)"

(S_2) A proposition P is defined by (ω, Γ) $\Big(\in \Omega \times 2^\Omega\Big)$.

(S_3) "A truth value of a proposition $P(= (\omega, \Gamma))$" $= \left\{\begin{array}{ll} \text{T} & (\omega \in \Gamma) \\ \text{F} & (\omega \notin \Gamma)\end{array}\right.$

This is all that matters. It is easy to understand Theorems 11.21 and 11.24 in the set theoretical formulation. In summary, I can say the following.

(T) "Wittgenstein's picture theory" = "The theory of Venn diagram in logical space Ω"

Thus, if he had used the element symbol "$\in$" (or "Venn diagram") in "Tractatus Logico-Philosophicus", this book would have been easily understood by everyone. He may have thought that using set theory to conclude that "logic is important" would be a circular argument. However, if his theory cannot be expressed clearly using mathematics, then I think his theory is "what we cannot speak about".

♠**Note 11.14.** (i): Wittgenstein's saying "The limits of my language mean the limits of my world is very famous. However, if so, quantum mechanical world is out of his world. That is because the argument in this section is restricted to classical measurements and not quantum measurements. However, his world is not so small. In fact, the world of equilibrium statistical mechanics fits best into his world (Ref. [46]).

(ii): TLP ends with the following sentence:

7 *"What we cannot speak about we must pass over in silence"*

Just before this, it says

6.54 *"He must, so to speak, throw away the ladder, after he has climbed up on it"*

I thought it was an impressive ending that could be interpreted in a variety of ways.

11.6.2 Which is more essential, quantum language or Wittgenstein's picture theory?

Now let's compare the discussion in Sec. 4.3.5 with the discussion in Sec. 11.6.1, that is,

Logic in quantum language (Sec. 4.3.5)	Logic in Wittgenstein's theory (Sec. 11.6.1)
Axiom 1 in Section 1.1 the linguistic Copenhagen interpretation	Definition 11.19 (Naive set theory?)
system, particle, object, the world, tomato	object, thing, the world, tomato
state space (state)	logical space (case, fact,, atomic fact)
measured value $\{1,0\}$	truth value $\{\mathrm{T},\mathrm{F}\}$
binary projection measurement $\mathsf{M}_{L^\infty(\Omega,\nu)}(\mathsf{O}^\Gamma \equiv (X(=\{1,0\}),2^X,F^\Gamma),S_{[\widehat{\omega}(t)]})$	proposition $\mathsf{P}_\Omega(\Gamma,S_{[\widehat{\omega}(t)]})$
Theorem 4.8 (Syllogism in measurements)	Theorem 11.21 (Syllogism in propositions)
elementary measurement	elementary proposition
Theorem 4.10 Elementary measurements are not fundamental	Theorem 11.24 Elementary propositions are fundamental

The most significant difference of the two is that

(A) the argument in Sec. 4.3.5 (Quantum language) is based on Axiom 1 and the linguistic Copenhagen interpretation (in Sec 1.1), which is connected to the world. Quantum language asserts that

- **in a class of classical binary projection measurements, measurement has properties like propositional logic**,

thus,

- measurement is the foundation of science, and logic is the foundation of mathematics

On the other hand,

(B) the argument in Sec. 11.6.1 (Wittgenstein's picture theory) is based on Definition 11.19, which is essentially the same as naive set theory (as discussed in Remark 11.25). Thus, his theory cannot be regarded as disconnected from the world as mathematics is. Wittgenstein's picture theory asserts that

- logic is the foundation of science as well as mathematics.

which is an idea inherited from Frege and Russell. That is, Wittgenstein's picture theory made Russell and Frege's dream come true.
(Whether their dream was on target is another matter. To be clear, Wittgenstein had replaced the importance of measurement (="there is no science without measurement") for the importance of logic (="there is no science without logic") .)

That is, I think that Wittgenstein's theory is, in a broad sense, a kind of mathematical theory, and thus, it cannot be regarded as a kind of worldview. If we have to use the word "worldview", $\Big($ We may sometimes call it the logical worldview (or, set theoretical worldview). But this is habitual, not formal.$\Big)$ Without the worldview (i.e., the world description method), logic (or mathematics) alone cannot produce anything but mathematical theory. I think that

(C) Quantum language has the power to solve philosophical unsolved problems, but Wittgenstein theory does not.

This has been shown throughout this text as follows.

List 11.26. (= List 1.6)

(D_1) **The list of quantum linguistic answers for philosophical unsolved problems**

- What is probability (or, measurement, causality) ? *cf.* Sec. 1.1.1)
- Zeno paradox (Flying arrow), (*cf.* Sec. 2.4.2)
- Zeno paradox (Achilles and a tortoise), (*cf.* Sec. 2.4.3)
- the measurement theoretical understanding of Plato's allegory of the sum , (*cf.* Sec. 3.3.2)
- Plato's Idea theory≈Zadeh's fuzzy theory≈Sausuure's linguistic theory (*cf.* Sec. 3.5.3)
- Syllogism holds in classical systems, but not in quantum systems (*cf.* Sec. 4.3.3)
- Only the present exists (*cf.* Sec. 6.1.2)
- What is the problem of universals? (*cf.* Sec. 6.5.1)
- What is Geocentrism vs. Heliocentrism? After all, the worldviewism (*cf.* Sec. 7.4.2)
- Two (scientific or non-scientific) interpretations of I think, therefore I am .(*cf.* Sec. 8.2.2)
- Leibniz-Clark correspondence (i.e., what is space-time?), (*cf.* Sec. 9.3)
- The problem of qualia (*cf.* Sec. 9.5.1)
- Brain in a vat argument (*cf.* Sec. 9.5.2)
- The solution of Hume's problem of induction (*cf.* Sec. 9.7.1)
- Grue paradox cannot be represented in quantum language (*cf.* Sec. 9.7.2)
- What is causality? (*cf.* Sec. 10.3)
- What is Peirce's abduction? (*cf.* Sec. 11.3.1)
- Five-minute hypothesis (*cf.* Sec. 11.4.1)
- McTaggart's paradox (*cf.* Sec. 11.4.2)
- quantitative representation of "Signifier" and "signified" (*cf.* Sec. 11.5.3)
- A scientific understanding of Wittgenstein's picture theory (*cf.* Sec. 11.6.2)

Current Point

- Wittgenstein's paradox 11.6.3
- Flagpole problem, (*cf.* Sec. 11.7.1)
- Hempel's raven paradox (*cf.* Sec. 11.8)
- the mind-body problem (i.e., How are mind and body connected?), (*cf.* Sec. 11.9.4)

(D_2) Also, for the solutions of unsolved problems in quantum mechanics, statistical mechanics, statistics and probability theory, see ref. [66]). Particularly, I think that the following three are important in physics:

- the discovery of Heisenberg's uncertainty relation (Ref. [32], or Sec. 4.3 in ref. [66])
- The clarification of the projection postulate (i.e., the wavefunction collapse) (ref. [59] , or Sec. 11.2 in ref. [66])
- The measurement theoretical characterizations of equilibrium statistical mechanics (Ref. [46], or Chap.17 in ref. [66])

It is known that Wittgenstein wrote "Tractatus Logico-Philosophicus" in order to solve all philosophical problems in one fell swoop. And he concluded that

(E_1) philosophical problems arise from insufficient attention to the variety of natural language use.

that is,

"Philosophical problems were not appropriate".

However. I think that his saying is not fair. I think that

(E_2) these unsolved problems in (D) arise from a lack of our understanding of dualistic idealism. (Recall my assertion that the formula (1.2), i,e., quantum language is the final goal of dualistic idealism.)

Thus, Wittgenstein's picture theory cannot solve unsolved problems in (D), since his theory is based on naive set theory (which is not related to dualistic idealism).

Some may assert that

(E_3) "Similar thoughts" in Wittgenstein's saing (D) in Sec. 11.6.1 might be an even greater thought than quantum language.

This may be a possible. However, it should be noted that quantum language solved many fundamental philosophical unsolved problems (in List 11.26), but Wittgenstein's picture theory has no power to slove such propbelms. Thus, I think he was going for such theory like quantum language.

11.6.3 * Wittgenstein's paradox

11.6.3.1 Without a worldview, we cannot say anything

Around the time I conceived of the quantum language (e.g., [35, 36, 37, 38, 40] in quantum mechanical worldview), almost all unsolved philosophical problems (philosophical paradoxes in dualistic idealism) can be solved within the framework of a quantum language, and I thought. In fact, they have been solved and are noted in List 11.26 (=1.6) [The list of our answers for philosophical problems]. As mentioned in Note 1.10, without a worldview, we have several paradoxes as follows.

(F_1) Zeno's paradoxes (in Sec. 2.4)

(F_2) Hume's problem of induction (and Grue paradox) (in Sec. 9.7)

(F$_3$) Wittgenstein's paradox

However, I believe I have solved (F$_1$) by regarding it as the philosophical problem concerning worldviews and not the mathematical problem concerning geometric series. Also, I solved (F$_2$), considering it within the framework of quantum language. However, I still think of (F$_3$) as a mere play on words and a completely unproductive problem. Next let us explain it.

11.6.3.2 Wittgebstein's paradox: The logical worldview is not a worldview

It might be reasonable to assume that analytical philosophy is a style of doing philosophy and has nothing to do with the logical worldview. If so, we can understand Wittgenstein's paradox as follows.

Problem 11.27. Wittgenstein's paradox (in Philosophical Investigations (1953))
Consider the following quiz:

[Quiz]:
$$1, \quad 2, \quad 3, \quad 4 \quad 5, \quad 6, \quad 7, \quad 8, \quad 9, \quad 10, \quad 11, \quad 12, \quad X, \cdots$$
What number is X ?

[Answer]: It may be natural to consider that $X = 13$. However, if I think of them as numbers on a clock face. there is a reason to consider that $X = 1$. Thus, this quiz is inappropriate as a question. □

Most readers will find this quiz trivial and boring. Thus, the true problem concerning Wittgenstein's paradox is as follows.

(G) Why did Wittgenstein mention a trivial problem like the quiz above? Why didn't Plato, Descartes and Kant consider these problems?

[Answer]: This is already answered in Remark 9.11. That is, Wittgenstein's paradox highlights the weaknesses of analytic philosophy (i.e., analytic philosophy didn't have a true worldview). That is, the above quiz cannot be solved without the condition: "the world is like this". Therefore, I think that Wittgenstein's paradox is important in the sense that it symbolizes the fact that analytic philosophy does not have a worldview. (I'm not sure if this will be generally approved as the answer to Wittgenstein's paradox, but I'll list it in List 1.6) □

♠**Note 11.15.** Readers may think that the above is too childish. However, it should be noted that the above is essentially the same as "Geocentrism vs. Heliocentrism" (*cf.* Chap. 7). That is, we can say nothing without a worldview.

11.6.4 Main character Wittgenstein in Linguistic turn

11.6.4.1 The power of Wittgenstein's word: Linguistic turn

However, in this paper, I want to assert that Wittgenstein is one of the greatest philosophers (Plato, Descartes, Kant, etc.). That is because he said the following sayings (H_1) - (H_3):

(H_1) "The limits of my language mean the limits of my world."

(H_2) "What we cannot speak about we must pass over in silence"

(H_3) "Language-game"

The above is just the spirit of quantum language. Instead of my poor explanation of the spirit of quantum language, I prefer to saying

(I) "The spirit of quantum language is represented by the above (H_1) - (H_3)"

Seeing the above (H_1)$\sim$ (H_3), I can understand "Why did Russell support Wittgenstein as his guardian?". Russell must have thought "Without Wittgenstein, we (i.e., Frege and Russell) cannot spread analytic philosophy alone." That is, I think Russell expected Newton to be an enlightener of the philosophy of language.

11.6.4.2 Wittgenstein advanced philosophy

I think that

(J_1) $\underset{\text{(Descartes, ..., Kant)}}{\boxed{\text{Epistemology}}} \xrightarrow[\text{regress (QL)}]{\text{Linguistic turn}} \underset{\substack{\text{Frege}\\ \text{Russell}}}{\boxed{\text{linguistic phil. analytic phil.}}} \xrightarrow{\text{mechanical turn}} \boxed{\text{Quantum Language}}$

where $\xrightarrow[\text{progress (QL)}]{}$ $\Big[$resp. $\xrightarrow[\text{regress (QL)}]{}$ $\Big]$ means "getting closer to quantum language" $\Big[$ resp. "getting away from quantum language"$\Big]$. That is because Frege and Russell's assertion was too mathematical. And so, their assertion is not related to dualistic idealism ($\approx$ quantum language).

However, I think that

(J_2) $\underset{\text{(Discourse on the Method)}}{\boxed{\text{Descartes}}} \xrightarrow[\text{progress (QL)①}]{\text{Copernican revolution}} \underset{\text{(Critique of Pure Reason)}}{\boxed{\text{Kant}}}$

$\xrightarrow[\text{progress (QL)②}]{\text{Linguistic turn}} \underset{\text{Tractatus Logico Philosophicus (=TLP)}}{\boxed{\text{Wittgenstein}}} \xrightarrow[\text{progress ③}]{\text{mechanical turn}} \boxed{\text{Quantum Language}}$

That is because

(K_1) "progress (QL)①" is proved in Problem 10.6 in Chap 10.

