Semigroup for linearized free surface problem of viscous fluid

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1. Problem and result

We consider the free surface problem of viscous incompressible fluid flowing down an inclined plane under the effect of gravity. Our main concern is the two dimensional disturbances from the laminar steady flow. After subtraction of the steady solution followed by change of coordinates, our problem is formulated as follows:

$$\partial_t \eta - u_2 = \eta^2 \partial_1 \eta \qquad \text{on } x_2 = 1, \tag{1}$$

$$\partial_t \boldsymbol{u} - \frac{1}{\mathcal{R}} \Delta \boldsymbol{u} + \frac{1}{\mathcal{R}} \nabla p + V_1 \partial_1 \boldsymbol{u} + \partial_2 V_1 \begin{pmatrix} u_2 \\ 0 \end{pmatrix} = F_0(\eta, \boldsymbol{u}, \nabla p) \quad \text{in } 0 < x_2 < 1, \tag{2}$$

$$\text{div}\boldsymbol{u} = 0 \qquad \text{in } 0 < x_2 < 1, \tag{3}$$

$$\boldsymbol{u} = \boldsymbol{0} \qquad \text{on } x_2 = 0, \tag{4}$$

$$\partial_2 u_1 + \partial_1 u_2 - 2\eta = F_1(\eta, \boldsymbol{u}) \qquad \text{on } x_2 = 1, \tag{5}$$

$$p - 2\partial_2 u_2 - \left(2 \cot \alpha - \sigma \csc \alpha \partial_1{}^2\right) \eta = F_2(\eta, \boldsymbol{u}) \quad \text{on } x_2 = 0. \tag{6}$$

x_1 and x_2 are stream-wise and cross-stream coordinates respectively. Here the unknowns $\eta, \boldsymbol{u}$ and p correspond to the unknown free surface, the disturbance of velocity vector and the scalar pressure respectively. $(V_1, V_2) = (-(1 - x_2)^2, 0)$ denotes the velocity of the basic laminar flow. The equation (1) on $x_2 = 1$ comes from the kinematic boundary condition. We assume that the unknowns are periodic in x_1 with period ℓ. The constant $\alpha (0 < \alpha < \pi/2)$ is the angle of inclination. The positive constant σ denotes the surface tension coefficient. $\mathcal{R}$ is the Reynolds number. The terms in F_1 , F_2 and $F_0(\eta, \boldsymbol{u}, \nabla p)$ are quadratic or higher. For this nondimensionalization and the derivation of (1) $\sim$ (6) see [3, 4] .

To study qualitative behavior of solutions to the full nonlinear problem (e.g., bifurcation problem, decay in time, etc.,), it is indispensable to investigate the properties of solutions to the linearized problem. To do so we write down the problem in the form of evolution equatio in appropriate function space by using an orthogonal projection parallel to some gradient space, and state that the operator arising in the linearized problem generates an analytic semigroup. We here present only the outline of the proof of our result.

We give some notations. Set $\Omega = \{(x_1,x_2) \in \mathbb{R}^2 \ ; \ 0 < x_1 < \ell \ , \ 0 < x_2 < 1\}$. Let $r \geq 0$. $H^r(\Omega)$ is the space of functions which are in $H^r_{loc}(\mathbb{R} \times (0,1))$ and are periodic in x_1 with period ℓ. Let $S_F = \partial\Omega \cap \{x_2 = 1\}$ and $S_B = \partial\Omega \cap \{x_2 = 0\}$. We identify S_F with the interval $(0 \ , \ \ell)$. $H^r(S_F)$ is the space of functions which are in $H^r_{loc}(\mathbb{R})$ and are periodic with period ℓ. H^0 denotes L^2. We set $H^r_0(S_F) = \left\{\varphi \in H^r(S_F) \, ; \ \int_0^\ell \varphi \, dx_1 = 0\right\}$.

To eliminate the pressure from (2) we use the orthogonal projection P onto the L^2 orthogonal complement of the following gradient space

$$\mathcal{G} = \left\{\nabla\phi \ ; \ \phi \in H^1(\Omega) \ , \ \phi = 0 \text{ on } x_2 = 1\right\} \ .$$

Applying P to (2) and using the boundary condition (6) we can write the linearized problem as follows

$$\frac{d}{dt}\begin{pmatrix}\eta \\ \boldsymbol{u}\end{pmatrix} - G\begin{pmatrix}\eta \\ \boldsymbol{u}\end{pmatrix} = \begin{pmatrix} g \\ \boldsymbol{f}\end{pmatrix} \tag{7}$$

where g_0 and $\boldsymbol{f}$ are arbitrarily given in $H_0^{\frac{3}{2}}(S_F)$ and $PL^2(\Omega)$ respectively. G denotes the 2×2 matrix of operator We give the detailed explanation of G in the next section.

We now announce our result.

Theorem 1 *There exists a* $\gamma > 0$ *such that, if* $\operatorname{Re}\lambda > \gamma$ *there exists the inverse* $(\lambda - G)^{-1}$ in X *with its operator norm satisfying*

$$\left|(\lambda - G)^{-1}\right|_X \leqq \frac{C}{|\lambda|}$$

where $X = H_0^{\frac{3}{2}}(0,\ell) \times PL^2(\Omega)$.

2. Formulation of the linear problem

We recall some properties of the orthogonal projection P.

Lemma 1 *Let* $r \geq 0$. i) P *is a bounded operator on* $H^r(\Omega)$. ii) *Suppose* $\phi \in H^1(\Omega)$. *Then* $P(\nabla\phi) = \nabla\psi$, *where* ψ *satisfies*

$$\psi = \phi \text{ on } S_F \ , \ \partial_2\psi = 0 \text{ on } S_B \ , \quad \Delta\psi = 0 \text{ in } \Omega \ .$$

See([1,page 369]) for the proof of Lemma 1.

We now formulate the linear problem

$$\partial_t \eta - u_2 = g_1 \qquad \text{on } x_2 = 1, \tag{8}$$

$$\partial_t \boldsymbol{u} - \frac{1}{\mathcal{R}}\Delta \boldsymbol{u} + \frac{1}{\mathcal{R}}\nabla p + V_1 \partial_1 \boldsymbol{u} + \partial_2 V_1 \begin{pmatrix} u_2 \\ 0 \end{pmatrix} = \boldsymbol{f}_0 \quad \text{in } 0 < x_2 < 1, \tag{9}$$

$$\mathrm{div}\boldsymbol{u} = 0 \qquad \text{in } 0 < x_2 < 1, \tag{10}$$

$$\boldsymbol{u} = \boldsymbol{0} \qquad \text{on } x_2 = 0, \tag{11}$$

$$\partial_2 u_1 + \partial_1 u_2 - 2\eta = g_2 \qquad \text{on } x_2 = 1, \tag{12}$$

$$p - 2\partial_2 u_2 - \left(2\cot\alpha - \sigma \csc\alpha {\partial_1}^2\right)\eta = g_3 \quad \text{on } x_2 = 0. \tag{13}$$

Applying P to (9) and taking (13) into account, by using Lemma 1 we have

$$\partial_t \boldsymbol{u} - P\frac{1}{\mathcal{R}}\Delta \boldsymbol{u} + P\left(V_1 \partial_1 \boldsymbol{u} + \partial_2 V_1 \begin{pmatrix} u_2 \\ 0 \end{pmatrix}\right) + \frac{1}{\mathcal{R}}\nabla p_1 + \frac{1}{\mathcal{R}}\nabla p_2 = P\boldsymbol{f}_0 - \frac{1}{\mathcal{R}}\nabla p_3$$

with

$$\Delta p_j = 0 \text{ in } \Omega\ , \qquad \partial_2 p_j = 0 \text{ on } S_B\ ,\ j = 1, 2, 3\ ,$$
$$p_1 = 2\partial_2 u_2\ , \quad p_2 = (2\cot\alpha - \sigma\csc\alpha\partial_1^2)\eta\ , \quad p_3 = g_3 \quad \text{on } x_2 = 1\ .$$

We collect the terms depending on $\boldsymbol{u}$, then define the operator A by

$$A\boldsymbol{u} = -P\frac{1}{\mathcal{R}}\Delta \boldsymbol{u} + P\left(V_1 \partial_1 \boldsymbol{u} + \partial_2 V_1 \begin{pmatrix} u_2 \\ 0 \end{pmatrix}\right) + \frac{1}{\mathcal{R}}\nabla p_1 \quad .$$

If we set $R : \boldsymbol{u} \to u_2|_{S_F}$ for $\boldsymbol{u} \in PL^2(\Omega)$, then the gradient $\nabla\psi$ of the solution

$$\Delta\psi = 0 \text{ in } \Omega\ , \quad \psi = \phi \text{ on } S_F\ , \quad \partial_2\psi = 0 \text{ on } S_B$$

for a given $\phi \in H^{1/2}(S_F)$ can be regarded as $R^*\phi$, where R^* is the formal adjoint of R with respect to L^2 inner product . (See [2].)