(K_2) "progress (QL)②"; Recall the argument in Sec. 4.3.5 ("Why does classical logic hold in classical systems?") and Sec. 11.6.2 (My scientific understanding of "Tractatus Logico-Philosophicus (=TLP)"), which say that Wittgenstein' picture theory can be completely understood in QL. Thus, we can conclude that TLP is closer to QL than to "Critique of Pure Reason, that is, progress (QL)②.

(K_3) "progress ③" is clear if you examine List 11.26 in Chap 11.

Therefore, in spite of (J_1), I assert that

$$(\mathrm{L})\quad \underset{\text{(Descartes, ..., Kant)}}{\boxed{\text{Epistemology}}} \xrightarrow[\text{progress (QL)}]{\text{Linguistic turn}} \overset{\text{Frege, Russell}}{\underset{\text{Saussure, Wittgenstein}}{\boxed{\text{linguistic phil, analytic phil.}}}} \xrightarrow[\text{progress}]{\text{mechanical turn}} \boxed{\text{Quantum Language}}$$

Therefore, from the quantum mechanical point of view, of the three (i.e., Frege, Russell, Wittgenstein), I can rate Wittgenstein the highest as a philosopher.

Even if that were the case, if quantum language were not good, everything would be meaningless. Of course, I believe that quantum language is going to get a lot of support.

Remark 11.28. (= Remark 11.13)): I will restate what I wrote at the beginning of this chapter as follows.

$$(\flat_{22})\begin{cases} (\flat_{22}^1) & \text{foundation of math.} \quad \text{(Boole, Frege, Russel, Cantor, Zermelo, Fraenkel, Gödel),...,} \\ (\flat_{22}^2) & \text{math.} \quad \text{(Bourbaki),} \\ (\flat_{22}^3) & \text{analytic philosophy, symbolic logic} \quad \text{(Frege, Russell, Wittgenstein,...)} \\ (\flat_{22}^4) & \text{scientific philosophy} \quad \text{(Hempel, Popper, ...)} \\ (\flat_{22}^5) & \text{linguistics, ordinary language} \quad \text{(Saussure, Wittgenstein (except logic))} \end{cases}$$

If so, the above formula (11.4) can be rewritten more precisely as follow:

$$(\sharp_2)\quad \boxed{\text{Descartes=Kant}} \xrightarrow[\text{progress}]{} \underset{\substack{\text{ⓐ: } (\flat_{22}^5)\text{:Saussure (Remark 11.12)} \\ \text{ⓑ: } (\flat_{22}^3, \flat_{22}^5)\text{: Wittgenstein (Sec. 11.6.3)} \\ \text{ⓒ: } (\flat_{22}^4)\text{:Hempel (Remark 11.42)}}}{\boxed{\text{linguistic phil. } (\flat_{22}^3), (\flat_{22}^4), (\flat_{22}^5)}} \to \boxed{\text{Quantum language}}$$

ⓒ will be discussed later.

♠**Note 11.16.** For each great discovery, an anecdote (or, a catch copy, stage effect) is left as follows.

($\sharp_1$) Archimedes$\cdots\cdots$golden crown, heureka! (*cf.* Sec.5.4)
($\sharp_2$) Galileo$\cdots\cdots$Leaning Tower of Pisa, "And Yet It Moves" (*cf.* Sec.7.3.4)
($\sharp_3$) Newton$\cdots\cdots$:Newton's apple, "Geocentrism vs. Heliocentrism"(*cf.* Note 7.8)
($\sharp_4$) Descartes$\cdots\cdots$:fly on the ceiling (*cf.* Note 8.2), : I think, therefore I am, (*cf.* Sec.8.2)
($\sharp_5$) Kant$\cdots\cdots$clock (*cf.* Note 10.2), dogmatic slumber (*cf.* Note 10.7)
($\sharp_6$) Wttgenstein$\cdots\cdots$primary school teacher, Guardian: Russell (*cf.* Sec.11.6.1)
($\sharp_7$) Einstein$\cdots\cdots$Elevator
($\sharp_8$) quantum mechanics$\cdots\cdots$Heisenberg's uncertainty principle (*cf.* Note 4.1 of ref. [66])

Here, the ($\sharp_8$) is my opinion.

Remark 11.29. [The key-words of the dualism] Now we can complete the table in Assertion 1.14. The idealistic worldview is the mind-matter dualism, which is composed of three key-words, that is, [A](= mind), [C](= matter) and [B](=body: something connecting [A:mind] and [C:matter]). Thus, we see that:

\	[A](= mind)	[B](Mediating of A and C)	[C](= matter)
Plato	actual world	Idea	/ [Idea world]
Thomas Aquinas	universale post rem	universale ante rem	/ [universale in re]
Descartes	I, mind, brain	body	/ [matter]
Locke	mind	secondary quality	primary quality [matter]
Berkeley	mind	secondary quality	/ [God]
Kant	phenomenon	recognition	/ [thing-in-itself]
Saussure Zadeh	/	signified [signifier] fuzzy set (= membership function)	/
Wittgenstein	truth value	[proposition]	logical space [object]
statistics	sample (space)	/ [trial]	parameter [population]
quantum mechanics	measured value	observable [measurement]	state [particle]
quantum language	**measured value**	**observable** [measurement]	**state [system]**

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11.6.5 Appendix: Linguistic turn (From epistemology to quantum language)

Kant's "Criticism of pure reason" may be given a severe evaluation in modern times, but it is revived by linguistic turn (by Wittgenstein and others), i.e.,

$$(\sharp_1) \qquad \underset{\text{Kant}}{\boxed{\text{epistemology}}} \xrightarrow[\text{linguistic turn}]{} \underset{\text{Wittgenstein}}{\boxed{\text{linguistic philosophy}}}$$

or

$$(\sharp_2) \qquad \underset{\text{Kant}}{\boxed{\text{Cognition}}} \xrightarrow[\text{linguistic turn}]{} \underset{\text{quantum language}}{\boxed{\text{Language}}}$$

In this section, I asset that

- the linguistic turn can be understood in $(\sharp_2)$ than $(\sharp_1)$

That is, it suffices to change "cognition" (in Critique of Pure Reason)) to "language", that is,

$$\boxed{\text{"cognition"}} \xrightarrow{\text{batch conversion}} \boxed{\text{"language"}}$$

by which "the spirit of Critique of Pure Reason" changes to "the spirit of quantum language" as follows.

Explanation 11.30. (Continued from Explanation 10.3 in Chap.10)
Explanation of the spirit of quantum language : i.e., from Critique of Pure Reason to quantum language (*cf.* [50])
That is, read such as

$$\overset{\textbf{[Explanation 10.3]}}{\textbf{[Explanation 11.30]}}, \qquad \text{i.e.,} \qquad \overset{\text{[the outline of "Critique of Pure Reason"]}}{\text{[the spirit of quantum language]}}$$

What was revealed in $\overset{\text{[Critique of Pure Reason]}}{\text{[Quantum language]}}$ is that human $\overset{\text{[perception ability]}}{\text{[linguistic ability]}}$ is not just to passively replicate things of the world. Rather it acts on the world actively and creates the subject of its $\overset{\text{[recognition]}}{\text{[language]}}$. Even though we make it, the world is not necessarily completed from nothing like God. The world is already there in some form. In order for $\overset{\text{[recognition]}}{\text{[description]}}$ to be established, information from this world that can be obtained through a sense is necessary as a material. However, this information is only disorganized confusing as it is. Human $\overset{\text{[perception ability]}}{\text{[linguistic ability]}}$ gives an orderly order to information of this confused sensation through a certain form that is inherent in human beings. It is necessary to compile the subject of unified $\overset{\text{[recognition]}}{\text{[language]}}$ for the first time. According to $\overset{\text{[Kant]}}{\text{[quantum language]}}$, its format (= $\overset{\text{[a priori synthetic judgment]}}{\text{[Axioms 1 and 2]}}$) is as follows.

(i) $\overset{\text{[Form of sensitivity(intuition)(space } (=\mathbb{R}\times\mathbb{R}^3))]}{\text{[Axioms 1 (measurement)]}}$

(ii) $\overset{\text{[Form of understanding(thinking)]}}{\text{[Axioms 2 (Causality)]}}$

If so, the proposition that "everything is in time and space" or "everything follows causality" cannot be proved empirically. Nevertheless, it will apply unconditionally to all subjects of experience. The reason is that the object is composed for the first time in a format such as $\overset{\text{[space, time, causality, etc.]}}{\text{[measurement, causality]}}$. For example, $\overset{\text{[if we wear green sunglasses]}}{\text{[we know the only term "green"]}}$, the remark that "the world is green" is similar to being regarded as a correct remark for all human beings.

Summary 11.31. Consider "language" and not "cognition". That is, consider the following linguistic turn:

$$\underset{\text{(Descartes·Kant)}}{\overset{\text{(cognition)}}{\boxed{\text{mind-matter dualism(=epistemology)}}}} \xrightarrow[\text{linguistic turn}]{} \underset{\text{(Wittgenstein)}}{\overset{\text{(language)}}{\boxed{\text{linguistic philosophy}}}}$$

Then, in several languages (ordinary language, mathematics, Newtonian mechanics, programing language, etc.), we say that (*cf.* Note 10.3)

($\sharp_1$) "ordinary language" is like tabula rasa (i.e.,British Empiricism)

($\sharp_2$) Mathematics is like Continental Rationalism

($\sharp_3$) **quantum language is like Kant's compromise**

In quantum language, Axioms (measurement and causality) are first declared, and thus, quantum language is not like "tabula rasa". However, the linguistic Copenhagen interpretation is going to be formed gradually through the experience, thus, quantum language is like Kant's compromise. As seen in the formula (1.1) in Chap. 1, we see that

$$\overset{\text{(=quantum language)}}{\boxed{\text{measurement theory}}} = \overset{\text{[Axiom 1]}}{\boxed{\text{measurement}}} + \overset{\text{[Axiom 2]}}{\boxed{\text{causal relation}}}$$

$$+ \underset{\text{[the manual to use Axioms 1 and 2]}}{\boxed{\text{linguistic Copenhagen interpretation}}}$$

♠**Note 11.17.** If "Why is our cognition possible?" is asked, then, we may have only answer such that "Wonder of the human recognition ability", which is represented by "transcendent" (due to Kant). And moreover, If "Why is our language possible?" is asked, then, we may have only answer such that "Wonder of the human language ability", which is represented by "language game" (due to Wittgenstein). That is,

$$\underset{\text{Kant (transcendental)}}{\overset{\text{Wonder of the human recognition ability}}{\boxed{\text{congnition}}}} \xrightarrow[\text{linguistic turn}]{\qquad} \underset{\text{Wittgenstein (language game))}}{\overset{\text{Wonder of the human language ability}}{\boxed{\text{language}}}}$$

11.7 Flagpole problem in the quantum mechanical worldview

This section was written with reference to the following.

- [70]:Ishikawa, S: Philosophy of science for scientists; The probabilistic interpretation of science Journal of quantum information science, Vol. 9, No.3 , 140-154, DOI: 10.4236/jqis.2019.93007 (https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

Remark 11.32. I mentioned Classification 11.2 as the classification of the logical worldview as follows:

Classification [The classification of the logical worldview] = Classification 11.2]

$(\flat_{22})$
- (A_0): Artificial intelligence, "Logic circuit may be more powerful than any worldview"
  - $(\flat_{22}^0)$ Computer; (Gödel, Turing, von Neumann, etc.)
- (A_1): mathematics: "Describe math. by logic (and set theory)!"
  - $(\flat_{22}^1)$ foundation of math. (Boole, Frege, Cantor, Russell, Gödel,...),
  - $(\flat_{22}^2)$ math. (Bourbaki),
- (A_2):**the logical worldview** (or precisely, the naive logical worldview) : "Describe science by logic (and set theory)!"
  - $(\flat_{22}^3)$ analytic philosophy, symbolic logic (Frege, Russell, Wittgenstein,...)
  - $(\flat_{22}^4)$ scientific philosophy (Hempel, Popper, ...)
- (A_3): linguistics: "Describe everything reasonably!"
  - $(\flat_{22}^5)$ linguistics, ordinary language (Saussure, Wittgenstein)

$(\flat_{23})$: the (classical, quantum) mechanical worldview

In this and the next sections, I will mention that C.G. Hempel pointed out that the logical worldview (A_2) is not fit for science. And I show that quantum language (i.e., the quantum mechanical worldview$(\flat_{23})$) is superior to the naive logical worldview (A_2).

11.7.1 * The quantum linguistic solution of Flagpole problem

Carl Gustav Hempel (1905-1997) was a German philosopher. He was a major figure in logical empiricism, a 20th-century movement in the philosophy of science. His studies of induction, explanation, and rationality in science exerted a profound influence upon a young generation of philosophers of science. He is also known for the raven paradox (also known as "Hempel's paradox")

As I have said many times in this text, philosophers like "logic" (or, the word "logic") too much. They always want to "be logical".

I think that he following fact is incomprehensible.

- **Philosophers are more fond of "logic" than mathematicians**

That is, I assert that

- the scientific explanation should not be described in the logical worldview, but in the quantum mechanical worldview.