We have now expressed (9) and (13) in the form

$$\partial_t \boldsymbol{u} + A\boldsymbol{u} + \frac{1}{\mathcal{R}}R^*(2\cot\alpha - \sigma\csc\alpha\partial_1^2)\eta = P\boldsymbol{f}_0 - \frac{1}{\mathcal{R}}R^* g_3\ .$$

Using an auxiliary solenoidal vector we can reduce (12) to the homogeneous case $g_2 = 0$. Thus, we can assume $g_2 = 0$ and $g_3 = 0$.

We now give the precise definition of the operator G. Set

$$G\begin{pmatrix}\eta\\ \boldsymbol{u}\end{pmatrix} = \begin{pmatrix}0 & R\\ -\frac{1}{\mathcal{R}}R^*(2\cot\alpha - \sigma\csc\alpha\partial_1^2) & -A\end{pmatrix}\begin{pmatrix}\eta\\ \boldsymbol{u}\end{pmatrix}$$

with the domain

$$\mathcal{D}(G) = \Big\{(\eta, \boldsymbol{u}) \in H_0^{\frac{3}{2}}(S_F) \times PL^2(\Omega)\,;$$
$$\eta \in H_0^{\frac{5}{2}}(S_F)\ ,\boldsymbol{u} \in H^2(\Omega)\ ,\ \boldsymbol{u} = \boldsymbol{0} \text{ on } S_B\ ,\ \partial_1 u_2 + \partial_2 u_1 - 2\eta = 0 \text{ on } S_F\Big\}\ .$$

The linearized problem can now be rewritten in the form (7).

In the following sections we shall construct the resolvent operator $(\lambda - G)^{-1}$ for $\lambda \in \mathbb{C}$ with sufficiently large real part. The resolvent equation can be written in the form of the stationary problem with parameter

$$\lambda\eta - u_2 = g_0 \qquad \text{on } x_2 = 1, \tag{14}$$

$$\lambda\boldsymbol{u} - \frac{1}{\mathcal{R}}\Delta\boldsymbol{u} + \frac{1}{\mathcal{R}}\nabla p + V_1\partial_1\boldsymbol{u} + \partial_2 V_1\begin{pmatrix}u_2\\ 0\end{pmatrix} = \boldsymbol{f} \qquad \text{in } 0 < x_2 < 1, \tag{15}$$

$$\mathrm{div}\boldsymbol{u} = 0 \qquad \text{in } 0 < x_2 < 1, \tag{16}$$

$$\boldsymbol{u} = \boldsymbol{0} \qquad \text{on } x_2 = 0, \tag{17}$$

$$\partial_2 u_1 + \partial_1 u_2 - 2\eta = 0 \qquad \text{on } x_2 = 1, \tag{18}$$

$$p - 2\partial_2 u_2 - \left(2\cot\alpha - \sigma\csc\alpha{\partial_1}^2\right)\eta = 0 \qquad \text{on } x_2 = 0. \tag{19}$$

We begin by solving the linear nonhomogeneous equations with homogeneous boundary conditions.

Proposiotion 1 *Let* $\mathrm{Re}\,\lambda > 0$. *Let* $\boldsymbol{f} \in PL^2(\Omega)$ *be arbitrarily given. Then there exist* $\boldsymbol{u}$ *and* p *satisfying*

$$\lambda\boldsymbol{u} - \frac{1}{\mathcal{R}}\Delta\boldsymbol{u} + \frac{1}{\mathcal{R}}\nabla p = \boldsymbol{f}\ ,\ \mathrm{div}\boldsymbol{u} = 0 \text{ in } \Omega\ ,$$
$$\boldsymbol{u} = \boldsymbol{0} \text{ on } S_B\ ,\ p - 2\partial_2 u_2 = 0\ ,\ \partial_2 u_1 + \partial_1 u_2 = 0 \text{ on } S_F$$

with

$$|\boldsymbol{u}|_{H^2} + \mathcal{R}|\lambda|\,|\boldsymbol{u}|_{L^2} \le C\mathcal{R}\,|\boldsymbol{f}|_{L^2}\ \ ,\ |p|_{H^1} \le C\mathcal{R}\,|\boldsymbol{f}|_{L^2}\ .$$

For the proof see [1, p371].