This will be discussed in Sec. 11.7.1.1 and Sec. 11.7.1.2.

11.7.1.1 Flagpole problem in Hempel's model (= the deductive-nomological model (DN model))

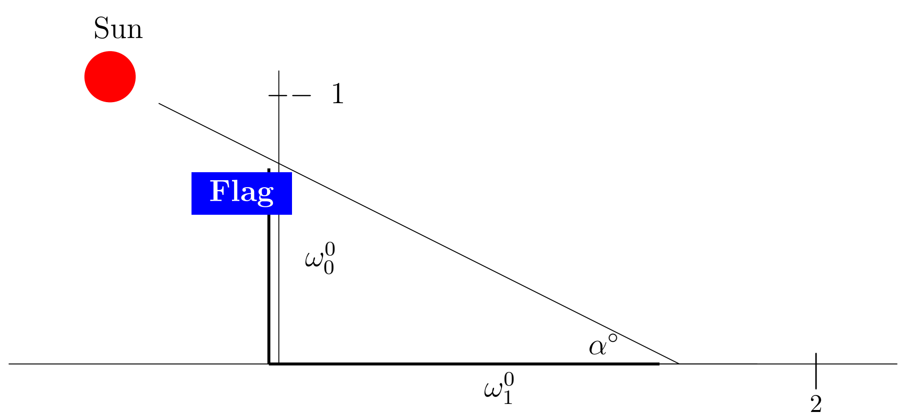


Figure 11.2: [Flagpole problem]

Let us explain the flagpole problem as follows. Suppose that the sun is at an elevation angle α° in the sky. Assume that $\tan\alpha^\circ = 1/2$. There is a flagpole which is ω_0^0 meters tall $(0 \le \omega_0^0 \le 1$. The flagpole casts a shadow ω_1^0 meters long. Suppose that we want to explain the length of the flagpole's shadow. On Hempel's model, the following explanation is sufficient.

(B$_1$) 1. The sun is at an elevation angle α° in the sky.
2. Light propagates linearly.
3. The flagpole is ω_0^0 meters high.
Then,
4. The length of the shadow is $\omega_1^0 = \omega_0^0/\tan\alpha^\circ = 2\omega_0^0$

This is a good explanation of "Why is that shadow $2\omega_0^0$ meters long?"
Similarly, we may consider as follows.

(B$_2$) 1. The sun is at an elevation angle $\alpha°$ in the sky.
2. Light propagates linearly.
3. The length of the shadow is ω_1^0
Then,
4. The flagpole is $\omega_0^0 (= (\tan\alpha°)\omega_1^0 = \omega_1^0/2)$ meters tall.

However, this is not sufficient as the explanation of "Why is the flagpole $\omega_0^0 (= \omega_1^0/2)$ meters tall?"

Thus we have the flagpole problem as follows:

(B$_3$) [**Flagpole problem**]
Why do we feel that the solution (B$_2$) is unnatural?

My opinion is as follows.

(C) the above explanations (B$_1$) and (B$_2$) rely on DN model (i.e., deductive-nomological model), which is due to a kind of the logical worldview (i.e., the spirit that science should be written logically). And thus, the most important concepts "measurement" and "causality" are not used in (B$_1$) and (B$_2$).

Therefore, in what follows, I will present the quantum linguistic explanation.

11.7.1.2 Flagpole problem by the quantum linguistic explanation

In what follows, we discuss the flagpole problem in terms of quantum language.

Consider two times $t = 0, \epsilon$ $(0 < \epsilon \ll 1)$. For simplicity, put $\epsilon = 1$. Consider a basic structure $[C(\Omega_t) \subseteq L^\infty(\Omega_t, \nu_t) \subseteq B(L^2(\Omega_t, \nu_t))]$, where $\Omega_0 = [0, 1]$ is the state space (in which the length of the flagpole is assumed to be represented) at time 0 (where the closed interval in the real line $\mathbb{R}$), $\Omega_1 = [0, 2]$ is the state space (in which the length of the shadow is assumed to be represented) at time 1 and the ν_t is the Lebesgue measure.

Since the sun is at an elevation angle $\alpha°$ in the sky, it suffices to consider to the causal map $\phi_{0,1} : \Omega_0 \to \Omega_1$ such that $\phi_{0,1}(\omega_0) = 2\omega_0$ $(\forall \omega_0 \in \Omega_0)$. Thus, we can define the causal operator $\Phi_{0,1} : L^\infty(\Omega_1) \to L^\infty(\Omega_0)$ such that

$$(\Phi_{0,1} f_1)(\omega_0) = f_1(\phi(\omega_0)) \;\; (\forall f_1 \in L^\infty(\Omega_1), \omega_0 \in \Omega_0)$$

Let $\mathsf{O}_e = (X, \mathcal{F}, F_e)$ be the exact observable in $L^\infty(\Omega_1, \nu_1)$ (*cf.* Example 1.19). That is, it satisfies that $X = \Omega_1, \mathcal{F} = \mathcal{B}_{\Omega_1}$ (i.e., the Borel field in Ω_1), $[F_e(\Xi)](\omega_1) = 1$ (if $\omega_1 \in \Xi$), $= 0$ (otherwise).

Thus, we have the measurement $\mathsf{M}_{L^\infty(\Omega_0,\nu_0)}(\Phi_{0,1}\mathsf{O}_e = (X, \mathcal{F}, \Phi_{0,1}F_e), S_{[\omega_0^0]})$. Then we have the following statement

(D$_1$) [Measurement]; the probability that the measured value $x(\in X)$ obtained by the measurement $\mathsf{M}_{L^\infty(\Omega_0,\nu_0)}$ $(\Phi_{0,1}\mathsf{O}_e = (X, \mathcal{F}, \Phi_{0,1}F_e), S_{[\omega_0^0]})$ is equal to $2\omega_0^0$ is given by 1.

which is the quantum linguistic representation of (B$_1$). That is, we consider that the (B$_1$) is the simplified form (or, the rough representation) of (D$_1$). Also,

(D$_2$) [Inference]; Assume that the measured value $\omega_1^0(\in X)$ is obtained by the measurement $\mathsf{M}_{L^\infty(\Omega_0,\nu_0)}$ $(\Phi_{0,1}\mathsf{O}_e = (X, \mathcal{F}, \Phi_{0,1}F_e), S_{[*]})$. Then, we can infer that $[*] = \omega_1^0/2$

which is the quantum linguistic representation of (B$_2$). That is, we consider that the (B$_2$) is the simplified form (or, the rough representation) of (D$_2$).

Thus, we conclude that "scientific explanation" is to describe by quantum language. Also, we have to add that the flagpole problem is not trivial but significant, since this is never solved without Axiom 1 [measurement] (in Section 1.1)(measurement) and Axiom 2 (in Section 1.1)(causality) (i.e., the answers to the problems "What is measurement ?" and "What is causality ?").

Remark 11.33. Recall the following the mainstream of western philosophy:

$$\underset{\text{(Descartes, ..., Kant)}}{\overset{\text{①:keywords [cognition, causality]}}{\boxed{\text{Epistemology}}}} \xrightarrow[\text{progress}]{\text{Linguistic turn}} \overset{\text{②:keyword [logic]}}{\boxed{\text{Logical worldview}}} \xrightarrow[\text{progress}]{\text{mechanical turn}} \overset{\text{③:keywords [measurement, causality]}}{\boxed{\text{Quantum Language}}}$$

where we see that

"①: keywords [cognition, causality]" $\approx$ "③: keyword [measurement, causality]"

Thus, "②:logic" is too specific. Thus, I am skeptical of the logical worldview. As mentioned often in this text, I believe that

($\sharp_1$) quantum language is the language of science (*cf.* Assertion 1.3).

($\sharp_2$) logic is the language of mathematics.

Also, recall Remark 11.3.

11.8 * Hempel's raven problem

11.8.1 Is "the set of all tyrannosaurus" meaningful? : the set theoretical worldview

Let us explain "the set theoretical worldview", (which is a kind of "the logical worldview"). Logic and set theory are similar, and thus, it usually believed that set theory as well as logic are considered reliable.
As mentioned Sec. 11.1, the following is the greatest history of mathematics (i.e., the beginning of modern mathematics):

$$\underset{(1891)}{\overset{\text{naive set theory}}{\boxed{\text{Cantor}}}} \longrightarrow \underset{(1921)}{\overset{\text{axiomatic set theory}}{\boxed{\text{ZFC-Axiom}}}} \longrightarrow \boxed{\text{modern math.}}$$

However, the difference between naive set theory and axiomatic set theory is negligible for most mathematicians (except mathematicians specializing in foundations of mathematics). Therefore, for simplicity, let's assume that "set theory" = "naive set theory" in this section (*cf.* Note 1.12). Therefore, we assume that "set" is defined by "a collection of things". This is the same as the use of "set" in everyday language.

General people (including philosophers) may think

($\sharp_1$) Set theory is very reliable because it is used to lay the foundation for mathematics. Therefore, if we use sets to describe the concept of this world, we will not fall into a mistake.

In this text, this is called the set theoretical worldview.

However, I am skeptical of the set theoretical worldview ($\sharp_1$). That is, I think that

($\sharp_2$) It is true that set theory is a very reliable and solid discipline. However, we must be cautious in using sets to describe the world.

For example,

($\sharp_3$) Is "the set of all tyrannosaurus" meaningful? Or, is "the set of all raven" meaningful? If it is not meaningless, how do we represent "All ravens are black"?

This ($\sharp_3$) is Hempel's raven problem.

11.8.2 Hempel's raven problem in the set theoretical worldview

First, let us review the traditional arguments concerning Hempel's raven problem (*cf.* refs. [27, 28]). Thus, we start from the followings:

(A_0) Let U be the set of all birds. Let $B(\subseteq U)$ be a set of all black birds. Let $R(\subseteq U)$ be a set of all ravens.

Although these should be doubtful (since these are as ambiguous as "the set of all tyrannosaurs"), we advance towards the next argument. The statement: "Every raven is black" is logically denoted by

(A_1) "Every raven is black" : $(\forall x)[x \in R \longrightarrow x \in B]$ $\quad$ *i.e.*, $R \subseteq B \subseteq U$,

Also, this is logically equivalent to the following contraposition:

(A_2) "Every non-black bird is a nonraven" : $(\forall x)[x \in U \setminus B \longrightarrow x \in U \setminus R]$
i.e., $U \setminus B \subseteq U \setminus R$

However, if these are equivalent, then we have the following problems (i.e., raven problem):

(A_3) Why is the actual verification of (A_2) much more difficult than the actual verification of (A_1)?

(A_4) Why can the truth of "(A_1): any raven is black" be known by (A_2), i.e., without seeing a raven also at once?

(A_5) Is it possible to experimentally verify "Every raven is black"?

These may be so called Hempel's raven paradox. However, there is a reason to consider that "the set of all ravens" is as ambiguous as "the set of all tyrannosaurs". If so, that is, if the above (A_0) is ambiguous, all other (A_1)-(A_5) are also ambiguous. That is, (A_3)-(A_5) are not scientific problems.

Now we think that the most essential problem concerning Hempel's raven problem is as follows:

(B) What is the scientific meaning of "Every raven is black"?

In order to study this problem, we must prepare the quantum linguistic formulation of ornithology, under which the meaning of "Every raven is black" will be clarified in this section. We believe that the above problems cannot be solved without measurement theory since the above problems includes the terms "actual verification" and "experimentally verify" which are closely related to measurement.

Remark 11.34. Just to be sure, in this paper we assume that the followings are the same:

$$\text{"any raven is black"} = \text{"every raven is black"} = \text{"all ravens are black"}.$$

This is the same as the usage in mathematics (i.e., "any" = "every" = "all" = "$\forall$").

11.8.3 Hempel's raven problem in the quantum mechanical world-view

In this section we slightly improve our result in

- Ref. [70]: Ishikawa, S., (2019) Philosophy of science for scientists; The probabilistic interpretation of science, Journal of quantum information science, Vol. 9, No.3 , 140-154, (`https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447`)

We think that Hempel's raven problem raises the problem of "What is the scientific meaning of 'Every raven is black'?". In order to answer this problem, we must prepare the quantum linguistic formulation of ornithology

The arguments below are essentially the same as Saussure's "signifier" and "signified" in Sec. 11.5.3.

Definition 11.35. [Membership function (= Fuzzy set, *cf.* [35, 36, 37, 38, 109]), Observable] Let Ω be a state space. For simplicity, we always assume that Ω is compact. A continuous function $m : \Omega \to [0, 1]$ (i.e., the closed interval in $\mathbb{R}$). is called a membership function. Also, we consider the following correspondence:

$$\underset{\text{menbership function}}{m : \Omega \to [0,1]} \quad \underset{\text{correspondence}}{\longleftrightarrow} \quad \underset{\text{observable}}{\mathsf{O} = (\{y, n\}, 2^{\{y,n\}}, F)} \tag{11.5}$$

such that

$$F(\{y\})](\omega) = m(\omega) \qquad F(\{n\})](\omega) = 1 - m(\omega), \qquad\qquad (\forall \omega \in \Omega)$$

where "y" and "n" respectively means "yes" and "no".

Definition 11.36. [Membership functions of black birds and ravens] Let Ω be a state space. A continuous function $m : \Omega \to [0, 1]$ (i.e., the closed interval in $\mathbb{R}$) is called a membership function. Assume that the state (i.e., quantitative property) of any bird can be expressed by a point in the state space Ω. Define the membership functions $m_B : \Omega \to [0,1]$ of black birds and the membership function $m_R : \Omega \to [0,1]$ of ravens as follows. Suppose that there are 100 specialists of ornithology and the following question is made them.

(C) Is this bird with the sate ω_1 ($\in \Omega$) a black bird or not?

The answer is as follows.

$$\text{(D)} \begin{cases} \text{70 specialists say that this bird is a black bird.} \\ \text{30 specialists say that this bird is not a black bird.} \end{cases}$$

Then the value of $m_B(\omega_1)$ is defined by 0.7. For many birds with the state ω_i $(i = 2, 3, ...N)$, repeating the experiment in the same way, the value of $m_B(\omega_i)$ $(i = 2, 3, ...N)$ is determined. And the membership function $m_B : \Omega \to [0,1]$ of black birds is defined by the interpolation method (which may be rather subjective). Similarly we get the membership function $m_R : \Omega \to [0,1]$ of ravens.

Definition 11.37. (i): [Raven state class, Black bird state class]: Put

$$\Omega_B := \{\omega \in \Omega \mid m_B(\omega) = 1\}, \qquad \Omega_R := \{\omega \in \Omega \mid m_R(\omega) = 1\}$$

which is respectively called a *black bird state class* and a *raven state class* (see Fig. 11.3 below).

(ii): [Raven, Black bird]: If the state of a certain bird belongs to Ω_R [resp. Ω_B], this bird of a certain is called a raven [resp. a black bird]. It is not asked whether this bird exists really. This bird may be extirpated like tyrannosaurs. Moreover, this bird may be a biology newly made by genome edit.

(iii): ["Every raven is a black bird"]: We say "Every raven is black", if it holds that $\Omega_R \subseteq \Omega_B$. (see Fig. 11.6 later).

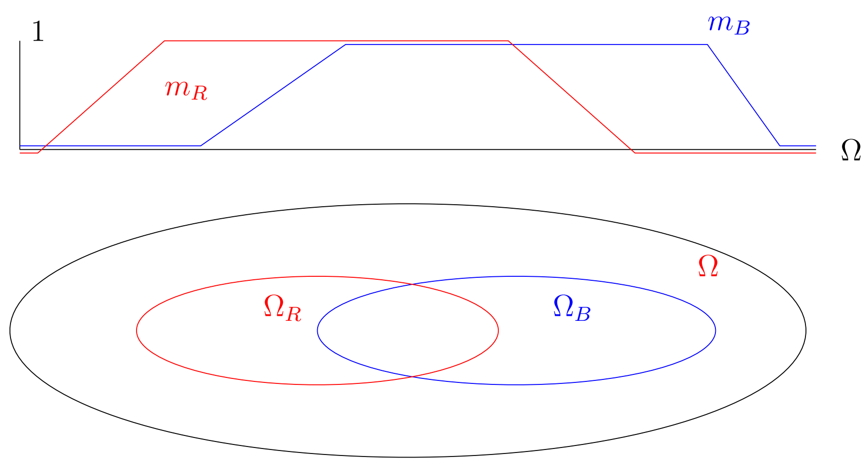


Figure 11.3: [Raven state class Ω_R, black bird state class Ω_B]

Definition 11.38. [Raven observable, Black bird observable]: Using the above membership functions, we define two observables (i.e., Black bird observable, Raven observable) $\mathsf{O}_B = (\{y,n\}, 2^{\{y,n\}}, F_B)$, $\mathsf{O}_R = (\{y,n\}, 2^{\{y,n\}}, F_R)$ in $C(\Omega)$, such that

$$
\begin{aligned}
&F_B(\{y\})](\omega) = m_B(\omega) \qquad F_B(\{n\})](\omega) = 1 - m_B(\omega), \\
&F_R(\{y\})](\omega) = m_R(\omega) \qquad F_R(\{n\})](\omega) = 1 - m_R(\omega) \qquad\qquad (\forall \omega \in \Omega)
\end{aligned}
$$

where "y" and "n" respectively means "yes" and "no". Thus, a membership function can be identified with a binary observable.

Since we assume that any bird is characterized by a certain point in the state space Ω, it is natural to consider that systematic ornithology is formulated as follows.

Formulation [I] [The quantum linguistic formulation of systematic ornithology [I]]:

(E$_1$) Ravens are characterize by the membership function $m_R : \Omega \to [0,1]$ (or, equivalently, the observable $\mathsf{O}_B = (\{y,n\}, 2^{\{y,n\}}, F_B)$). If the state ω of a bird belongs to Ω_R, then the bird is called a raven.

(E$_2$) Black birds are characterize by the membership function $m_R : \Omega \to [0,1]$. (or, equivalently, the observable $\mathsf{O}_R = (\{y,n\}, 2^{\{y,n\}}, F_R)$). If the state ω of a bird belongs to Ω_B, then the bird is called a black bird.

Interpretation 11.39. [The probabilistic interpretation of membership functions] We add the following probabilistic interpretation to this formulation [I]: For example, again consider Definition 11.35, and moreover, the statement (D). i.e.,

$$(\mathrm{D}') \begin{cases} \text{70 specialists say that this bird with a state } \omega_0 \text{ is a black bird.} \\ \text{30 specialists say that this bird with a state } \omega_0 \text{ is not a black bird.} \end{cases}$$

If we choose one person from the 100 specialists at random, the probability that he/she says that this bird is black is given 0.7. Such a measurement is represented by the symbol $\mathsf{M}_{C(\Omega)}(\mathsf{O}_B = (\{y,n\}, 2^{\{y,n\}}, F_R), S_{[\omega_0]})$. Therefore, we can use Axiom 1 [measurement] (in Section 1.1) as follows.

(F_1) for a bird with a state $\omega_0(\in \Omega_R)$, the probability that the measured value y [resp. n] is obtained by the measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}_R = (\{y,n\}, 2^{\{y,n\}}, F_R),\ S_{[\omega_0]})$ is equal to $[F_R(\{y\})](\omega)$ [resp. $[F_R(\{n\})](\omega)$].

(F_2) for a bird with a state $\omega_1(\in \Omega_B)$, the probability that the measured value y [resp. n] is obtained by the measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}_B = (\{y,n\}, 2^{\{y,n\}}, F_B),\ S_{[\omega_1]})$ is equal to $[F_B(\{y\})](\omega_1)$ [resp. $[F_B(\{n\})](\omega_1)$].

11.8.4 A priori proposition: "Any small black bird is black"

Next consider the following figure:

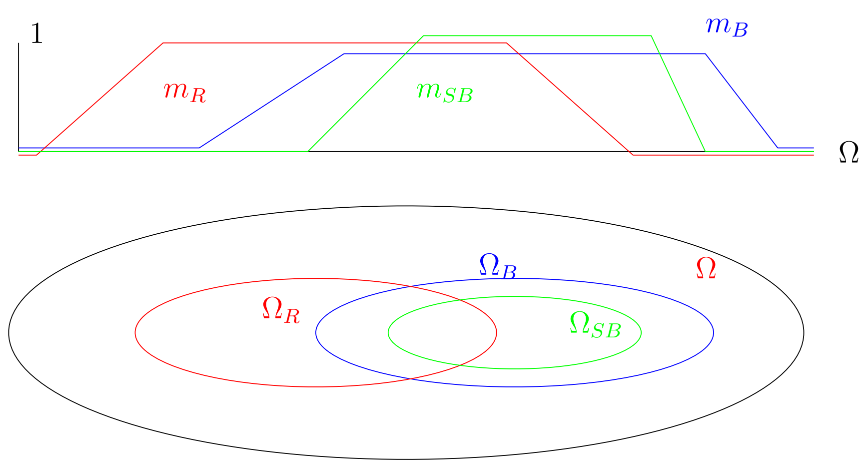


Figure 11.4: [Raven state class Ω_R, black bird state class Ω_B,small black bird state class Ω_{SB}]

That is, we add the small black bird observable:

Definition 11.40. (i): [Membership function of small black birds]: Define the membership function $m_{SB} : \Omega \to [0,1]$ of small black birds such as Definition 11.35.
(ii):[Small black bird state class] The small black bird state class Ω_{SB} is defined by $\{\omega \in \Omega \mid m_{SB}(\omega) = 1\}$.

(iii): [Small black bird]: If the state of a certain bird belongs to Ω_{SB}, this bird is called a small black bird.
(iv): [”Every small black bird is black”]: We say ”Every small black bird is black” if it holds that

$$\Omega_{SB} \subseteq \Omega_B$$

Note that this necessarily holds without actual verification since it is assumed that a small black bird is defined by a black bird such that it is small. Thus, ”Every small black bird is black” is a priori proposition.
(v): [Small black bird observable]: And define Small black observable $\mathsf{O}_{SB} = (\{y, n\}, 2^{\{y,n\}}, F_{SB})$ such that $[F_{SB}(\{y\})](\omega) = m_{SB}(\omega)$, $[F_{SB}(\{n\})](\omega) = 1 - m_{SB}(\omega)$, $(\forall \omega \in \Omega)$.

Thus, we have the new formulation, which is a development of Formulation [I] (i.e., The quantum linguistic formulation of systematic ornithology [I]]):

Formulation [II] [The quantum linguistic formulation of systematic ornithology [II]]:

(G_1) Ravens are characterize by the membership function $m_R : \Omega \to [0, 1]$. A bird with a state ω_R such that $\omega_R \in \Omega_R$ is called a raven.

(G_2) Black birds are characterize by the membership function $m_R : \Omega \to [0, 1]$. A bird with a state ω_B such that $\omega_B \in \Omega_B$ is called a black bird.

(G_3) Small black birds are characterize by the membership function $m_{SB} : \Omega \to [0, 1]$. A bird with a state ω_{SB} such that $\omega_{SB} \in \Omega_{SB}$ is called a small black bird.

(G_4) It holds that $\Omega_{SB} \subseteq \Omega_B$, i.e., Every small black bird is black. This is a priori statement, which is directly derived from Definition 11.40 (iv).

Exercise 11.41. It is easy to see that the above (G_4):$\Omega_{SB} \subseteq \Omega_B$ says that

(H) Let $u_i (i = 1, 2, ..., N)$ be a small black bird with the state $\omega_i (\in \Omega_{SB})$, which is denoted by $\widetilde{\omega}(u_i)$. For each small black bird u_i, the probability that the measured value y is obtained by the measurement $\mathsf{M}_{C(\Omega)}(\mathsf{O}_B = (\{y, n\}, 2^{\{y,n\}}, F_B), S_{[\widetilde{\omega}(u_i)]})$ is equal to 1.

According to the linguistic Copenhagen interpretation (E_4) in Sec. 1.1: ”Only one measurement is permitted”, the above (H_1) is formally written as follows.

(I) the probability that the measured value $(\underbrace{y, y, y, ..., y}_{N})$ is obtained by the parallel measurement $\bigotimes_{i=1}^N \mathsf{M}_{C(\Omega)}$ $(\mathsf{O}_B = (\{y, n\}, 2^{\{y,n\}}, F_B), S_{[\widetilde{\omega}(u_i)]})$ is equal to 1.

11.8.5 A posteriori proposition: "Every raven is black"

11.8.5.1 Popper's falsificationism in measurement theory

♠**Note 11.18.** Karl Popper (1902-1994) was one of the 20th century's most influential philosophers of science. Popper claims that, in order for something to be considered science, it must be falsifiable. If it is false, it can be shown through observation and experiment to be false. However, I think that the meaning of "observation and experiment" is ambiguous. That is, it must be "observation and experiment that is described by quantum language". This will be done below.

In the previous section, we discussed "$\Omega_{SB} \subseteq \Omega_B$" (i.e., Every small black bird is black). Since this is a priori statement, we can accept this statement without verification by experiment.

In this section we will discuss the statement "$\Omega_R \subseteq \Omega_B$" (i.e., Every raven is black), which is not a priori proposition but a posteriori proposition.

Hence, our problem is as follows:

(J) How can we be sure of $\Omega_R \subseteq \Omega_B$ (i.e., "Every raven is black")?
i.e., What should we do to be sure of "$\Omega_R \subseteq \Omega_B$"?.

In order to do it, we obey Popper's falsificationism (*cf,* ref. [89]) such that

(K) "$\Omega_R \subseteq \Omega_B$" should be accepted, if many experiments which deny "$\Omega_R \subseteq \Omega_B$" are conducted and "$\Omega_R \subseteq \Omega_B$" still cannot be denied.

For instance, we mention the following two tests ([Test I] and [Test II])

[Test I]: In order to deny "$\Omega_R \subseteq \Omega_B$",

(L) we try to find a bird with the state ω_0 such that $\omega_0 \in \Omega_R \setminus \{\omega \mid m_B(\omega) = 0\}$ (See Figure 11.5 below)

This test is quite natural, and thus, we should try this first.