3. Model problem in the half-space

In this section we first consider the following problem in $\left\{(x_1,x_2)\in\mathbb{R}^2\,;\,x_2>0\,\right\}$ with periodicity in x_1 with period ℓ.

$$\lambda\eta - v_2 = b_1 \qquad \text{on } x_2=0, \tag{20}$$

$$\lambda\boldsymbol{v} - \frac{1}{\mathcal{R}}\Delta\boldsymbol{v} + \frac{1}{\mathcal{R}}\nabla q = \boldsymbol{0} \qquad \text{in } x_2>0\ , \tag{21}$$

$$\mathrm{div}\boldsymbol{v} = 0 \qquad \text{in } x_2>0\,, \tag{22}$$

$$\partial_2 v_1 + \partial_1 v_2 = b_2 \qquad \text{on } x_2=0\,, \tag{23}$$

$$-q + 2\partial_2 v_2 + \sigma_0 {\partial_1}^2\eta = b_3 \qquad \text{on } x_2=0\,. \tag{24}$$

This problem retains only principal terms.

Proposiotion 2 *Let $\gamma>0$ be arbitrarily fixed. Let $b_1\in H_0^{3/2}(S_F)$ and let $b_2\,,\,b_3\in H^{1/2}(S_F)$. Then, for $\mathrm{Re}\,\lambda\ge\gamma$, there exist $\eta\,,\,\boldsymbol{v}$ and q satisfying* (20) $\sim$ (24) *with*

$$|\eta|_{5/2} + |\lambda|\,|\eta|_{3/2} \le C\left(|b_1|_{3/2} + |b_2|_{1/2} + |b_3|_{1/2}\right)\ , \tag{25}$$

$$|\boldsymbol{v}|_2 + |\lambda|\,|\boldsymbol{v}|_0 \le C\left(|b_1|_{3/2} + |\lambda|^{\frac14}\,|b_2|_{1/2} + |b_3|_{1/2}\right)\ . \tag{26}$$

The outline of the proof . We follow [5]. Set $y=x_2$. We decompose the unknowns into each Fourier mode:

$$\eta = \sum_{n\neq 0}\eta^{(n)}\exp(i\xi x_1)$$

$$\boldsymbol{v} = \sum_n \begin{pmatrix} v_1^{(n)}(y) \\ v_2^{(n)}(y)\end{pmatrix}\exp(i\xi x_1)\ ,\ q = \sum_n q^{(n)}(y)\exp(i\xi x_1)\ ,$$

Here $\xi = \dfrac{2\pi n}{\ell}$, $n\in\mathbb{Z}$. Then we arrive at a boundary value problem of ordinary differential equations for $v_1^{(n)}, v_2^{(n)}$ and $q^{(n)}$.

$$\text{(27)} \qquad \mathcal{R}\lambda v_1 - \left(\left(\frac{d}{dy} \right)^2 - |\xi|^2 \right) v_1 + i\xi q = 0 \ ,$$

$$\text{(28)} \qquad \mathcal{R}\lambda v_2 - \left(\left(\frac{d}{dy} \right)^2 - |\xi|^2 \right) v_2 + \frac{dq}{dy} = 0 \ ,$$

$$\text{(29)} \qquad i\xi v_1 + \frac{dv_2}{dy} \ , \qquad \text{in } y > 0$$

$$\text{(30)} \qquad \lambda\eta - v_2 = b_1 \ ,$$

$$\text{(31)} \qquad i\xi v_2 + \frac{dv_1}{dy} = b_1 \ ,$$

$$\text{(32)} \qquad -q + 2\frac{dv_2}{dy} - \sigma_0 |\xi|^2 \eta = b_3 \qquad \text{on } y = 0 \ .$$