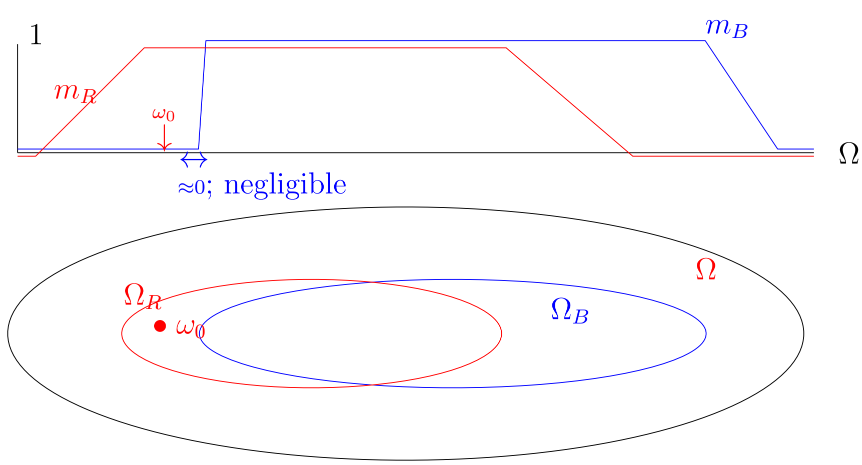


Figure 11.5: [$\omega_0 \in \Omega_R \setminus \{\omega \mid m_B(\omega) = 0\}$, $\rho_0(\{\omega \in \Omega \mid 0 < m_B(\omega) < 1\} \bigcap \Omega_R\}) \approx 0$, i.e., negligible.]

[Test II]: In order to deny "$\Omega_R \subseteq \Omega_B$",

(M$_0$) we try to confirm the hypothesis that there are non-black ravens by 3 percentages in 100 ravens. That is, we take the parallel mixed measurement $\bigotimes_{i=1}^{100} \mathsf{M}_{C(\Omega)}(\mathsf{O}_B := (\{y,n\}, 2^{\{y,n\}}, F_B), S_{[*]}(\rho_0))$, where a mixed state ρ_0 $(\in \mathfrak{S}^m(C(\Omega)^*))$ satisfies $\rho_0(\Omega_R) = 1$ and $\rho_0(\Omega_R \setminus \Omega_B) = 0.03$. Here, we, for simplicity, assume that $\rho_0(\{\omega \in \Omega \mid 0 < m_B(\omega) < 1\} \bigcap \Omega_R\}) \approx 0$, i.e., negligible. (See Figure 11.5 above.)

And assume that

(M$_1$) as the result of the (M$_0$), we get that one hundred ravens were black continuously

which is written in terms of quantum language as follows:

(M$_2$) By the parallel mixed measurement $\bigotimes_{i=1}^{100} \mathsf{M}_{C(\Omega)}(\mathsf{O}_B := (\{y,n\}, 2^{\{y,n\}}, F_B), S_{[*]}(\rho_0))$, a measured value $(\underbrace{y, y, y, ..., y}_{100})$ is obtained.

Then, we calculate, by Axiom$^{(m)}$ 1 (mixed measurement) in Chapter 1,

(M$_3$) the probability that a measured value $(\underbrace{y, y, y, ..., y}_{100})$ is obtained by the parallel mixed measurement
$\bigotimes_{i=1}^{100} \mathsf{M}_{C(\Omega)}$ $(\mathsf{O}_B := (\{y,n\}, 2^{\{y,n\}}, F_B), S_{[*]}(\rho_0))$ is given by $(97/100)^{100} (< 0.048)$. That is, the probability that (M$_2$) is realized (i.e., we meet one hundred black ravens continuously) is less than 0.048 $(> (97/100)^{100})$.

Thus, if we believe (M_0), a very rare thing (i.e., (M_3)) happened since probability 0.048 is quite rare. Therefore, we should doubt the hypothesis (M_0). That is, we couldn't deny "$\Omega_R \subseteq \Omega_B$ (i.e., any raven is black)". When we can't do such test many times and still deny "$\Omega_R \subseteq \Omega_B$ (i.e., any raven is black)", according to Popper's falsificationism, we will believe this.

If we believe in "$\Omega_R \subseteq \Omega_B$ (i.e., any raven is black)", we can propose the following new formulation:

Formulation [III] [The quantum linguistic formulation of systematic ornithology [III]]:

(N_1) Ravens are characterize by the membership function $m_R : \Omega \to [0,1]$. The definition of ravens is given by a raven state class Ω_R as shown in Definition 11.37 (ii).

(N_2) Black birds are characterize by the membership function $m_R : \Omega \to [0,1]$. The definition of black birds is given by a raven state class Ω_B as shown in Definition 11.37 (ii).

(N_3) Small black birds are characterize by the membership function $m_{SB} : \Omega \to [0,1]$. The definition of small black birds is given by a raven state class Ω_{SB} as shown in Definition 11.40 (iii).

(N_4) It holds that $\Omega_{SB} \subseteq \Omega_B$, i.e., Every small black bird is black. This is a priori statement, which is directly derived from Definition 11.40 (iv).

(N_5) It holds that $\Omega_R \subseteq \Omega_B$, i.e., Every raven bird is black, This is a posteriori statement, which is guaranteed in the sense of Popper's falsificationism (or, statistical hypothesis testing)

Thus we see the progress of ornithology (i.e., Formulation [I] $\overset{\text{progress}}{\Longrightarrow}$ Formulation [II] $\overset{\text{progress}}{\Longrightarrow}$ Formulation [III]).

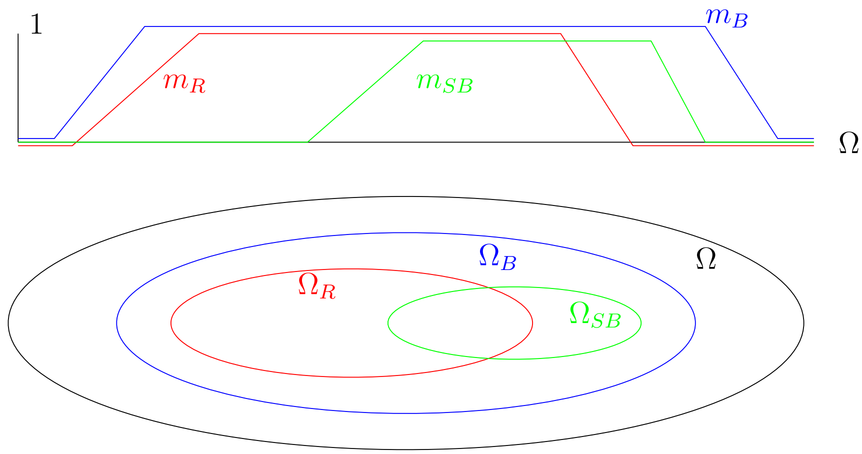


Figure 11.6: [All ravens are black: $(\Omega_R \subseteq \Omega_B)$]

Remark 11.42. Let's compare analytic philosophy $(\flat_{22}^3)$ and scientific philosophy $(\flat_{22}^4)$ in Classification 11.2.

(A$_1$): mathematics Describe math. by logic (and set theory)!

$\begin{cases} (\flat_{22}^1) & \text{foundation of math.} \quad \text{(Cantor, Russell, Gödel),} \\ (\flat_{22}^2) & \text{math.} \quad \text{(Bourbaki),} \end{cases}$

(A$_2$):the logical worldview: Describe science by logic (and set theory)!

$\begin{cases} (\flat_{22}^3) & \text{analytic philosophy, symbolic logic} \quad \text{(Frege, Russell, Peirce, Wittgenstein,...)} \\ (\flat_{22}^4) & \text{scientific philosophy} \quad \text{(Hempel, Popper, ...)} \end{cases}$

(A$_3$): linguistics: Describe everything carefully!

$(\flat_{22}^5)$ linguistics, ordinary language (Saussure, Wittgenstein)

Both analytic philosophy $(\flat_{22}^3)$ and scientific philosophy $(\flat_{22}^4)$ should have argued for the logical worldview. However, scientific philosophy $(\flat_{22}^4)$ has the following had the following advantages:

($\sharp_1$) The science was simple and therefore easy to think about.

($\sharp_2$) Scientists were more likely to be interested.

But, as a result, this advantages have become weaknesses od scientific philosophy. Thas is because the flagpole problem (in Sec. 11.7) and Hempel's raven problem (in Sec. 11.8) and so on highlighted the weaknesses of the logical worldview. It has become the scientific philosophy that scientists are not interested in. It would have been a bit better if art (aesthetics, music, etc.) had been the subject of the research. I do, however, have a lot of respect for the philosophy of science. This is because it exposed the weaknesses of the logical worldview and showed me the way to a quantum mechanical worldview. That is, I assert that

$$(\sharp_2)\quad \boxed{\text{Descartes=Kant}} \xrightarrow[\text{progress ①}]{} \boxed{\text{Analytic phil. } (\flat_{22}^3, \flat_{22}^4, \flat_{22}^5)} \xrightarrow[\text{progress ②}]{\text{List 11.26}} \boxed{\text{Quantum language}}$$

($\flat_{22}^5$):Saussure (Remark 11.13)
($\flat_{22}^3, \flat_{22}^5$): Wittgenstein (Sec. 11.6.3)
($\flat_{22}^4$):Hempel (Remark 11.42)

See the top page in this chapter.

11.9 Three approaches to the mind-body problem

The mind-body problem is the most famous problem in Descartes philosophy. There are two standing positions on whether we see this as a problem of science or a problem of philosophy.

(A) [Science]:If we are in the position of existential monism (i.e., the scientific position), i.e., if we expect a scientific solution, then the mind-body problem is a problem of brain science, AI, and cognitive science.

(B) [Philosophy]: if we consider the mind-body problem as a philosophical problem (i.e., dualistic idealism), it is a question of proposing a worldview with "mind" and "body" as the key words.

Of course, our interest is focused on (B).

This section was written with reference to the following.

(C) [62] Ishikawa,S., A Final solution to mind-body problem by quantum language, *Journal of quantum information science*, Vol. 7, No.2 , 48-56, 2017, DOI: 10.4236/jqis.2017.72005 (`http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=76391`)

If quantum language is the only scientifically successful theory in dualistic idealism, it is natural to study the mind-body problem in quantum language. This will be discussed in Section 9.4.4 (The third approach).

11.9.1 The mind-body problem

Now let us introduce the mind-body problem, which is said to be the greatest unsolved problem in dualistic idealism.

In spite that the cogito proposition "I think, therefore I am" is non-sense (*cf.* Sec. 8.2), Descartes used it in order to propose Descartes philosophy (i.e., mind-matter dualism). That is, his argument is as follows.

(D) If the existence of "I" is deduced from the cogito proposition, the existence of "matter" (which is perceived by "I") is accepted. And further, the medium of "I" and "matter" is automatically accepted as "body (= sensory organ)".

Therefore, the key-words of Descartes philosophy (= mind-matter dualism) is

(E) "I"(="mind"), "body"(="sensory organ"), "matter"

Here, we have the following problem:

(F): The mind-body problem in dualistic idealism

How are "mind" and "body" connected?

(or more generally, how are "mind", "body" and "matter" connected?)

This is generally considered to be the most important problem in Descartes philosophy.

11.9.2 The first approach; Cognitive scientific approach

As mentioned in Note 9.8, Dr. Click (the most noted for being a co-discoverer of the structure of the DNA molecule in 1953 with James Watson) said in his book "The astonishing hypothesis" [11]) as follows.

(G_1) *You, your joys and your sorrows, your memories and your ambitions, your sense of personal identity and free will, are in fact no more than the behavior of a vast assembly of nerve cells and their associated molecules.*

From the scientific point of view, I agree to his opinion (G_1). (i.e., the denial of the substance dualism). Therefore, I believe that the following will be realized.

(G_2) $\underset{\text{(Kant,Husserl)}}{\boxed{\text{Descartes}}} \xrightarrow[\text{progress}]{} \underset{\text{(based on cognitive science)}}{\boxed{\text{Philosophy of mind}}} \xrightarrow[\text{progress}]{} \boxed{\text{Brain science}} \xrightarrow[\text{progress}]{} \underset{\text{(the mind-body problem)}}{\boxed{\text{Solution}}}$.

This may imply that

(G_3) the mind-body problem will be solved in science

However, it should be noted that the (H_1) (i.e., the denial of the substance dualism) and the dualistic idealism (i.e., quantum language) do not contradict each other. That is because quantum language says:

(H) *Describe any monistic phenomenon (such as* (G_1) *) by dualistic language (=quantum language) !*

11.9.3 The second approach; Illusory problem?

It should be noted that

(I_1) the term "mind" and "body" in the mind-body problem (F) is ambiguous in Descartes=Kant epistemology.

That is, the sentence "How are 'mind' and 'body' connected?" is meaningless in Descartes=Kant epistemology. Thus, there may be a reason to consider that the mind-body problem is just "what we cannot speak about". Therefore, according to Wittgenstein's famous saying "What we cannot speak about we must pass over in silence" (in [107]), some may conclude that we must speak nothing about the problem (F). That is, the mind-body problem is an illusory problem. However, I think, by (J) and (K) mentioned in the following section, that this second approach is not only non-productive but also wrong. As mentioned before, I think that the Wittgenstein's next assertion is non-productive:

(I_2) philosophical problems arise from insufficient attention to the variety of natural language use.

11.9.4 * The third approach; Quantum linguistic solution to the mind-body problem

It should be noted that

(J) the demarcation problem (i.e., how to distinguish between "what we cannot speak about" and "what we can speak about") depends on language.

For example, the proposition "the earth goes around the sun" cannot be written in mathematics but in the Newtonian mechanical language. Note that both "the limits of my language mean the limits of my world" and "the limits of your language mean the limits of your world" are true. Therefore,

(K) in order to solve the mind-body problem in dualistic idealism, we should create the language in which the mind-body problem can be regarded as "what we can speak about"

Without this challenge (K), we cannot obtain the solution to the mind-body problem (F). In this sense, the second approach in Section 9.4.3 may be shallow.

Concerning the causality problem (i.e., What is causality?), we already answered it in Problem 1,1 (and Note 1.2), that is,

$$\text{"The solution to the causality problem"} \Leftrightarrow \text{"Axiom 2"} \tag{11.6}$$

Similarly, I can give the solution to the mind-body problem (i.e., How are "mind" and "body" connected?) as follows.

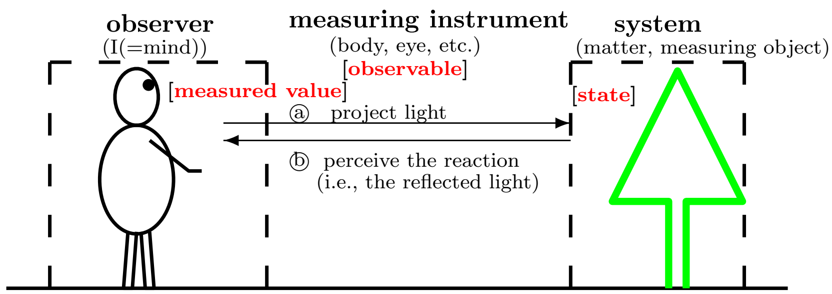
Answer 11.43. [The solution to the mind-body problem]; The correspondence of the key-words (Assertion 1.14) says that

$$\underset{\text{(mind, body)}}{\boxed{\text{Descartes}}} \xrightarrow[\text{linguistic turn}]{\text{the correspondence of the key-words}} \underset{\text{(measured value, measuring instrument)}}{\boxed{\text{Quantum language}}}$$

This says:

$$\begin{array}{c}\text{the epistemological mind-body problem}\\ \boxed{\text{Descartes}}\\ \text{How are "mind" and "body" connected?}\end{array} \xrightarrow[\text{linguistic turn}]{} \begin{array}{c}\text{the epistemological mind-body problem}\\ \boxed{\text{quantum language}}\\ \text{How are "measured value" and "measuring instrument" connected?}\end{array} \tag{11.7}$$

Also, recall Figure 1.2 as follows:



If so, we can easily solve the mind-body problem (in the right-hand side of formula (11.7) that is,

The solution to the mind-body problem

The solution to the mind-body problem is just Axiom 1 (in Sec.1.1.1), that is, the relation between "measured value" and "measuring instrument" is given as follows.

- With any system S, a basic structure $[\mathcal{A} \subseteq \mathcal{N} \subseteq B(H)]$ can be associated in which measurement theory of that system can be formulated. When the observer takes a measurement of an observable (or, by a measuring instrument) $\mathsf{O}{=}(X, \mathcal{F}, F)$ for a system $S_{[\rho]}$ i.e., a system S with a state ρ), the probability that a measured value x $(\in X)$ obtained by the measurement belongs to Ξ $(\in \mathcal{F})$ is given by $\rho(F(\Xi))(\equiv {}_{\mathcal{A}^*}(\rho, F(\Xi))_{\mathcal{N}})$.

Therefore,

(L) "The solution to the mind-body problem"⇔"Axiom 2"

Hence, by this (L) and the formula (11.6), we have the following equivalences:

to propose quantum language

⇔to propose Axiom 1 (measurement) and Axiom 2 (causality)

$\Leftrightarrow$to solve the mind-body problem and the causality problem

and further, we want to add:

$$\Leftrightarrow \text{to build a firm theory in dualistic idealism (i.e., metaphysics)} \tag{11.8}$$

If so, then my next desire has been fulfilled.

- **"mind-body problem" should be the greatest problem in philosophy.**

Chapter 12

Postscript: Philosophy is One

12.1 Philosophy (of worldviews) has progressed towards quantum language

Recently I proposed "quantum language" (or, "QL", "measurement theory", "the linguistic Copenhagen interpretation of quantum mechanics", "the quantum mechanical worldview"), which was not only characterized as the metaphysical and linguistic turn of quantum mechanics but also the linguistic turn of Descartes=Kant epistemology (≈ dualistic idealism).

I assert that this theory is located as follows in the history of world-descriptions.

Assertion 12.1. (= Assertion 0.1 in Preface= Assertion 1.5) [The location of quantum language in the history of world-description (*cf.* ref.[43, 66])]

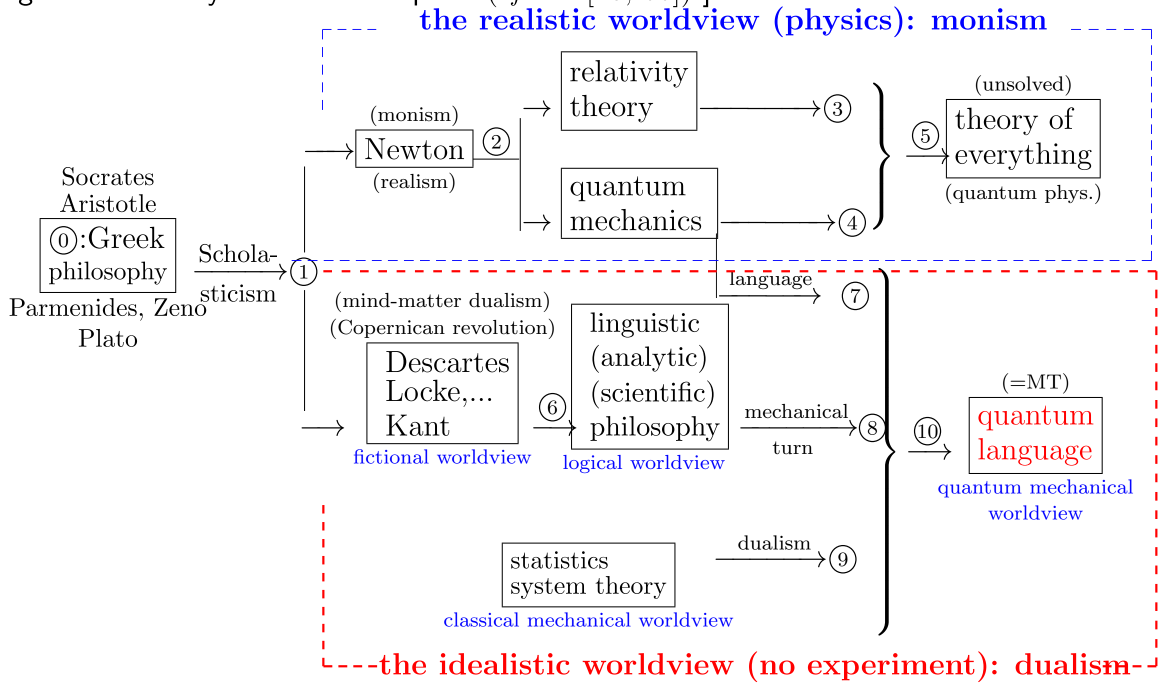


Figure 0.1: The history of the world-descriptions

Thus, quantum language, roughly speaking, has the three aspects as follows.

The three aspects of quantum language

⑦: the standard interpretation of quantum mechanics
(i.e., the true colors of the Copenhagen interpretation)
thus, in this paper we consider that
"the linguistic Copenhagen interpretation"= "the linguistic interpretation"
="the Copenhagen interpretation"
⑧: the final goal of the dualistic idealism (Descartes→Kant→Wittgenstein)
epistemology = measurement theory
"Generalized logic" in the sense that Wittgenstein's picture theory $\subset$ QL

⑨: theoretical statistics of the future
(i.e., the dualistic foumulation of statistics)

Thus, I assert that **QL is the unified theory of (scientific) metaphysics**

♠**Note 12.1.** These pioneers are as follows.

(a;⑦) Born's discovery "the probabilistic interpretation of quantum mechanics" in [6] (1926)

(b;⑧) Tractatus Logico-Philosophicus by L. Wittgenstein in [107] (1921)

(c;⑨) Fisher's great book "Statistical Methods for Research Workers" in [20] (1925)

These three are among the major works of the top ten most influential writings of the 20th century. Thus, it is surprising that the three (Born, Wittgenstein, Fisher) investigated the same thing in the different fields in the same age. That is, I have summarized these as one theory (= quantum language) as follows.

(a;⑦) → ref. [32]: Uncertainty relation ...
(b;⑧) → ref. [35]: Fuzzy inferences ..
(c;⑨) → ref. [36, 38]: Statistics in measurements...
⟶ (= Measurement theory) Quantum language refs. [32],...., [40] ,..... [61, 66]

The above is the starting point of my work about quantum language.

////

I believe, from the scientific point of view, that

(A_1) **quantum language is the final destination of the genealogy of Western philosophy.**

And

(A_2) **a scientific perfection of dualism and idealism is realized by quantum language**

In order to assert the (A) (=(A_1), (A_2)), we proved the following proposition throughout this paper:

Proposition 12.2.

$$
\text{(B)} \quad \boxed{\text{Plato}} \xrightarrow[\text{progress(QL)}]{\text{Cogito turn (dualism)}} \overset{\text{mind-matter dualism}}{\boxed{\text{Descartes=Locke}}} \xrightarrow[\text{progress(QL)}]{\text{Copernican revolution}} \overset{\text{transcendental idealism}}{\boxed{\text{Kant}}}
$$

$$
\xrightarrow[\text{progress(QL)}]{\text{linguistic turn}} \overset{\text{linguistic philosophy}}{\boxed{\text{Wittgenstein}}} \xrightarrow[\text{progress(see (D) later)}]{\text{quantum mechanical turn}} \underset{\text{quantitative argument is possible}}{\overset{\text{the quantum mechanical worldview}}{\boxed{\text{Quantum language}}}} \tag{12.1}
$$

if "$\xrightarrow[\text{progress(QL)}]{}$" is defined by "getting closer to quantum language".

[Remark]:

- As mentioned in Sec. 10.4.3, the definition of "progress" varies.
- For the $\xrightarrow[\text{progress(see (D) later)}]{}$, see Sec. 11.6.4.2 or, the (D) below.

12.2 QL has the power to solve almost all unsolved problems concerning dualistic idealism

12.2.1 The list of my answers of unsolved problems

If (A_1), (A_2) and (B) are true, it is natural to consider:

(C) **many unsolved problems raised in the 2500 year history of dualistic idealism can be solved within the framework of quantum languages.**

My results concerning quantum language are summarized in the following two texts

$$
\text{(D)} \begin{cases} (D_1)\text{: This text: History of western philosophy from the quantum theoretical point of view; [Ver. 3]} \\ (D_2)\text{: Ref. [66]: The linguistic Copenhagen interpretation of quantum mechanics: Quantum language [Ver 5]} \end{cases}
$$

The (C) is shown throughout this paper as follows.