We have omitted the upper indices and also have used same b_j $j = 1, 2, 3$ to denote its Fourier coefficients. From (30) $\eta = \lambda^{-1}(v_2 + b_1)$. We substitute this into (32). As Solonnikov did in [5, pages 200 – 206] , we can obtain the solution of the system which decays as $y \to \infty$. The explicit form can be written as follows

$$\begin{aligned}
\text{(33)} \quad & v_1(\xi, y, \lambda) = -\frac{e^{-ry}}{r} b_2 \\
& + \frac{\mathcal{R}}{r(r+|\xi|)\mathcal{P}} \left\{ \sigma_0 i\xi |\xi|^2 r(r-|\xi|) b_1 + \left[(3r-|\xi|)|\xi|^2\lambda + \sigma_0|\xi|^4 \right] b_2 + i\xi r(r-|\xi|)\lambda b_3 \right\} e^{-ry} \\
& \qquad + \frac{\mathcal{R}}{(r+|\xi|)\mathcal{P}} \left\{ -\sigma_0 i\xi|\xi|^2(r^2+|\xi|^2) b_1 - |\xi|^2(2r\lambda + \sigma_0|\xi|^2) b_2 \right. \\
& \qquad \qquad \left. - i\xi\lambda(r^2+|\xi|^2) b_3 \right\} \frac{e^{-ry} - e^{-|\xi|y}}{r - |\xi|} \ ,
\end{aligned}$$

$$\begin{aligned}
\text{(34)} \quad & v_2(\xi, y, \lambda) \\
& = \frac{\mathcal{R}}{r(r+|\xi|)\mathcal{P}} \left\{ -\sigma_0|\xi|^3 r(r+|\xi|) b_1 - i\xi r(r-|\xi|)\lambda b_2 - |\xi| r(r+|\xi|)\lambda b_3 \right\} e^{-ry} \\
& \qquad + \frac{\mathcal{R}}{(r+|\xi|)\mathcal{P}} \left\{ \sigma_0|\xi|^3(r^2+|\xi|^2) b_1 - i\xi|\xi|(2r\lambda + \sigma_0|\xi|^2) b_2 \right. \\
& \qquad \qquad \left. + |\xi|\lambda(r^2+|\xi|^2) b_3 \right\} \frac{e^{-ry} - e^{-|\xi|y}}{r - |\xi|} \ ,
\end{aligned}$$

$$\begin{aligned}
\text{(35)} \quad & q(\xi, y, \lambda) \\
& = \frac{\mathcal{R}}{\mathcal{P}} \left\{ -\sigma_0|\xi|^2(r^2+|\xi|^2) b_1 + (2i\xi r\lambda + i\sigma_0\xi|\xi|^2) b_2 - \lambda(r^2+|\xi|^2) b_3 \right\} e^{-|\xi|y} \ ,
\end{aligned}$$

where $r = \sqrt{\mathcal{R}\lambda + |\xi|^2}$ and

$$\mathcal{P} = \left(r^2 + |\xi|^2\right)^2 - 4r|\xi|^3 + \mathcal{R}\sigma_0|\xi|^3 \quad .$$

From (30) we can recover η :

$$\eta = \frac{1}{\lambda}\left(1 - \frac{\mathcal{R}\sigma_0|\xi|^3}{\mathcal{P}}\right) b_1 - i\xi\frac{(r-|\xi|)^2}{\mathcal{P}} b_2 - \mathcal{R}\frac{|\xi|}{\mathcal{P}} b_3 \quad . \tag{36}$$

To obtain (25) and (26) we need

Lemma 2 *Let $\gamma > 0$ be arbitrarily fixed. For λ with $\operatorname{Re}\lambda \geq \gamma$ it holds that*

$$\begin{gathered} |\mathcal{P}| \geq \mathcal{R}^2\gamma^2 \quad , \quad |\mathcal{P}| \geq 2\mathcal{R}|\lambda||\xi|^2 \ , \\ \mathcal{R}\sigma_0|\xi|^3 \leq \left(\frac{7}{2} + \frac{\sigma_0}{2}\sqrt{\frac{\mathcal{R}}{\gamma}}\right)|\mathcal{P}| \ , \\ \left(\mathcal{R}|\lambda|\right)^2 \leq \left(3 + \frac{\sigma_0}{2}\sqrt{\frac{\mathcal{R}}{\gamma}}\right)|\mathcal{P}| \ . \end{gathered}$$

This lemma is proved in [5, Lemma 2.5]. For the rest of the proof we only have to estimate each terms in (33) ,(34) and (36) by using Lemma 2.