(D_1) **The list of my answers for philosophical unsolved problems in (D_1)**

- What is probability (or, measurement, causality) ? *cf.* Sec. 1.1.1)
- Zeno paradox (Flying arrow), (*cf.* Sec. 2.4.2)
- Zeno paradox (Achilles and a tortoise), (*cf.* Sec. 2.4.3)
- the measurement theoretical understanding of Plato's allegory of the sum , (*cf.* Sec. 3.3.2)
- Plato's Idea theory≈Zadeh's fuzzy theory≈Sausuure's linguistic theory (*cf.* Sec. 3.5.3)
- Syllogism holds in classical systems, but not in quantum systems (*cf.* Sec. 4.3.3)
- Only the present exists (*cf.* Sec. 6.1.2)
- What is the problem of universals? (*cf.* Sec. 6.5.1)
- What is Geocentrism vs. Heliocentrism? After all, the worldviewism (*cf.* Sec. 7.4.2)
- Two (scientific or non-scientific) interpretations of I think, therefore I am .(*cf.* Sec. 8.2.2)
- Leibniz-Clark correspondence (i.e., what is space-time?), (*cf.* Sec. 9.3)
- The problem of qualia (*cf.* Sec. 9.5.1)
- Brain in a vat argument (*cf.* Sec. 9.5.2)
- The solution of Hume's problem of induction (*cf.* Sec. 9.7.1)
- Grue paradox cannot be represented in quantum language (*cf.* Sec. 9.7.2)
- What is causality? (*cf.* Sec. 10.3)
- What is Peirce's abduction? (*cf.* Sec. 11.3.1)
- Five-minute hypothesis (*cf.* Sec. 11.4.1)
- McTaggart's paradox (*cf.* Sec. 11.4.2)
- quantitative representation of "Signifier" and "signified" (*cf.* Sec. 11.5.3)
- A scientific understanding of Wittgenstein's picture theory (*cf.* Sec. 11.6.2)
- Wittgenstein's paradox (*cf.* Sec. 11.6.3)
- Flagpole problem, (*cf.* Sec. 11.7.1)
- Hempel's raven paradox (*cf.* Sec. 11.8)
- the mind-body problem (i.e., How are mind and body connected?), (*cf.* Sec. 11.9.4)

(♯) Also, for the solutions of unsolved problems in quantum mechanics, statistical mechanics, statistics and probability theory, see ref. [66]). Particularly, I think that the following three are important in physics:

- the discovery of Heisenberg's uncertainty relation (Ref. [32], or Sec. 4.3 in ref. [66])
- The clarification of the projection postulate (i.e., the wavefunction collapse) (ref. [59] , or Sec. 11.2 in ref. [66])
- The measurement theoretical characterizations of equilibrium statistical mechanics (Ref. [46], or Chap.17 in ref. [66])

(D_2) **The list of my answers for scientific unsolved problems in (D_2)**
ref. [66]; Linguistic Copenhagen interpretation of quantum mechanics; Quantum Language [Ver 5], Research Report, Dept. Math. Keio University,
KSTS/RR-19/003 (2019); 473 p (http://www.math.keio.ac.jp/academic/research_pdf/report/2019/19003.pdf)

- Kolmogorov's extension theorem in quantum language (Sec.4.1 in ref. [66]) (Sec.4.1 in ref. [66])
- The law of large numbers in quantum language (Sec.4.2 in ref. [66])
- the true discovery of Heisenberg's uncertainty relation (Sec. 4.3 in ref. [66])
- Bell's inequality holds in both classical and quantum systems (Sec. 4.5.2 in ref. [66])
- Measurement theoretical formulation of measurement, inference, control (Sec. 5.2 in ref. [66])
- Monty-Hall problem in quantum language (non-bayesian approach) (Sec.5.5 in ref. [66])

- Two envelope problem in quantum language (non-bayesian approach) (Sec.5.6 in ref. [66])
- Confidence interval and statistical hypothesis test (Chapter 6 in ref. [66])
- Analysis of variance (Chapter 7 in ref. [66])
- Syllogism holds in classical systems, but not in quantum systems (Sec.8.6 and Sec.8.7 in ref. [66])
- Mixed measurement theory (Bayesian measurement theory) (Chap. 9 in ref. [66])
- The measurement theoretical characterization of the wave-function collapse (= projection pustulate) (Sec.11.2 in ref. [66])
- The measurement theoretical characterizations of de Broglie's paradox, quantum Zeno effect, Schrödinger cat, Wigner's friend, Wheeler's delayed choice experiment, Hardy Paradox, quantum eraser (Sec.11.3∼Sec.11.8 in ref. [66])
- The measurement theoretical characterizations of double-slit experiment, Wilson cloud chamber (Sec.12.2, Sec.12.3 in ref. [66])
- The measurement theoretical characterizations of regression analysis (Sec.13.2 in ref. [66])
- The measurement theoretical characterizations of Brownian motion, Zeno's paradox (Sec.14.2 , Sec.14.4 in ref. [66])
- The measurement theoretical characterizations of least-squares method (Chap.15 in ref. [66])
- The measurement theoretical characterizations of Kalman filter (Chap.16 in ref. [66])
- The measurement theoretical characterizations of equilibrium statistical mechanics (Chap.17 in ref. [66])
- The measurement theoretical characterizations of psychological tests (Chap.18 in ref. [66])
- The measurement theoretical characterizations of belief (Chap.19 in ref. [66])
- The mathematical foundation of science (Hempel's raven paradox) (Chap.20 in ref. [66])

12.2.2 In the end

The late Hawking (1942–2018) said in his best seller book [23]:

(E) *Philosophers reduced the scope of their inquiries so much that Wittgenstein the most famous philosopher this century, said "The sole remaining task for philosophy is the analysis of language." What a comedown from the great tradition of philosophy from Aristotle to Kant!*

Thus, Dr. Hawking was concerned about the divergence between philosophy and science. I would not have extended quantum language into the realm of philosophy had I not read this sentence by Hawking. My dream was to have Hawking see this text, but it is now an unfulfilled dream. I'm sorry to hear about Hawking's passing.

I am not an expert in philosophy. However, I have a confidence in only the insistence (A_1) and (A_2).

(A_1) **quantum language is the final destination of the genealogy of Western philosophy (dualistic idealism).**

(A_2) **a scientific perfection of dualism and idealism is realized by quantum language**

Thus I hope that readers will improve this text based on the (A_1) and (A_2). Also, readers will be able to easily solve numerous unsolved problems outside of the lists (D_1) and (D_2) in Sec. 12.2.1.

Also, if you find my mistake in the lists (D_1) and (D_2) in Sec. 12.2.1, please let me know it. I will correct the mistake and make it public (probably, in the next version of this text). Particularly, if there is another more excellent interpretation of Wittgenstein's picture theory, I would be interested to know about this.

The history of western philosophy in this text is my own fiction. Naturally, there may be some differences from the accepted theory. However, even if there are such differences, I optimistically believe that these are acceptable to many like-minded philosophy enthusiasts under Nietzsche's words: *there are no facts, only interpretations*. I wanted to make the interrelationship of unsolved problems in (D_1) clear in my own fiction. In other words, I want to believe "Philosophy is One."

It is my hope that many philosophy enthusiasts and philosophers will write improved versions of this text.

November 2020

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[1]For the further information concerning quantum language,
see home page: `http://www.math.keio.ac.jp/~ishikawa/indexe.html`

References

[1] Alexander, H. G., ed. *The Leibniz-Clarke Correspondence,* Manchester University Press, 1956.

[2] Arthurs, E. and Kelly, J.L.,Jr. *On the simultaneous measurement of a pair of conjugate observables,* Bell System Tech. J. **44**, 725-729 (1965)

[3] Aspect, A, Dallibard, J. and Roger, G. *Experimental test of Bell inequalities time-varying analysis,* Physical Review Letters 49, 1804–1807 (1982)

[4] Bell, J.S. *On the Einstein-Podolosky-Rosen Paradox,* Physics 1, 195–200 (1966)

[5] Bohr, N. *Can quantum-mechanical description of physical reality be consbluBreIdeaomplete?,* Phys. Rev. (48) 696-702 1935

[6] Born, M. *Zur Quantenmechanik der Stoßprozesse (Vorläufige Mitteilung),* Z. Phys. (37) 863–867 1926

[7] Busch, P. *Indeterminacy relations and simultaneous measurements in quantum theory,* International J. Theor. Phys. **24**, 63-92 (1985)

[8] G. Caella, R.L. Berger, *Statistical Inference,* Wadsworth and Brooks, 1999.

[9] D.J. Chalmers, *The St. Petersburg Two-Envelope Paradox,* Analysis, Vol.62, 155-157, 2002.

[10] Clauser,J.F., Horne M.A., Shimony,A, Holt,R.A., Proposed experiment to test local hidden variable theories, *Phys,Rev,Lett,* 23(15), 880-884 (1969)

[11] F. Click, *The Astonishing Hypothesis: The Scientific Search For The Soul,* New York: Charles Scribner's Sons., 1994.

[12] Conant, J. and Diamond,C. *On Reading the Tractatus Resolutely,*: "On Reading the Tractatus Resolutely: to Meredith Williams and Peter Sullivan" in Max Kölbel and Bernhard Weiss, The Lasting Significance of Wittgenstein's Philosophy, Routledge 2004,

[13] Davies, E.B. *Quantum theory of open systems,* Academic Press 1976

[14] de Broglie, L. *L'interpretation de la mechanique ondulatoire,* Journ. Phys. Rad. 20, 963 (1959)

[15] Dunford, Nelson; Schwarz, Jacob T. *Linear Operators part I:general theory (Wiley Classics ed.),* John Wiley and Sons (1988)

[16] Einstein, A., Podolosky, B. and Rosen, N. *Can quantum-mechanical description of reality[red completely?* Physical Review Ser 2(47) 777–780 (1935)

[17] Everett, Hugh *Relative State Formulation of Quantum Mechanics* Reviews of Modern Physics. Ser 29(3) 454-462 (1957)

[18] R. P. Feynman *The Feynman lectures on Physics; Quantum mechanics* Addison-Wesley Publishing Company, 1965

[19] G.A. Ferguson, Y. Takane, *Statistical analysis in psychology and education* (Sixth edition). New York: McGraw-Hill. (1989)

[20] R.A.Fisher *Statistical Methods for Research Workers* Oliver and Boyd (1925) 1970

[21] Goodman, Nelson (1983) Fact, fiction, and forecast, Harvard University Press

[22] L. Hardy, *Quantum mechanics, local realistic theories, and Lorentz-invariant realistic theories,* Physical Review Letters 68 (20): 2981-2984 1992

[23] Hawking, Stephen *A brief History of Time,* Bantam Dell Publishing Group 1988

[24] Heisenberg, W. *Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik,* Z. Phys. 43, 172–198 (1927)

[25] Halmos, P. R. *Naive set theory,* (1960) Princeton, NJ: D. Van Nostrand Company. Reprinted by Springer-Verlag, New York, 1974. Harper

[26] Harari, Y.N. (2014) Sapiens: A Brief History of Humankind, Harper

[27] Hempel, C.G. (1945) Studies in the Logic of Confirmation(I), Mind, 54 1-26

[28] Hempel, C.G. (1965) Aspects of Scientific Explanations and other Essays in the Philosophy of Science, Free Press

[29] Holevo, A.S. *Probabilistic and statistical aspects of quantum theory,* North-Holland publishing company (1982)

[30] D.Howard, Who invented the "Copenhagen Interpretation"? A study in mythology, Philosophy of Science, 71 2004, 669-682

[31] Isaac, R. *The pleasures of probability,* Springer-Verlag (Undergraduate texts in mathematics) 1995

[32] S. Ishikawa, *Uncertainty relation in simultaneous measurements for arbitrary observables,* Rep. Math. Phys., **9**, 257-273, 1991
doi: 10.1016/0034-4877(91)90046-P

[33] Ishikawa, S. *Uncertainties and an interpretation of nonrelativistic quantum theory,* International Journal of Theoretical Physics 30 401–417 (1991) doi: 10.1007/BF00670793

[34] Ishikawa, S., Arai, T. and Kawai, T. *Numerical Analysis of Trajectories of a Quantum Particle in Two-slit Experiment,* International Journal of Theoretical Physics, Vol. 33, No. 6, 1265-1274, 1994
doi: 10.1007/BF00670793

[35] Ishikawa,S. *Fuzzy inferences by algebraic method,* Fuzzy Sets and Systems 87, 181–200 (1997) doi:10.1016/S0165-0114(96)00035-8

[36] Ishikawa,S. *A Quantum Mechanical Approach to Fuzzy Theory,* Fuzzy Sets and Systems, Vol. 90, No. 3, 277-306, 1997, doi: 10.1016/S0165-0114(96)00114-5

[37] Ishikawa,S. *Fuzzy logic in measurements,* Fuzzy Sets and Systems, Vol. 100, 291-300, 1998, doi:10.1016/S0165-0114(97)00154-1

[38] Ishikawa,S. *Statistics in measurements*, Fuzzy sets and systems, Vol. 116, No. 2, 141-154, 2000
doi:10.1016/S0165-0114(98)00280-2

[39] S. Ishikawa, T. Arai, T. Takamura, *A dynamical system theoretical approach to Newtonian mechanics*, Far east journal of dynamical systems 1, 1-34 (1999)
(`http://www.pphmj.com/abstract/191.htm`)

[40] S. Ishikawa, *Mathematical Foundations of Measurement Theory,* Keio University Press Inc. 335pages, 2006, (`http://www.keio-up.co.jp/kup/mfomt/`) (This was often revised e.g., refs. [64, 65, 66])

[41] S. Ishikawa, *Dynamical systems, measurements, quantitative languages and Zeno's paradox,* Far East Journal of Dynamical Systems, Volume 10, Issue 3, Pages 277 - 292 (October 2008) (`http://www.pphmj.com/abstract/3595.htm`)

[42] S. Ishikawa, *A New Interpretation of Quantum Mechanics,* Journal of quantum information science, Vol. 1, No. 2, 35-42, 2011, doi: 10.4236/jqis.2011.12005
(`http://www.scirp.org/journal/PaperInformation.aspx?paperID=7610`)

[43] S. Ishikawa, *Quantum Mechanics and the Philosophy of Language: Reconsideration of traditional philosophies,* Journal of quantum information science, Vol. 2, No. 1, 2-9, 2012
doi: 10.4236/jqis.2012.21002
(`http://www.scirp.org/journal/PaperInformation.aspx?paperID=18194`)

[44] S. Ishikawa, *A Measurement Theoretical Foundation of Statistics,* Applied Mathematics, Vol. 3, No. 3, 283-292, 2012, doi: 10.4236/am.2012.33044
(`http://www.scirp.org/journal/PaperInformation.aspx?paperID=18109&`)