As a consequence of Proposition 2 we can show

Proposiotion 3 *Let $b_1 \in H_0^{3/2}(S_F)$. There is a $\gamma_1 > 0$ such that , for $\operatorname{Re}\lambda \geq \gamma_1$, there exist η , $\boldsymbol{v}$ and q satisfying*

$$\lambda\eta - v_2 = b_1 \qquad \text{on } x_2 = 0, \tag{37}$$

$$\lambda\boldsymbol{v} - \frac{1}{\mathcal{R}}\Delta\boldsymbol{v} + \frac{1}{\mathcal{R}}\nabla q = \mathbf{0} \qquad \text{in } x_2 < 1 \ , \tag{38}$$

$$\operatorname{div}\boldsymbol{v} = 0 \qquad \text{in } x_2 < 1 \, , \tag{39}$$

$$\partial_2 v_1 + \partial_1 v_2 - 2\eta = 0 \qquad \text{on } x_2 = 1 \, , \tag{40}$$

$$q - 2\partial_2 v_2 - \left(2\cot\alpha - \sigma\csc\alpha{\partial_1}^2\right)\eta = 0 \qquad \text{on } x_2 = 1 \, . \tag{41}$$

with

$$|\eta|_{5/2} + |\lambda|\,|\eta|_{3/2} \leq C\,|b_1|_{3/2} \ , \tag{42}$$

$$|\boldsymbol{v}|_2 + |\lambda|\,|\boldsymbol{v}|_0 \leq C\,|b_1|_{3/2} \ . \tag{43}$$

4. Outline of the proof of Theorem 1

We finally explain how to solve

$$\lambda\eta - u_2 = g_0 \qquad \text{on } x_2 = 1, \tag{44}$$

$$\lambda \boldsymbol{u} - \frac{1}{\mathcal{R}}\Delta \boldsymbol{u} + \frac{1}{\mathcal{R}}\nabla p = \boldsymbol{f} \qquad \text{in } 0 < x_2 < 1, \tag{45}$$

$$\mathrm{div}\boldsymbol{u} = 0 \qquad \text{in } 0 < x_2 < 1, \tag{46}$$

$$\boldsymbol{u} = \boldsymbol{0} \qquad \text{on } x_2 = 0, \tag{47}$$

$$\partial_2 u_1 + \partial_1 u_2 - 2\eta = 0 \qquad \text{on } x_2 = 1, \tag{48}$$

$$p - 2\partial_2 u_2 - \left(2\cot\alpha - \sigma\csc\alpha{\partial_1}^2\right)\eta = 0 \qquad \text{on } x_2 = 1. \tag{49}$$

Here $g_0 \in H_0^{3/2}(S_F)$ and $\boldsymbol{f} \in PL^2(\Omega)$. Note that, for our purpose, it is enough to consider (45) because (45) contains the principal terms of (15) and the terms in (15) which are not in (45) can be regarded as lower order purturbations. Further we can assume $\boldsymbol{f} = \boldsymbol{0}$ because of Proposition 1 , using the fact that $u_2 \in H_0^{3/2}(S_F)$ if $\boldsymbol{u} \in H^2(\Omega)$ satisfies $\boldsymbol{u} = \boldsymbol{0}$ and $\mathrm{div}\boldsymbol{u} = 0$.

Since we solve the model-problem in the half-space $\{(x_1, x_2)\,;\, x_2 < 1\}$ by Proposition 3 , we have to cut off this solution. Then we adjust the solenoidal condition by solving the boundary value problem of Poisson equation. This can be done by use of [1, Lemma 2.8]. Finally we adjust the boundary conditions (47), (48) , (49) and the right hand side of the equations (45). Then we have $\eta, \boldsymbol{u}$ and p satisfying (45) – (49). Instead of (44) this soution satisfies the equation of the form

$$\lambda\eta - u_2 = g_0 - \mathcal{M}g_0$$

where $\mathcal{M}$ is some linear operator on $H_0^{3/2}(S_F)$ which becomes the contraction map on this space if $\mathrm{Re}\,\lambda$ is large enough. Hence if we start to solve the problem above with $(Id - \mathcal{M})^{-1}g_0$ in the right hand side of (44), we can get the desired solution. By combining these results we can show Theorem 1.

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