[45] S. Ishikawa, *Monty Hall Problem and the Principle of Equal Probability in Measurement Theory, Applied Mathematics*, Vol. 3 No. 7, 2012, pp. 788-794, doi: 10.4236/am.2012.37117.
(`http://www.scirp.org/journal/PaperInformation.aspx?PaperID=19884`)

[46] S. Ishikawa, *Ergodic Hypothesis and Equilibrium Statistical Mechanics in the Quantum Mechanical World View, World Journal of Mechanics*, Vol. 2, No. 2, 2012, pp. 125-130. doi: 10.4236/wim.2012.22014.
(`http://www.scirp.org/journal/PaperInformation.aspx?PaperID=18861#.VKevmiusWap`)

[47] S. Ishikawa, *The linguistic Copenhagen interpretation of quantum mechanics,* arXiv:1204.3892v1[physics.hist-ph],(2012) (`http://arxiv.org/abs/1204.3892`)

[48] S. Ishikawa, *Zeno's paradoxes in the Mechanical World View,* arXiv:1205.1290v1 [physics.hist-ph], (2012)

[49] S. Ishikawa, *What is Statistics?; The Answer by Quantum Language,* arXiv:1207.0407 [physics.data-an] 2012. (`http://arxiv.org/abs/1207.0407`)

[50] S. Ishikawa, *Measurement Theory in the Philosophy of Science,* arXiv:1209.3483 [physics.hist-ph] 2012. (`http://arxiv.org/abs/1209.3483`)

[51] S. Ishikawa, *Heisenberg uncertainty principle and quantum Zeno effects in the linguistic interpretation of quantum mechanics,* arxiv:1308.5469[quant-ph],(2013)

[52] S. Ishikawa, *A quantum linguistic characterization of the reverse relation between confidence interval and hypothesis testing,* arxiv:1401.2709[math.ST],(2014)

[53] S. Ishikawa, *ANOVA (analysis of variance) in the quantum linguistic formulation of statistics,* arxiv:1402.0606[math.ST],(2014)

[54] S. Ishikawa, *Regression analysis in quantum language,* arxiv:1403.0060[math.ST],(2014)

[55] S. Ishikawa, K. Kikuchi: *Kalman filter in quantum language,* arXiv:1404.2664 [math.ST] 2014. (`http://arxiv.org/abs/1404.2664`)

[56] S. Ishikawa, *The double-slit quantum eraser experiments and Hardy's paradox in the quantum linguistic interpretation,* arxiv:1407.5143[quantum-ph],(2014)

[57] S. Ishikawa, *The Final Solutions of Monty Hall Problem and Three Prisoners Problem,* arXiv:1408.0963 [stat.OT] 20 14. (`http://arxiv.org/abs/1408.0963`)

[58] S. Ishikawa, *Two envelopes paradox in Bayesian and non-Bayesian statistics* arXiv:1408.4916v4 [stat.OT] 2014. (`http://arxiv.org/abs/1408.4916`)

[59] S. Ishikawa, *Linguistic interpretation of quantum mechanics; Projection Postulate,* Journal of quantum information science, Vol. 5, No.4 , 150-155, 2015, DOI: 10.4236/jqis.2015.54017
(`http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=62464`)
Or see the following preprint;
(`http://www.math.keio.ac.jp/academic/research_pdf/report/2015/15009.pdf`)

[60] S. Ishikawa, *History of Western Philosophy from the quantum theoretical point of view*, Research Report (Department of mathematics, Keio university, Yokohama), (KSTS-RR-16/005, 2016, 142 pages)
(`http://www.math.keio.ac.jp/academic/research_pdf/report/2016/16005.pdf`)

[61] S. Ishikawa, *History of Western Philosophy from the quantum theoretical point of view [Ver. 2]*, Research Report (Department of mathematics, Keio university, Yokohama), (KSTS-RR-17/004, 2017, 132 pages)
(`http://www.math.keio.ac.jp/academic/research_pdf/report/2017/17004.pdf`)

[62] Ishikawa,S., A Final solution to mind-body problem by quantum language, *Journal of quantum information science*, Vol. 7, No.2 , 48-56, 2017, DOI: 10.4236/jqis.2017.72005 (http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=76391)

[63] Ishikawa,S., Bell's inequality should be reconsidered in quantum language , Journal of quantum information science, Vol. 7, No.4 , 140-154, 2017, DOI: 10.4236/jqis.2017.74011
(http://www.scirp.org/Journal/PaperInformation.aspx?PaperID=80813)

[64] S. Ishikawa, Linguistic interpretation of quantum mechanics; Quantum Language, Research Report, Dept. Math. Keio University, (http://www.math.keio.ac.jp/en/academic/research.html)
[Ver 1]; KSTS/RR-15/001 (2015); 416 p (http://www.math.keio.ac.jp/academic/research_pdf/report/2015/15001.pdf)

[Ver 2]; KSTS/RR-16/001 (2016); 426 p (http://www.math.keio.ac.jp/academic/research_pdf/report/2016/16001.pdf)

[65] S. Ishikawa, Linguistic Copenhagen interpretation of quantum mechanics; Quantum Language, Research Report, Dept. Math. Keio University, (http://www.math.keio.ac.jp/en/academic/research.html)
[Ver 3]; KSTS/RR-17/007 (2017); 434 p (http://www.math.keio.ac.jp/academic/research_pdf/report/2017/17007.pdf)
[Ver 4]; KSTS/RR-18/002 (2018); 449 p (http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18002.pdf)

[66] S. Ishikawa, Linguistic Copenhagen interpretation of quantum mechanics; Quantum Language, Research Report, Dept. Math. Keio University, (http://www.math.keio.ac.jp/en/academic/research.html)
[Ver 5]; KSTS/RR-19/003 (2019); 473 p (http://www.math.keio.ac.jp/academic/research_pdf/report/2019/19003.pdf)

[67] S. Ishikawa, Linguistic Interpretation of Quantum Mechanics –Towards World-Description in Quantum Language– Shiho-Shuppan Publisher, 411 p. (2017)
(http://www.shiho-shuppan.com/index.php?LIQM)

[68] Ishikawa,S., Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language , Open Journal of philosophy, Vol. 8, No.5 , 466-480, 2018, DOI: 10.4236/ojpp.2018.85032
(https://www.scirp.org/Journal/PaperInformation.aspx?PaperID=87862)

[69] Ishikawa,S., Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language [Revised version] , (https://philpapers.org/rec/ISHLCB)
(http://www.math.keio.ac.jp/academic/research_pdf/report/2018/18001.pdf)

[70] Ishikawa, S., (2019) Philosophy of science for scientists; The probabilistic interpretation of science, Journal of quantum information science, Vol. 9, No.3 , 140-154, DOI: 10.4236/jqis.2019.93007
(https://www.scirp.org/Journal/paperinformation.aspx?paperid=95447)

[71] Ishikawa, S., Wittgenstein's picture theory in the quantum mechanical worldview, (In preparation) . For the information, see my homepage

[72] K. Kikuchi, S. Ishikawa, *Psychological tests in Measurement Theory, Far east journal of theoretical statistics,* 32(1) 81-99, (2010) ISSN: 0972-0863

[73] K. Kikuchi,, *Axiomatic approach to Fisher's maximum likelihood method,* Non-linear studies, 18(2) 255-262, (2011)

[74] Kalman, R. E. *A new approach to linear filtering and prediction problems,* Trans. ASME, J. Basic Eng. 82, 35 (1960)

[75] I. Kant, *Critique of Pure Reason (Edited by P. Guyer, A. W. Wood),* Cambridge University Press, 1999

[76] A. Kolmogorov, *Foundations of the Theory of Probability (Translation),* Chelsea Pub Co. Second Edition, New York, 1960,

[77] U. Krengel, "Ergodic Theorems," Walter de Gruyter. Berlin, New York, 1985.

[78] Kremer, Michael *The purpose of Tractarian Nonsense,* Noûs, Volume35, Issue 1, March 2001, Pages 39-73

[79] Lee, R. C. K. *Optimal Estimation, Identification, and Control,* M.I.T. Press 1964

[80] G. Lüders, Über die Zustandsänderung durch den Messprozess, Ann. Phys. (Leibzig) (6)8,322-328, 1951

[81] Magee, Bryan *Confessions of a Philosopher* Random House 1997

[82] J. M. E. McTaggart, *The Unreality of Time,* Mind (A Quarterly Review of Psychology and Philosophy), Vol. 17, 457-474, 1908

[83] G. Martin, *Aha! Gotcha: Paradoxes to Puzzle and Delight* Freeman and Company, 1982

[84] B. Misra and E. C. G. Sudarshan, *The Zeno's paradox in quantum theory,* Journal of Mathematical Physics 18 (4): 756-763 (1977)

[85] N.D. Mermin, *Boojums all the way through, Communicating Science in a Prosaic Age,* Cambridge university press, 1994.

[86] Ozawa, M. *Quantum limits of measurements and uncertainty principle,* in Quantum Aspects of Operational Communication edited by Bendjaballah et all. Springer, Berlin, 3–17, (1991)

[87] M. Ozawa, *Universally valid reformation of the Heisenberg uncertainty principle on noise and disturbance in measurement,* Physical Review A, Vol. 67, pp. 042105-1–042105-6, 2003,

[88] Peirce. C.S. (1958) Collected Papers of Charles Sanders Pierce, Harvard Univ. Press.

[89] Popper, K.R. (1959) The logic of Scientific Discover, London Hutchinson

[90] Popper, K.R. (1982) Quantum mechanics and the schism in physics, London Hutchinson

[91] Prugovečki, E. *Quantum mechanics in Hilbert space,* Academic Press, New York. (1981).

[92] Putnam, H. *Reason, Truth, and History,* Cambridge University Press. (1982).

[93] Redhead, M. Incompleteness, nonlocality, and realism, Oxford University Press, Oxford (1987)

[94] Robertson, H.P. *The uncertainty principle,* Phys. Rev. 34, 163 (1929)

[95] D. Ruelle, "Statistical Mechanics, Rigorous Results," World Scientific, Singapore, 1969.

[96] Russell. B. *The Problems of Philosophy,* London: Williams and Norgate. (1912).

[97] Russell. B. *The Analysis of Mind,* London: George Allen and Unwin. (1921).

[98] Russell. B. *A History of Western philosophy,* Simon and Schuster. (1945).

[99] Russell. B. *The autobiography of Bertrand Russell,* London: George Allen and Unwin. (1956).

[100] Sakai, S. *C^*-algebras and W^*-algebras,* Ergebnisse der Mathematik und ihrer Grenzgebiete (Band 60), Springer-Verlag, Berlin, Heidelberg, New York 1971

[101] Selleri, F. *Die Debatte um die Quantentheorie,* Friedr. Vieweg&Sohn Verlagsgesellscvhaft MBH, Braunschweig (1983)

[102] Shannon, C.E., Weaver. W *A mathematical theory of communication,* Bell Syst. Tech.J. 27 379–423, 623–656, (1948)

[103] von Neumann, J. *Mathematical foundations of quantum mechanics* Springer Verlag, Berlin (1932)

[104] S. P.Walborn, et al. *"Double-Slit Quantum Eraser,"* Phys.Rev.A 65, (3), 2002

[105] Weinberg, Steven *"To Explain the World: The Discovery of Modern Science"* HarperCollins Publishers (2015), (2015)

[106] J. A. Wheeler, *The 'Past' and the 'Delayed-Choice Double-Slit Experiment',* pp 9-48, in A.R. Marlow, editor, Mathematical Foundations of Quantum Theory, Academic Press (1978)

[107] Wittgenstein, L *Tractatus Logico-philosophicus,* Oxford: Routledge and Kegan Paul, 1921

[108] Yosida, K. *Functional analysis,* Springer-Verlag (Sixth Edition) 1980

[109] Zadeh, L.A. *Fuzzy sets, Information and Control,* Vol. 8, 338-353 1965

Index

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Research Report

2018

[18/001] Shiro Ishikawa,
Leibniz-Clarke correspondence, Brain in a vat, Five-minute hypothesis, McTaggart's paradox, etc. are clarified in quantum language,
KSTS/RR-18/001, September 6, 2018 (Revised October 29, 2018)

[18/002] Shiro Ishikawa,
Linguistic Copenhagen interpretation of quantum mechanics: Quantum Language [*Ver. 4*],
KSTS/RR-18/002, November 22, 2018

2019

[19/001] Sumiyuki Koizumi,
Contribution to the N. Wiener generalized harmonic analysis and its application to the theory of generalized Hilbert transforms,
KSTS/RR-19/001, August 2, 2019
(Second edition, September 20, 2019)
(Third edition, December 6, 2019)

[19/002] Shiro Ishikawa,
Hempel's raven paradox, Hume's problem of induction, Goodman's grue paradox, Peirce's abduction, Flagpole problem are clarified in quantum language,
KSTS/RR-19/002, September 12, 2019 (Revised November 1, 2019)

[19/003] Shiro Ishikawa,
Linguistic Copenhagen interpretation of quantum mechanics: Quantum Language [*Ver. 5*],
KSTS/RR-19/003, December 26, 2019

2020

[20/001] Shiro Ishikawa,
History of Western Philosophy from the quantum theoretical point of view; [*Ver. 3*],
KSTS/RR-20/001, November 15, 2020